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Big Bang Nucleosynthesis with $f(R)$ Gravity Scalarons and Astrophysical Consequences

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Abstract

$f(R)$ gravity is one of the serious alternatives of general relativity with a large range of astronomical consequences. In this work, we study Big Bang nucleosynthesis (BBN) in $f(R)$ gravity theory. We consider a modification to gravity due to the existence of primordial black holes (PBHs) in the radiation era that introduce additional degrees of freedom known as scalarons. We calculate the light element abundances by using the BBN code `PArthENoPE`. It is found that for a range of scalaron mass $(2.2-3.5)\times 10^4$ eV, the abundance of lithium is lowered by 3−4 times the value predicted by general relativistic BBN, which is a level desired to address the cosmological lithium problem. For the above scalaron mass range, the helium abundance is within the observed bound. However, the deuterium abundance is found to be increased by 3−6 times the observed primordial abundance. It calls for a high efficiency of stellar formation and evolution processes for the destruction of primordial deuterium, which is suggested as possible in scalaron gravity. A novel relation between scalaron mass and black hole mass has been used to show that the above scalaron mass range corresponds to PBHs of subplanetary mass ($\sim 10^{19}$ g) serving as one of the potential candidates of nonbaryonic dark matter. We infer Big Bang equivalence of power-law $f(R)$ gravity with PBHs that are detectable with upcoming gravitational wave detectors.

Unified Astronomy Thesaurus concepts: Big Bang nucleosynthesis (151); Cosmology (343); Dark energy (351); Gravitation (661)

1. Introduction

With only six free parameters—(1) cosmic baryon density (ρ_b), (2) dark matter density ($\rho_{\rm DM}$), (3) Hubble parameter (H_0), (4) spectral index (n_s) measuring departure from scale invariance of primordial density perturbations, (5) amplitude of primordial density perturbation (δ_i), and (6) optical depth (τ) for scattering of the cosmic microwave background (CMB) photons by intergalactic medium during cosmic reionization—the standard ΛCDM cosmology has successfully explained cosmic expansion history and origin of the large-scale structures of the Universe (Efstathiou et al. 1990; Peebles & Ratra 2003; Peebles 2015). It is based on flat spatial sections of the Friedmann–Lemaître–Robertson–Walker (FLRW) geometry. It predicts that the Universe started with a hot Big Bang (HBB) singularity. One of the remarkable achievements of the HBB model is its ability to predict abundances of the light elements, deuterium (D), helium (^{3}He, ^{4}He), and lithium (^{7}Li), which formed through a sequence of capture reactions between primordial protons and neutrons (Peebles 1966a, 1966b; Wagoner et al. 1967). After 1 s of the Big Bang when the Universe cooled down to $\sim 10^{10}$ K, the rate of interactions keeping protons and neutrons in equilibrium fell below the cosmic expansion rate (Weinberg 2008). This is the "freeze-out" epoch. As the Universe cooled down to $\sim 10^9$ K, nuclear reactions started locking free protons and neutrons, which were frozen in a fixed proportion. The Big Bang nucleosynthesis (BBN) of the ΛCDM cosmology assumes standard laboratory physics of nuclear reactions, three neutrino species, and no additional relativistic degrees of freedom (Peebles 2015).

Cosmic inflation (Guth 1981; Linde 1983) driven by a primordial scalar field has greatly supported the ΛCDM cosmology. Quantum fluctuation of energy of the scalar field generates seed density fluctuations. These fluctuations were stretched and later amplified by gravity leading to cosmic structures. Primordial black holes (PBHs) are the earliest structures formed by large amplitude random density fluctuations (Misner 1968; Carr & Hawking 1974). The formation of the PBHs depends on the details of the inflation model. It might not be a natural process in the standard cold inflationary scenario. However, the production of PBHs is found to be quite natural in the warm inflation scenario where the scalar field has nonnegligible interactions with the preexisting matter fields (Arya 2020; Bastero-Gil & Subías Díaz-Blanco 2021). Correa et al. (2022) advocated for the production of high-mass PBHs by considering shift-symmetry-protected natural inflation under a warm inflationary paradigm. A PBH of mass M forms if the cosmological density of the early Universe, $\rho = 3/32\pi G t^2$, achieves the density required to form a black hole, $3M/4\pi R_s^3$, where $R_s = 2GM/c^2$ is the Schwarzschild radius. This implies linear growth of the PBH mass, $M \sim c^3 t/G$. Hawking (1971) advocated for PBHs with masses of 10^{-5} g upward. This minimum bound arises from the classical nature of gravity—that the Schwarzschild radius of a black hole must not be smaller than the Planck length ($l_{\rm Pl} \sim 10^{-33}$ cm), the scale of the Universe when it was 10^{-43} s old. These PBHs, however, undergo Hawking evaporation. The evaporation timescale is shorter for smaller black holes (Hawking 1974). Extremely low-mass PBHs evaporate long before the onset of BBN. PBHs with masses around 10^{13} g evaporated by the epoch of matter–radiation decoupling. PBHs with mass above 10^{15} g survive the Hubble time. These PBHs formed around 10^{-24} s after the Big Bang. This is long before the era of BBN, which constrains the amount of baryonic matter in the Universe. Therefore, these PBHs act as potential candidates of nonbaryonic dark matter. Mass windows of PBHs that can contribute to the mass budget of nonbaryonic dark matter have been succinctly reported in

Green & Kavanagh (2021). It has been found that PBHs with mass $10^{17}-10^{22}$ g are eligible candidates for nonbaryonic dark matter.

The standard cosmology is, however, not free from problems. Cosmological singularity at the beginning of the Universe demands quantum corrections to general relativity (GR). This calls for alternative theories of gravity. The ΛCDM cosmology relies on unknown physics of the cosmological constant and cold dark matter. There are alternatives to the ΛCDM paradigm. These include time-varying dark energy components (Ratra & Peebles 1988; Caldwell et al. 1998; Zlatev et al. 1999) within GR and modification of GR on the cosmological scales (Capozziello 2002; Nojiri & Odintsov 2003; Starobinsky 2007). Distinguishing between these two is one of the major goals of the ongoing Dark Energy Survey (The Dark Energy Survey Collaboration 2005) and the upcoming Euclid mission (Laureijs et al. 2011). In addition, the standard cosmology has recently experienced a sequence of serious tensions. The notable ones are the Hubble tension (Freedman 2021) and the σ_8 tension (Böhringer et al. 2013; Bhattacharyya et al. 2019). These call for serious alternatives to the paradigm (Di Valentino et al. 2021).

The BBN is plagued by the cosmological lithium problem. Whereas abundances of D, ^{4}He, and ^{3}He match with those observed in the intergalactic medium, interstellar medium, and solar system (Izotov et al. 1994; Linsky et al. 1995; Olive & Steigman 1995; Niemann et al. 1996; Peimbert et al. 2002; Kirkman et al. 2003), the ^{7}Li abundance has an anomaly. The predicted relative abundance of lithium (^{7}Li/H) is $(4.68 \pm 0.67) \times 10^{-10}$ (Cyburt et al. 2016), while the observed relative abundance is reported as $^7\mathrm{Li/H} = (1.58^{+0.35}_{-0.28}) \times 10^{-10}$ (Sbordone et al. 2010). This discrepancy of a factor of 3−4 is known as the lithium problem. Neither new observation nor nuclear physics consideration of the BBN process has been able to remove this discrepancy (Alcaniz et al. 2021). This casts doubt on one or more of the following—the success of the BBN, the thermal history of the Universe, and the cosmology of the early Universe. Available attempts to address the problem include a variation of fundamental physics during BBN (Ichikawa & Kawasaki 2002; Landau et al. 2006; Berengut et al. 2010; Hou et al. 2017; Vattis et al. 2019).

Once the physics of the BBN such as cosmic expansion rate and nuclear reaction cross-sections are fixed, the predicted abundances of the light elements depend only on one parameter—the baryon-to-photon ratio ($\eta = n_B/n_\gamma$). Assuming nuclear and other nongravitational physics to be accurate during the BBN era, we investigate the effect of modified gravity theory on the cosmological abundances of light elements. Here we consider the additional scalar degree of freedom of $f(R)$-modified gravity theories as an extra energy density component in the Friedmann evolution of the radiation era. Along with the baryon-to-photon ratio this serves as an input parameter in predicting modified elemental abundances.

The study of modified theories of gravity in the BBN era was initiated by Dicke (1968). He proposed that the astronomical test of the Brans–Dicke scalar-tensor theory of gravity is possible through the determination of helium abundance in Population II stars. This was found to be possible as the scalar field energy density of the modified theory enhances the expansion rate so much that helium production becomes forbidden during the BBN era. Abundances of all other light elements get affected as this theory changes the rate of cooling of the hot Universe. The result is the shift of the freeze-out epoch and hence a change in the frozen proportion of neutrons and protons. Greenstein (1968) and Barrow (1978) studied the constraints on temporal variation of Newton's constant G embedded in the Brans–Dicke theory through observed abundances of D and ^{4}He. In recent times viable cosmological models in alternative gravitation theories such as the Hořava–Lifshitz theory and $f(Q, T)$ gravity theory have been proposed through consistency with BBN (Dutta & Saridakis 2010; Bhattacharjee 2022). Observed abundances of D and ^{4}He are used by Kusakabe et al. (2015) to constrain the $f(R)$ gravity theory. These theories are used for building models of the accelerated cosmic expansion.

Black holes provide astrophysical environments for testing alternative theories of gravity (Borka et al. 2012; Hees et al. 2017). $f(R)$ theories constitute a class of alternatives to GR where the left-hand side of Einstein's field equations is altered (Nojiri & Odintsov 2011). They are often used for explaining the observed cosmic acceleration (Capozziello et al. 2003) and the dark matter phenomena (Capozziello et al. 2007) without incorporating exotic fields or particles (Nojiri & Odintsov 2008). It has been shown (Kalita 2018, 2020) that gravitational correction (polynomial functions of Ricci scalar R) to quantum fluctuation in empty space around black holes naturally produces such alteration of the theory of gravity. The empty space field equations are derived from the modified Einstein–Hilbert action:

$$A_{E-H} = \frac{c^4}{16\pi G} \int f(R) \sqrt{|\det g_{\alpha\beta}|}\, d^4x. \tag{1}$$

Here $\det g_{\alpha\beta}$ represents a determinant of the space-time metric tensor and $f(R)$ is a function of the Ricci scalar curvature R. The scalar mode of gravity present in these theories is described by the scalaron field, $\psi = df(R)/dR$. Astronomical consequences of $f(R)$ gravity scalarons have been extensively studied by using stellar orbits near the Galactic center black hole and the shadow size of the black hole (Kalita 2020, 2021; Paul et al. 2023, 2024). The mass of the scalaron is computed from ultraviolet (UV) and infrared (IR) cutoff scales of curvature-corrected vacuum fluctuations near black holes (Kalita 2020) as

$$m_\psi = \frac{2\pi}{\lambda_{\rm IR}\lambda_{\rm UV}} \sqrt{\frac{\lambda_{\rm IR}^2 - \lambda_{\rm UV}^2}{12\ln(\lambda_{\rm IR}/\lambda_{\rm UV})}}, \tag{2}$$

where $\lambda_{\rm UV}$ and $\lambda_{\rm IR}$ represent UV and IR cutoff scales, respectively. The UV scale is chosen as the Schwarzschild radius, $\lambda_{\rm UV} = R_s = 2GM/c^2$. The IR scale is chosen as the one that presents vacuum thermal energy density corresponding to the Gibbons–Hawking temperature of black holes and is written as $\lambda_{\rm IR} = (hc/a_B)^{1/4} T_H^{-1}$, where T_H is the Hawking temperature of a black hole (see Kalita 2020 for details of such calculations). The scalaron mass (m_ψ) is found to be inversely proportional to the black hole mass (M), and in natural units, $c = 1 = \boldsymbol{h}$, it is given by (Talukdar et al. 2024),

$$m_\psi \approx 10^{-10}\,\mathrm{eV}\left(\frac{M_\odot}{M}\right). \tag{3}$$

In a recent study, it has been reported that this naïve relation ensures the existence of stationary and asymptotically flat black hole solutions in $f(R)$ gravity (Paul et al. 2024).

In this work, we consider scalarons associated with PBHs in the BBN era for which the density parameter of scalarons falls below closure density ($\rho_{\rm closure} \sim \rho_r \sim T^4$). The existence of PBHs from subplanetary mass ($10^{17} - 10^{22}$ g) to $100 M_{\odot}$ in the early Universe is found to be possible in hybrid inflationary cosmology with isocurvature density perturbation (Ahmed et al. 2022). First, we prove that scalarons in the early Universe constitute an additional dark energy component with constant density and negative pressure in a power-law $f(R)$ gravity theory. The elemental abundances have been estimated by adding scalaron dark density in the BBN code PArthENoPE (Gariazzo et al. 2022), which uses the latest data on nuclear reaction rates (Pisanti et al. 2021). It has been found that scalarons with masses around 10 keV are eligible to reduce the lithium abundance by a factor ∼3 relative to the GR-based Big Bang prediction without harming abundances of helium inferred from several independent astronomical probes. However, the deuterium abundance for these scalarons has been found to be elevated relative to the observed bounds.

The paper is organized as follows. Section 2 demonstrates that $f(R)$ gravity scalarons act as a constant density dark energy component. In Section 3, the cosmological density parameter of scalarons is expressed in terms of scalaron mass. Section 4 contains the effect of scalarons on the BBN abundances of light elements. Section 5 contains the results and discussions. Section 6 concludes.

2. Scalaron as a Dark Energy Component

We consider the $f(R)$ gravity field equation (Amendola & Tsujikawa 2010)

$$f'(R)R_{\alpha\beta} - \frac{f(R)}{2}g_{\alpha\beta} = \nabla_\alpha\nabla_\beta f'(R) - (\nabla^\mu\nabla_\mu f'(R))g_{\alpha\beta}. \quad (4)$$

We write the above equation in Einstein form:

$$R_{\alpha\beta} - \frac{R}{2}g_{\alpha\beta} = T_{\alpha\beta}, \quad (5)$$

where $T_{\alpha\beta}$ is the energy momentum tensor of the scalaron field and is given by

$$T_{\alpha\beta} = \frac{\nabla_\alpha\nabla_\beta\psi}{\psi} - \frac{\Box\psi}{\psi}g_{\alpha\beta} + \frac{g_{\alpha\beta}}{2}\left(\frac{f(R)}{\psi} - R\right). \quad (6)$$

The energy momentum tensor for a fluid is given by

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta - p g_{\alpha\beta}, \quad (7)$$

where, ρ, p, and u represent the density, pressure, and velocity of the fluid, respectively.

Comparing Equations (6) and (7), we obtain the pressure and density of the scalaron field as

$$p_\psi = \frac{2\,\Box\,\psi + R\psi - f(R)}{2\psi}, \quad (8a)$$

$$\rho_\psi = \frac{f(R) - R\psi}{2\psi}. \quad (8b)$$

This gives the equation of the state of the scalarons as

$$\omega_\psi = \frac{p_\psi}{\rho_\psi} = \frac{2\,\Box\,\psi}{f(R) - R\psi} - 1. \quad (9)$$

We call this "scalaron fluid."

2.1. Massless Scalarons as Cosmological Constant

It is seen that the scalaron field acts like a dark fluid similar to the cosmological constant $\omega = -1$, if the scalaron field ψ satisfies the following two conditions simultaneously:

$$\Box\psi = 0, \quad (10a)$$

$$f(R) \neq R\psi. \quad (10b)$$

Equation (10a) is the Klein–Gordon equation of a massless scalaron field.

In flat FLRW cosmology, the metric tensor components are $g_{\mu\nu} = {\rm dia}(1, -a^2(t), -a^2(t)r^2, -a^2(t)r^2\sin^2\theta)$. With this metric, the first condition becomes

$$\Box\psi = \ddot{\psi} + 3H\dot{\psi} = 0, \quad (11)$$

where the dot represents the derivative with respect to cosmic time and $H = \dot{a}/a$ is the Hubble parameter. This gives the following solution:

$$\dot{\psi} \sim a^{-3}. \quad (12)$$

Writing $\dot{\psi} = (d\psi/da)(da/dt)$ and assuming a power-law expansion of the Universe as $a(t) \sim t^\alpha (\alpha > 0)$, we obtain

$$\frac{d\psi}{da} \sim a^{(1/\alpha)-4}. \quad (13)$$

This gives the scalaron field as a function of the scale factor:

$$\psi(a) = \frac{df(R)}{dR} \sim a^{(1/\alpha)-3}. \quad (14)$$

The Ricci scalar is considered as reciprocal of the square of the length scale ct. Therefore,

$$R \sim a^{-2/\alpha}. \quad (15)$$

This gives $df(R)/dR \sim R^{(3\alpha-1)/2}$. The outcome is a power-law $f(R)$ gravity theory:

$$f(R) \sim R^{(3\alpha+1)/2} \sim R^m, \quad (16)$$

where $m = (3\alpha + 1)/2 > 0$.

The second condition (Equation (10b)) can be parametrized as

$$f(R) = \chi R\psi,\ \chi \neq 1. \quad (17)$$

Therefore,

$$\frac{df(R)}{f(R)} = \frac{1}{\chi}\frac{dR}{R}. \quad (18)$$

This has a power-law gravity solution:

$$f(R) \sim R^{1/\chi}. \quad (19)$$

The massless scalaron field in power-law $f(R)$ gravity behaves as the cosmological constant. We generalize this result for massive scalarons in the following manner.

2.2. Massive Scalarons as General Dark Energy Fluid

With the help of Equation (8b), Equation (9) can be written as

$$\omega_\psi = \frac{\frac{\Box\psi}{\psi}}{\rho_\psi} - 1. \quad (20)$$

For a constant equation of state different from the cosmological constant ($\omega \neq -1$) and for constant dark density, we must satisfy the following two conditions simultaneously:

$$\frac{\Box\psi}{\psi} = c_1', \tag{21a}$$

$$\rho_\psi = c_1, \tag{21b}$$

where c_1' and c_1 are constants. Equations (21a) and (21b) are to be satisfied separately so that the scalaron field acts as a massive Klein–Gordon field and presents a constant dark density component. In flat FLRW cosmology, the Klein–Gordon equation (Equation (21a)) for massive scalaron field ψ is given as

$$\ddot{\psi} + 3H\dot{\psi} - c_1'\psi = 0. \tag{22}$$

For the power-law expansion model of the Universe $a(t) \sim t^\alpha (\alpha > 0)$, we follow the procedure similar to the massless case and obtain the differential equation for the evolution of the scalaron field:

$$t^{2\alpha-2}\left(\frac{d^2\psi}{da^2}\right) + 4t^{\alpha-2}\left(\frac{d\psi}{da}\right) = c_1'\frac{df(R)}{dR}. \tag{23}$$

For $R \sim t^{-2}$, the above equation can be written as

$$R^m\left(\frac{d^2\psi}{da^2}\right) + 4R^n\left(\frac{d\psi}{da}\right) = c_1'\frac{df(R)}{dR}, \tag{24}$$

where $m = 1 - \alpha$ and $n = (2 - \alpha)/2$. We consider the following two cases.

Case I: Here we consider a slow variation of the scalaron field and make a Taylor expansion of $\psi(a)$ around an initial epoch $a = a_i$. It gives

$$\psi(a) \approx \psi(a_i) + \left(\frac{d\psi}{da}\right)_i (a - a_i) + \frac{1}{2}\left(\frac{d^2\psi}{da^2}\right)_i (a - a_i)^2. \tag{25}$$

Using Equation (25) in (24) we get the following condition:

$$R^m c_3 + 4R^n(c_2 + c_3 a) = c_1'\frac{df(R)}{dR}, \tag{26}$$

where

$$c_2 = \left(\frac{d\psi}{da}\right)_i, \tag{27a}$$

$$c_3 = \left(\frac{d^2\psi}{da^2}\right)_i. \tag{27b}$$

The cosmic scale factor is related to the Ricci scalar as $a \sim R^{-\alpha/2}$. This gives Equation (26) as

$$\frac{df(R)}{dR} \sim R^m + R^n + R^q, \tag{28}$$

where $q = n - (\alpha/2)$.

This produces power-law gravity:

$$f(R) \sim R^s. \tag{29}$$

Case II: Here we consider the power-law evolution of the scalaron field, $\psi(a) \sim a^u$, similar to the massless case (see Equation (14)). The power-law variation of the scalaron field is motivated by inflationary and quintessence scalar field models (Ratra & Peebles 1988). Hence, Equation (24) takes the form

$$\frac{df(R)}{dR} \sim R^m a^{u-2} + R^n a^{u-1}. \tag{30}$$

This condition also permits a power-law gravity $f(R) \sim R^w$.

Therefore, it is a general consequence that both massless and massive scalarons behave as a constant dark density fluid ($\omega = -1$, $\omega = \text{constant} \neq -1$, $\rho_\psi = \text{constant}$) in a power-law gravity theory.

Hence, scalarons in a power-law $f(R)$ gravity theory are eligible to act as a constant dark density fluid. It is to be noted that power-law $f(R)$ gravity has an important impact in cosmology. It has been used as a viable model for cosmic inflation. The most notable $f(R)$ gravity model for inflation is the quadratic gravity theory $f(R) = R + \alpha R^2$ (Starobinsky 1980). It has been proven to be a reliable model of inflation after the release of the Planck 2018 data (Planck Collaboration et al. 2020b). Power-law $f(R)$ gravity is found to connect early-time (inflation) and late-time (dark energy) accelerated expansion (Odintsov et al. 2023). Kusakabe et al. (2015) used BBN abundances to constrain the power-law $f(R)$ gravity.

3. Scalaron Density in a Flat Universe

Considering scalarons as an additional constant dark density component in the flat FLRW Universe, the Friedmann equation for the expansion rate of the radiation-dominated Universe is written as

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_{m_\psi}), \tag{31}$$

where ρ_r and ρ_{m_ψ} are the mass densities of radiation and scalaron, respectively.

We simplify the Friedmann equation (Equation (31)) as

$$H^2 = H_{\rm GR}^2 m_r. \tag{32}$$

Here, $H_{\rm GR} = \sqrt{(8\pi G\rho_r/3)}$ is the Hubble expansion rate in GR and $m_r = 1 + (\rho_{m_\psi}/\rho_r)$. The critical density of the Universe in scalaron gravity is written as

$$\rho_c^{f(R)} = \rho_c^{\rm GR} m_r, \tag{33}$$

where $\rho_c^{f(R)} = 3H^2/8\pi G$ and $\rho_c^{\rm GR}$ are the critical densities in $f(R)$ and GR scenarios, respectively. The dimensionless cosmological density parameter of scalarons is written as

$$\Omega_{m_\psi} = \frac{\rho_{m_\psi}}{\rho_c^{f(R)}}. \tag{34}$$

In this study, we take scalarons as the nonrelativistic degree of freedom. It has been found that for nonrelativistic scalarons the scalaron density parameter is dependent on scalaron mass (see below), and therefore, it is possible to constrain scalaron mass by using the density parameter in the BBN code. Assuming scalarons to be in thermal equilibrium with radiation and applying Bose–Einstein statistics in the radiation era, the scalaron mass density has been obtained as (Talukdar et al. 2024)

$$\rho_{m_\psi} = \frac{g}{\sqrt{2}\pi^2\hbar^3}(m_\psi k_B T)^{3/2}\zeta(3/2)\Gamma(3/2), \tag{35}$$

where m_ψ and T represent scalaron mass and temperature, respectively, and $\zeta(\nu)$ and $\Gamma(\nu)$ represent Riemann zeta function and gamma function for a number ν, respectively. It has been found earlier that the scalaron density parameter is independent of scalaron mass in the relativistic case (Talukdar et al. 2024). Therefore, the BBN calculation cannot be used for constraining scalaron mass for the relativistic case. The BBN abundances and interpretations in this study are confined to nonrelativistic scalarons. The radiation density is given by

$$\rho_r = N a_{\rm B} T^4/c^2, \tag{36}$$

where N is the number of effective relativistic degrees of freedom and is expressed as

$$N = \frac{g_{\rm boson}}{2} + \left(\frac{7}{16}\right) g_{\rm fermions} \left(\frac{4}{11}\right)^{4/3} \approx 1.68. \tag{37}$$

Density parameter for scalarons is, therefore, obtained as

$$\Omega_{m_\psi} \approx \frac{1}{1 + (2 \times 10^{-99}) T^{5/2} m_\psi^{-5/2}}. \tag{38}$$

We have considered scalaron mass in the range $(1 - 10^5)$ eV. This mass range is enclosed by $(10^{-16} - 10^5)$ eV, which was obtained earlier from the observed shift of freeze-out temperature (Talukdar et al. 2024). In this work, the higher end of scalaron mass is considered for obtaining the lower bound on the PBH mass (see Equation (3)). The scalaron density parameter is estimated at the freeze-out temperature ($T \sim 9 \times 10^9$ K). For the scalaron mass range considered in this work, the scalaron density parameter is found to be in the range 10^{-16}–10^{-3}.

4. Scalarons and BBN Abundances

Several astrophysical probes have been used to estimate the abundances of the light elements. Helium (^{4}He) abundance is a reliable probe of new physics of the early Universe. It is because the primordial abundance of ^{4}He depends on the amount of neutron fraction at the freeze-out epoch. This amount is sensitive to the expansion rate of the Universe, which in turn is governed by departure from standard physics. Therefore, accurate measurement of ^{4}He abundance is necessary for understanding the physics of the early Universe (Cyburt et al. 2016). Primordial ^{4}He abundance is obtained from the spectroscopic study of the metal-poor H II region NGC 346 in the Small Magellanic Cloud as $Y_p = 0.2451 \pm 0.0026\,(1\sigma)$ (Valerdi et al. 2019). Another independent measurement of ^{4}He was reported from a pristine intergalactic gas cloud at cosmological redshift $z_{\rm abs} = 1.724$ (containing about less than 30% less metal content than the most metal-poor H II regions) as $Y_p = 0.250^{+0.033}_{-0.025}\,(1\sigma)$ (Cooke & Fumagalli 2018).

Deuterium abundance is an accurate estimator of cosmological baryon density (Wagoner 1973). Any observed abundances of deuterium give the upper bound on the baryon-to-photon ratio η. Measurements of the D/H ratio by using high-redshift absorbing clouds along the line of sight to distant quasars were reported earlier (Tytler et al. 1996; Burles & Tytler 1998). For more than two decades the D/H ratio was measured by using high-resolution spectrographs on large telescopes employed for observing absorption lines in the quasar spectra (Tytler et al. 1996; Noterdaeme et al. 2012; Cooke et al. 2014, 2018). The most precise measurement of deuterium abundance by using low-hydrogen column density of one of the absorber clouds at redshift $z_{\rm abs} = 3.572$ in the line of sight of quasar PKS1937-101 was reported by Riemer-Sørensen et al. (2017). It is found as $\mathrm{D/H} = (2.62 \pm 0.05) \times 10^{-5}\,(1\sigma)$. Incorporating all prior measurements Kislitsyn et al. 2024 reported the best estimate of deuterium abundance as $\mathrm{D/H} = (2.533 \pm 0.024) \times 10^{-5}\,(1\sigma)$. Another observation of primordial D/H is taken from an absorption system along the line of sight of quasar Q1243+307 located at redshift $z_{\rm abs} = 2.52564$, which is found out to be $\mathrm{D/H} = (2.527 \pm 0.030) \times 10^{-5}\,(1\sigma)$ (Cooke et al. 2018). Since there is no known process of creation of deuterium nuclei, these abundances are considered as the primordial ones produced in the BBN. There is astrophysics associated with the observed deuterium abundance. There is a trend of monotonic decrease of deuterium abundance over time. This indicates that galactic chemical evolution has affected the interpretation of local measurements D/H (Cyburt et al. 2016).

Metal-poor halo stars of the Galaxy are potential probes of primordial ^{7}Li abundance. Halo main sequence stars, which are cooler than $T_{\rm eff} = 6000$ K, destroy much of the primordial ^{7}Li as they have a thick convection zone. However, the hot halo stars with $T_{\rm eff} > 6000$ K have a thin convection zone due to which ^{7}Li does not go down to layers where it gets depleted. Consequently, these stars preserve primordial ^{7}Li. Spite & Spite (1982) observed that ^{7}Li abundance remains flat for very low-metallicity stars ([Fe/H] = −3 to −1.5). The ^{7}Li abundance for halo stars with this metallicity is known as the "Spite plateau" (Cyburt et al. 2016) and provides the primordial abundance. It has been found that ^{7}Li abundance is less than the standard prediction of the Big Bang model. The observed abundance of lithium is taken as $^7\mathrm{Li/H} = (1.58^{+0.35}_{-0.28}) \times 10^{-10}\,(2 - 3\sigma)$ (Sbordone et al. 2010). This is derived from observations of metal-poor halo stars.

The theoretical description of the BBN process is pretty well known (Wagoner et al. 1967; Wagoner 1969, 1973; Kawano 1992; Smith et al. 1993). A number of public and private codes like `PArthENoPE` (Pisanti et al. 2008; Consiglio et al. 2018; Gariazzo et al. 2022), `AlterBBN` (Arbey 2012; Arbey et al. 2020), and `PRIMAT` (Pitrou et al. 2018) have been in development for detailed calculation of BBN abundances. In this work, we have chosen the latest version of the code `PArthENoPE`, which is `PArthENoPE 3.0`. This code allows users to calculate the abundances of elements in the standard BBN as well as in alternative cosmologies. `PArthENoPE` takes different parameters of BBN such as baryon-to-photon ratio ($\eta_{10} = \eta/10^{-10}$), number of additional neutrino species (ΔN_ν), neutron β-decay timescale (τ_n), and dimensionless dark density component (ρ_Λ in the code. which is $\boldsymbol{\Omega}_{m_\psi}$ in our case) as input and computes the detailed evolution of elemental abundance of light elements from nuclear statistical equilibrium condition. To execute this, we have considered Ω_{m_ψ} as an additional dark density parameter in the Friedmann–Lemaître expansion law. This component increases the expansion rate and consequently affects the BBN abundances.

To calculate the abundances using the BBN code the neutron lifetime is taken as $\tau_n = (879.4 \pm 0.6)$ s (Particle Data Group et al. 2020). Baryon density determined by the Planck CMB observation is taken as $\Omega_b h^2 = (0.02242 \pm 0.00014)$ (Planck Collaboration et al. 2020a). The corresponding baryon-to-photon ratio is $\eta_{10} = (6.13815 \pm 0.04239)$ (see Steigman (2006) and Cooke et al. (2018) for the conversion formula between $\Omega_b h^2$ and η_{10}). The variation of abundances D/H, ^{4}He/H, and ^{7}Li/H as a function of scalaron mass m_ψ is shown in Figure 1 (left). The results of this study are discussed below.

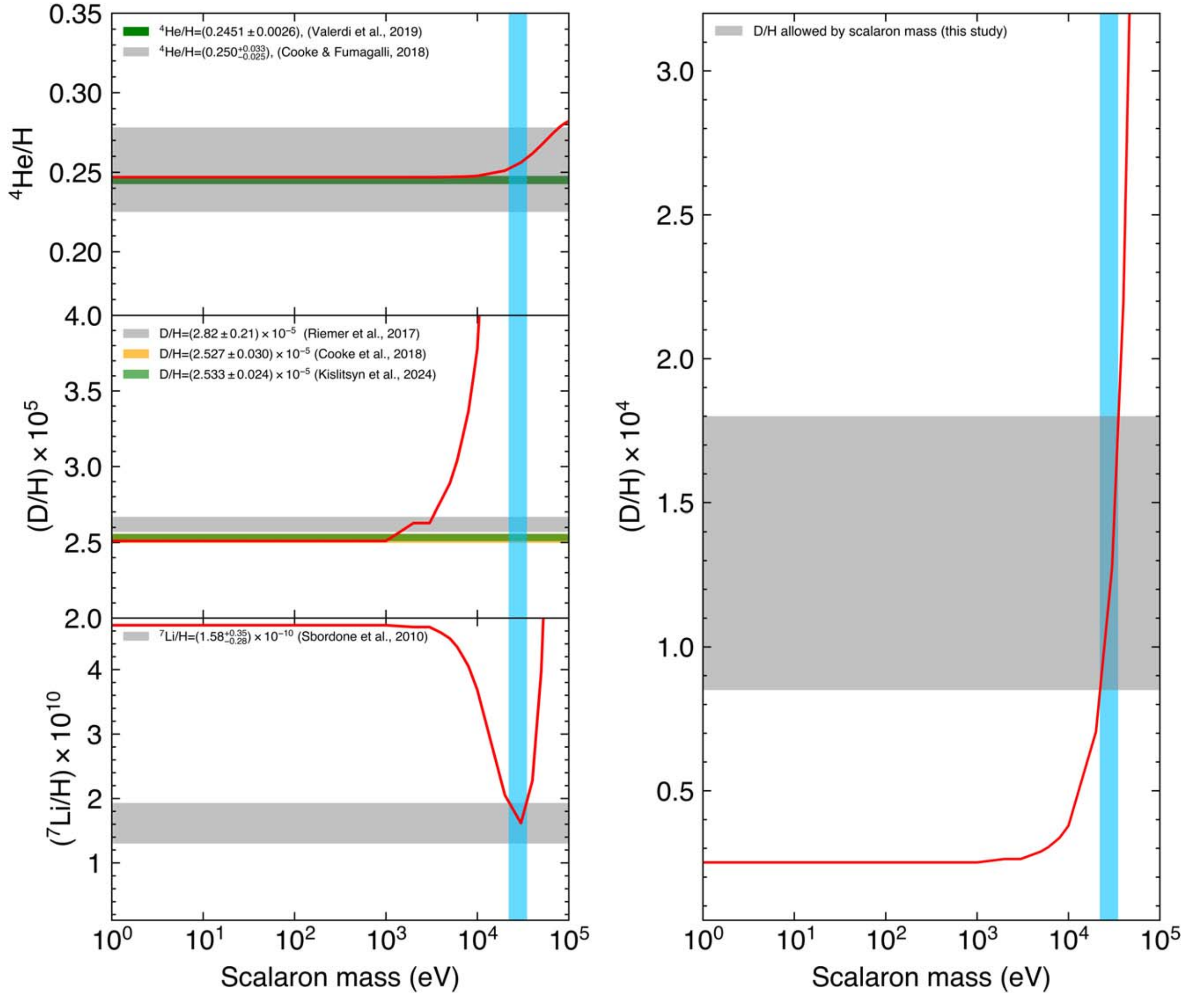

Figure 1. Left: abundances of light elements with scalaron mass evaluated at $T \sim 9 \times 10^9$ K. Right: D/H abundance for scalaron mass showing the enhancement.

5. Results and Discussions

In this work, we have investigated the effect of $f(R)$ gravity scalarons on the abundances of D, ^{4}He, and ^{7}Li using the BBN code PArthENoPE. In flat FLRW cosmology, power-law $f(R)$ gravity theory is found to present scalarons as a constant density dark energy component including the cosmological constant for massless scalarons. We take the dark density of the scalarons in the Friedmann equation of the radiation era and study the variation of light element abundances with scalaron mass at the freeze-out epoch.

It has been found that scalarons are eligible to bring down lithium abundance by a factor of 3−4 from that of standard BBN. It addresses the lithium problem. The mass range of scalarons for which this occurs is $(2.2 - 3.5) \times 10^4$ eV. Whereas helium abundance is found to be compatible with observed bounds, deuterium abundance for these scalarons is found to be elevated with respect to the observed abundance by a factor of 3−6.

The primordial helium abundance falls in the range of 0.250 −0.257 and is compatible with the abundance estimated from the observation of metal-poor H II regions and pristine intergalactic gas clouds. The deuterium abundance shows interesting behavior with further prospects. The calculated abundance is higher than the observed abundance by a factor of 3−6 (D/H $\sim 8.5 \times 10^{-5} - 1.8 \times 10^{-4}$; see Figure 1 (right)). In the late 1990s, there were reports of a high primordial D/H ratio ($\sim 10^{-4}$; Fuller & Cardall 1996; Rugers & Hogan 1996; Songaila et al. 1997; Webb et al. 1997). A low deuterium abundance ($\sim 2.6 \times 10^{-5}$) measured with absorbers of quasar spectra is the result of observations in specific locations in space and is considered as a standard primordial abundance. A discordant D/H ratio is thought to be an indicator of cosmological inhomogeneity (Webb et al. 1997). Tytler et al. (2000) critically examined this discordance with the possibility of hydrogen contamination in the absorbers, which yields a high D/H ratio. Hydrogen absorption looks much like that of deuterium giving rise to apparent enhancement of the D/H ratio in some absorbers. Depletion of early high deuterium is a possibility. Although global depletion by a factor of 4 may seem unlikely (Tytler et al. 2000), stellar formation and evolution is a well-appreciated source of reduction of D/H. Deuterium is extremely fragile and gets destroyed at or above 10^6 K. Stellar atmospheres (corona) and supernova explosions easily exceed this threshold. As more and more stars form and evolve, the D/H ratio goes down in the clouds enriched by stellar ejecta. It is suggested that stellar processes can destroy deuterium only by a maximum of 1% (Tytler et al. 2000). The problem may not be closed yet. It is believed that star formation history and model of galactic chemical evolution may need reevaluation after the new era of the James Webb Space Telescope (JWST), which has observed dusty galaxies in the high-redshift Universe. JADES (JWST Advanced Deep Extragalactic Survey) JWST/NIRSpec spectroscopy has reported the very rarely seen N III]λ 1748 line, which suggests unusually high N/O abundance in galaxy GN-z11 at redshift $z > 10$ (Bunker et al. 2023), possibly indicating the formation of high-metallicity stars. It calls for high star formation efficiency in the early Universe, which is not expected in the standard ΛCDM cosmology (Boylan-Kolchin 2023; Robertson et al. 2023). The rapid star formation process indicates the destruction of a high primordial deuterium abundance (Tosi et al. 1998).

The presence of PBHs can accelerate the gravitational collapse process relative to that expected in the standard ΛCDM model (Carr & Kuhnel 2020). It leads to the early growth of massive galaxies. It has been found that PBHs with $10^4 M_\odot$ can induce large star formation efficiency ($\epsilon = 32\%$), thereby accounting for the existence of high-redshift galaxies with high-stellar-mass content, $10^{10} - 10^{11} M_\odot$ (Liu et al. 2022; Colazo et al. 2024). It naturally ameliorates the recently realized puzzle of JWST's massive high-redshift galaxies with large stellar masses (Gouttenoire et al. 2024). Scalarons associated with the PBHs provide a natural means to accelerate gravitational instabilities. They provide an additional attractive gravitational force with Yukawa potential, known as the "scalaron fifth force" (Kalita 2018, 2020). Equation (3) suggests that as PBHs grow in the mass, scalaron mass decreases. A $10^4 M_\odot$ PBH corresponds to 10^{-14} eV scalarons. A $10^6 M_\odot$ black hole corresponds to 10^{-16} eV scalarons. These scalarons present the stellar scale range ($\sim$0.0014 – 0.14 au) of the Yukawa fifth force. Therefore, an accelerated growth of stellar mass can be naturally expected with scalarons. This may likely provide a seed for the realistic stellar mechanism of destroying primordial deuterium. Taking the uncertainty of cosmic star formation history we infer that if primordial deuterium abundance is high, lithium abundance is low by a factor of 3–4 relative to the standard BBN, and if ^{4}He abundance is compatible with all existing measurements, then $f(R)$ gravity theory with scalaron masses of $(2.2–3.5) \times 10^4$ eV can have a Big Bang equivalence.

According to the relation between scalaron mass and black hole mass (see Equation (3)), our derived scalarons correspond to PBHs of masses around 10^{19} g. These are subplanetary scales including the asteroids (smaller than Ceres) and some moons of the solar system. They formed around 10^{-20} s after the Big Bang and survived the Hubble time against Hawking evaporation. They are eligible to be candidates of nonbaryonic dark matter. This mass falls within the range $10^{17}–10^{22}$ g, which is unconstrained and hence is open as a window that provides all of the dark matter density in the Universe (Capela et al. 2013a, 2013b). PBHs within the above range are produced through isocurvature density perturbations in specific inflationary models and thereby contribute to scalar-induced gravitational wave (SIGW) signals with frequencies ranging from nHz to kHz (Ahmed et al. 2022). Ghosh & Mishra (2024) calculated the SIGW spectrum of these PBHs and found that GW detectors such as eLISA, BBO, and DECIGO are capable of detecting these signals. Detection of PBHs with 10^{19} g, therefore, will hint toward a Big Bang equivalence of $f(R)$ gravity theory. Growth of PBHs with a mass of 10^{19} g (this work) to massive PBHs naturally calls for a dynamical scalaron field with cosmological coupling of scalaron mass. This calls for further investigation of scalarons in the early Universe. We infer that $f(R)$ gravity with a Big Bang equivalence allows subplanetary mass PBH dark matter.

6. Conclusion

A power-law $f(R)$ gravity theory allows its scalar gravitational mode to act as a constant density dark energy fluid. This dark (scalaron) density is found to be compatible with the BBN abundance of helium. Its presence reduces the lithium abundance to the level desired to address the cosmological lithium problem. However, deuterium abundance is found to increase beyond observed bounds for the scalaron mass range addressing the lithium problem. If a high deuterium abundance in the early Universe is treated as an open problem we infer that a power-law gravity theory may call for exceptionally high efficiency of stellar formation and evolution in the early stages of the Universe. We wait for further improvement in the JWST's measurements of early galactic evolution. Masses of the PBHs derived from the scalarons addressing the lithium problem are found to be within the mass window that is eligible to act as nonbaryonic dark matter. We wish to conclude that a power-law gravity theory with PBHs of subplanetary mass has a Big Bang equivalence.

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