

# PRELIMINARY MEASUREMENT OF 4D BEAM PHASE SPACE DISTRIBUTION USING A SLIT EMITTANCE METER SYSTEM

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## Abstract

Transverse emittances are generally measured in horizontal and vertical phase space separately as 2D projections of 4D phase space without any correlations between them. It is true only if their degrees of freedom are independent. Recent studies show that there exists correlation across conjugate pairs. This correlation can affect beam dynamics and cause beam loss. In our study, we sought to measure 4D beam phase space distribution with possible correlations. For this purpose, we used a direct method of measuring the 4D phase space distribution using slits. A set of 4 slits is used to slice the beam into a specific volume of the 4D phase space, and the charge inside each volume is measured. KOMAC has a test bench called BTS (Beam Test Stand) which consists of a microwave ion source, LEPT, a 200 MHz RFQ and two beamlines. At one of the beamlines, we have just installed slit emittance meter system to measure 4D beam phase space distribution. This paper presents our slit emittance meter system and shows preliminary experimental results thereof.

## INTRODUCTION

Beam diagnostics research for measuring high power beam distribution becomes more important and is studied in several laboratories [1–4]. Especially in high power accelerators, nonlinear space charge forces cause beam loss which can lead to radio-activation of the accelerator components. It is important to understand the beam phase space distribution fully to reduce uncontrolled losses for the safe hands-on maintenance. In this paper, we present works of measuring 4D beam phase space distribution specified below:

- Slit emittance meter and its installation in BTS.
- Measurements of 4 D beam phase space distribution with all possible correlations across conjugate pairs  $x - x'$ ,  $y - y'$ ,  $x - y'$ ,  $y - x'$ ,  $x - y$  and  $x' - y'$ .
- Reconstruction of  $x$ ,  $y$ ,  $x - x'$ ,  $y - y'$ ,  $x - y'$  and  $y - x'$  beam distributions from the 4 D phase space distribution  $f(x, x', y, y')$  measurement.
- Comparison between measured and reconstructed beam distributions.
- Calculation of 4D emittance.

## SLIT EMITTANCE METER

Slit emittance meter uses two pairs of slits installed at a distance  $L = 0.5$  m. Fig. 1 (a) shows the cross-sectional view of the chamber with a pair of vertical and horizontal slits.

These slits are driven by a set of linear motion manipulator and a stepper motor. The beam is directed to first slit pair (slit 1 and 2 for  $x1$  and  $y1$  positions) and chopped into flat beamlets. This beamlet is spread with initial angular spreads in  $x$  and  $y$ , and then passes through the second slit pair (slit 3 and 4 for  $x2$  and  $y2$  positions). After passing through all the slits, the final beamlet is measured as the beam current at the collector. By repeating these measurements at various combinations of slit positions, the entire phase space  $f(x, x', y, y')$  can be directly measured as  $x = x1$ ,  $y = y1$ ,  $x' = (x2 - x1)/L$  and  $y' = (y2 - y1)/L$ . The slit emittance meter is installed in the straight beamline of the BTS. BTS consists of a microwave ion source, a LEPT (low energy beam transport) system, a RFQ (Radio Frequency Quadrupole) accelerator, quadrupole magnets and beam diagnostics tools such as current transformer, faraday cups and wire scanners as shown in Fig. 1 (b). The slit emittance meter is installed as seen in Fig. 1 (c).

## 4D PHASE SPACE MEASUREMENT

The slit emittance meter installed in the BTS (Fig. 1 (c)) is set to measure multi-dimensional phase space distributions with certain combinations of slits. When measuring  $x' - y'$ , all 4 slits are used, which took more than 50 hours-long measurement time. And the  $x' - y'$  data can be used to reconstruct its sub-dimensional distributions, that is, the  $x - x'$ ,  $y - y'$ ,  $x - y'$  and  $y - x'$  distributions. In this section, we show experimental results and compare directly measured distributions with distributions reconstructed from  $x' - y'$  data.

### Measurements of Beam Phase Space Distribution

$x - x'$ ,  $y - y'$  and  $x - y$  beam phase space distributions are measured using the  $x1 - x2$ ,  $y1 - y2$  and  $x1 - y1$  slit pairs with the moving step of 0.5 mm respectively. For  $x - y'$  and  $y - x'$  beam phase space measurements, we used three slits, i.e.  $x1 - y1 - y2$  and  $x1 - y1 - x2$  pairs. For  $x' - y'$  beam phase space measurements, all four slits are used. These measurement results are shown in Fig. 2 with phase space ellipses (yellow dotted lines) of their emittances and twiss parameters obtained from the beam phase distribution. The emittance and twiss parameters are summarized in Table 1.

### Reconstruction of Beam Phase Space Distribution

$x' - y'$  measurement data is a function of beam intensity with respect to  $x1$ ,  $y1$ ,  $x2$  and  $y2$  which can be written as,

$$I(x1, y1, x2, y2) = I(x, y, x', y') \quad (1)$$

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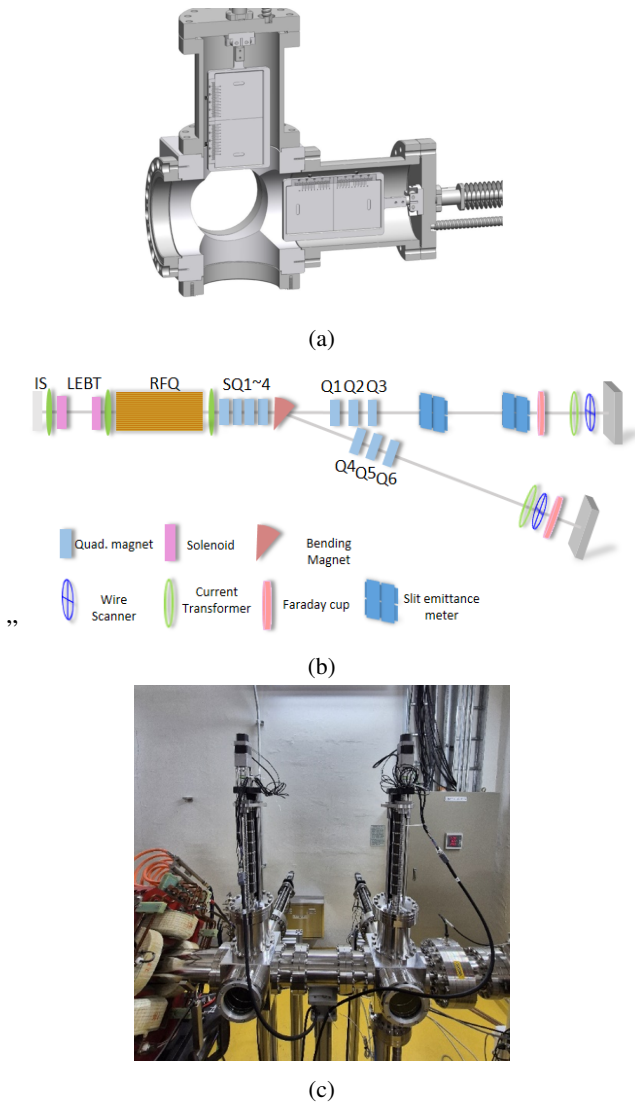


Figure 1: (a) Slit emittance meter; (b) BTS with beam diagnostics; (c) picture of the installed slit emittance meter in the BTS.

Table 1: Emittance and Twiss Parameters:  $\epsilon$  and  $\beta$  of  $x-x'$ ,  $y-y'$ ,  $x-y'$  and  $y-x'$  are in  $\pi$  mm mrad and mm/ $\pi$  mrad and  $\epsilon$  of  $x-y$  are in  $\pi$  mm<sup>2</sup> and  $\pi$  mrad<sup>2</sup> respectively. Others are unitless

	norm. rms $\epsilon$	$\alpha$	$\beta$
$x-x'$	0.201	4.304	4.258
$y-y'$	0.181	2.857	3.912
$x-y'$	0.579	-0.260	1.333
$x-y$	0.702	0.054	1.065
$y-x'$	0.736	-0.100	0.889
$x'-y'$	0.756	0.220	1.636

Using the  $x'-y'$  measurement data,  $x$ ,  $y$ ,  $x-x'$ ,  $y-y'$ ,  $x-y'$  and  $y-x'$  phase space distributions are reconstructed and compared with the directly measured phase space distribu-

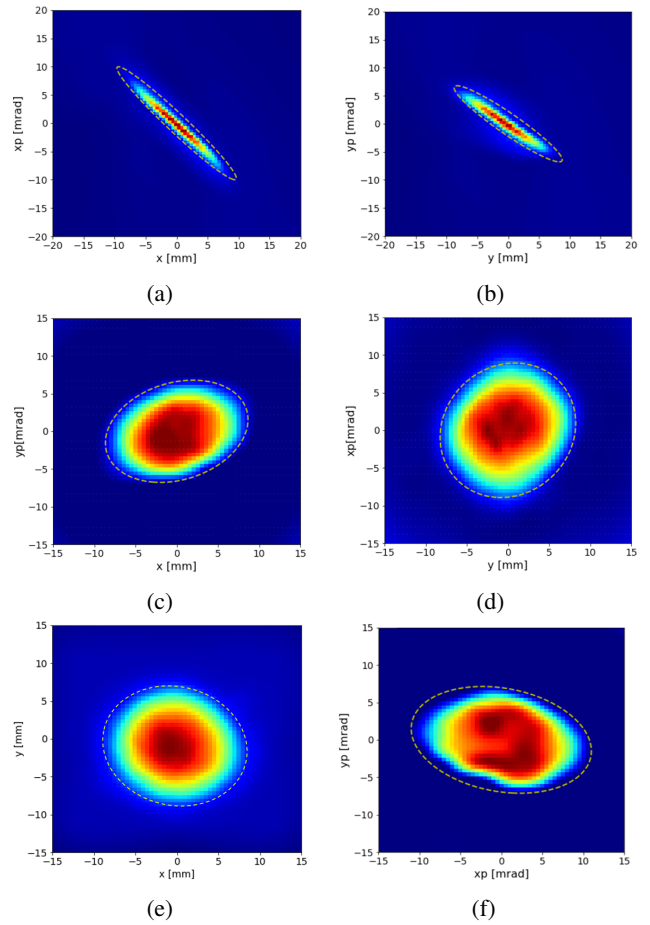


Figure 2: Measured beam phase space distribution of (a)  $x-x'$ , (b)  $y-y'$ , (c)  $x-y'$ , (d)  $y-x'$ , (e)  $x-y$  and (f)  $x'-y'$ .

tions shown in the previous subsection. Reconstructed beam phase space distributions are calculated using the equations

$$I(x, y') = \sum_y \sum_{x'} I(x, y, x', y'), \quad (2)$$

$$I(y, x') = \sum_x \sum_{y'} I(x, y, x', y'), \quad (3)$$

$$I(x, x') = \sum_y \sum_{y'} I(x, y, x', y'), \quad (4)$$

$$I(y, y') = \sum_x \sum_{x'} I(x, y, x', y'), \quad (5)$$

$$I(x) = \sum_{x'} \sum_y \sum_{y'} I(x, y, x', y'), \quad (6)$$

$$I(y) = \sum_x \sum_{x'} \sum_{y'} I(x, y, x', y'). \quad (7)$$

These reconstruct  $x-x'$ ,  $y-y'$ ,  $x-y'$  and  $y-x'$  phase spaces are shown in Fig. 3 (a)-(d) with phase space ellipses (magenta dotted lines). The phase space ellipses (yellow dotted lines) from the measured distributions are plotted together. Compared to Fig. 2, the data details in the beam core part of Fig. 3 (a)-(d) are lacking due to a wider step size for shorter measurement time. However, the emittances and twiss parameters are agreeable. We also compared the x1

and  $y_1$  slit measurements (plotted in blue and red) with the reconstructed ones (in sky blue and yellow) shown in Fig. 3 (e) and (f).  $x$  and  $y$  distributions are fitted with Gaussian function and their rms beam radii are obtained (see Fig. 3 (e) and (f)).

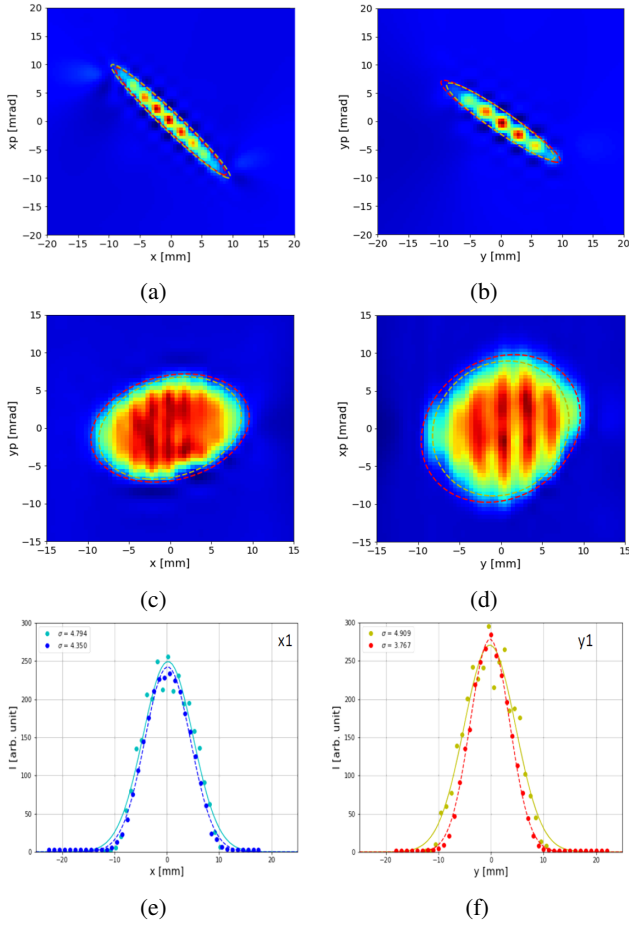


Figure 3: Reconstructed beam phase space distribution of (a)  $x - x'$ , (b)  $y - y'$ , (c)  $x - y'$ , (d)  $y - x'$ , (e)  $x$  and (f)  $y$ .

## 4D EMITTANCE

The 4D symmetric beam matrix is expressed as

$$\Sigma = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle xx' \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'y' \rangle \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy}^T & \sigma_{yy} \end{bmatrix}.$$

If at least one of elements in the off-diagonal sub-matrix is nonzero, the beam is  $x$ - $y$  coupled.

The eigen emittances  $\epsilon_1, \epsilon_2$  are yielded by diagonalization of the  $\Sigma$  matrix. They are found to be

$$\Sigma \rightarrow M \Sigma M^T. \quad (8)$$

4D emittance,  $\epsilon_{4D}$  is

$$\epsilon_{4D} = |\Sigma|^T = \epsilon_1 \epsilon_2 \leq \epsilon_x \epsilon_y, \quad (9)$$

where  $|\dots|$  means the determinant.

Eigen emittance is given by [5]

$$\epsilon_{1,2} = \frac{1}{2} \sqrt{-\text{tr}[(\Sigma U)^2] \pm \sqrt{\text{tr}^2[(\Sigma U)^2] - 16|\Sigma|}}, \quad (10)$$

where  $U$  is the unit symplectic matrix,

$$U = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

$\Sigma$  matrix is found to be

$$\Sigma = \begin{bmatrix} 18.533 & -18.733 & -0.818 & 3.262 \\ -18.733 & 19.957 & 1.597 & -3.594 \\ -0.818 & 1.597 & 15.332 & -11.199 \\ 3.262 & -3.594 & -11.199 & 9.182 \end{bmatrix}.$$

Eigen emittances are  $\epsilon_{1,2}$  are 0.222 and 0.119  $\pi$  mm mrad. 4D emittance,  $\epsilon_{4D} = 0.571 \pi$  mm<sup>2</sup> mrad<sup>2</sup> is obtained while  $\epsilon_x \times \epsilon_y = 0.788 \pi$  mm<sup>2</sup> mrad<sup>2</sup>.

## CONCLUSION

A slit emittance meter was designed and manufactured to measure 4 D beam phase space distribution. We successfully measured  $x - y$ ,  $x - x'$ ,  $y - y'$ ,  $x - y'$ ,  $y - x'$  and  $x' - y'$  phase space distribution using the slit emittance meter installed in the BTS beamline at KOMAC. From  $x' - y'$  phase space distribution, we reconstructed all the sub-dimensional distributions and compared them with the directly measured data. 4D emittance is obtained from the  $\Sigma$  matrix.

## ACKNOWLEDGEMENTS

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