

# STUDY ON SPHERICAL ABERRATION CORRECTION OF SOLENOID LENS IN ULTRAFAST ELECTRON DIFFRACTION

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## Abstract

High electron beam quality is required in Ultrafast Electron Diffraction (UED) to achieve high spatial resolution. However, aberrations mainly induced by solenoid lens will deteriorate the beam quality and limit the resolution. Spherical aberration introduces the largest distortion which is unavoidable in the case of static cylindrically symmetric electromagnetic fields on the basis of Scherzer's theorem. In order to reduce the spherical aberrations, different models have been designed which are composed of three symmetrical lens and one asymmetrical lens. We obtain the magnetic field distribution and calculate the aberration of each model by OPERA, and the result is that the solenoid without poles has the minimum aberration and meets the design requirement best.

## INTRODUCTION

Ultrafast Electron Diffraction (UED) is the primary tool for material science, chemistry, physics, biology and industry with nanometer and sub picosecond resolution. The electron beam is accelerated to 3 MeV in microwave electron gun. After passing through the solenoid lens, the electron beam diffracts on the sample and finally produces the diffraction pattern on the fluorescent screen. By analyzing the diffraction pattern, the information of the material microstructure with atomic resolution can be obtained.

Solenoid magnetic lenses are usually used for transverse focusing of electron beams, but the aberration inevitably exists, which leads to the emittance growth and degrades performance in the system. In this paper, the beam emittance growth caused by aberration is theoretically derived and four models including three symmetric lens and one asymmetric lens are designed guided by this theory. We obtain the magnetic field distribution and calculate the aberration of each model by OPERA, and the result is that the solenoid without iron yokes has the minimum aberration and meets the design requirement best.

## EMITTANCE GROWTH CAUSED BY ABERRATION

### Magnetic Field Distribution of Solenoid Lens

Axisymmetric magnetic field generated by electrified coils can be used for focusing electron beams. The magnetic lenses with and without ferromagnetic material outside the coil are called electromagnetic and ironless magnetic lens respectively [1]. The magnetic field distribution of the electromagnetic lens is more concentrated, and the

focusing ability is stronger when the current is the same as shown in Fig. 1. The distribution of magnetic field is related to the shape of the yoke and poles, which determines the aberration. All the magnetic lenses discussed in this paper are electromagnetic lenses.

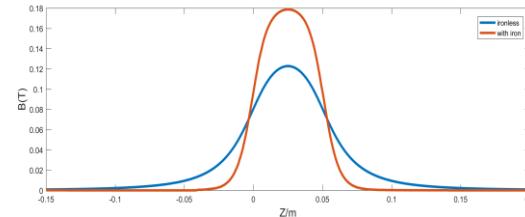


Figure 1: Magnetic field distribution on the axis with and without yoke.

### Emittance Growth Caused by Lens Aberration

The magnetic field in a solenoid lens is axisymmetric. In cylindrical coordinates, the magnetic vector can be expanded in series to

$$A_\theta(r, z) = \frac{1}{2} B'(z)r - \frac{1}{16} B''(z)r^3 + \dots \quad (1)$$

where  $B(z)$  is the magnetic induction on the axis.

According to  $B = \nabla \times A$ , we have

$$\begin{cases} B_r(r, z) = -\frac{1}{2} B'(z) + \frac{1}{16} B'''(z)r^3 - \dots \\ B_z(r, z) = B(z) - \frac{1}{4} B''(z)r^2 + \dots \end{cases} \quad (2)$$

The Gauss trajectory equation of electrons in the paraxial approximation of an axisymmetric magnetic field is

$$r'' = -\frac{e}{8mV} B^2(z)r \quad (3)$$

where  $m$  is the mass of electrons,  $V$  is the potential representing the kinetic energy of electrons.

Assume  $K = e/8mV$ , combining formulas (2) and (3), preserving small quantities of three or less times in the equation

$$r'' = -K[B^2(z)r - \frac{B(z)}{2} B''(z)r^3 + \dots] \quad (4)$$

Under the approximation of thin lens, when the electron passing through the magnetic lens we can approximately assume that the radius remains constant, only the slope changes abruptly. Therefore, the slope changes caused by the nonlinear term can be expressed as

$$\Delta_3 r' = \frac{K r^3}{2} \int B(z) B''(z) dz \quad (5)$$

When the nonlinear term is not considered, the focal length is expressed as [2]

$$\frac{1}{f} = K \int B^2(z) dz \quad (6)$$

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We can define the spherical aberration coefficient as

$$C_s = -\int B(z)B''(z)dz / \int B^2(z)dz \quad (7)$$

According to the conclusion of Y. YANG et al [3], the effective emittance of the beam is

$$\epsilon_{eff} = \sqrt{\epsilon^2 + K(C_s R^4 / 2f)^2} \quad (8)$$

Therefore, in order to improve beam quality and minimize emittance growth, the aberration must be reduced.

## SYMMETRIC MAGNETIC LENS

### Glaser Model

Glaser model is the most commonly used physical model for the magnetic field generated by a symmetric magnetic lens on the axis.

$$B_z(z) = B_0 \cdot \frac{a^2}{z^2 + a^2} \quad (9)$$

where  $B_0$  is the maximum magnetic field on the axis,  $a$  is the full width at half maximum (FWHM) of the curve.

The empirical formulas available under this model are

$$\begin{cases} B_0 = \frac{u_0 NI}{b} \\ b = \sqrt{s^2 + 1.8r^2} \\ \frac{a}{b} \approx 0.485 \end{cases} \quad (10)$$

where  $NI$  is the number of ampere-turns of the coil,  $s$  is the length of air gap between yokes,  $r$  is the inner radius of the magnetic lens.

By synthesizing formulas (9), (6) and (7), we can get

$$\begin{cases} \frac{1}{f} = \frac{\pi}{2} K a B_0^2 \\ C_s = \frac{1}{2a^2} \end{cases} \quad (11)$$

It can be known from equation (11) that the FWHM of the magnetic field distribution curve should be increased as much as possible under the premise of keeping the focal length constant. In the limit case, the shape of the magnetic field distribution curve will become rectangle, and the aberration coefficient at this time is the smallest. Specifically, in the design of the magnetic lens, the measures taken are to increase the air gap length while keeping the inner radius constant to limit the magnetic field divergence and increase the full width at half maximum.

### Electromagnetic Lenses without Poles

We have designed a kind of electromagnetic lenses without poles. The coil is wound by a type of hollow wire which is square outside and the hollow inside is circular. Considering the insulating layer outside the conductor, the cross-sectional area of each turn is 8 mm×8 mm and the diameter of the inner circle is 4 mm. The coil consists of three layers and each with 12 turns. The material of yoke is DT4 which is 5 mm thick, and the inner radius of the lens is 15 mm. Ferromagnetic materials are generally unsaturated in the design of magnetic lenses. For the material DT4, the magnetic induction should not exceed 1.5 T as Fig. 2 shows.

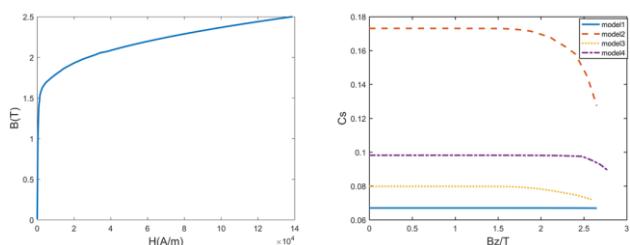


Figure 2: (a) BH curve of DT4; (b) Aberration changes when the yoke is saturated.

We have established models in opera and obtained the magnetic field distribution. The result is that when the current density is 6.24 A/mm<sup>2</sup>, the magnetic induction in yoke is less than 1.2 T. At this time, the number of ampere-turns of the magnetic lens is 13824 At. The distribution of magnetic field in yoke and on axis is shown in Fig. 3.

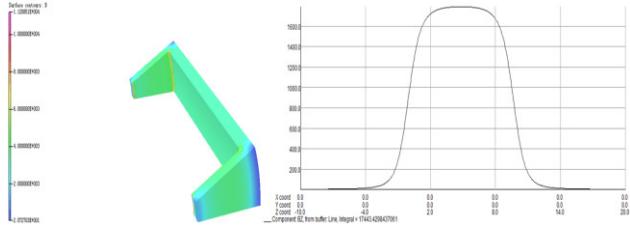


Figure 3: Distribution of magnetic induction in yoke and on axis.

Adjust the current density to change the saturation of the yoke and observe the aberration coefficient as model1 shows in Fig. 2(b).

### Electromagnetic Lenses with Poles

The electromagnetic lens with poles is a typical type of Glaser lens whose magnetic field distribution on the axis is inverted bell shaped. The structure is designed according to formula (9) and the empirical formula (10).

The shape of the pole is a structure with smaller head and wider tail, which makes the distribution of radial magnetic field more uniform and the ferromagnetic material more difficult to saturate. The coil consists of two layers and each with 12 turns. The thickness of the pole and the outside yoke are 10mm, and the inner radius of the lens is 15 mm. The length of air gap is 42 mm.

We have established models in opera and obtained the magnetic field distribution. The result is that when the current density is 6.8 A/mm<sup>2</sup>, the maximum magnetic induction at the pole is 1.54 T which is less than 1.4 T on the yoke. At this time, the number of ampere-turns of the magnetic lens is 10444.8 At. The distribution of magnetic field in yoke and on axis is shown in Fig. 4.

Adjust the current density to change the saturation of the yoke and observe the aberration coefficient as model2 shows in Fig. 2 (b).

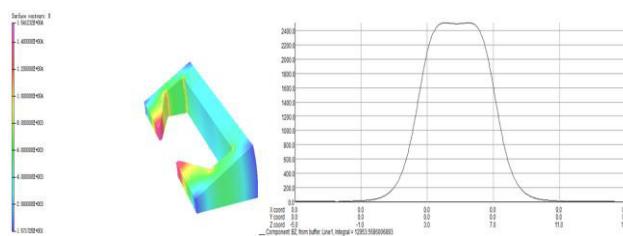


Figure 4: The distribution of magnetic induction in yoke and on axis.

### Electromagnetic Lenses with Multiple Coils

From the empirical formula (13), it can be known that the air gap length should be increased to obtain larger FWHM. But when the length of air gap exceeds 42 mm in this model, the magnetic field distribution on the axis will change, and the depression will appear at the flat top of the curve as shown in Fig. 5(a). Therefore, a magnetic lens model with multiple coils is designed, as shown in Fig. 5(b). Increase the current of the central coil to obtain the required flat top field distribution.

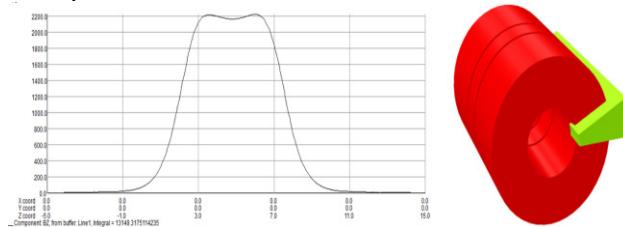


Figure 5: (a) Magnetic field distribution along the axis when  $s$  is large; (b) Electromagnetic lenses with multiple coils.

The coil consists of two edge coils and one central coil. Each coil is divided into three layers. The edge coils are 5 turns per layer and the central coil is 2. The thickness of the pole is 2 mm while the outside yoke is 10 mm. The length of air gap is 90 mm and the inner radius of the lens is 15 mm.

We have obtained the magnetic field distribution. The result is that when the current density is  $7.35 \text{ A/mm}^2$  in central coil and  $5.65 \text{ A/mm}^2$  in edge coils, the maximum magnetic induction at the pole is 1.45 T which is less than 1.4 T on the yoke. At this time, the number of ampere-turns of the magnetic lens is 13670 At. The distribution of magnetic induction in yoke and on axis is shown in Fig. 6.

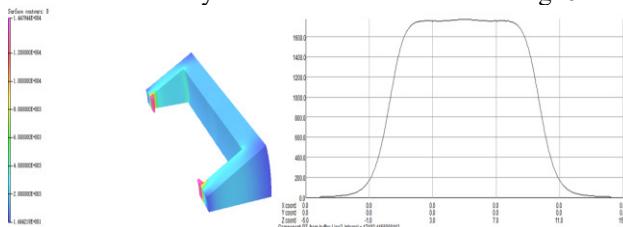


Figure 6: The distribution of magnetic induction in yoke and on axis.

Adjust the current density to change the saturation of the yoke and observe the aberration coefficient as model3 shows in Fig. 2(b).

### MC5: Beam Dynamics and EM Fields

### D01 Beam Optics - Lattices, Correction Schemes, Transport

## ASYMMETRIC LENS

For the asymmetric lens, the inner diameter of the lower pole is smaller than that of the upper one, and the magnetic field distribution on the axis is no longer symmetrical with respect to the center of the lens [4]. When the ferromagnetic material is unsaturated, the distribution of magnetic field depends on two parameters:  $D1/D2$  and  $(D1 + D2)/2s$ , where  $D1, D2$  are the inner diameter of upper and lower poles, and  $s$  is the length of air gap.

The coil consists of two layers and each with 12 turns. The thickness of the pole and the outside yoke are 10mm and the length of the air gap is 60 mm. The inner radius of the upper pole is 35 mm and that of the lower pole is 15 mm.

We have established models in opera and obtained the magnetic field distribution. The result is that when the current density is  $8.2 \text{ A/mm}^2$ , the maximum magnetic induction at the pole is 1.65 T which is less than 1.5 T on the yoke. At this time, the number of ampere-turns of the magnetic lens is 12595.2 At. The distribution of magnetic induction in yoke and on axis is shown in Fig. 7.

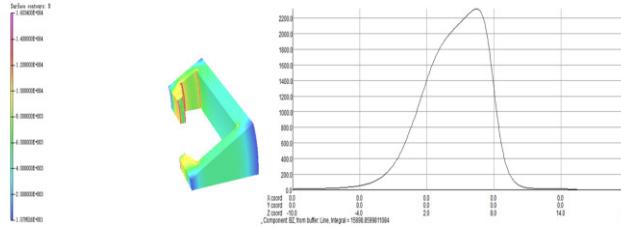


Figure 7: The distribution of magnetic induction in yoke and on axis.

Adjust the current density to change the saturation of the yoke and observe the aberration coefficient as model4 shows in Fig. 2(b).

## CONCLUSION

By comparing the four kinds of lenses mentioned above, it can be concluded that the aberration of the electromagnetic lens without pole is the smallest as shown in Fig. 2(b) when the inner diameter and total length are limited, but the number of ampere-turns required for this model is the largest under the same focal length. When the saturation is enhanced, the aberration of each model decreases, and the electromagnetic lens with poles decreases most significantly. The setting of pole can reduce the ampere-turns and improve the focusing ability, but the aberration will increase too. In the process of design, we need to balance the relationship between reducing current density and reducing the aberration, and choose the model that meets the requirements best. For the UED designed by Huazhong University of Science and Technology, electromagnetic lenses without pole is better to be selected.

## REFERENCES

[1] L. N. Sheng, M. T. Song. "Beam optics of axially symmetric magnetic lens and its applications in ECR ion source." *High Power Laser & Particle Beams* 19.7(2007):1211-1215.

- [2] Kumar, Vinit. "Understanding the focusing of charged particle beams in a solenoid magnetic field." *American Journal of Physics* 77.8(2009):737.
- [3] Y. Yang et al., "Study on beam emittance growth induced by spherical aberration of solenoid lens." *Atomic Energy Science & Technology* 47.12(2013):2336-2340.
- [4] W. X. Chen, "The Electron Optical Properties of Asymmetrical Condenser-objective Lenses." *Chinese Journal of Scientific Instrument* (1984).