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COMPARISON OF FLAT BEAM PWFA ANALYTIC MODEL WITH PIC SIMULATIONS

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Abstract

This paper explores the phenomenon of asymmetric blowout in plasma wakefield acceleration (PWFA), where the transversely asymmetric beam creates a transversely asymmetric blowout cavity in plasma. This deviation from the traditional axisymmetric models leads to unique focusing effects in the transverse plane and accelerating gradient depending on the transverse coordinates. We extend our series of studies on PWFA by comparing our recently developed analytic model on the blowout cavity shape created by transversely asymmetric long beams, with Particle-in-Cell (PIC) simulations. The analysis focuses on validating the model's ability to predict the behaviors of different beam profiles in this regime.

INTRODUCTION

Plasma wakefield acceleration (PWFA) in the nonlinear regime [1] employs a driver beam that is much denser than the plasma density to displace surrounding electrons and create an ion column, facilitating the acceleration of particles with a high energy gradient. A flat beam, characterized by a highly asymmetric transverse spot size, plays a crucial role in potentially reducing beam-beam effects at the interaction points of future colliders [2]. Such beams are utilized at facilities like AWA [3] and could be potentially employed at FACET-II [4]. Recent analyses have begun to explore the flattop regime, building on the previous work that focused on the limit where the bubble size is much smaller than the plasma wavelength [5]. A recent phenomenological model explores the flat beam regime over a broader parameter scan, formulating its wake potential, ψ, and the transverse wake fields, W_x and W_y, using a simplified elliptical approximation [6]. This paper extends the investigation and verification of this novel model, examining the effects of varying driver beam profiles, densities, and ellipticities on the resulting blowout structure, particularly where the blowout is larger or comparable to the plasma wavelength.

THEORY

In this paper, all physical quantities are normalized to plasma units as followed: time is normalized to ω_p^{-1}, with the plasma frequency $\omega_p = \sqrt{n_0 e^2/m_e\epsilon_0}$; distance to the plasma skin depth $k_p^{-1} = c/\omega_p$; velocities to the speed of light c; masses to the electron mass m_e; magnitude of particle charge to the electron charge magnitude e. The quasi-static approximation allows the coordinate transformation, $(x, y, z, t) \longrightarrow (x, y, \xi \equiv t - z, s \equiv z)$, to be employed for the following analysis, where ξ represents longitudinal position respect to the driver beam and (x, y) represents the transverse coordinate.

In the newly proposed model, plasma wakefield acceleration (PWFA) using a flat beam driver creates an elliptical blowout boundary. The electron sheath, formed by electrons expelled by the denser driver beam, is modeled as a delta function confined to the radial elliptical coordinate [6]. This simplification can be used for determining the blowout boundary created by a flat beam driver in the long beam limit, where $r_\perp \ll \gamma\sigma_z$, where $r_\perp$ is the transverse blowout size, γ is the Lorentz factor, and σ_z is the bunch length of the beam. In this scenario, the longitudinal variation of the fields can be neglected, allowing for the derivation of the following boundary condition:

$$\left[(1+v_z)\vec{\boldsymbol{W}}_\perp - v_z\vec{\boldsymbol{E}}_{\boldsymbol{i}\perp} + (1-v_z)\vec{\boldsymbol{E}}_{\boldsymbol{b}\perp}\right]\Big|_{\partial\Omega} = 0\,, \quad (1)$$

where $\vec{\boldsymbol{W}}_\perp$ is the transverse plasma wake field and $\vec{\boldsymbol{E}}_{\boldsymbol{i}\perp}$ and $\vec{\boldsymbol{E}}_{\boldsymbol{b}\perp}$ are the transverse electric fields for ion column and the driver beam, respectively. The velocities of plasma electrons can be approximated by assuming that the return current sheath extends to one plasma skin depth, k_p^{-1}. Consequently, the longitudinal velocity, v_z, is given by $v_z = \lambda_b/(\pi(a_p+1)(b_p+1))$, where λ_b is the beam charge per unit length.

The solution for transverse wake fields of the elliptical blowout were found to be linear and the x-component is given by:

$$W_x(x, y; \xi) = E_x - B_y = \frac{b_p(\xi)^2 x}{a_p(\xi)^2 + b_p(\xi)^2} = \frac{x}{1+\alpha_p(\xi)^2}\,. \quad (2)$$

This formulation clearly delineates the dependence of the wake fields on the ellipticity of the plasma blowout, $\alpha_p(\xi) = a_p/b_p$, and the transverse coordinates (x, y). a_p and b_p are the major and minor axes of the elliptical blowout boundary, respectively.

The x-component of the transverse electric field due to the elliptical ion column can be derived as follows [7]:

$$E_{i,x} = \frac{b_p x}{a_p + b_p} = \frac{x}{1+\alpha_p}\,. \quad (3)$$

The x-component of the transverse electric field of the elliptical flat-top beam can be calculated using the formula provided in [8]:

$$E_{\text{flattop},x} = \frac{n_b x}{4(a^2+t_1)^{1/2}((a_p^2+t_1)^{1/2}+(b_p^2+t_1)^{1/2})} \quad (4)$$

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Table 1: Simulation Parameters

Parameter	Value	Unit
Beam density, n_b	10/15/20/40	n_0
Bunch length, σ_z	10	k_p^{-1}
Flattop Hard Edge-x, a_b	0.4/0.5657/0.8	k_p^{-1}
Flattop Hard Edge-y, b_b	0.08/0.05657/0.04	k_p^{-1}
Gaussian spot size-x, σ_x	0.28285/0.4/0.5657	k_p^{-1}
Gaussian spot size-y, σ_y	0.05657/0.04/0.028285	k_p^{-1}

where $t_1 = ((x^2 + y^2 - a_b^2 - b_b^2)^2/4 + (x^2 b_b^2 + y^2 a_b^2 - a_b^2 b_b^2))^{1/2} + (x^2 + y^2 - a_b^2 - b_b^2)/2$. a_b and b_b are the major and minor axis of the flat-top driver. n_b is the beam density for flat-top beam and maximum beam density for Gaussian beam.

All the fields in the y-direction, W_y, $E_{i,y}$ and $E_{\text{flattop},y}$, follow a same formulation with the parameters a_p, b_p, a_b, b_b and with x and y interchanged.

The transverse electric fields of an elliptical driver beam with a Gaussian distribution ($\sigma_x \neq \sigma_y$, different spot sizes in the x and y components) were determined numerically using FFTs (fast Fourier transforms). We began by transforming Poisson's equation, $\nabla_\perp^2 \phi_{\text{gaussian}} = -\rho_{\text{gaussian}}(x, y)$, into Fourier space, where ϕ_{gaussian} is the electric potential of the Gaussian driver beam and ρ_{gaussian} is the driver beam charge distribution. Spatial coordinates are converted into wave numbers in the frequency domain, k_x and k_y. This results in the direct solution in frequency space:

$$\hat{\phi}(k_x, k_y) = \frac{\hat{\rho}_{\text{gaussian}}(k_x, k_y)}{k_x^2 + k_y^2} . \quad (5)$$

The edge case $k_x = k_y = 0$ coincides with a constant term which we choose to be zero, enforcing a zero potential boundary condition.

With both analytical and numerical solutions available for all fields referenced in Eq. 1, we then identified the optimal pair of (a_p, b_p) that nullifies the left side of Eq. 1. This facilitated the subsequent computation of the wake field as described in Eq. 2.

SIMULATION RESULTS

Simulations are conducted using the particle-in-cell simulation tool Osiris [9]. We explore both Gaussian and flattop driver beams across a variety of beam ellipticities and densities, with specific parameters outlined in Table 1. The study involves 24 simulations, permuting four different beam densities ($n_b = 10, 15, 20, 40$), three beam ellipticities ($\alpha_b = a_b/b_b = 5, 10, 20$ for flattop drivers and $\alpha_b = \sigma_x/\sigma_y = 5, 10, 20$ for Gaussian drivers), and two beam profiles (flattop and Gaussian). The spot sizes specified in Table 1 are carefully selected to ensure that λ_b is consistent at each longitudinal coordinate for all simulations within the same beam density. This consistency allows for a direct comparison of the impact of driver profiles on the blowout shape, isolating profile effects from other variables.

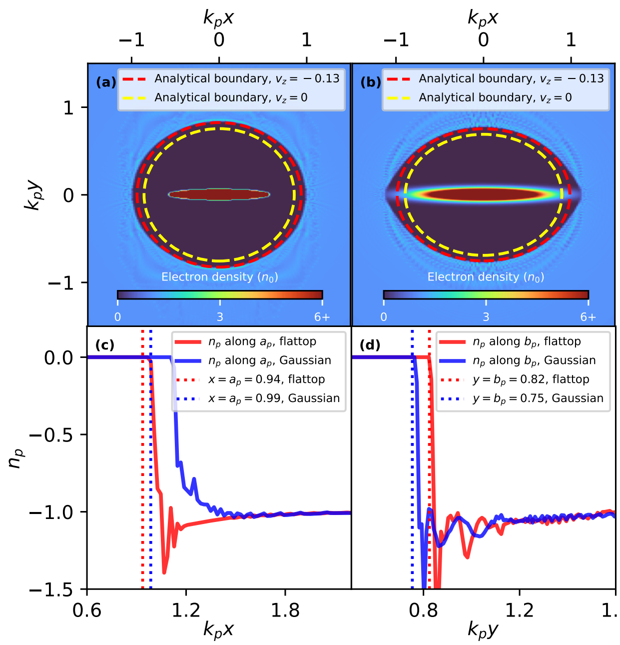

Figure 1: Elliptical plasma wakefield created by a asymmetric driver. Flattop driver with $a_b = 0.5657$ and $b_b = 0.05657$ (a), Gaussian driver with $\sigma_x = 0.4$ and $\sigma_y = 0.04$ (b), simulated transverse plasma electron line-outs of both cases along the major axis of the blowout (c) and the minor axis of the blowout (d).

For each simulation, transverse slices are taken at different longitudinal positions, ξ. Particularly, the middle transverse slice of the longbeam ($\xi = 0$), for both flat-top and Gaussian cases, along with the line-outs of the plasma electron sheaths, are illustrated in Fig. 1. It is observed that a key structural difference between the flat-top and Gaussian beam cases occurs around $x = \pm a_p$, where the lower density part of the Gaussian driver meets the blowout boundary.

This interaction results in, in the Gaussian driver case, an electron sheath along the a_p direction which gradually decays back to the uniform plasma electron density of -1, unlike the abrupt drop observed with the flattop driver, as shown in Fig. 1(c). The gradual decay in the Gaussian case corresponds to a linear response from the plasma electrons [10], which also influences the W_x generated by the Gaussian driver, as detailed in Fig. 2. To assess the accuracy of our model, we calculated the relative root mean square error (RRMSE),

$$RRMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{\sum_{i=1}^{n}(\hat{x}_i^2)}} ,$$

at various longitudinal positions, where $\hat{x}_i$ and x_i represent the simulation result and analytical prediction, respectively.

This metric provides a measure of the model's precision by comparing simulated results against expected values. Our findings, depicted in Figure 3, demonstrate that the analytical model predicts the simulated transverse wakefield with

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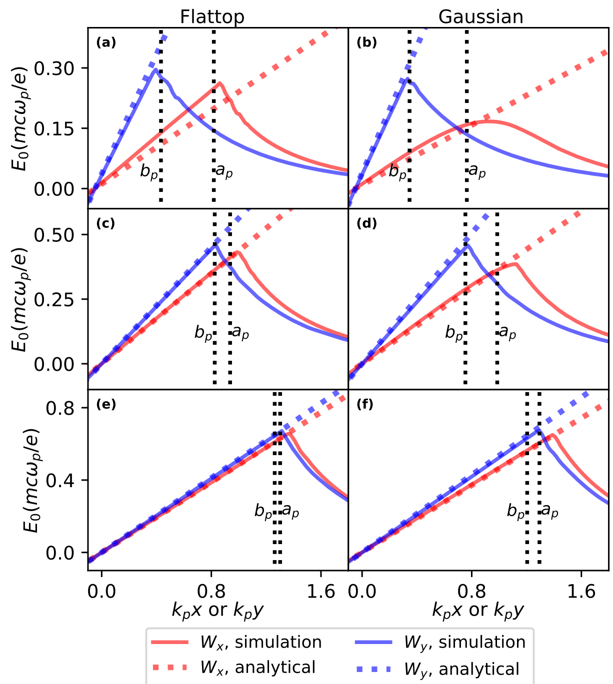

Figure 2: Comparison between the transverse wakefield created by flattop and Gaussian drivers at different beam densities and ellipticities. Flattop driver (a) and Gaussian driver (b) with $n_b = 10, \alpha_b = 20$, Flattop driver (c) and Gaussian driver (d) with $n_b = 20, \alpha_b = 10$, Flattop driver (e) and Gaussian driver (e) with $n_b = 40, \alpha_b = 5$.

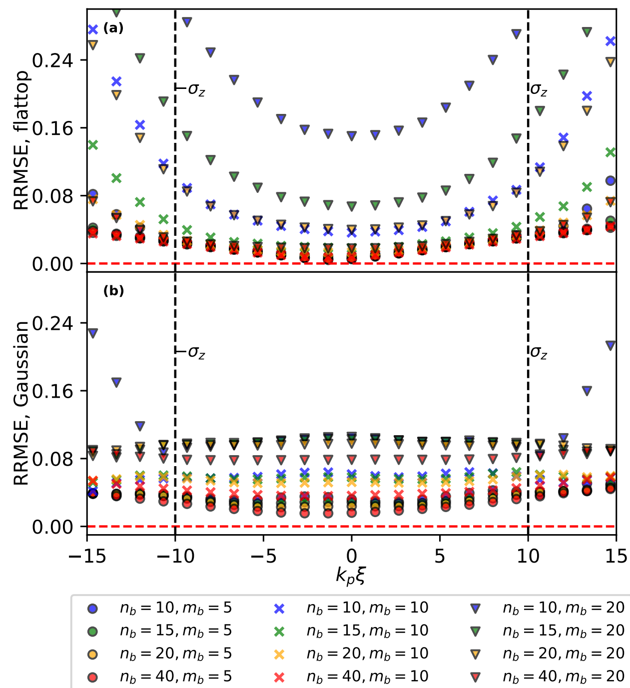

Figure 3: Relative Root Mean Squared Error (RRMSE) between the analytical transverse wakefield and the simulation of different beam densities and ellipticities of flattop (a) and Gaussian case(b).

minimal error. The correlation between theoretical and simulated results improves as the driver beam becomes denser and less elliptical.

CONCLUSION

In this paper, we further validated the proposed phenomenological model of flat beam plasma wakefield acceleration (PWFA) through particle-in-cell (PIC) simulations utilizing different driver beam profiles. Observations indicate that the theoretical predictions align well with the PIC simulations for both flat-top and Gaussian drivers across various values of beam density (n_b) and ellipticity (α_b). To enhance the accuracy of this model and obtain simplified algebraic expression for blowout boundary, further analysis of the linear responses of the plasma electrons by considering the exterior expansion of the driver beam would be useful. Additionally, to analytically calculate the blowout boundary without relying on the longbeam approximation, further assumptions and supplementary information are required to close the system effectively. A plasma wakefield experiment involving beams with asymmetric emittances and spot sizes has been proposed at AWA [11, 12]. Our analysis of the focusing forces created in the elliptical blowout cavity can be used to determine the matching conditions for stable propagation of the flat beam driver [13].

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