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10 Tessellation Approach in Modeling Properties of Physical Vacuum and Fundamental Particles

E.G. Dmitrieff ⋆

Irkutsk State University, Russia

Abstract. The approach of representing fundamental particles by defects in the periodical tessellations built of small electrically-charged domains is discussed in this paper. We give reasons for its use, enumerate the assumptions underlying it, formulate the main tasks that arise with this approach and provide some of solutions for them that we found.

Povzetek. Avtor predstavi svoj model za opis lastnosti osnovnih gradnikov snovi – kvarkov, leptonov in njihovih antidelcev ter interakcij med njimi. Elementarne delce predstavi kot defekte v periodični teselaciji prostora, ki jih določajo majhna električno nabita območja. Pove od kod je črpal vzpodbudo za svoj model, našteje privzetke, na katerih je model zdrajen ter napovedi, ki jih model ponuja.

Keywords: tessellation, bit graph, particle, defect, triple-periodical, satori

10.1 Introduction

The *Tessellation approach* is the denotation for using some analogy between fundamental particles, on one hand, and structure defects in periodical spatial tessellations, on another hand, in calculation of particle properties and speculations about particle physics problems.

We do not know exactly, how deep this analogy is, and what causes such a correspondence, but we found this approach useful and productive, and also found it interesting to explore its limits, trying to extend them.

We formulated assumptions of the approach while developing several particle models based on bit graphs, aiming to get digital, more calculable by computers, representation of particles instead of usual quantum mechanical one [1].

There are approaches, that have some correspondences to our approach, among them are the Spin-charge-family theory [2], the Cellular automata interpretation [3], [4], and the ether hypothesis [5].

The bit graphs generalize the idea of *numbers as bit sequences* by allowing not just ordered, i.e. sequential, but also non-ordered and partially-ordered bit combinations. For instance, three bit organized in a closed loop appeared suitable to describe both the color charge of quarks or anti-quarks, and the color absence, characteristic of leptons and anti-leptons [1].

⋆ E-mail: elia@sr.isu.ru

Two three-bit loops was found enough to represent also gluons, weak bosons, and the electrical charge for all the particles. Adding two more bits to the graph, we get a model suitable to describe three fermion families, triplet- and singlet-states of bosons, Higgs scalar and the photon.

All these models provide the correct quantum numbers of the corresponding particles as combinations of their bit's values. The only thing one must assume is that the bit's values are not 0 and 1 but $+\frac{1}{6}$ or $-\frac{1}{6}$ and they have the physical sense of electric charge.

The weak points of our bit-graph models, including the most advanced one, was that they were completely **C**-symmetrical, and therefore they did not provide the representation of the handedness and the parity asymmetry. To overcome this obstacle, we modified the principal three-bit loop graph, assuming it *directed*, and, therefore, we get the whole model chiral and **CP**-symmetrical. The charge conjugation **C**, meaning exchange of all bit values from 0 to 1 ($+\frac{1}{6}$ to $-\frac{1}{6}$) or back, and **P**, meaning the reverse of all loops' directions, being applied together, turn the model back to the original state.

This trick helped, but the *bits* looked this time rather less like binary digits because they must somehow carry, in addition to the electric charge, some extra information about the direction.

According to our eight-bit model, there must be two different versions for all the bosons, one of them more, and another one less symmetrical, which we associated with the triplet and single state of them. The scalar Higgs boson H took, in this model, the place of the singlet Z. Because of **CP**-symmetry, the same thing also happened to the photon representation, predicting some new scalar chargeless particle taking place of the singlet, or longitudinal, photon.

Stacking, like children's blocks, several copies of our Higgs boson model graphs with each other, we found that an electrically and color-neutral filling of space with unlimited size is easily obtained in this way. We associate it with the vacuum condensate. It is chiral because its **CP**-symmetrical partner is another condensate, which is produced the same way by stacking with each other the copies of longitudinal photon model graphs.

We recognized that it can be very effective to consider this condensate as vacuum background, instead of empty free space. It looks like regular periodic directed bit graph, infinite or big enough, consisting of multiple copies of the background bit combination, either Higgs or longitudinal photon. Some of these copies can be easily replaced with other model graphs corresponding to any of known particles, so particles will be just defects in the regular structure, with one or more bits with inverted charge.

Since the background is chiral, the left- and right-handed configurations become completely different. As an example, the photon and Z boson, that were **CP**-partners, went far one from another. Heavy and short-living Z has 6 defect bits in respect to the background while the light-weight and stable photon has only two defected bits.

The Higgs boson manifests itself as a scalar neutral particle on the background of longitudinal photon condensate. On its own background it would be non-distinguished from it, and thus experimentally not observable, i.e. non-existent

- the same way as the longitudinal photon does not exist on the background of itself.

That was the first time we think about the space as filled with the regular structure so it can be treated as a start point of our tessellation approach. In contrast to the purely mathematical structure of the bit graph, filling of the space with regions of different charge is a picture that can be called physical. It can be explored to find out what laws can exist in this 'world' and under which of them it will be more similar to ours.

10.2 Assumptions

The assumptions we listed below constitute an essential part of the approach. Changing them, we usually get a model that significantly differs from the observables.

Generally, they are as follows:

- the idea of tessellation,
- the statement of electrical charge carried by domains in it, and
- grouping of charged domains into triplets and pairs.

10.2.1 The ground state is a domain tessellation

The principal assumption of the *tessellation approach* is to treat the vacuum not as an empty space, either with fluctuations or without them, but, instead, as a *dense filling of small regions*, or *domains*. The domains can be either similar or different from each other, and may be either separated or not separated by some kind of walls. These are details that can vary in particular models.

This filling, or tessellation, is assumed to be the ground state, so that all fluctuations, defects, geometric distortions should be considered against this background.

In principle, the tessellation can be assumed global, crystal-like, or local, similar to some fluid, and even finite, looking like gas of domain clusters. In the last case, though, it is not the tessellation, but something more close to the classical empty space with free distinct particles in it. The liquid tessellation, with just near order of domains, should have some secondary unordered walls separating these ordered regions from each other, that, on our opinion, contradicts to the observations. So we assume the long-ranged, up to the infinity, and, in the first approximation, strictly periodical crystal-like space filling as the basic object for our model. In fact, each defect is the local violation of the periodicity, and the vicinity of a defect also can be slightly distorted. Also, there can be waves of the distortion, but all this is considered as excitations of the ideally periodical ground state.

10.2.2 Domains are electrically charged

As the second assumption, we take the statement that the principal difference between domains, and probably the only one, is the difference in their *electric charge*. We assume it to be *either* $+1/6$ *or* $-1/6$ in units of proton charge e.

Fulfillment of this requirement is necessary to ensure that all the particles will have their electrical charges proportional to $\pm\frac{1}{3}$ only. In this case, the tessellation model gets compatible with the bit graph models we studied before, so we can use the bit values 1 and 0, converted to the charges $+1/6$ or $-1/6$, to represent the particle that we want to explore.

In fact, it is not mandatory for absolutely all domains to carry these charges: it is only required for those domains that can change their charges individually or along with another domains of the same charge.

In case a pair of domain can participate just in mutual charge exchanges, or in case of individual domains that can not change its charge at all, these domains could have any charge as long as they keep compensating each other.

However, this is a kind of complification, that we try to avoid. Our 8-bit graph model allows exchange between *any* pair of bits, so the tessellation, that is compatible to it, must have all the domains charged with either $+1/6$ or $-1/6$ only.

On our opinion, the scalar electric potential of these charges can play the role of Higgs field in explanation of particle masses, so we *do not assume an extra Higgs field* for this purpose. The electric field of domains is the only primary field assumed [7].

This hypothetical unification of both fields allows to estimate the domain radius:

$$r \approx \frac{\alpha}{6^2\eta} \approx \frac{1}{36 \cdot 137 \cdot 246 GeV} \approx 8,2 \cdot 10^{-7} GeV^{-1} \approx 1.7 \cdot 10^{-20} cm. \tag{10.1}$$

The whole picture of vacuum as a scalar field, non-zero almost everywhere (excepting walls), looks now close to the vacuum domain model [6] with the difference that domain sizes are not on cosmological but on sub-particle scale. This could, in our opinion, explain the paradox of absence of domain observations while they are predicted as a consequence of symmetry break in the electroweak theory.

10.2.3 Triplets and Pairs

It is well known that all the fundamental particles have their electrical charge values in the range from -1 to 1. All the multi-charged particles are considered composite, as bound states. In the tessellation approach we consider this limit as an evidence in favor of assumption that the *count of domains* that are able to possess simultaneous inversion in the same direction, is *exactly three*. We suppose them to reside in the tessellation in the close vicinity of each other, most likely being the *immediate neighbors*.

In other words, the tessellation consists of multiple positive- and negative-charged domain triplets, each carrying electric charge of $\pm\frac{1}{2}$, and one, two of three domains in a triplet can be defected, i.e. have the charge inverted.

This assumption immediately leads to the phenomena of the *isotopic symmetry*, because it stipulate existence of two variants for each defect configuration, depending on place where it occurs: either instead of positive triplet in the ground state tessellation, or instead of negative one. The difference of electrical charge between them is exactly 1, so defects in the positive triplets are down particles; the same defects in negative triples are up.

The relocation of the defected triplet from originally negative place to the positive one, is, in fact, its exchange with the positive triplet resided in its place. It causes, besides the transformation of the up particle into down one, the appearance of new positive-charged triple defect in the negative place. It corresponds to the weak boson, so all this exchange should be considered as an example of weak interaction, for instance: $u^{\frac{2}{3}} \rightarrow d^{-\frac{1}{3}} + W^{+}$. This defect can migrate, exchanging its place with triplets in negative places, or cause the relocation of some defected triplet from the positive place into negative one.

In addition to triplets, we assume the possibility of domain pairs. It is the artificial construction, serving as the simplest way to represent several different particles with the same charge. The exchange between domains in a pair affects neither color nor electric charge, but the result combination differs from the original.

10.3 Objectives of the tessellation approach

To be applied to problems in particle physics, the tessellation approach requires the concrete suitable tessellation. To calculate energies, including masses, it is necessary to figure out, what is the energy in this case. For the dynamic processes, including interactions, the way of defect migration also should be identified.

So the determining of the most optimal structure, obtaining the appropriate Hamiltonian and definition of dynamic may be considered as main objectives for the research.

Also it is possible that there are some physical systems, analogous to the tessellations, for instance foams and liquid-liquid mixtures, so the approach could be applied to them, and some observations and experiments with these systems can improve the knowledge of this subject.

10.3.1 Finding the optimal structure

There are a lot of mathematically possible different spatial fillings that, in principle, can be used in the tessellation approach. Each of them provides, as its defect combinations, the spectrum of possible fundamental particles. Some of them are better than others, i.e. their defect combinations looks more similar to the particles found in the real world. So there should be one or several tessellations that provide the best correspondence to experimental data. So, **Determining of the optimal structure** is the first and main task of the tessellation approach.

We examined five structures, in the following order:

- 1-dimensional probe tessellation of 8-bit 'V' bit graphs
- simple cubic grid (**NaCl** type),
- body-centered cubic grid (**CsCl**),
- Weaire-Phelan [8] structure, or A15 phase [9] (β-**W**, $\mathbf{Nb_3Sn}$) [10], and

- 4-dimensional 'Satori' structure [11], built as alternation of two modified A15 grids.

All the structures are compatible with, but not limited by, our 8-bit model.

In all these cases we considered electrically-neutral grids containing equal quantities of positive- and negative-charged domains in their nodes.

In the simple cubic grid, to ensure both the neutrality, and also the **CP**-symmetry, we used as node's *charge* its *parity*, calculated as product of its row's, column's and layer's parities.

Since all subsequent grids can be produced from the (hyper-)cubic grid performing shifts of its rows, columns and/or layers, the parity is still defined for their nodes so we distribute the charge in the same way.

To obtain the domain structure from the grid, we use the Voronoi diagram [12] built for the nodes. In case of simple cubic grid, the Voronoi diagram is also simple cubic, dual to the original. In case of body-centered grid, the Voronoi diagram is the Kelvin structure, the tessellation of equal tetrakaidecahedra, each of them is truncated octahedron.

In both cases, the structure is not chiral, so both even and odd domains have identical shape and spatial orientation.

The key difference of the Weaire-Phelan structure in respect to simple and body-centered grids is that in it the domains of different parity have different orientation, being mirror reflections of each other. Moreover, there are two different kinds of domains: for each three tetrakaidekahedra of three different orientations, there is one dodecahedron. Each translation unit consist of two equilateral triangles built from tetrakaidecahedra and two dodecahedra of opposite parities. So it is obviously compatible with the 8-bit graph model, while the first two are not.

The last tessellation that is 4-dimensional, now it is constructed but not well-studied yet. We needed the four-dimensional structure in order to have any model of three-dimensional defect motion (see below). Like A15, from which it is derived, it has minimal wall pro cell ratio, but, in contrast to it, is built of the domains having the same shape.

10.3.2 Constructing the Hamiltonian

The electrical charge of particles, factorized into ones' complement bit representation, define most of the quantum numbers as bit combinations: weak charge, hyper-charge, baryon- and lepton-numbers, and matter type (matter-or-anti-matter bit). The unary triplet-bit-loop represents the color charge. So, it is easy to determine bit combinations and corresponding defects for the properties that influence on the electric charge.

It is more difficult to guess the possible combinations, that would represent the equal-charged particles of both handedness-es, different spin, members of three (or more) families, or possessing boson and fermion kind of statistic. For instance, they are up, charm, and top quarks, or W^- boson, tau, muon, and electron. However, it can be done, following the symmetry of the tessellation structure.

But the problem of particle masses, which are very different, very special, and do not manifest any dependence on the particle's charge, on our opinion, can be solved in the tessellation approach just by applying some additional assumption about mass origin.

Since there is nothing in the model but spatially distributed electrical charge, the mass of particle, which appears as some difference in the distribution structure in respect to background one, should depend on this difference, that can be expressed analytically in geometric terms.

We start with choosing of the suitable definition for mass. The best one, on our opinion, is to treat as the particle's mass, the part of energy, associated with it regardless of its state of motion and of its interactions with other particles. It is preferable to the inertial mass definition, because it does not depend on motion, and to the gravitational one, since it does not require more than one particle.

It means that if we prepare the model containing one non-moving defect, corresponding to a particle, in the infinite periodic tessellation, and calculate the difference in energy between pure and defected models, we should get the particle's mass.

The tensor field of tessellation distortion, that might emerge around the defect, as we suppose, should be associated with the gravitational field of the particle. In this approach, the field of gravity is not created by mass nor by energy, but it is an essential part of the energy, and particularly, of mass. Following it, we should consider the total mass as split in two different parts: one of them is connected to the changes of not only sizes but also of the topology of domain walls, that is occurred in the place of defect; while another part is connected to the minor residual changes in shape of domains around it, that retain the tessellation topology, but can spread on rather bigger distances. Both parts are supposed to be able to exchange their energy and minimize it.

So, **obtaining the appropriate Hamiltonian** is the second task of the approach, essential for its application to mass and energy prediction. The energy function could depend on domains' and walls' volume, area, curvature, thickness, charge density and so on. To check it, we calculate the Hamiltonian for the sample pure background tessellation (that should be as large as possible, ideally infinite). After that we figure out how the appearance of the particular defect rearranges the tessellation components in-place and in the vicinity, and calculate the Hamiltonian again, this time for the defected tessellation. The difference we treat as defect energy, which should be equivalent to the particle mass in absence of interactions and movements (in the reference frame where the domain centers are motionless).

In addition to the mass calculation, the Hamiltonian can play another significant role. Both the initial assumption about existing of the domain tessellation, and choosing concrete structure for it, need some physical grounding for them, aside of their usefulness in explaining or predicting the particle and vacuum properties. We suppose that the energy depends on the structure shape so that it has the locally or globally minimum corresponding to the tessellation in the ground state.

Taking the Hamiltonians gradient as analogue of tension force, we can allow the model to relax under it, and do not care anymore about maintaining of correct form of domains.

In the most preferable case, we can omit the step of choosing the shape of tessellation, allowing the Hamiltonian minimization to self-assemble the tessellation.

This task looks rather real because, for instance, the tessellation A15 is an example of extremal case: it has minimal known wall area to given domain volume ratio among all 3-dimensional equal-volumed tessellations.

Nevertheless, the use of just such a Hamiltonian is not necessary: for simplicity, tessellation can be given imperatively, by the coordinates of points, or analytically, for example by a trigonometric or exponential function.

We have considered some simple rules of calculating energy, as follows:

- The simplest hypothesis is to estimate the energy as being proportional to the count of bits or domains that are inverted with respect to the ground state. Its advantage is that it can be applied to infinite or even to the finite bit graphs regardless of their structure.
  The results are mostly qualitative, and can only be considered valid for a few cases. For instance, the smallest but non-zero masses must correspond to the photon and neutrino because they are represented with just two inverted bits. The most heavy particle should be Higgs boson, built from eight defects. Z corresponds to six defects while triplet-W does with five ones. So the mass ratios should be $\frac{m_W}{m_H} = \frac{5}{8} = 0,625$, $\frac{m_Z}{m_H} = \frac{6}{8} = 0,75$, while experimental values are 0.643 and 0.728.
- Considering two kinds of bits, that reside in triplets and in pairs, as different, and treating solo changes of domain in pairs as having no influence on the mass, we could improve these results. This caused us to move from bit models to tessellations, where we can take in account the geometric properties.
- In the polyhedral approximation of A15 structure, constructed from domains of two parities (and, of course, two corresponding charges), there are three kinds of faces of different area, and they can separate domains of either equal or opposite charge.
  We supposed that the energy is proportional to area of the domain walls and it is different for two types of wall: for double-layered walls between opposite-charged domains, containing zero-charged film in their core, and for walls between domains of the same charges: these walls supposed to have another structure, without zero surface inside.
  The particle, as combinations of several defects, define the configuration of walls, that can be calculated manually, even without computer simulation, just by counting faces of particular type.
  For A15 model, this energy calculation leads to existence of massless, low-massive, and highly massive particles. The massless particles correspond to inversions in dodecahedra, that have six equal pentagonal faces of each type, and after recharging they have six equal faces of each type, again.
  Since the changes can be in both directions, and the difference between arithmetic mean of two face's area and the third face is very small, the particles containing combinations, compensating each other, are lite-weight. Others are massive.
  We could not reproduce all the known masses in this simple scheme, but slightly varying the tessellation geometry, we found some defect combinations, that simultaneously give correct quantum numbers and also correct masses, for the photon, neutrinos, electron, weak bosons and Higgs.

Gluon threads in mesons, supposed as 1-dimensional condensates of diagonal ($r\tilde{r}$, $g\tilde{g}$, $b\tilde{b}$) gluons, also appear massless excepting their ends. The solo gluons, not stacked in threads, have in this schema sufficient masses on GeV scale, so the conception of threads is preferable. Quarks do not look like individual particles, but as indispensable ends of diagonal gluon thread or, for closed non-diagonal one, as sites where it changes its direction.
Some mass values, for example $105.65MeV$ for the muon, could not be represented this way unless we allow not just even but also odd count of changed faces, even though they always appear in pairs. This can mean that the second family should be considered in dynamics only, as oscillation or combination of two forms, having both even but different changed faces count, producing odd arithmetic mean.

So by now we have not suggested the Hamiltonian that we could call ultimate nor close to it. The task seems to be complex because it should allow to take in account the particle's motion, including relativistic case.

10.3.3 Dynamics, time and motion

To be able to represent dynamic effects we needed at least the tessellation that can get changed. However, we did not see that such an ability is present in any of the three-dimensional tessellations that we considered. Both the ground state, and the defects, manifest their tendency to be stable, motionless, especially under the Hamiltonian minimization. Nothing forces the defects to jump into another locations and also nothing causes them to keep jumping conserving their momentum or velocity.

Cellular automaton as 4d tessellation One thing we could do is to consider consequential 'snapshots' of the same tessellation, where the defects took different places, 'moving' in the same sense as 'move' the motionless frames on a film. By assuming some external, additional rules of the jumps we could get the working model that would be a kind of cellular automaton.

Geometrically, the cellular automaton build on the basis of three-dimensional Kelvin or A15 structure is the infinite four-dimensional tessellation with the dedicated direction, that is the direction of computation, orthogonal to the other three. Each 3-dimensional domain turns in it into the 4-dimensional cylinder or prism.

From the viewpoint of the tessellation approach, there is no reason to believe that the shape of prism or cylinder is the best shape for the domain in the tessellation, suitable for the modeling. Instead, we should get one step back and suggest some 4-dimensional tessellation that would be 'good' or may be 'the best' according to its abilities to reproduce the phenomena we want in our model.

Cross-sections with 'moving' domains On another hand, observing the 2-dimensional cross-sections[1] of the 3-dimensional A15 model, we found out that its

[1] In the trigonometrical approximation, with $\rho = \frac{1}{4}(\sin z(1-\cos x)(1+\cos y) + \sin x(1-\cos y)(1+\cos z) + \sin y(1-\cos z)(1+\cos x))$

sequential cross-sections, that can be taken continually, look like a cartoon film, showing perpetually moving two-dimensional domains, even in the pure non-defected tessellations. The character of movement could be described as kind of oscillation or rotation, but since the similar-charged domains are indistinguishable, when they meet, they can exchange, so the movement also can be treated as directed relocation of domains on any distance and in any direction with the limited velocity.

Any defect, occurred in this tessellation, in order to conserve its charge, must participate in its neighborhood's movement. Otherwise, it would overlay with other domain of the same charge, causing the double-charged domain, or mutually cancel the domain of opposite charge, forming the domain with reduced or zero charge. Both cases violate the principal assumption of the domain behavior, postulating their constant charge. So the charge conservation can be treated as the cellular automaton law, determining domain migration into the appropriate place on the each step.

Hypothetical speculations about modeling movements and time Each time when the defected domain meets two neighbors of the opposite charge, it must choose, which place to take. Manipulating with this choice, we can control the movement: if it happens predominantly in one direction, than the defect moves there; otherwise it moves randomly or oscillating, keeping close to the point of origin.

The small distortions of the domain's walls shape, caused by last choice made, can play role of the short-term memory, keeping some information about it, and make influence on the next upcoming choice. This possibility turns the process to be analog of Markov chain and allows keeping the movement direction, for instance, with the mechanism similar to the Bresenham's line algorithm.

We also supposed that the number of situations of making some choice of direction, can play role of the own time for the particle, that influences on the probability of the particle's decay. Propagating with high velocities, close to the limit, defected domains have less freedom in choosing direction, that can be treated by the low-velocity observer as the time dilation of the quickly propagating particle.

Unfortunately, the effect of 'moving' domains could not be used directly to represent the movement in the 3-dimensional model, because it reduces the dimension count by one, so in each temporal moment, i.e. cross-section, the model space is flat.

Combining the idea of cellular automaton, as a 4-dimensional tessellation, with the observations of movement-like behavior of domains in flat cross-sections, we supposed that there exists a 4-dimensional tessellation allowing cross-sections, which in turn are 3-dimensional tessellations, able to represent known set of particles, and the movement observed from within 3d sections is a certain process in 4d one, equivalent to sampling successive sections in some direction with strict conservation of charge for each domain in the section.

So the third task of the tessellation approach can be formulated as to **find the appropriate 4-dimensional tessellation**. It must offer the same possibilities as 3-dimensional ones, but, additionally, provide the way to represent momentum and, ideally, the law that causes domains to conserve it.

4d tessellation 'Satori' Since the most successive 3-dimensional model was the *optimal* space tessellation, we looked for the references to optimal tessellation in the 4-dimensional space, but did not find any. So we analyzed the way how the optimal tessellations in 2 and 3 dimensions are build, and found out that they are relaxed Voronoi diagrams of square or cubic point grids, with some nodes shifted on the unit half-size along the rows, columns or through layers.

We noted that the optimal 3d node grid is produced from two isomeric optimal 2d grids (in one of them each second row is shifted while in another one the points are shifted in each second column). Being placed in alternating adjacent layers, they offer possibility to perform additional shift of $\frac{1}{4}$ points along the straight lines orthogonal to the layers, so the ratio of shifted points raises from 0 in 1 dimension through $\frac{1}{2}$ in 2d up to $\frac{3}{4}$ in 3d, and the calculated value of the optimality criterion[2] was reduced, which meant compaction.

This procedure also produces two 3d-isomers, depending on selection of even-odd or odd-even order of 2d isomers used.

Following this way, we repeated the same operation once more, placing two alternating isomeric 3d grids in adjacent spaces. Doing so, we got all the remaining non-shifted $\frac{1}{4}$ points disposing on straight lines perpendicular to the spatial layers, so we could perform the ultimate shift along these lines.

Calculating the Voronoi diagram (using the qhull package [13]), we found out that it consists of all the regions having the same size and the same shape. They are 78-verticed polytopes, with 26 3d faces, two of which are distorted dodecahedra while the remaining 24 are nonahedra. They have 4 orthogonal orientations, that can be defined by the vector connecting centers of their dodecahedral 3d-faces. Polytopes of the same orientation stack together sharing dodecahedral 3d-faces along each of four orthogonal axis. Even and odd polytopes are alternating along the stack, being the mirror reflections of each other.

Calculating the optimality criterion, we found it[3] $\approx 4.9\%$ less than in 3d, so since all the points are yet shifted, it is impossible to get more compact tessellation with the same way. It means that, probably, this 4d tessellation that we called 'Satori' is the most compact one in all the Euclidean spaces.

[2] The optimality criterion we calculate as $c = \frac{D_{d-1}}{d \cdot \sqrt[d]{N D_d^{d-1}}}$ where d is the space dimension count, D_{d-1} is the hyper-area of walls in the sample of N domains, and D_d is the hyper-volume of the sample. It has the value of 1 for simple hyper-cubic grids in all dimensions. The optimal flat honeycomb has $c = \sqrt{\frac{3}{2\sqrt{3}}} \approx 0.93060$ while non-relaxed A15 has $c \approx 0.882825$.

[3] $c = \frac{1}{8}\left(1 + 7\sqrt{\frac{2}{3}}\right) \approx 0.83943$

Checking the cross-sections[4], we made sure that they keep the 'moving' behavior of domains, now in three dimensions. The section is to be made orthogonal to one of the axis . In contrast, when the section is performed orthogonal to the diagonal of the Cartesian reference frame, the 'movement' looses its stochastic character, keeping all domains in 4-beat oscillating near points close to their centers.

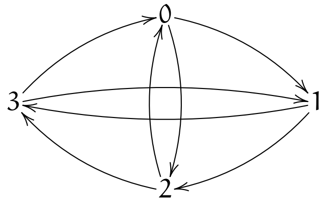
The new structure is made of equal domains so it is supposed to be stable under the relaxation with the tension applied with suitable Hamiltonian.

4d Cylinder tessellation With all its advantages, the Satori structure has at least two drawbacks that make us look for improvements. First, there is no more D-type domains that had equal count of neighbors of both parities, which allowed us to easily build models for massless particles using them. Now each domain shares two dodecahedral 3d-faces with two its neighbors, so even in mutual charge exchange between two neighbors the opposite 3d-faces would change their kind, that we usually treat as a sign of some mass connected with such a defect.

Second, the tessellation looks having the lack of causality from the viewpoint of observer inside 3d cross-section. Propagating in some direction, the process of cross-sectioning can meet regions, containing other defects, that for the 3d observer would be miracle artifacts, appearing from nowhere and violating the conservation laws.

We see that the possible solution for both problems listed above is the restriction in one of four dimensions *with only one translating unit*, turning the tessellation into the 4-dimensional cylinder, infinite in three dimensions but periodical in the fourth one.

In this case two domains of opposite parity lying along the *periodical* axis would share both dodecahedral 3d-faces, so they both will remain intact in the mutual charge exchanges. The process of cross-sectioning is limited now with only four domain layers, so it cannot meet anything that does not exists in these layers. That ensures the same reality for both 4d and 3d observers. The sectioning process degenerates to the directed oscillation or rotation between four 3d-spaces, schematically shown below:



in which states of domains in each space depend only on states of domains in two previous spaces, and also influence only on the state of domains in two subsequent spaces (rules for even and odd spaces are different due to their different structures).

[4] In the trigonometric approximation of the 'Satori' structure that we constructed having extremal points in the domain centers: $\rho = \frac{1}{4}(\sin x(\cos y - \cos z + \cos t - \cos y \cos z \cos t) + \sin y(\cos z - \cos x + \cos t - \cos z \cos x \cos t) + \sin z(\cos x - \cos y + \cos t - \cos x \cos y \cos t) - \sin t(\cos x + \cos y + \cos z + \cos x \cos y \cos z))$

10.4 Discussion

It is not obvious whether the tessellation approach is compatible with the known 'no-go' theorems. For instance, it should not be considered as deterministic because it is based on bit graphs, which are multivalent, producing multiple eigenvalues as result of the serialization, which corresponds to the quantum measurement. Also, it offers some combination of spatial and internal degrees of freedom so it is interesting to check against the Coleman-Mandula theorem.

10.5 Conclusion

The tessellation approach that we define and discuss in this paper allow us to formulate and solve problems of the particle modeling. Some of them have also the general mathematical meaning, for instance the problem of multi-dimensional filling optimality and measurement of information that tessellation holds.

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