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DYNAMICS STUDY OF THE CRAB CROSSING AT THE ELECTRON ION COLLIDER USING SQUARE MATRIX AND ITERATIVE METHODS*

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Abstract

Crab crossings are designed to increase the luminosity of accelerators by ensuring beam interactions are closer to a head on collision. The scheme has been realized in KEKb and will be implemented at the Electron Ion Collider (EIC) at Brookhaven National Laboratory and the LHC High Luminosity upgrade in CERN. It is then important to examine how the crab cavity will affect beam dynamics at the EIC. Methods such as Frequency Map Analysis (FMA) have been shown to be helpful in examining the phase space of accelerators in order to find properties such as resonances and the dynamic aperture. An alternative to such methods is an iterative method based on square matrix method that has been shown to reveal similar properties as FMA while reducing the computational power needed [1]. This method has been applied to the crab crossing scheme in order to find and explain effects of the higher order mode of crab cavities on the particle dynamics of the EIC.

THEORETICAL MODEL

The model used for this study was a 4-D system looking at the transverse and longitudinal components of the crab cavity (CC) crossing. The lattice consists of two crab cavities placed a phase of $\pi/2$ apart from one another on either side of the interaction point (IP). A simple nonlinear lattice in the longitudinal direction and a linear transverse lattice were used for the rest of the accelerator. It is assumed that there are minimal longitudinal effects from the lattice between the two crab cavities. The crab cavities provide a sinusoidal kick to a proton beam as well as a quadrupole and sextupole kick of the following form [2]:

$$\Delta p_x = \frac{-\tan\theta_c \sin(k_c z)}{k_c\sqrt{\beta_{cc}\beta_{IP}}} + b_2 x\sin(k_c z) + b_3 x^2 \sin(k_c z) \ . \tag{1}$$

To preserve the symplectic nature of the cavity, a longitudinal kick is also imparted on the beam of the following form:

$$\Delta p_z = \frac{-x\tan\theta_c \cos(k_c z)}{\sqrt{\beta_{cc}\beta_{IP}}} + \frac{b_2 k_c}{2}x^2\cos(k_c z) + \frac{b_3 k_c}{3}x^3\cos(k_c z) \ . \tag{2}$$

where θ_c is the half crossing angle, $k_c = 2\pi f_c/c$ is the wave number of the crab cavity, and b_2, b_3 are the integrated quadrupole and sextupole strengths respectively.

The parameters of the model are given in Table 1. Note that the sextupole strength is unrealistically high. This is due to the rest of the transverse lattice being linear and therefore a larger value was needed in order to see its effects. The rest of the lattice has the following form longitudinally

$$z' = z - \frac{2\pi h c\eta\beta_s}{f_{\rm rf}} p_z' \ , \tag{3}$$

where

$$p_z' = p_z + \frac{eV_{\rm rf}}{E\beta_s^2}(\sin\phi - \sin\phi_s) \ , \tag{4}$$

η is the slip factor, ϕ is the longitudinal phase, and β_s is the ratio of the longitudinal velocity and the speed of light.

Table 1: Parameters of Crab Cavity (CC) Model

Parameter	Symbol	Value
Half crossing angle	θ_c	25 [mrad]
CC wave number	k_c	
Transverse Beta Function at CC	β_{cc}	1300 [m]
Transverse Beta Function at IP	β_{IP}	90 [cm]
Transverse Position RMS	σ_x	120 [μm]
Longitudinal Position RMS	σ_z	7 [cm]
Longitudinal Momentum Deviation RMS	σ_{p_z}	6.6×10^{-4}
Integrated Quadrupole Strength	b_2	0 [1/m]
Integrated Sextupole Stregnth	b_3	500,000 [1/m^2]
Synchronous Phase	ϕ_s	0
Beam Energy	E	275 [GeV]
RF Voltage	$V_{\rm rf}$	15.8 [MV]
RF Frequency	$f_{\rm rf}$	591 [MHz]
Harmonic Number	h	7560
Momentum Compaction Factor	α_c	1.5×10^{-3}
Linear Tunes (Transverse / Longitudinal)	ν_x/ν_z	0.310 / 0.015

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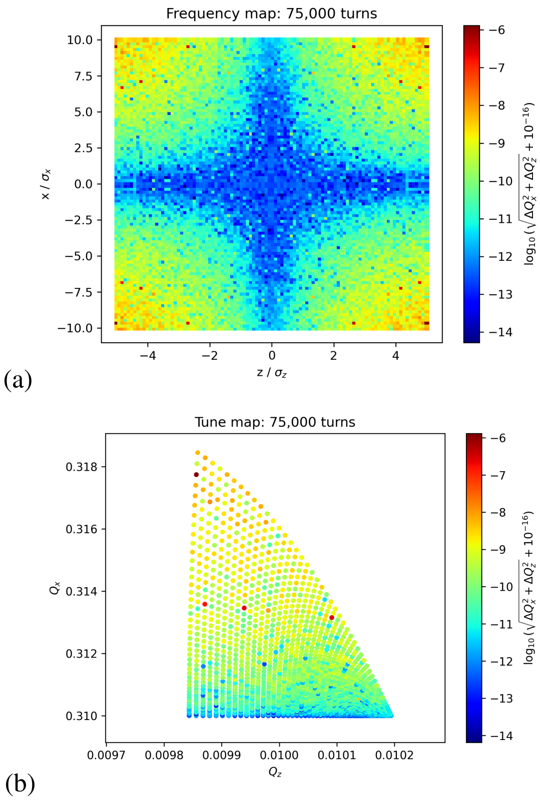

Figure 1: Frequency map (a) and tune map (b) of the crab cavity crossing at the EIC using the parameters in Table 1.

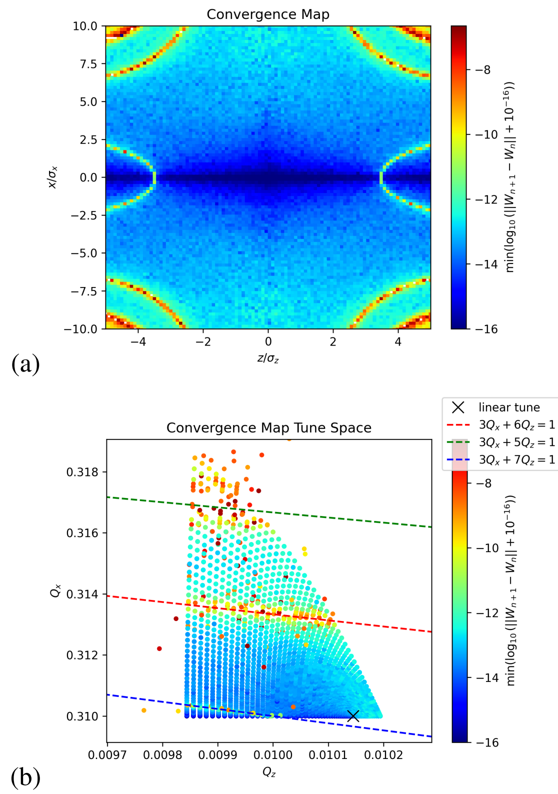

Figure 2: Convergence map (a) and tune map (b) of the crab cavity crossing at the EIC using the parameters in Table 1.

RESULTS

The details of the iteration method are described thoroughly in another conference paper [1]. But briefly here, the iteration method aims to find a diffeomorphism that transforms the phase space coordinates to a space where each iteration is a pure rotation of an angle equal to the tune of the particle. It does this by solving Eqs. 5 and 6 in frequency space iteratively. Since the left hand side of Eqs. 5 and 6 are the differences of h/g, the first Fourier component should vanish, so we choose our tunes on the right hand side to ensure this. The tunes are used to calculate the higher order Fourier components of h/g. Finally, the initial conditions are used to get the first Fourier component of h and g and the process is repeated with the updated values:

$$h^{(n+1)}(\alpha+\rho_x^{(n+1)},\beta+\rho_z^{(n+1)}) - h^{(n+1)}(\alpha,\beta) = \eta_x(\rho_x^{(n+1)},\alpha,\beta,h^{(n)}(\alpha,\beta),g^{(n)}(\alpha,\beta)) \quad (5)$$

$$g^{(n+1)}(\alpha+\rho_x^{(n+1)},\beta+\rho_z^{(n+1)}) - g^{(n+1)}(\alpha,\beta) = \eta_z(\rho_z^{(n+1)},\alpha,\beta,h^{(n)}(\alpha,\beta),g^{(n)}(\alpha,\beta))\ , \quad (6)$$

where the (n) in the superscript denotes iteration number, not turn number, and h and g are defined by the following

$$\theta_x = \alpha + h\left(\alpha,\beta\right) \quad (7)$$

$$\theta_z = \beta + g\left(\alpha,\beta\right)\ , \quad (8)$$

and

$$\theta_j = -i \log z_j\ , \quad (9)$$

where j is either the longitudinal or transverse dimension and z in Eq. 9 refers to eigenvectors of the linear map.

Figure 1(a) shows frequency map analysis using 75,000 turns and NAFF [3] to calculate the tunes of the parameters in Table 1. We can compare this to the same parameters and initial conditions but using the iteration method in Fig. 2(a). The color bar on this map shows the base 10 log of the minimum error that the method converges to, the error being $||\mathbf{X}_{n+1} - \mathbf{X}_n||$ where n is the iteration number and $\mathbf{X}$ is the phase space vector.

We can see when we plot the same points in tune space in Fig. 2(b), that the higher errors correspond to the resonance lines of $3Q_x + mQ_z = 1$, where $m = 5, 6, 7$ which correspond to an 8th, 9th, and 10th order resonance respectively. The 8th order resonance corresponds to the points on the corners of the map in x, z space. The next lines in from those corners correspond to the 9th order resonance line. Finally, the lines near $x = 0$ correspond to the 10th order resonance lines.

This $m = 7$ line also appears when we turn off the sextupole as shown in Fig. 5. This is because with no sextupole, the iteration method only shows a tune shift in the Q_z which still crosses the $3Q_x + 7Q_z = 1$ resonance line. This suggests

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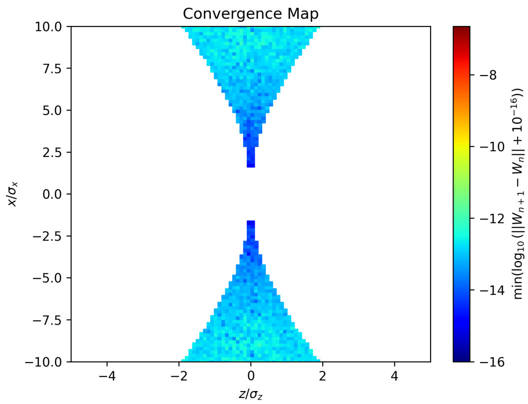

Figure 3: Area where we see an increase in the longitudinal tunes compared to the linear tune from Fig. 2.

that the crab cavity with a sextupole will result in an increase in horizontal tunes that then encounter the 8th and 9th order resonances described before.

Figure 3 shows the points where we see an increase in the longitudinal tunes. This further shows how the crab cavity causes the tune shifts to be dependent on the particles transverse and longitudinal positions.

We can also gather more information by removing the time dependency. Figure 4 shows the convergence map when there is no time dependency, i.e. just a normal sextupole instead of a crab cavity with time dependent sextupole. This removes the coupling between the transverse and longitudinal coordinates. However we still see some small traces of the resonances which we do not expect due to the lack of coupling. FMA does not show any coupling either. This would suggest that this $m = 7$ resonance is most likely a numerical effect from the iteration method. More specifically, this could be that during one of the iterations the predicted tunes land very close to this resonance. This would cause the corresponding Fourier component to explode as being close to a resonance gives a near zero denominator when we update it. I would also think this is why we see the lines in Fig. 5. But this wouldn't necessarily discount their appearance in Fig. 2 as the color scales are different.

Figure 1(a) shows instability near the corners where we see the $m = 5, 6$ resonances in Fig. 2(a). This could mean that the instabilities we see in the frequency map are caused by the resonance lines that are more clearly shown in the convergence map. Since resonances appear more clearly in the convergence maps, the iteration method could be useful for finding resonances that FMA does not show as clearly.

The shape of the tune space in Fig. 4(b) shows that the increase in the longitudinal tunes, as well as the curved horizontal and longitudinal tune boundary we see on the right in Fig. 1(b), are due to the coupling caused by the crab cavity. The crab cavity also seems to strengthen the resonances discussed previously.

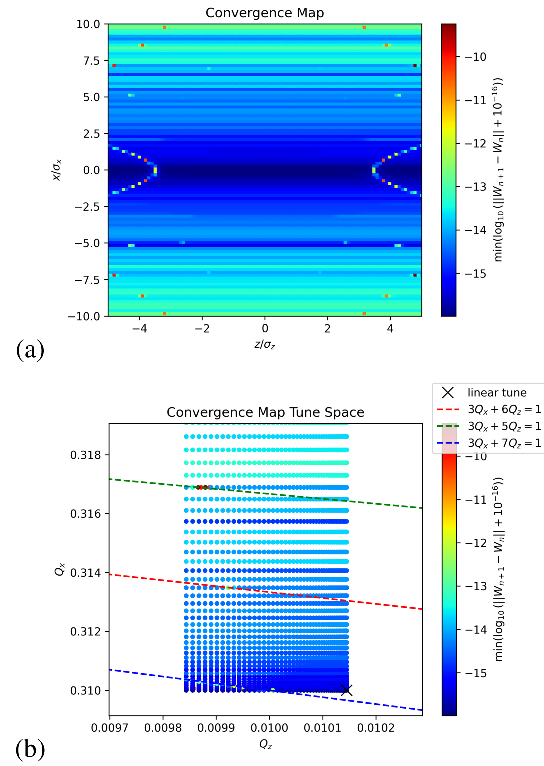

Figure 4: Convergence map (a) and tune map (b) with a time independent sextupole and no crab cavity.

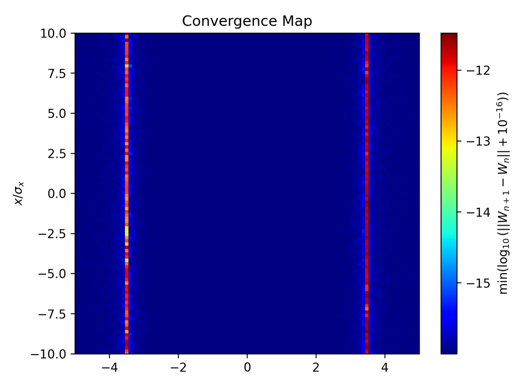

Figure 5: Convergence map using the parameters in table 1 but with no sextupole ($b_3 = 0$).

CONCLUSION

The iterative method has proven to be able to analyze the crab cavity crossing at the Electron Ion Collider. It has identified similar tune shifts as FMA such as the increase in longitudinal tune caused by the crab cavity. It has also identified resonances encountered due to tune shifts caused by the crab cavity and its time dependent sextupole. Some of these resonances seem to coincide with areas where FMA shows instability. Future work to analyze the full 6-D phase with this method is being conducted.

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