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Phase Structure in Bosonic String Theory

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ABSTRACT

We carefully re-analyse a lattice model of oriented closed bosonic strings in light cone gauge due to Klebanov and Susskind, completing the procedure suggested by these authors for solving for the spectrum. Our results enable us to relate the concept of minimum distance in bosonic string theory in un-compactified space to the phase structure of strings on tori, and in particular their duality symmetry. Generalisations to a theory of arbitrary dimension are given.

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I. Introduction

In the realm of bosonic and superstring theories, many of the questions of physical principle seem as difficult as they ever were. To name but one example, the tachyon which forced the construction of superstring theories originally, reappears unbidden in the canonical partition function of finite temperature superstring theories [1,2]. Inevitably one is led to consider the possibility of phase transition to a new vacuum at the appearance of such tachyons. In order to study such phenomena it is necessary to have a non-perturbative framework for string theory. This requires both a second quantized formulation and a non-perturbative regularization, in conventional language. A possible basis for such a framework in fact appears in an ingenious paper by Klebanov and Susskind [3], where they study a light-cone lattice hamiltonian originally proposed for investigating lattice gauge theory. It is a matrix model on a $D-2$ dimensional transverse lattice, in which string excitations arise for a broad range of parameters.

In fact lattice regularization of transverse space in the light cone formulation of bosonic string theory is a key to revealing the wild spatial properties of strings and their lack of short distance structure. It is these properties that we concentrate upon in this paper. In the following section 2 we give a short review of the curious spatial properties of the bosonic string. In sections 3 and 4 we shall study a version of the light cone hamiltonian proposed in ref.[3], which in a certain limit describes free strings without the necessity of tuning a spatial lattice spacing to zero. The spectrum given in ref.[3] in fact appears to be double that of free string theory. By imposing hitherto neglected Hilbert space constraints we will see that the spectrum is indeed that of a free string. In section 5 we go on to discuss the phase structure of the lattice model to the extent that this is possible for the unrealistic free theory. It is found that the spectrum in un-compactified space is unchanged when one varies the lattice spacing between zero and πR_o, where $R_o = 1/\sqrt{2}$ is the self dual radius of circle compactification. At these limits there exist phenomena corresponding to 'deconfinement' and '(de)roughening' transitions, respectively. We view the space-time approach to critical behaviour in this paper as complimentary to the regularized random surface approach.

For convenience, we will start in one transverse dimension (i.e. a linear lattice) as in ref.[3]. A lattice scheme for more transverse dimensions is given in section 6.

II. Spatial Properties of Strings

The 2-dimensional scalar fields X^μ giving the string worldsheet location in space-time are not well defined fields. They are not Wightman fields. They have unremovable IR divergences associated with them. Of course, one could regard this behaviour as merely a nuisance and construct formal procedures for taming it. We prefer to regard it as an important clue to the understanding of string theory. Such is the unruly nature of X^μ that it is probably impossible to give any sensible definition of a local physical quantity in string theory, except through the trivial procedure of calculating a global quantity and dividing the answer by the volume of space. The spectrum, S matrices and vacuum energy are classic results. They are global results. When one tries to localize the string, for example in calculating the vacuum energy in a finite box [4], the result is non-sensical. In this case one is breaking conformal invariance explicitly. The lack of UV divergences in the standard calculation of the total vacuum energy is a consequence of modular invariance, which cannot be maintained in the localization procedure.

The physical, gauge-invariant modes of the string are most easily analysed in transverse space of the light cone system. Free string oscillations in each independent direction are equivalent to an infinite tower of harmonic oscillators. Zero-point motion of these oscillators causes delocalisation of the string, in transverse space, so that the wavefunctional Ψ of ground and finite excited states describes strings with divergent length expectation [5]:

$$< \Psi(X(\sigma)) \mid \int d\sigma \mid \partial_\sigma X \mid \; \mid \Psi(X(\sigma)) > = \infty \tag{2.1}$$

These strings fill transverse space many times over (having infinite naïve Hausdorf dimension). A sensible regularization of this behaviour, which one might regard as an IR property, involves UV regularization on the string. For example one could discretize the string into a line of 'partons' or links. According to the particular method involved, there might be a concomitant UV regularization of space. In the model discussed in this paper, this is the case, with the result [3] that as the continuum limit on the string is taken the UV space regularization remains, intact.

III. Light Cone Lattice Strings

Consider D-dimensional space-time in light cone co-ordinates (X^+, X^-, X^i) where i labels $D-2$ transverse directions. Replace transverse space by a lattice. For simplicity, if we work in $D=3$ then we have a linear lattice L. On each directed link $\pm l$ of the lattice L there are general $N_c \times N_c$ complex matrices $M_{ab}(X^+, X^-, l)$ which satisfy

$$M_{ab}(X^+, X^-, l) = M^*_{ba}(X^+, X^-, -l) \tag{3.1}$$

Explicit reference to X^+, X^- in the argument will be dropped. Following ref.[3] we consider the following light cone hamiltonian, where X^+ is regarded as 'time',

$$\begin{aligned} P^- &= \frac{\Omega}{N_c a^2} \int dX^- \sum_l \mathrm{Tr}\{\mu M(l) M^\dagger(l) - \lambda M(l) M^\dagger(l) M(l) M^\dagger(l) \\ &\quad - M(l) M^\dagger(l) M(l+1) M^\dagger(l+1)\} \end{aligned} \tag{3.2}$$

which can be derived from a Lagrangian with kinetic term $2(\partial M/\partial X^+)(\partial M/\partial X^-)$. Here a is the lattice spacing while Ω, μ, λ are dimensionless constants (in general dependent upon a), and throughout this paper we set the string theory $\alpha' = 1/2$. The three terms in P^- shown in Fig1 basically correspond to traces around all possible loops of length 2 or 4. Upon quantization equation (3.1) translates into

$$M_{ab}(l) = M^\dagger_{ba}(-l) \tag{3.3}$$

where the dagger symbol here does not act on indices but has a purely quantum meaning. Link fields may be decomposed into creation and annihilation operators

$$M_{ab} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk^+}{\sqrt{2k^+}} (A_{ab}(k^+) e^{-ik^+X^-} + B^\dagger_{ab}(k^+) e^{ik^+X^-}) \tag{3.4}$$

which obey commutation relations

$$[A_{ab}(k^+), A^\dagger_{cd}(q^+)] = [B_{ab}(k^+), B^\dagger_{cd}(q^+)] = \delta_{ac}\delta_{bd}\delta(k^+ - q^+) \tag{3.5}$$

Each link is assigned a direction and $A^\dagger_{ab}(k^+)$ creates a string bit which carries longitudinal momentum k^+ and points from index a to index b along the link's direction. Similarly, $B^\dagger_{ab}(k^+)$ creates a bit pointing from b to a and opposite to the assigned direction. An oriented closed string state is defined as the trace over creation operators around a loop, acting on the vacuum, such that the sum of momenta k^+ carried by the bits is the total longitudinal momentum of the string P^+:

$$\mathrm{Tr}(\text{Product of } A^\dagger \text{ and } B^\dagger \text{ operators}) \mid 0 > \quad . \tag{3.6}$$

These are loops invariant under a local $U(N)$ symmetry of the hamiltonian analogous to lattice gauge theory. The collection of all such oriented loops on the lattice L forms the Hilbert space. For the $N_c = \infty$ case to be studied here the theory is free i.e. the hamiltonian propagates loops on the lattice without splitting or joining them.

In order to reproduce the expected characteristics of bosonic strings in transverse space, namely divergent length, a sufficiently negative value of μ is taken. That this gives the desired instability can be seen by looking at the term in P^-:

$$< \mu \int dX^- \sum_l \mathrm{Tr}\left(M(l)M^\dagger(l)\right) > \sim \mu \sum_{j=1}^{N} \frac{1}{k_j^+} \tag{3.7}$$

where the expectation value is taken in a state of total N links. For μ sufficiently negative it is favourable to have a large number of links, each carrying a small fraction of the total longitudinal momentum. In light cone formalism the longitudinal momentum between two points on the string is proportional to the amount of σ-space between these points. For μ sufficiently negative each string bit carries an infinitesimal portion of σ-space[2] and we have a continuum limit on the string. The spatial lattice spacing a remains untouched. The spectrum with respect to the ground state can now be solved for by evaluating the expectation of the other terms in P^- in a basis of states with total number of links $N \to \infty$.

IV. Spectrum

On a linear lattice a string of length N can be represented by a series of N pluses and minuses. A label for the centre of mass motion is also needed. It is therefore convenient to think of these configurations as a series of Ising spins together with an overall phase factor, for which a typical string state would be denoted by

$$\exp(ipx_0)\ |\uparrow\downarrow\downarrow\uparrow \cdots \downarrow\uparrow> \tag{4.1}$$

[2]Following ref.[3] we regularize so as to introduce a minimum amount of longitudinal momentum P^+/N

where x_0 is the centre of mass in space of the given spin configuration and, for the given string configuration, ranges over a lattice of spacing a. When all spin configurations are included, and $N \to \infty$, the string centre of mass may take any real value. It is straightforward to show that, to leading order in N_c, the action of the second term in the hamiltonian P^- on states in the spin representation is equivalent to

$$-\frac{\Omega' \lambda N}{2P^+a^2} \sum_{j=1}^{N} (1 - \sigma_3(j)\sigma_3(j+1)) \tag{4.2}$$

while the action of the third term in P^- is

$$-\frac{\Omega' N}{2P^+a^2} \sum_{j=1}^{N} \frac{1}{2} (1 + \sigma_3(j)\sigma_3(j+1)) + \sigma_+(j)\sigma_-(j+1) + \sigma_-(j)\sigma_+(j+1) \tag{4.3}$$

where Ω' is some constant fixed by α' and a, and the standard Pauli matrices have been used. In order to eliminate the lattice momentum p from the problem for the time being we shall compactify the transverse lattice on a circle, i.e. mod out by some sublattice. If we mod out by the full lattice we end up with a single link lattice with periodic boundary conditions, thus eliminating the need for p. It is again straightforward to check that equations (4.2) and (4.3) are unaffected by this trick, to leading order in N_c.[3] Later we will argue that the fundamental length involved in the model is in fact two links.

The full hamiltonian now reads

$$P^- = -\frac{N\Omega'}{2P^+a^2} \sum_{j=1}^{N} \sigma_+(j)\sigma_-(j+1) + \sigma_-(j)\sigma_+(j+1) - 2\Delta\, \sigma_3(j)\sigma_3(j+1) \tag{4.4}$$

up to a quadratically divergent constant (as $N \to \infty$) and Δ is related to λ. Equation (4.4) is proportional to the XXZ hamiltonian of a linear quantum spin chain with periodic boundaries. The finite size scaling spectrum with respect to the groundstate energy as $N \to \infty$ for a state of conformal spin S is [6]

$$E = h + \overline{h} + r + \overline{r} \quad , \quad S = h + r - \overline{h} - \overline{r} \tag{4.5}$$

where $h, \overline{h}$ are Virasoro highest weights of representations of two commuting $U(1)$

[3] In fact this is a general consequence of large N_c reduction of lattice matrix models. Because the theory is free only the zero mode is affected by compactification to one link.

Kač-Moody algebras in charge sector Q taking the possible values

$$(h,\bar{h}) = \left(\frac{[Q+4g\alpha]^2}{16g}, \frac{[Q-4g\alpha]^2}{16g}\right) \tag{4.6}$$

$$g = \frac{\pi}{4(\pi - \arccos\Delta)} \tag{4.7}$$

$$Q = \frac{1}{2}\sum_{j=1}^{N}\sigma_3(j) \tag{4.8}$$

and α can be any integer. The integers $r, \bar{r}$ are the usual string mode excitation numbers. Q has integer (half-integer) eigenvalues for N even (odd) respectively. The model is continuously critical in the range $-1 < \Delta < 1$, so that the above equations are appropriate for $1/4 < g < \infty$ $(0 < \arccos\Delta < \pi)$.

The string interpretation of this is as follows. Equation (4.4) is reckoned to represent the lattice model on a single periodic link, which one can think of as circle compactification of radius $R = a/2\pi$. Thus $2Q$ should be winding number n, and we must at least make the identification

$$32g = \left(\frac{2\pi}{a}\right)^2 \tag{4.9}$$

in order that winding makes the correct contribution to the spectrum, which is now of the form

$$\frac{P^+P^-}{2} = n^2R^2 + \frac{\alpha^2}{16R^2} + r + \bar{r}\ . \tag{4.10}$$

The map between matrix model states and spin states is one-to-many since all spin states obtained from one another by shifting the spin configurations around by a certain number of sites, are equivalent to the same matrix model state. Thus one should apply the constraint of zero conformal spin (σ-space momentum) to project out the translation invariant states. It is noteworthy that this constraint, which appears in standard treatments of light cone strings, arises naturally in this matrix model approach. For N odd n is odd and we have immediately that

$$Q\alpha = \frac{n\alpha}{2} = \bar{r} - r \tag{4.11}$$

must be an integer, implying that we should project onto even α. In fact the situation is more subtle. The σ-space momentum $\dot{S}$ given here is the generator of shifts in the spins by an even number of sites. To see this recall the standard method of solution of the XXZ chain [7] in terms of fermions via a Jordan–Wigner transformation

$$\psi(j) = i^j \prod_{k<j} \sigma_3(k)\sigma_-(j) \tag{4.12}$$

$$\psi^\dagger(j) = i^{-j} \prod_{k<j} \sigma_3(k)\sigma_+(j) \tag{4.13}$$

For N even it is meaningful to stagger these fermions in doublets.

$$\Psi(j) \sim \sqrt{N} \begin{pmatrix} \psi(j) \\ \psi(j+1) \end{pmatrix} \tag{4.14}$$

These Ψ are then Dirac fermions in a Thirring model, which can be bosonized to yield the spectrum. For staggered fermions ordinary continuum σ-space translations correspond to translations by an even number of sites, generated by

$$S \sim i \sum_j \psi^\dagger(j)\psi(j+2) - \psi^\dagger(j+2)\psi(j) \tag{4.15}$$

Thus, imposing S=0 in the present problem still leaves an additional projection to be done. String/spin states should be invariant under shifts by an odd number of sites also; annihilated by

$$\sum_j \psi^\dagger(j)\psi(j+1) + \psi^\dagger(j+1)\psi(j) \tag{4.16}$$

which is the axial charge in fact. For N odd this is not actually an extra condition since equation (4.15) suffices to generate all translates from a given spin configuration. If N is even, n is even, and equations (4.15) (4.16) require

$$\frac{n\alpha}{2} = \bar{r} - r \quad , \quad \alpha \text{ even} \tag{4.17}$$

That α even is needed is most easily seen by introducing the conventional Kaluza-Klein number $m = \alpha/2$ and seeing that equation (4.16) acts in transverse space as a rotation once around the compactification circle. This operation will have eigenvalues ± 1 for the two types of state invariant under equation (4.15), corresponding to m integer or half-integer. We want invariance, so m is integer. Hence equation (4.10) becomes the bosonic string spectrum on the circle of Fig2(a).

If more links are now added to the lattice in transverse space we must reintroduce the lattice momenta p of equation (4.1). These range over the Brillouin zone

$-\pi/a < p \leq \pi/a$ and are in addition discretized according to the appropriate boundary conditions. Following ref.[3] we may modify the hamiltonian (4.4) to incorporate p by making the substitutions

$$\begin{aligned}\sigma_+(j)\sigma_-(j+1) &\longrightarrow \sigma_+(j)\sigma_-(j+1)\exp(2ipa/N) \\ \sigma_-(j)\sigma_+(j+1) &\longrightarrow \sigma_-(j)\sigma_+(j+1)\exp(-2ipa/N)\end{aligned} \tag{4.18}$$

since whenever $\sigma_+(j)\sigma_-(j+1)$ acts on a string state the centre of mass moves $2/N$ lattice units to the right. Similarly the conjugate term moves the centre of mass to the left. The Pauli matrix algebra is preserved under the transformation

$$\begin{aligned}\sigma_-(j) &\longrightarrow \sigma_-(j)\exp(-2ipaj/N) \\ \sigma_+(j) &\longrightarrow \sigma_+(j)\exp(2ipaj/N)\end{aligned} \tag{4.19}$$

which reduces the hamiltonian to the XXZ one again, but with boundary conditions

$$\sigma_\pm(N+1) = \sigma_\pm(1)\exp(\pm 2ipa) \tag{4.20}$$

The spectrum becomes [6]

$$E = \frac{Q^2}{8g} + 2g(\alpha + pa/\pi)^2 + r + \bar{r} \tag{4.21}$$

and we should take α even. If transverse space has L links then the winding number is $2Q/L$, being an integer for closed strings, and we have compactification at radius $R = aL/2\pi$. Once again we require winding modes to make the correct contribution to the spectrum. Reworking the analysis one finds again equation (4.9). We thus have $R \to \infty$ as $L \to \infty$ with a unaffected (and fixed and non-zero through (4.9)). As $L \to \infty$ the momentum zero mode becomes continuous and we have free strings in un-compactified space.

In the case of un-oriented closed strings we should mod out the Hilbert space by the transformation $\sigma \to -\sigma$, of the σ co-ordinate on the string, which reverses the counting of spins in a given configuration of the XXZ chain. This operation can be realised in the original light cone lattice model by using real link matrices M_{ab}. One can also orbifold the model by modding out by $X^i \to -X^i$, achieved in the spin model by requiring invariance under the operation of spin flip and $x_0 \to -x_0$ for the centre of mass variable. Thus, the model appears to incorporate all the features of free closed string theory.

V. Phase Structure

It was claimed in ref.[3] that the phase structure of the lattice field theory in un-compactified space was governed solely by that of the XXZ model. We therefore remind the reader of these properties, which may also be phrased in the language of the lattice string model on a single periodic link. (Note that the L link periodic lattice may also be regarded, for fixed zero mode momentum p, as a lattice with a single periodic link $R = a/2\pi$ and twisted boundary conditions as follows from equations (4.9) (4.20) (4.21)). The XXZ chain renormalises onto a Gaussian fixed line for $-1 < \Delta < 1$. At $\Delta = -1$ there is a natural boundary at which there is a first order transition to a phase with $< \sigma_3 >= 1$ and corresponds to zero lattice spacing (compactification circumference). We are prevented from making a arbitrarily large due to a Kosterlitz-Thouless type transition at $\Delta = 1$, where $< (-1)^j \sigma_3(j) >$ becomes non-zero. This staggered spin operator is a mass term. The reason for this transition is the presence in the $\sigma_3\sigma_3$ term of equation (4.4) of a spin-wave vertex operator which becomes marginal at a lattice compactification radius corresponding to $R = 1/2\sqrt{2}$, i.e. half the self-dual radius. In fact it is the first, of all possible spin-wave operators, to become relevant as one increases the radius up from zero. The three phase structure exhibited here is not uncommon in other physical systems. The Gaussian phase is the roughened region, where translation symmetries are restored at finite spatial lattice cutoff, and $\Delta = 1$ marks the roughening transition. The point $\Delta = -1$ may be thought of as a 'deconfining' transition since, formally speaking, it signals the relevance of vortices of infinite winding number (from (4.8) as $N \to \infty$).

It may seem surprising that the critical region is $0 < R < 1/2\sqrt{2}$ and not $0 < R < 1/\sqrt{2}$ since one is used to the 6-Vertex model, which has the XXZ 'hamiltonian', mapping onto the physically inequivalent circle compactifications of a free boson. However we should recall the peculiarity of the present case (equation (4.17)) where we have to omit α odd. If we had compactified onto two links (Fig2(b)) then the spectrum appropriate for this could have been gotten by forgetting the momenta p and including α odd as a prescription instead, in this case. The equivalence of the full XXZ model and a circular boson such that $0 < R < 1/\sqrt{2}$ is then clear [6]. In fact one should regard the two link length as the 'fundamental length', at least in the present description of closed bosonic strings.[4] The string co-ordinate must consist of right and left movers , naturally relating to the right and left vector current densities of the fermion solution to the XXZ model. These current densities are spread over two sites on account of the staggered nature of the fermions. We believe that the calculations presented here go some way to quantifying the often repeated dictum that duality symmetry on the circle has something to do with ambiguity in defining short distances. Or more precisely the relation of short distance ambiguity to the line of physically inequivalent circle vacua.

VI. More Than One Transverse Dimension

The most natural generalisation of P^- to greater than 1 transverse dimension is to take a $D-2$ dimensional lattice and consider terms in P^- which are again traces around all possible loops of length 2 or 4. In order to prove equivalence to $D-2$ uncoupled XXZ models we find it convenient to work, not on a hyper-cubic lattice, but on a lattice of body diagonals of the latter (i.e. the lattice generated by the (overcomplete) set of vectors $\{(\pm1, \pm1, \ldots, \pm1)\}$). In this way each string bit on such a diagonal can be labelled by a plus or a minus (Ising spin) by projection onto each of the $D-2$ Cartesian axes of the hyper-cube. The Hilbert space then consists of $D-2$ spin models, and the problem is to show that terms in P^- which couple these models are irrelevant in the $N \to \infty$ continuum limit.

The length 2 loops in P^- provide the instability as before, and the length 4 loops can be of zero Z or non-zero NZ area (not necessarily planar), illustrated for $D-2=2$ in Fig3. We have to study the action of these operators on all possible configurations of successive string bits $(j, j+1)$. The Z terms act as both kinetic $\sigma_+\sigma_-$ and potential $\sigma_3\sigma_3$ operators, while the NZ terms are only of kinetic type. By studying the effects of these matrix model operators on simple string configurations it is not hard to see

[4]The present lattice model seems only suited to closed bosonic strings. The open strings one may construct do not have the standard open bosonic string spectrum.

that the following spin operators

$$A_j = \sum_i \sigma_+^i(j)\sigma_-^i(j+1) + \sigma_-^i(j)\sigma_+^i(j+1) \tag{6.1}$$

$$B_j = \sum_i \sigma_3^i(j)\sigma_3^i(j+1) \tag{6.2}$$

are sufficient to analogue any behaviour, on appropriate spin states, where the sum over i in this form is dictated by cyclic symmetry $i \to i+1$. The potential hamiltonian will be some polynomial in variable (6.2) and the kinetic hamiltonian will be a polynomial in the variable (6.1) . Upon transformation to fermion fields via a Jordan–Wigner map (4.12) (4.13) we obtain as $N \to \infty$

$$N\sum_j A_j \longrightarrow \sum_i \int d\sigma \Psi_i^\dagger \gamma_5 \partial_\sigma \Psi_i \tag{6.3}$$

$$N\sum_j B_j \longrightarrow \sum_i \int d\sigma \, (\Psi_i^\dagger \gamma_0 \Psi_i)^2 \tag{6.4}$$

Thus we need only keep terms up to and including the linear ones in our polynomials, the higher terms will be irrelevant operators and are supressed by powers of N in the continuum limit on the string. The hamiltonian is then, up to a quadratically divergent constant, a sum of $D-2$ uncoupled XXZ hamiltonians with the coefficient of the kinetic part set to the fundamental string tension by adjusting a parameter analogous to Ω appropriately. We can make the coefficients of the potential parts variable, analogous to Δ, by giving special consideration to the only purely potential Z term illustrated for $D-2=2$ in Fig3(c). In general there should be $D-2$ such terms each with an independent coefficient. These $D-2$ degrees of freedom set the radii of a simple torus compactification.

VII. Conclusions

By completing the solution for the spectrum of Klebanov and Susskind's light-cone lattice string model as a compactification problem we have been able to establish a quantitative relationship between the ambiguity of short distances in closed bosonic string theory and the line of physically inequivalent circle vacua in moduli space. Indeed, we have shown that the lattice is hidden behind continuum free string theory (for a certain range of parameters) providing that the lattice spacing a is *non-zero* and the fundamental length $2a$ is less than the self-dual circumference $\pi\sqrt{2}$ for circle compactification. If $2a$ is increased beyond the self-dual circumference the model suffers a de-roughening transition to a lattice dominated phase. This gives an intuitive understanding of the nature of duality symmetry in background space, at least in short distance regularizations of that space investigated here.

Many important questions now arise. Here, as in ref.[3], the vacuum energy has remained quadratically divergent (with the opposite sign to the conventional light-cone gauge divergence) enabling a simplification of the model. It has not been proved whether this can be renormalised away to yield a Lorentz invariant spectrum, and at the same time leave all the considerations about lack of short distance structure intact. A supersymmetric version of the lattice model, if indeed one exists yielding a superstring spectrum, would be an important test of the logic applied in this paper, especially pertinent in view of the effects that worldsheet fermions might be expected to have on critical properties. Lastly there is the question of string interactions. At finite N_c there are 3-string interactions with coupling $1/N_c$, and one might hope that a large N_c expansion would yield the dual amplitudes. As far as we know, this has not yet been shown. In any case, once the interactions are turned on (however small), one would expect to expose the full richness of the phase structure of the compactified model, such as Polyakov line condensates [8] and Kaluza-Klein condensates.

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Fig1 The 2 and 4 link loops on a linear lattice

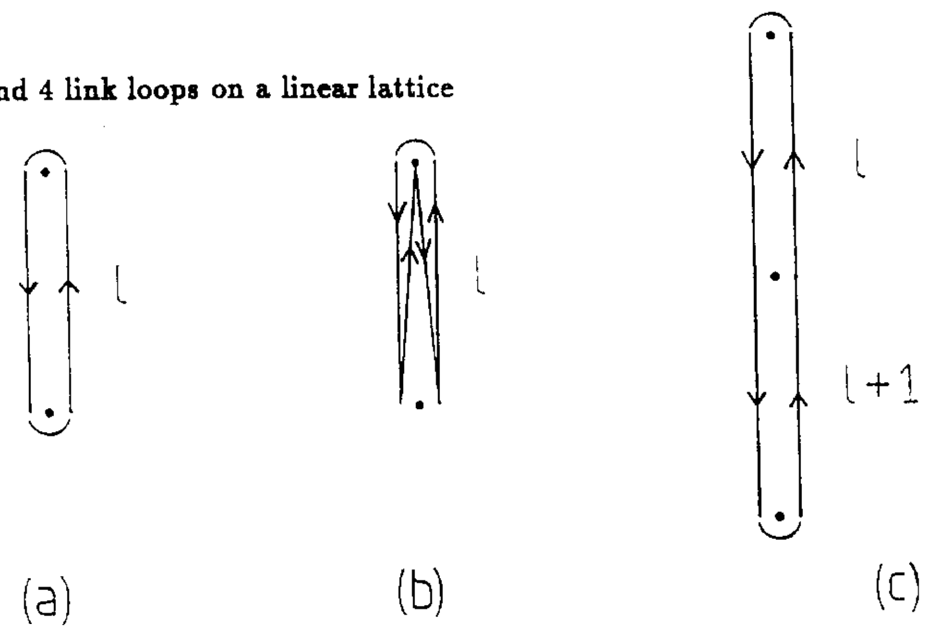

Fig2 Compactification on a) one link b) two links

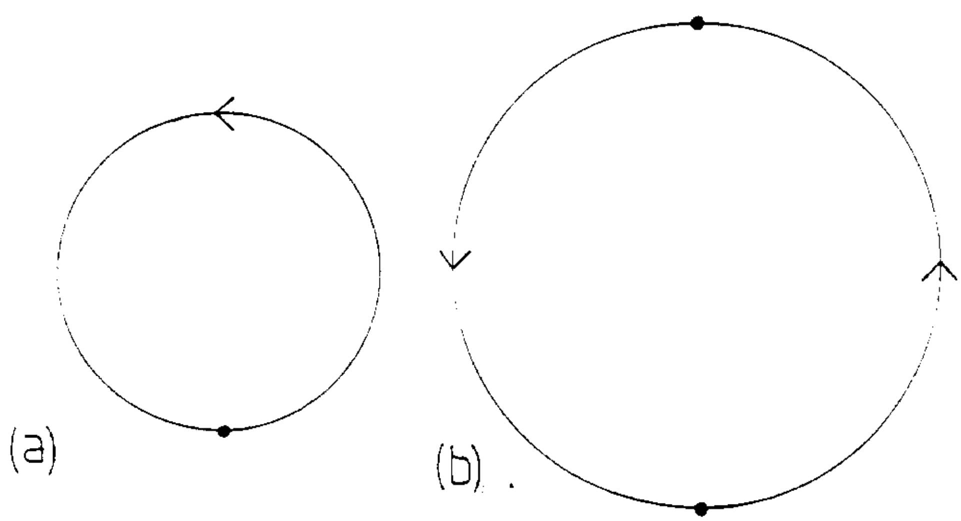

Fig3 Some 4 link loops in D-2 = 2