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Supersymmetric SU(5) model with four light Higgs doublets

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Abstract

We show that a supersymmetric SU(5) grand unified model with four light Higgs doublets, a light colour octet, an electroweak triplet and two singlets predict the correct value of $\sin^2\theta_W$. Moreover it is possible to obtain the rations $m_b(m_b)/m_\tau$ and $m_s(1\mathrm{GeV})/m_\mu$ in agreement with experiments provided the Yukawa coupling constants for the second generations are relatively big.

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There has recently been renewed interest in grand unified business related with the recent LEP data which allow to measure $\sin^2(\theta_W)$ with unprecedented accuracy. Namely, the world averages of LEP data mean that the standard non-supersymmetric SU(5) model [1] is ruled out finally and forever (the fact that the standard non-supersymmetric SU(5) model is in conflict with experiment was well known [2] before LEP data). But maybe the most striking and impressive lesson from LEP is that the supersymmetric extension of the standard SU(5) model[3,4,5] predicts the Weinberg angle θ_W in very good agreement with experiment. The remarkable success of the supersymmetric SU(5) model is considered by many physicists as the first hint in favour of the existence of low energy broken supersymmetry in Nature. However, as it has been shown in [6,7,8] it is possible to construct non-supersymmetric generalisations of the standard SU(5) model which give good predictions for $\sin^2(\theta_W)$ and don't have problems with proton decay by the introduction of the additional split $5+\bar{5}$ and $10+\overline{10}$ multiplets in the minimal three families $\bar{5}+10$ of SU(5) [6,7] or by the introduction of the additional light coloured octets and electroweak triplets which originate from 24 adjoint multiplets of SU(5) [8]. Several non-supersymmetric and supersymmetric SU(5) models with light coloured octets and electroweak triplets have been constructed. The typical feature of the models proposed in [8] is the existence of $H \leq 4$ light Higgs doublets. On the other hand besides problems with proton decay and $\sin^2\theta_W$ the grand unified model have the problem with fermion masses. Namely the standard SU(5) grand unified theories (both supersymmetric and non-supersymmetric) predict equal masses of lep tons and down-type quarks at the GUT scale [9]. This prediction could be successful for the third generation provided the top Yukawa coupling constant is relatively big [10] and fails for the first and second generations. There are several ways to overcome these troubles by postulating new nonrenormalizable interactions [11] or by the introduction of a 45 Higgs multiplet with nonzero vacuum expectation value [12]. As it has been shown recently in [13] it is possible to explain m_d/m_e ratio for the first generation in the standard SUSY-SU(5) model with radiative fermion masses of the first generation through soft supersymmetric breaking terms.

In this paper we consider the SU(5) grand unified model with four light Higgs doublets, light colour octet, electroweak triplet and two singlets which has been proposed in [8]. We show that the model predicts the correct value of $\sin^2\theta_W$. Moreover we find that it is possible to obtain the ratios $m_b(m_b)/m_\tau$ and $m_s(1GeV)/m_\mu$ in agreement with experiment provided the Yukawa coupling constants for the second and the third generations are relatively big. The key observation is very simple: in the model with four light Higgs isosinglets we can have relatively big Yukawa coupling constants both for the second and third generations. In the calculation of the Yukawa coupling evolution from the electroweak scale to the GUT scale we have to take into account the h^3-term for the Yukawa β-function which is positive and decreases the evolution of the Yukawa coupling from the electroweak scale to the GUT scale compared to the case when we take into account the QCD evolution only. Unfortunately for the first generation we can't explain the m_d/m_e ratio (or to be more precise: if we obtain the right value for m_d/m_e then we have troubles with the explanation of the m_s/m_μ ratio for the second generation). So we shall assume the mechanism proposed in [14] for m_d/m_e ratio within the standard SU(5) model also for in our model.

So consider the supersymmetric $SU(5)$ model [8] with three generations, four light Higgs superdoublets, one colour octet superhiggs, one triplet superhiggs and two singlet superhiggses $2((1,1,2)+(1,1,-2))$. The representation $(1,1,2)$ can originate from a $\overline{10}$-plet of SU(5). The $SU(3)\times SU(2)\times U(1)$ content of the 10-representation is $10=(3,2,-1/2)+(\bar{3},1,4/3)+(1,1,-2)$. We assume that due to the "missing partner" mechanism [14] only particles with quantum numbers $(1,1,-2)+(1,1,2)$ remain light inside $10+\overline{10}$.

For the $SU(3)\times SU(2)\times U(1)$ effective supersymmetric gauge theory the β-functions up to two loops are given by

$$\beta_i = \frac{g_i^3}{16\pi^2}(b_i + \sum_{j=1}^{3} b_{ij}\frac{g_j^2}{16\pi^2}) \tag{1}$$

For the considered model we have $b_3 = 0, b_2 = 4, b_1 = 9.6$ and

$$b_{ij} = N_g \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} + H \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{144}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

where N_g is the number of generations and H is the number of Higgs doublets. Here we neglected the contribution of the Yukawa coupling constants to the two loop gauge β-functions. At one loop level the solutions for the effective coupling constants are

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_G} + b_i t \tag{3}$$

where $t = \frac{1}{2\pi}\ln(m/\mu)$. Here we also assume that the electromagnetic coupling constant α and the strong coupling constant α_s are given at the electroweak scale $\mu = M_Z$. The use of eq. (3) allows one to determine the grand unified scale M_{GUT}, the unified coupling constant α_G and the Weinberg angle θ_W. At one loop level the following basic formulae take place [5,9]

$$\begin{aligned} \sin^2(\theta_W) &= \frac{1}{\alpha}(b_2 - b_3 + (b_1 - b_2)c^2\frac{\alpha}{\alpha_S}) \\ t &= \frac{1}{d}(\frac{1}{\alpha} - \frac{1+c^2}{\alpha_3}) \\ \frac{1}{\alpha_G} &= \frac{1}{d}(-\frac{b_3}{\alpha} - \frac{b_2 + c^2 b_1}{\alpha_3}) \\ d &= b_2 - b_3 + c^2(b_1 - b_3) \\ c^2 &= \frac{5}{3} \end{aligned} \tag{4}$$

In the assumption that all additional light particles have masses $O(M_Z)$ for $\alpha(M_Z) = 1/128$ and $\alpha_S(M_Z) = 0.115 \pm 0.01$ one can find that [8]

$$\begin{aligned} \sin^2(\theta_W) &= 0.2317^{-0.0025}_{0.0030} \\ M_{GUT} &= 2.0^{+1.6}_{-1} 10^{16}\mathrm{GeV} \\ \frac{1}{\alpha_G} &= 8.7^{-0.7}_{+0.8} \end{aligned} \tag{5}$$

Note that according to LEP data [15] the value of $\sin^2(\theta)$ in the $\overline{MS}$-scheme [16] at the M_Z energy is [17]

$$\sin^2(\theta_W) = 0.2333 \pm 0.0008$$

It appears that two-loop corrections are not small. Our numerical results for the calculations of $\sin^2\theta_W, M_{GUT}$ and α_{GUT} are presented in figures 1a-c. So we see that an account of the two loop corrections leads to the increase by a factor of ≈ 5 of the grand unified scale which is a welcomed feature at least from the point of view of string theories [18].

In SU(5) grand unified theories with light Higgs doublets which come from the 5-plet representation of SU(5) the Yukawa term $10_M \bar{5}_M \bar{5}_H$ is responsible for the creation of masses for both down quarks and charged leptons. As a consequence at the GUT-scale we have the relations between charged leptons and down-quark Yukawa coupling constants

$$\bar{h}_d(M_{GUT}) = \bar{h}_e(M_{GUT}) \tag{6}$$

$$\bar{h}_s(M_{GUT}) = \bar{h}_\mu(M_{GUT}) \tag{7}$$

$$\bar{h}_b(M_{GUT}) = \bar{h}_\tau(M_{GUT}) \tag{8}$$

For the standard supersymmetric SU(5) model the boundary conditions (4) and (5) for the first two generations lead to the wrong predictions for the m_d/m_e and m_s/m_μ ratios. As for the case of the third generation one can obtain the experimental ratio m_b/m_τ only provided that the top Yukawa coupling constant is not small which seems to be the case in reality since the recent analysis of LEP-data, based on the account of radiative corrections, gives $m_t = 120 - 170$GeV [15]. In our case we have four light Higgs doublets so we can realize the situation when one pair of the Higgs doublets is coupled to the third generation and the second pair couples to the second generation whereas some linear combination of the first and second pair couples to the first generation. The masses of the fermions are equal to the corresponding vacuum expectation value times the Yukawa coupling constant. So in our model, unlike to the standard supersymmetric SU(5) model with two Higgs doublets we can realize a situation where the Yukawa coupling constants for both the second and third generations are not small and the h^3-terms in the β-functions decrease the evolution for the Yukawas from M_{GUT} to the electroweak scale compared to the case when only QCD-evolution is taken into account. Here for simplicity we shall neglect the mixing between the second and the third generation. Note that as it has been shown in [19] by postulating some discrete symmetries it is possible to solve the problem of flavour changing neutral currents in the case of models with more than two relatively light Higgs doublets.

The renormalization group equations for the Yukawa coupling constant of the third and second generation above the SUSY-threshold [‡] are [20,21]

$$\begin{aligned} Dh_{t(c)} &= (6h_{t(c)}^2 + h_{b(s)}^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2)h_{t(c)} \\ Dh_{b(s)} &= (h_{t(c)}^2 + 6h_{b(s)}^2 + h_{\tau(\mu)}^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{9}g_1^2)h_{t(c)} \\ Dh_{\tau(\mu)} &= (4h_{\tau(\mu)}^2 + 3h_{b(s)}^2 - 3g_2^2 - 3g_1^2)h_{\tau(\mu)} \end{aligned} \tag{9}$$

‡For simplicity we assume that the masses of the additional octets, triplets and singlets are $O(M_{SUSY})$.

with $D = 16\pi^2 \mu d/d\mu$. We also assume that for the scale $m_t < \mu < M_{SUSY}$ we have effectively the standard Weinberg Salam model with a single Higgs doublet and the corresponding renormalization group equation have the form

$$
\begin{aligned}
Dh_{t(c)} &= (\frac{9}{2}h^2_{t(c)} + \frac{3}{2}h^2_{b(s)} + h^2_{\tau(\mu)} - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2)h_{t(c)} \\
Dh_{b(s)} &= (\frac{3}{2}h^2_{t(c)} + \frac{9}{2}h^2_{b(s)} + h^2_{\tau(\mu)} - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{12}g_1^2)h_{t(c)} \\
Dh_{\tau(\mu)} &= (\frac{5}{2}h^2_{\tau(\mu)} + 3h^2_{b(s)} + 3h^2_{t(c)} - \frac{9}{4}g_2^2 - \frac{15}{4}g_1^2)h_{\tau(\mu)}
\end{aligned}
\qquad (10)
$$

The masses of the fermions are

$$
\begin{aligned}
&m_c(\mu) = \overline{h}_c(\mu)v_1 \qquad m_\mu(\mu) = \overline{h}_\mu(\mu)v_2 \\
&m_s(\mu) = \overline{h}_s(\mu)v_2 \qquad m_b(\mu) = \overline{h}_b(\mu)v_3 \\
&m_\tau(\mu) = \overline{h}_\tau(\mu)v_3 \qquad m_t(\mu) = \overline{h}_t(\mu)v_4
\end{aligned}
\qquad (11)
$$

where the v_i are the vacuum expectation values of the corresponding Higgs doublets. "Experimental values" for the quark masses are [22]

$$
\begin{aligned}
m_b(m_b) &= 4.25 \pm 0.15 \text{GeV} \\
m_s(1\text{GeV}) &= 180 \pm 50 \text{MeV} \\
m_d(1\text{GeV}) &= 8.9 \pm 2.6 \text{MeV}
\end{aligned}
\qquad (12)
$$

We found that it is possible to obtain reasonable values for the b- and s- quark starting from the GUT boundary conditions

$$
\begin{aligned}
&\overline{h}_b(M_{GUT}) = \overline{h}_\tau(M_{GUT}) \\
&\overline{h}_s(M_{GUT}) = \overline{h}_\mu(M_{GUT})
\end{aligned}
\qquad (13)
$$

and solving the renormalization group equations for the Yukawas from the GUT scale to the 1 GeV scale. Of course the results depend on the scale of supersymmetry breaking M_{SUSY}, on the value of α_s and on the initial value of $\overline{h}_b(M_{GUT})$ and $\overline{h}_s(M_{GUT})$. Our results are presented in figures 2a-d and 3a-d

In conclusion we would like to stress that for the grand unified models with more than two Higgs by playing with the Yukawa couplings it is possible to obtain the ratios $m_b(m_b)/m_\tau$ and $m_s(1\text{GeV})/m_\mu$ in agreement with experiment. For the first generation we can obtain the right value for the $m_d(1\text{GeV})/m_e$ ratio, but it appears that it is impossible to obtain simultaneously the right ratios for the first and second generation at least within the considered model.

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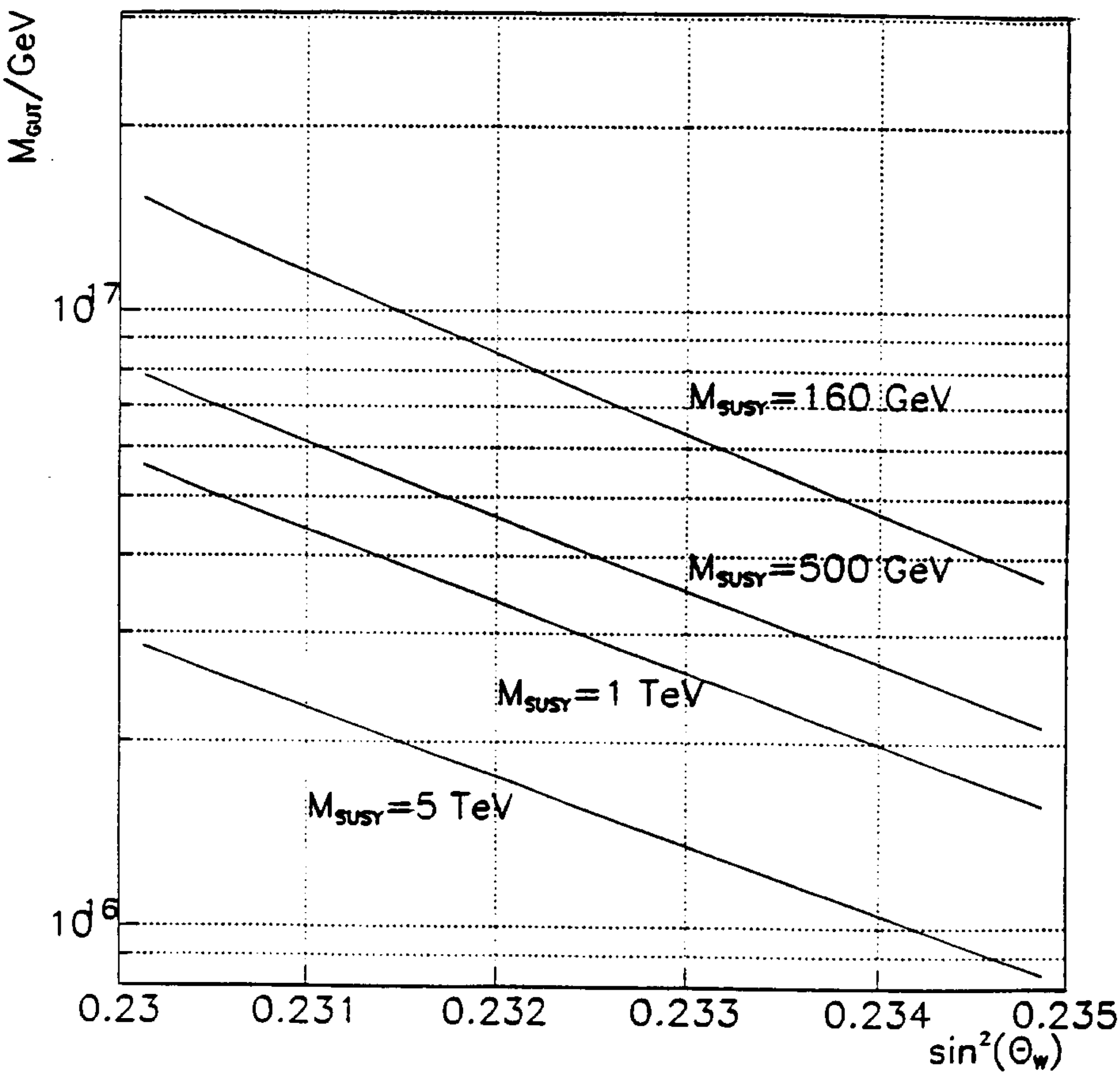
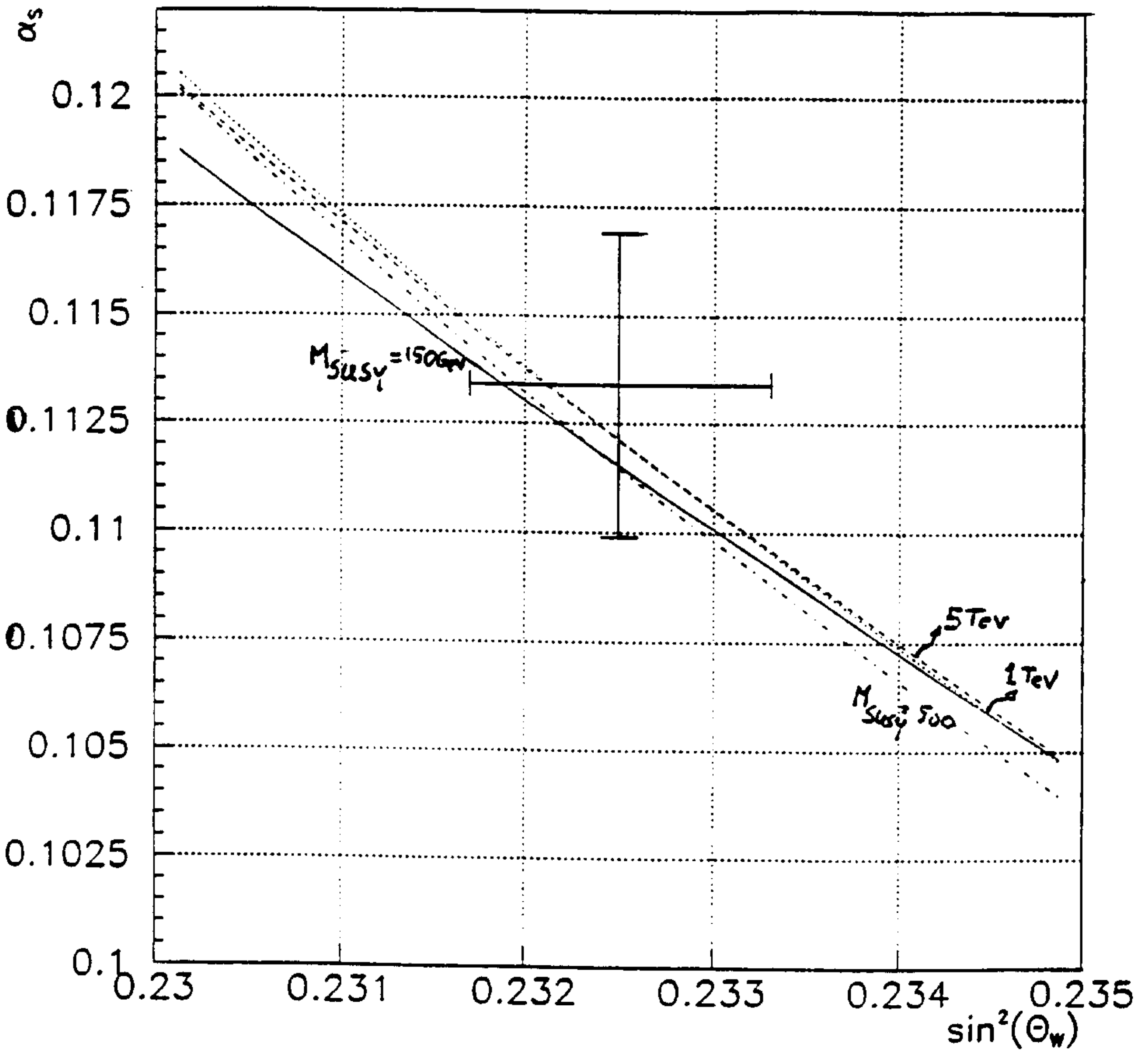
Figure captions

Fig.1: M_{GUT}, α_s, and α_{GUT} in our model as a function of $\sin^2\theta_W$ and M_{SUSY}.
Fig.2: Ratio of m_s and m_μ for various α_s and M_{SUSY}. $h(M_{GUT})$ is the common Yukawa coupling at the GUT scale.
Fig.3: Like Fig.2, but for m_b/m_τ .



M_{GUT}/GeV
10^7
10^6
M_{SUSY}=160 GeV
M_{SUSY}=500 GeV
M_{SUSY}=1 TeV
M_{SUSY}=5 TeV
0.23
0.231
0.232
0.233
0.234
0.235
$\sin^2(\Theta_W)$

Fig. 1. a



α_s
0.12
0.1175
0.115
0.1125
0.11
0.1075
0.105
0.1025
0.1
0.23
0.231
0.232
0.233
0.234
0.235
$\sin^2(\Theta_W)$
M_{SUSY}=150GeV
5TeV
1TeV
M_{SUSY}= 500

Fig.1.b

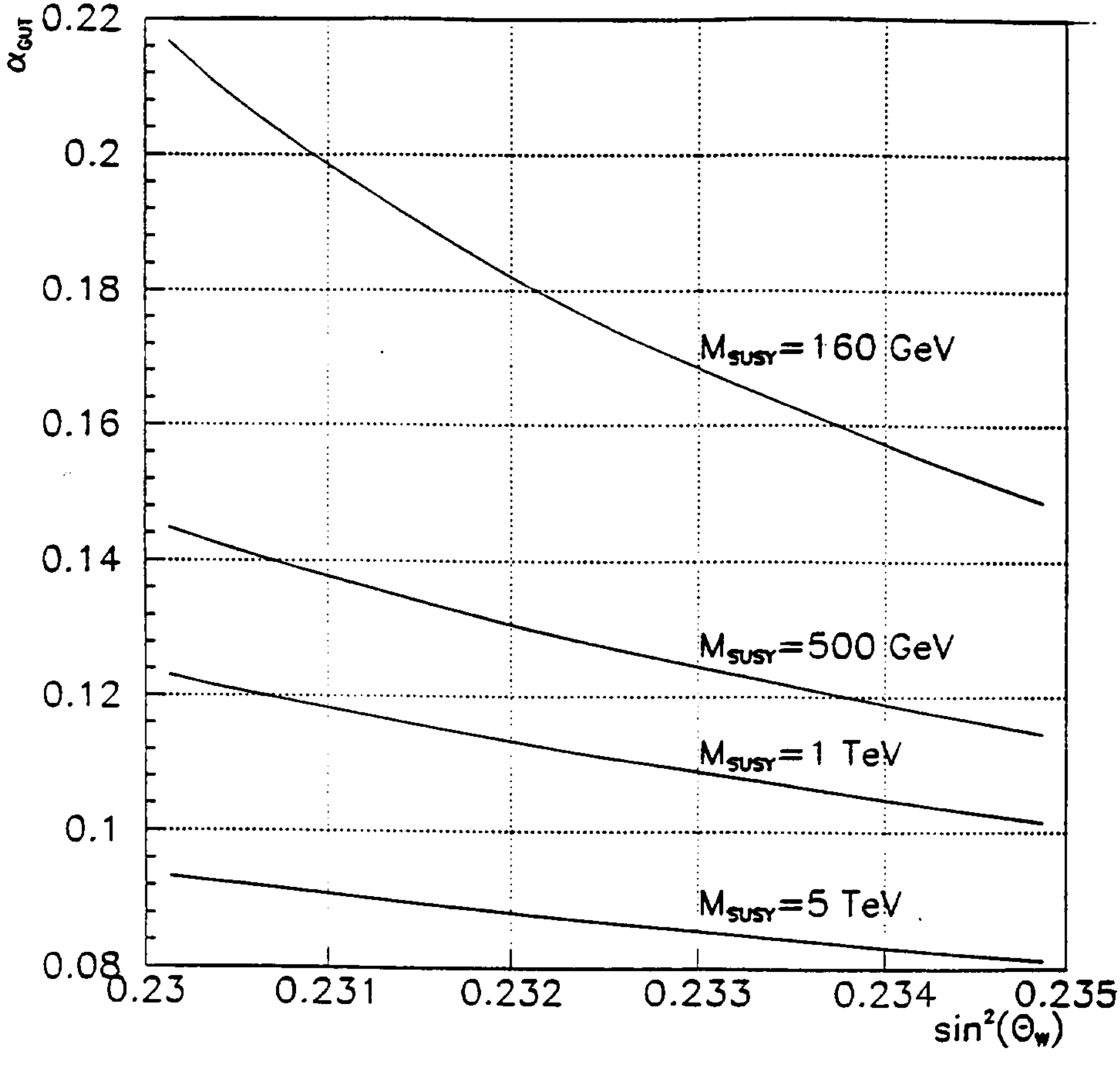
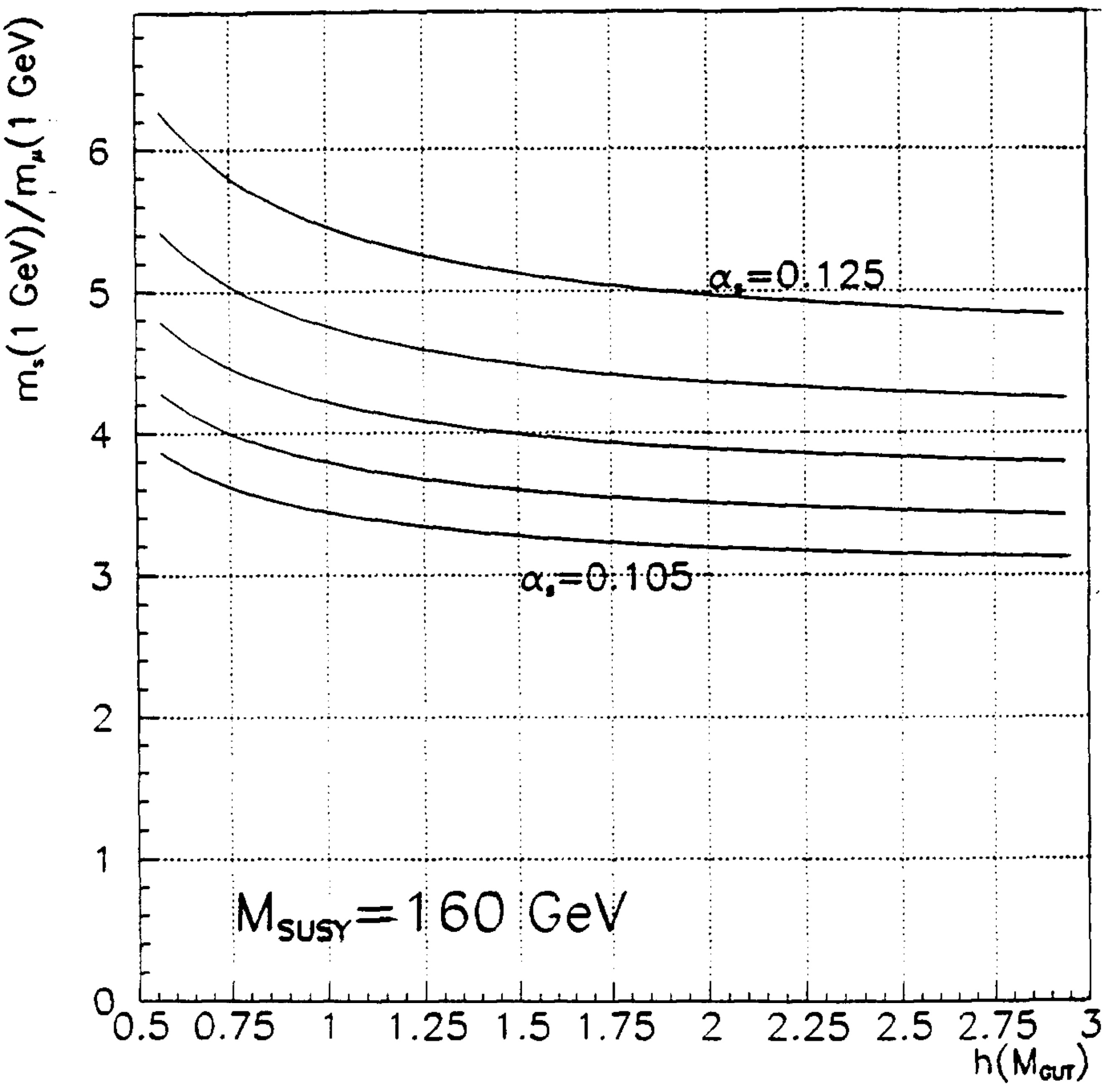
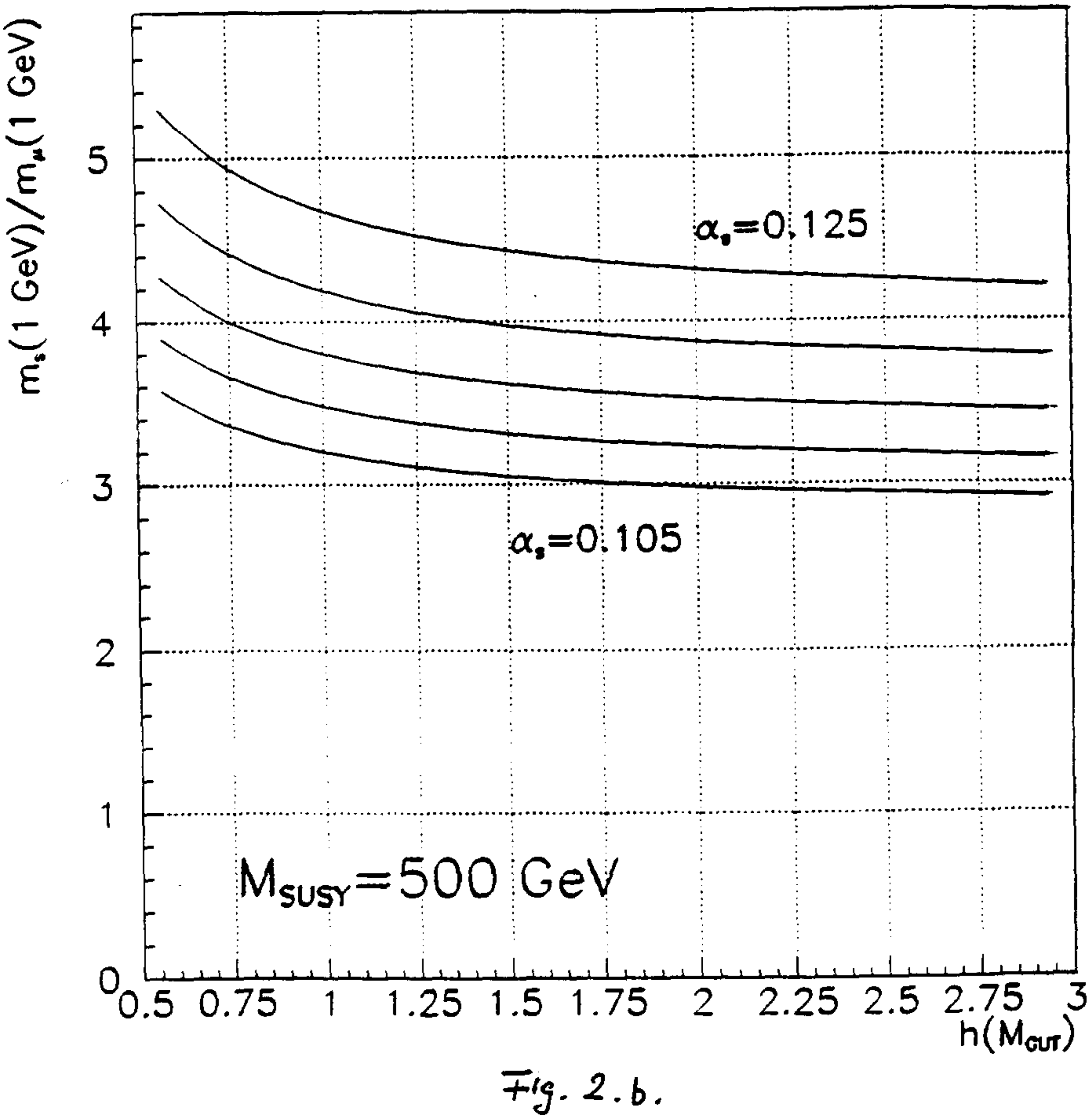

α_{GUT}
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.23
0.231
0.232
0.233
0.234
0.235
$\sin^2(\Theta_w)$
M_{SUSY}=160 GeV
M_{SUSY}=500 GeV
M_{SUSY}=1 TeV
M_{SUSY}=5 TeV

Fig. 1c.



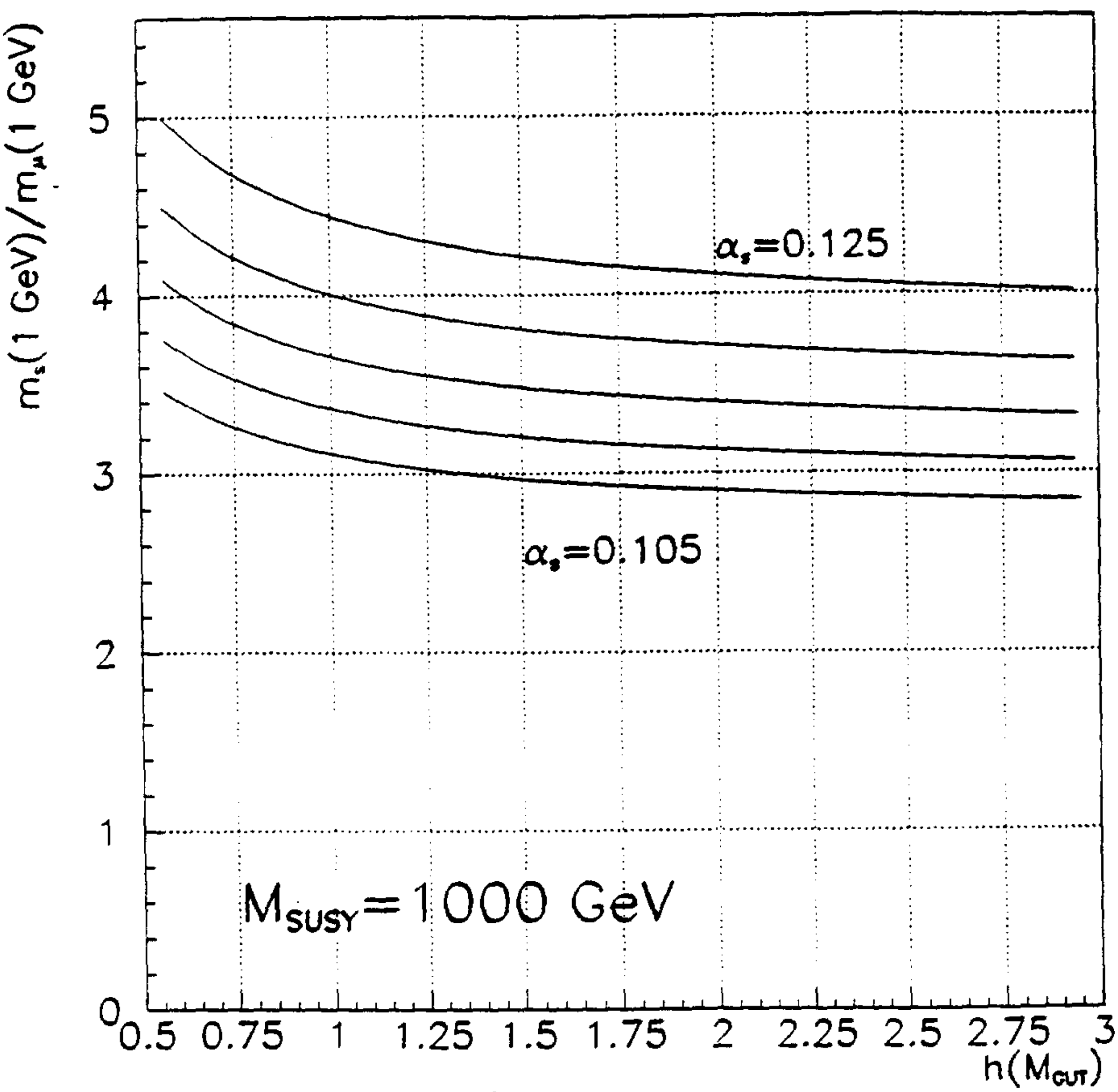
m_s(1 GeV)/m_μ(1 GeV)
6
5
4
3
2
1
0
0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3
h(M_{CUT})
α_s=0.125
α_s=0.105
M_{SUSY}=160 GeV

Fig. 2.a.



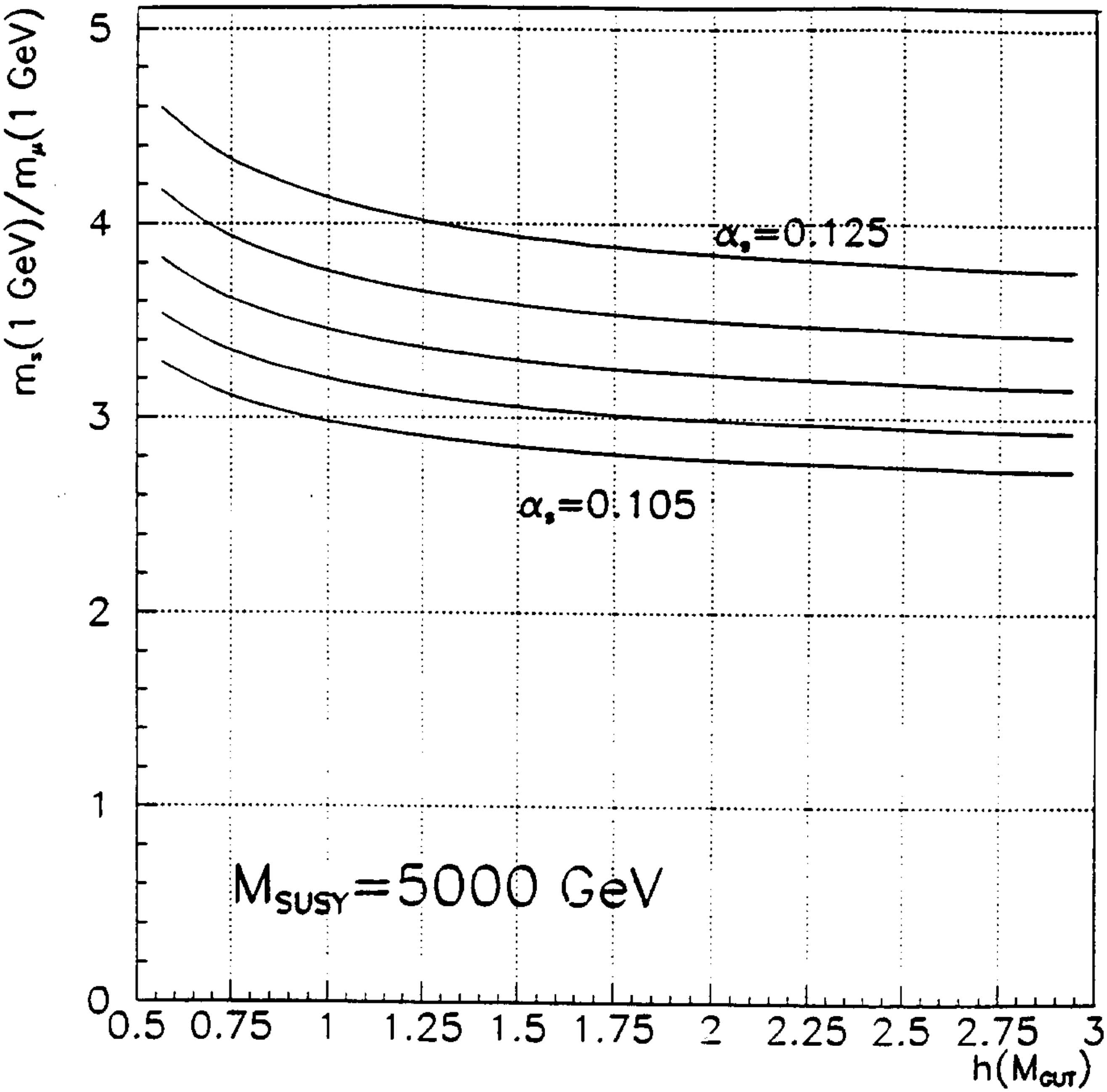
$m_s(1\ \mathrm{GeV})/m_\mu(1\ \mathrm{GeV})$
α_s=0.125
α_s=0.105
M_{SUSY}=500 GeV
0
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
1
2
3
4
5
$h(M_{GUT})$

Fig. 2.b.



$m_s(1\ GeV)/m_\mu(1\ GeV)$
$\alpha_s=0.125$
$\alpha_s=0.105$
$M_{SUSY}=1000$ GeV
0
1
2
3
4
5
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
$h(M_{GUT})$

Fig. 2.c.



$m_s(1\text{ GeV})/m_\mu(1\text{ GeV})$
5
4
3
2
1
0
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
$h(M_{GUT})$
α_s=0.125
α_s=0.105
M_{SUSY}=5000 GeV

Fig. 2.d.

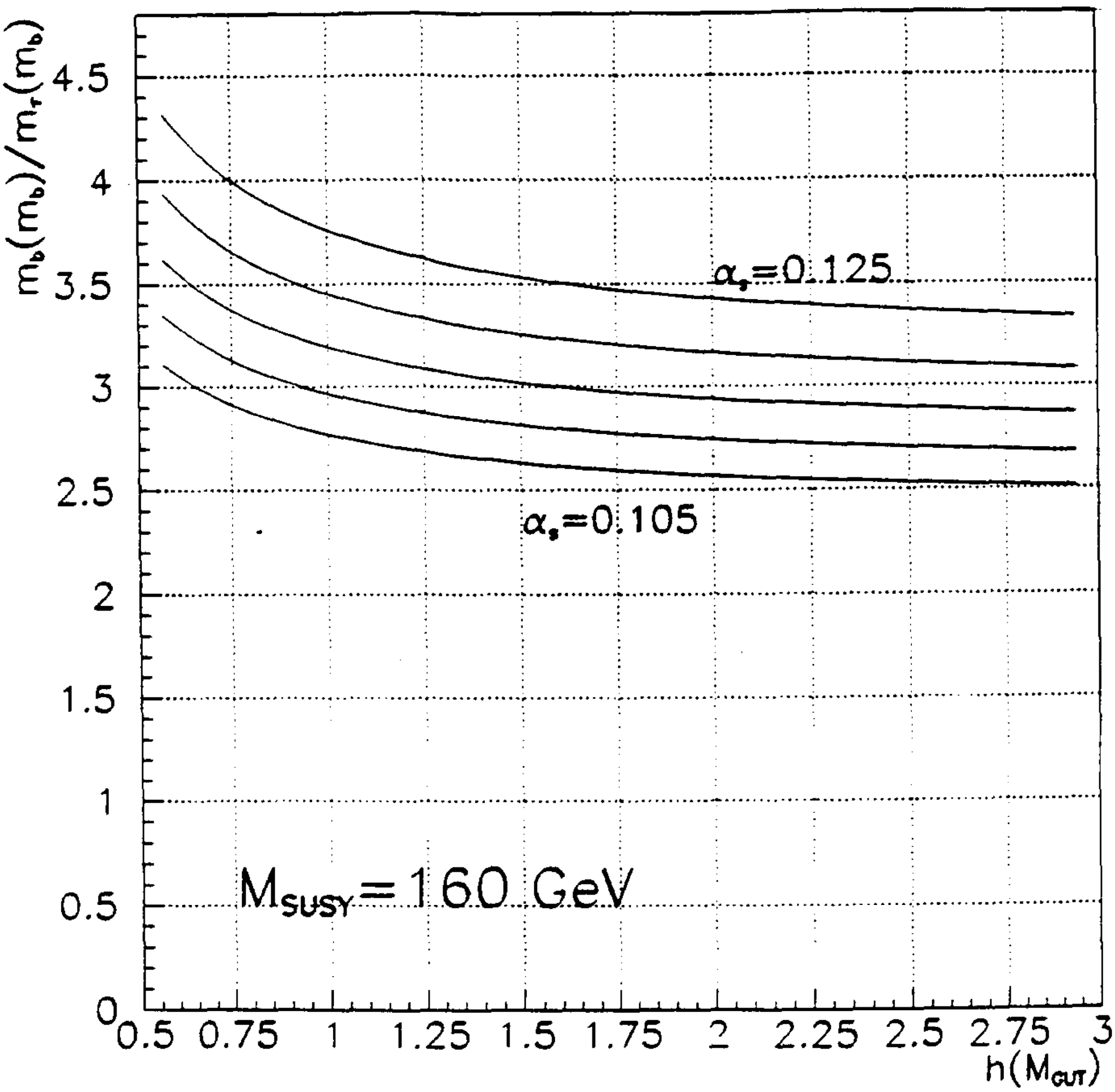
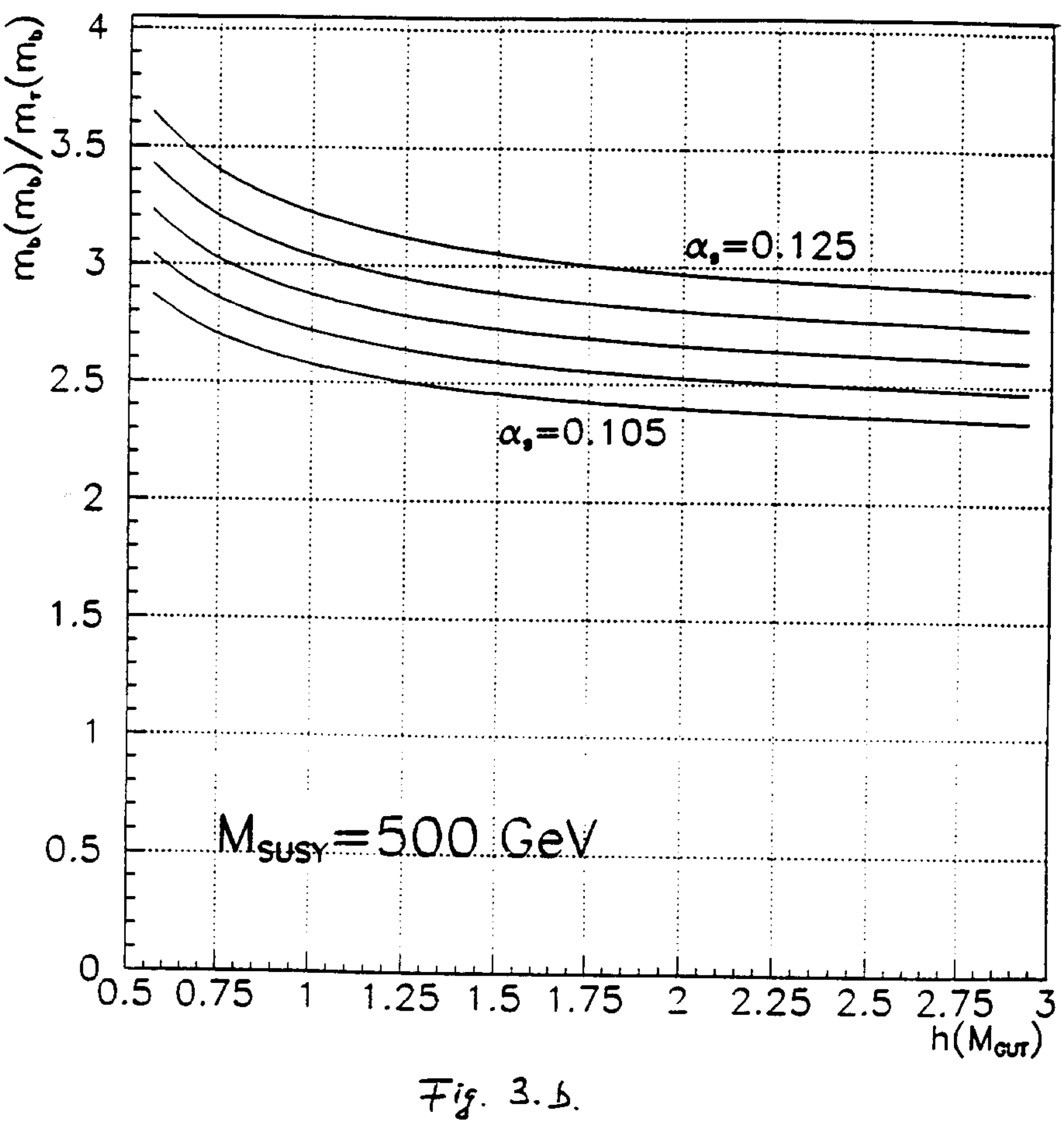
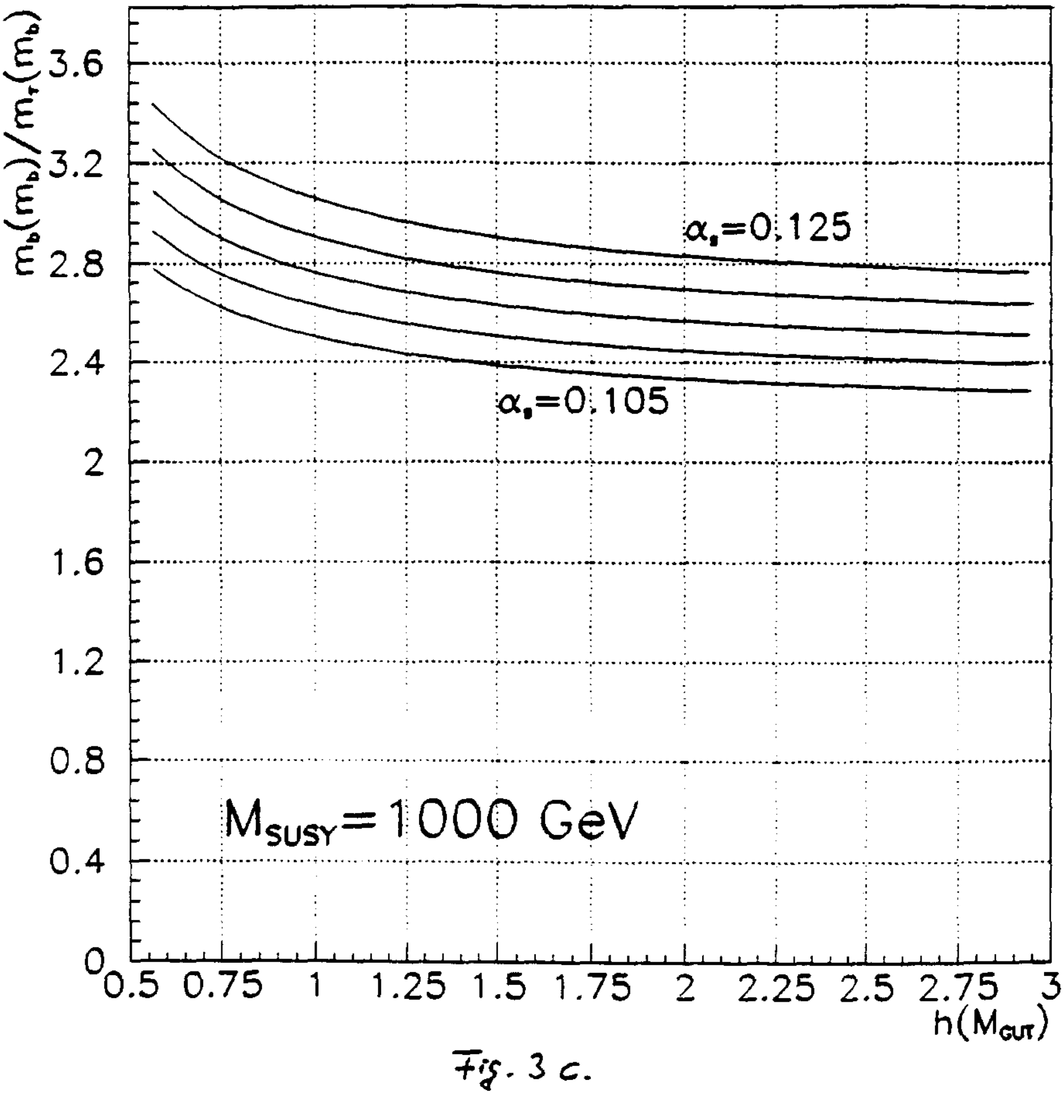

$m_b(m_b)/m_\tau(m_b)$
α_s=0.125
α_s=0.105
M_{SUSY}=160 GeV
$h(M_{GUT})$
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3

Fig. 3.a.



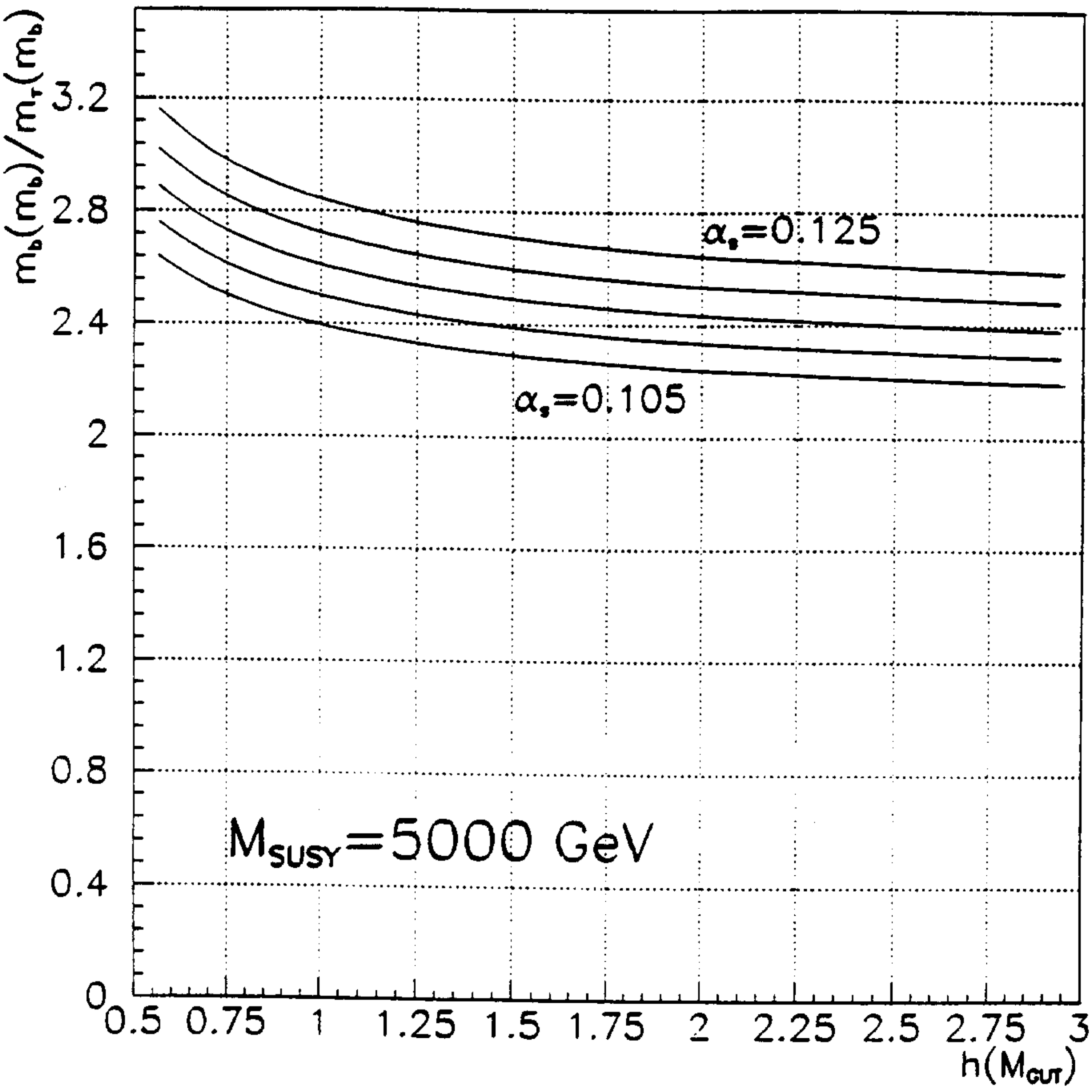
$m_b(m_b)/m_\tau(m_b)$
4
3.5
3
2.5
2
1.5
1
0.5
0
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
$h(M_{GUT})$
α_s=0.125
α_s=0.105
M_{SUSY}=500 GeV

Fig. 3.b.



$m_b(m_b)/m_\tau(m_b)$
3.6
3.2
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
$h(M_{GUT})$
α_s=0.125
α_s=0.105
M_{SUSY}=1000 GeV

Fig. 3 c.



$m_b(m_b)/m_\tau(m_b)$
3.2
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
$h(M_{GUT})$
α_s=0.125
α_s=0.105
M_{SUSY}=5000 GeV

Fig. 3. d.