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Soft Gluons in the Finite Momentum Wave Function and the BFKL Pomeron

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1 Introduction

Our object in this paper is to construct the small-x infinite momentum wavefunction of a hadron in QCD for those soft gluons reasonably well localized in a small transverse area. To make the problem of transverse spatial localization simple we choose the large-x part of our hadron to be a heavy quark-antiquark state, an "onium" state. (This device has previously been used by Balitsky and Lipatov[1] in their work on the Balitsky, Fadin, Kuraev, Lipatov (BFKL)[1-3] pomeron.) The radius of the onium state then naturally furnishes an infrared cutoff, and if this cutoff is sufficiently large perturbation theory applies. The accuracy of our approximation is leading logarithmic. That is, for the component of the wavefunction having n soft gluons, with momentum between z_0p and p, where p is the onium momentum, we calculate only the $(\alpha \ \ell n \ 1/z_0)^n$ contribution to the square of the infinite momentum onium wavefunction.

In our construction of the square of the onium wavefunction having n soft gluons, we find it convenient to label the gluons by a longitudinal momentum $z_i p$ and a transverse coordinate $\underline{x}_i$ with $i = 2, 3 \cdots n+1$. The transverse coordinate representation is especially useful because one can view the i^{th} gluon as being emitted from the heavy quark-antiquark pair and gluons $g_2, g_3, \cdots g_{i-1}$ with the spatial coordinates of these "sources" being frozen during the emission of the gluon g_i.

The motivations for trying to construct the soft gluon part of the infinite momentum wavefunction are severalfold. (i) The infinite momentum wavefunction, at leading logarithmic level, should allow one to reproduce results usually obtained from the BFKL equation when one uses that wavefunction to calculate a scattering amplitude. (ii) Two, and more, pomeron exchange amplitudes should be easily accessible from the infinite momentum wavefunction. (iii) When used in a scattering the infinite momentum wavefunction, at leading logarithmic level, should give a good estimate of the production of very larger numbers of minijets, numbers far above the average in a high energy event. (iv) If the wavefunction could be constructed at next-to-leading logarithmic accuracy one would be able to extract corrections[4-6], of higher order in α to the BFKL pomeron trajectory.

We have succeeded in writing a nonlinear equation for the generating functional of the n-gluon component of the square of the soft gluon wavefunction at leading logarithmic accuracy in the large N_c limit. This equation is given below in eq.(20). Also, in sec.4, we have shown that (20) yields results corresponding to the BFKL pomeron when the gluon density is calculated.

Eq.(20) has a form remarkably close to that which occurs in the decay of a jet[7,8]. Indeed, eq.(20) defines a branching process which could be implemented by a Monte Carlo simulation much as is done in jet physics. It is quite remarkable that there exists a classical branching process which gives, *exactly*, the leading logarithmic soft gluon wavefunction, including arbitrary numbers of soft gluons.

Our derivation of the BFKL equation,(29), seems much simpler than previous derivations appearing in the literature. Our form for the BFKL kernel K given in eq.(30), as $\rho \to 0$, also apears different from the usual forms used, although (34) shows that indeed it is the ususal kernel.

2 Lowest Order Onium Wavefunction

In this section, we shall derive the small-x part of an onium wavefunction in the approximation where only one soft gluon is present. We shall first exhibit this wavefunction in momentum space and then in a mixed representation using transverse coordinate and longitudinal momentum variables.

2.1 The Onium Wavefunction Without Soft Gluons

To set notation and normalization, we begin with the onium wavefunction containing no soft gluons. The light-cone wavefunction is $\psi_{\alpha\beta}(\underline{k}, z)$ where $z_1 = k_{1+}/p_+$ and α and β are heavy quark and antiquark spinor indices. ψ is illustrated in Fig.1. It will be convenient to use transverse coordinate variables which are introduced according to

$$\psi^{(0)}_{\alpha\beta}(\underline{x}_1, z_1) = \int \frac{d^2k_1}{(2\pi)^2} e^{i\underline{k}_1\cdot\underline{x}_1} \psi^{(0)}_{\alpha\beta}(\underline{k}_1, z_1) \tag{1}$$

where we may suppose that the quark, p-k, has transverse coordinate $\underline{x} = 0$ while the antiquark has $\underline{x} = \underline{x}_1$. Our normalization is

$$\int \frac{d^2k_1}{(2\pi)^2} \int_0^1 dz_1 \Phi(\underline{k}_1, z_1) = \int d^2x_1 \int_0^1 dz_1 \Phi(\underline{x}_1, z_1) = 1 \tag{2}$$

where

$$\Phi^{(0)}(\underline{k}_1, z_1) = \sum_{\alpha\beta} \mid \psi^{(0)}_{\alpha\beta}(\underline{k}_1, z_1) \mid^2 \tag{3}$$

and

$$\Phi^{(0)}(\underline{x}_1, z_1) = \sum_{\alpha\beta} \mid \psi^{(0)}_{\alpha\beta}(\underline{x}, z_1) \mid^2 \tag{4}$$

$\Phi^{(0)}$ is illustrated in Fig.2, where the two vertical lines indicate that two energy denominators, one for ψ and one for ψ^*, are included, distinguishing $\Phi^{(0)}$ from a simple time-ordered-product.

2.2 The Momentum Space Wavefunction with One Soft Gluon

The order α momentum space wavefunction with one soft gluon has contributions which are illustrated in Fig.3. Call $z_2 = k_{2+}/p_+$. We suppose z_2/z_1 and $z_2/(1-z_1)$ are both much less than 1. This defines the gluon k_2 as a soft gluon. It is straightforward to calculate the contributions of the graphs in Fig.3 and one finds

$$\psi^{(1)a}_{\alpha\beta}(\underline{k}_1, \underline{k}_2; z_1, z_2) = \; -\; 2gT^a[\psi^{(0)}_{\alpha\beta}(\underline{k}_1, z_1) - \psi^{(0)}_{\alpha\beta}(\underline{k}_1 + \underline{k}_2, z_1)] \; \frac{\underline{k}_2 \cdot \underline{\epsilon}_2^\lambda}{\underline{k}_2^2} \tag{5}$$

where a is the color index of the emitted gluon, T^a a color matrix, and $\underline{\epsilon}_2^\lambda$ the polarization vector of the soft gluon with helicity λ. For SU(3) color $T^a = \lambda^a/2$. At this one gluon level Φ takes the form

$$\Phi^{(1)}(\underline{k}_1, z_1) = \frac{1}{(2\pi)^3} \int d^2k_2 \int_{z_0}^{z_1} \frac{dz_2}{2z_2} \sum_{\substack{\lambda a \\ \alpha\beta}} \mid \psi_{\alpha\beta}^{(1)a}(\underline{k}_1, \underline{k}_2; z_1, z) \mid^2 . \tag{6}$$

We note that the integration over the region $\underline{k}_2 \to 0$ is convenient due to the color neutrality of the onium state. The $\underline{k}_2-$ integral in (6) is ultraviolet divergent, as expected, though this divergence will disappear when the light-cone wavefunction is used to calculate observables. We shall later insert a cutoff to regulate ultraviolet divergences. We have put a cutoff, z_0, on the z_2- integration in (6). Again this cutoff will not appear in physical quantities. We always work in the leading logarithmic approximation in z-integrals in which spirit we have set the upper limit of the z_2-integral to be z_1 in (6).

2.3 The Coordinate Space Wavefunction with One Soft Gluon

Although one can deal with the one soft gluon part of the onium wavefunction in momentum space it is extremely difficult to deal with more than one soft gluon in the momentum representation. For that reason we now introduce the transverse coordinate representation in the one gluon approximation. The key to the simplicity of Φ in coordinate space is to recall that the transverse coordinate positions of the quark and antiquark are frozen during the time of emission of the soft gluon, k_2. If we describe the wavefunction in terms of a quark at $\underline{x} = \underline{x}_0 = \underline{0}$, an antiquark at $\underline{x} = \underline{x}_1$ and a gluon at $\underline{x} = \underline{x}_2$, then the wavefunction corresponding to (5) is

$$\psi_{\alpha\beta}^{(1)a}(\underline{x}_1, \underline{x}_2; z_1, z_2) = \int \frac{d^2k_2}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} e^{i\underline{k}_1\cdot\underline{x}_1 + i\underline{k}_2\cdot\underline{x}_2} \psi_{\alpha\beta}^{(1)a}(\underline{k}_1, \underline{k}_2; z_1, z_2). \tag{7}$$

Using the explicit form given in (6) we find

$$\psi_{\alpha\beta}^{(1)a}(\underline{x}_1, \underline{x}_2; z_1, z_2) = -\frac{igT^a}{\pi} \psi_{\alpha\beta}^{(0)}(\underline{x}_1, z_1)\left(\frac{\underline{x}_{20}}{\underline{x}_{20}^2} - \frac{\underline{x}_{21}}{\underline{x}_{21}^2}\right) \cdot \underline{\epsilon}_2^\lambda \tag{8}$$

where $\underline{x}_{20} = \underline{x}_2 - \underline{x}_0$ and $\underline{x}_{21} = \underline{x}_2 - \underline{x}_1$. The $\underline{x}_{20}/\underline{x}_{20}^2$ term in (8) corresponds to graph A of Fig.3 while the $\underline{x}_{21}/\underline{x}_{21}^2$ term corresponds to graph B.

Corresponding to $\Phi^{(1)}$ in momentum space we can calculate $\Phi^{(1)}$ in transverse coordinate space. Thus

$$\Phi^{(1)}(\underline{x}_1, z_1) = \int d^2x_2 \int_{z_0}^{z_1} \frac{dz_2}{2z_2} \sum_{\substack{a\lambda \\ \alpha\beta}} \mid \psi_{\alpha\beta}^{(1)a}(\underline{x}_1, \underline{x}_2; z_1 z_2) \mid^2 \tag{9}$$

with a trace over color indices implicit in (9). We find

$$\Phi^{(1)}(\underline{x}_1, z_1) = \int d^2x_2 \int_{z_0}^{z_1} \frac{dz_2}{z_2} \frac{\alpha C_F}{\pi^2} \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \Phi^{(0)}(\underline{x}_1, z_1). \tag{10}$$

The simplicity of (10), as compared to (5) and (6), is that the soft gluons factorize more completely from the quark and antiquark in transverse coordinate space.

It will turn out convenient to write d^2x_2 in terms of dx_{10} and dx_{20}, where $x_{ij} = |\underline{x}_{ij}|$. If one writes

$$d^2x_2 \;=\; x_2\, dx_2\, d\phi = J dx_{12}\, dx_{20} \tag{11}$$

with ϕ the angle between $\underline{x}_{20}$ and $\underline{x}_{10}$, then

$$J(x_{21}, x_{20}) = \frac{4\; x_{21}x_{20}}{\sqrt{[(x_{21}+x_{20})^2 - x_{10}^2][x_{10}^2 - (x_{21}-x_{20})^2]}} \tag{12}$$

where an extra factor of 2 is included in (12) to take into account the region $\pi < \phi < 2\pi$ as well as $0 < \phi < \pi$.

3 The Onium Wavefunction to All Orders of Soft Gluons

In this section we shall generalize (10) to include n soft gluons $k_2, k_3 \cdots k_{n+1}$ ordered according to $z_2 >> z_3 >> z_4 \cdots >> z_{n+1}$ where, as usual, z_i is the fraction of the original momentum p_+ carried by the i^{th} gluon. We shall also evaluate $\Phi^{(n)}$ only in the large N_c limit. It is not clear whether $1/N_c$ corrections take a simple form.

To see how to treat color factors return to the expression on the right-hand side of (10). We may rewrite the integrand as

$$\frac{\alpha\, C_F}{\pi^2}\, \frac{x_{10}^2}{x_{20}^2\, x_{21}^2} \;=\; \frac{\alpha\, C_F}{\pi^2}\, \left(\frac{1}{x_{21}^2} \;-\; \frac{2\, \underline{x}_{21}\cdot\, \underline{x}_{20}}{x_{21}^2\, x_{20}^2} \;+\; \frac{1}{x_{20}^2}\right). \tag{13}$$

The first term on the right-hand side of (13) comes from the square of term A of Fig.3 while the second term comes from the interference between A and B, and the third term comes from the square of term B. Viewing the color part of the gluon line as a quark-antiquark pair the first, second and third terms on the right-hand side of (13) are illustrated in Fig.4 as A, B and C respectively. From these figures it is clear that each term has the same color factor, $N_c = 2C_F$. The minus sign for the second term on the right-hand side of (13) corresponds to interference between emission off a particle and off an antiparticle.

In order to see what happens at higher orders we now consider the two soft gluon component of the onuim wavefunction. If the second soft gluon, g_3, is emitted off the first soft gluon, g_2, it is useful to consider the emission as consisting of two parts corresponding to emission off the quark or antiquqark "components" of g_2. Of course the only difference between these two terms is in the color structure, but a proper organization of the color factors is crucial in obtaining a simple form for the final answer. Due to the large N_c limit the number of graphs is strongly limited.

For example, when g_3 is emitted off the original heavy quark or the antiquqark part of g_2 it must be re-absorbed (the complex conjugate amplitude in the formula for $\Phi^{(2)}$) only by the original heavy quark or by the antiquark part of g_2. But emission off the antiquark part of g_2 is *exactly* the same as an emission of an actual antiquark when $z_3/z_2 << 1$. Thus, the net result, in calculating $\Phi^{(2)}$, for emission off the original heavy quark and the antiquark part of g_2 is simply the factor

$$\frac{\alpha\, C_F}{\pi^2}\, \frac{x_{20}^2}{x_{30}^2\; x_{32}^2}\, \frac{dz_3}{z_3}\; J(x_{30}, x_{32})\; dx_{30} dx_{32}. \tag{14}$$

A similar result comes from emission off the original heavy antiquark and the quark component of g_2. The sum of both terms gives the following expression for $\Phi^{(2)}$.

$$\Phi^{(2)}\; (\underline{x}_1, z_1) \;=\; (\frac{4\alpha C_F}{\pi^2})^2 \frac{1}{2} \ell n^2\; (z_1/z_0)\; \Phi^{(0)}(\underline{x}_1, z_1) x_{01}^2 \int dx_{20}\; dx_{21}\; \frac{J(x_{20}, x_{21})}{x_{20}^2 x_{21}^2}$$

$$[\int dx_{30} dx_{32}\; \frac{J(x_{30}, x_{32})\; x_{20}^2}{x_{30}^2 z_{32}^2} \;+\; \int dx_{32} dx_{31}\; \frac{J(x_{32}, x_{31})\; x_{21}^2}{x_{32}^2 z_{31}^2}]. \tag{15}$$

It is convenient to view the sequential emission of soft gluons pictorially as shown in Fig.5. Term A of that figure shows first the gluon g_2 being emitted off the heavy quark, 0 in the figure, and off the heavy antiquark, 1 in the figure. Then the gluon g_3 is emitted off the heavy quark and the antiquark part of g_2. Term B in the figure illustrates gluon g_3 being emitted off the heavy antiquark and the quark part of g_2.

The formula for further emissions is now straightforward to write down. In order to organize the expressions it is useful to write an integral equation for the generating functional of the light-cone wavefunction of the onium. To that end, define $\Phi(\underline{x}_1, z_1, u(\underline{x}, z))$ by the equation

$$\frac{\delta}{\delta u(\underline{x}_2, z_2)}\, \frac{\delta}{\delta u(\underline{x}_3, z_3)} \cdots \frac{\delta}{\delta u(\underline{x}_{n+1}, z_{n+1})}\; \Phi(\underline{x}_1, z_1, u(\underline{x}, z))|_{u=0}$$

$$= \Phi^{(n)}(\underline{x}_1, \underline{x}_2, \cdots \underline{x}_{n+1}; z_1, z_2 \cdots z_{n+1}) \tag{16}$$

where $\Phi^{(n)}$ is the square of the wavefunction for the heavy quark-antiquark pair along with n gluons having transverse coordinates $\underline{x}_2, \underline{x}_3 \cdots \underline{x}_{n+1}$ and momentum fractions $z_2, z_3, \cdots z_{n+1}$. Let

$$\Phi(\underline{x}_1, z_1, u) = \Phi^{(0)}(\underline{x}_1, z_1) Z(\underline{x}_1, \underline{x}_0, z_1, u) \tag{17}$$

then Z obeys

$$Z(\underline{x}_1, \underline{x}_0, z_1, u) = 1 + \frac{\alpha\, C_F}{\pi^2}\, x_{01}^2 \int \frac{d^2\underline{x}_2}{x_{20}^2\; x_{21}} \int_{z_0}^{z_1} \frac{dz_2}{z_2}\; u(\underline{x}_2, z_2) Z(\underline{x}_2, \underline{x}_1, z_2, u) Z(\underline{x}_2, \underline{x}_0, z_2, u). \tag{18}$$

In (18) we have, temporarily, gone back to using $d^2\underline{x}_2$ as the coordinate measure, though when actual calculations are done we shall use (11) and (12) to evaluate $d^2\underline{x}_2$.

Eq.(18) is not quite complete, even in the leading logarithmic aproximation. Our normalization is such that Φ should obey

$$\int d^2\underline{x}_1 \int_0^1 dz_1\ \Phi(\underline{x}_1, z_1, u)|_{u=1} = 1 \tag{19}$$

while it is easy to check that (17) and (18) do not give such a Φ. The problem is apparent. Eqs.(17) and (18) give the probabilities for real gluon emissions, but they do not account for the leading logarithmic virtual corrections. The virtual corrections are determined most easily by enforcing (19), order by order in perturbation theory. In order to do this it is now important to cut off the ultraviolet divergences present in (18) when x_{20} and/or x_{21} go to zero. We define a region $R(x_{20}, x_{10})$ by simply requiring that $x_{20} \geq \rho$ and $x_{10} \geq \rho . \rho$ is a fixed quantity taken to be small compared to the average radius of the onium state. Then the complete formula for Z is

$$Z(\underline{x}_1, \underline{x}_0, z_1, u) = exp\{-\frac{4\ \alpha\ C_F}{\pi}\ell n(\frac{x_{10}}{\rho})\ell n(z_1/z_0)\} + \frac{\alpha\ C_F}{\pi^2}\int_{z_0}^{z_1}\frac{dz_2}{z_2}\int_{R(x_{20},x_{10})}$$

$$exp\{-\frac{4\ \alpha\ C_F}{\pi}\ell n(\frac{x_{10}}{\rho})\ell n(z_1/z_2)\}\frac{d^2\underline{x}_2 x_{10}^2}{x_{20}^2\ x_{21}^2}\ u(\underline{x}_2, z_2)Z(\underline{x}_2, \underline{x}_1, z_2, u)Z(\underline{x}_2, \underline{x}_0, z_2, u). \tag{20}$$

The form of the virtual corrections in (20) is not difficult to determine. When u=1 it must be true that Z=1 since the production of soft gluons cannot change the *inclusive* probability of having a heavy quark-antiquark pair with relative separation $\underline{x}_1$ and anti-quark momentum fraction z_1. Thus $\Phi(\underline{x}_1, z_1, u=1) = \Phi^{(0)}(\underline{x}_1, z_1)$ and by (17) this implies $Z(x_{10}, z_1, 1) = 1$. It is straightforward to check that the form of the virtual corrections we have taken satisfies this conservation of probability constraint.

The form of the generating functional, (17) and (20), is very very close to that which occurs in the decay of a jet[9]. In each case one can follow the perturbative evolution through a "classical" cascade of sequential parton decays. There are, however, essential differences. In the present circumstance one is constructing the square of the onium wavefunction, including an arbitrary number of *virtual* gluons, while in the jet cascades the final multiparton system is a system of essentially free partons which undergo a simple hadronization to become actual physical particles. A key issue in the present case is how to free the virtual partons. Also, the time-like coherent cascades are built around the leading double logarithmic approximation. Next-to-leading terms, the modified leading logarithmic approximation (MLLA), can also be included in time-like partonic cascades but only for quantities where azimuthal averaging of the parton splittings is assumed[9]. In the present case the series is a leading single logarithmic series, in the longitudinal momentum, and the classical partonic cascade, determined by (17) and (20), gives the series exactly in the large N_c approximation.

4 The Balitsky, Fadin, Kuraev, Lipatov (BFKL) Pomeron

In order to illustrate the fact that (20) contains the physics of the BFKL Pomeron we shall calculate the unintegrated gluon distribution[10,11], F, of the onuim state. We take the magnitude of the transverse momentum of the unintegrated gluon to be Q with $\rho << 1/Q \sim$ R with R the radius of the onuim state. At lowest order in α

$$zF(z,Q^2) = \frac{2\alpha\, C_F}{\pi} \int v(Qx_{10}) d^2\underline{x}_1 \int_0^1 dz_1 \Phi^{(0)}(\underline{x}_1, z_1) \tag{21}$$

where the exact form of v is not of concern here, and where we take z to be the longitudinal momentum fraction to be consistent with our previous notation. (Our notation is such that in case $Q^2R^2 >> 1$ F is given simply by

$$F(z,Q^2) = Q^2 \frac{\partial}{\partial Q^2} G(z,Q^2) \tag{22}$$

with G the usual gluon distribution in an onium state. $v(QR) \to 1$ when $QR \to \infty$.)

Then F can be written in terms of an amplitude T according to

$$zF(z,Q^2) \;=\; \int\, d^2\underline{x}_1 \,\int_0^1 dz_1\; \Phi^{(0)}(\underline{x}_1, z_1)\; T(x_{10}, z_1; Q, z) \tag{23}$$

where

$$T(x_{10}, z_1; Q, z) = \frac{2\alpha\, C_F}{\pi} v(Qx_{10}) exp\{-\frac{4\;\alpha\; C_F}{\pi}\; \ell n(\frac{x_{10}}{\rho})\ell n(z_1/z)\} + \frac{4\;\alpha\; C_F}{\pi} \int_z^{z_1} \frac{dz_2}{z_2}$$

$$\int_{R(x_{20},x_{10})} exp\{-\frac{4\;\alpha\; C_F}{\pi} \ell n(\frac{x_{10}}{\rho})\ell n(z_1/z_2)\} \tilde{K}(x_{10}, x_{12}) dx_{12} T(x_{12}, z_2; Q, z) \tag{24}$$

with

$$\tilde{K}(x_{10}, x_{12}) \;=\; \frac{1}{2\pi} \int_R \frac{x_{10}^2}{x_{12}^2\; x_{20}^2}\; J(x_{21},\; x_{20})\; dx_{20}. \tag{25}$$

The form of K is evident from a comparison of the kernel in (20) with (11). Eq.(24) reflects the fact that the measured gluon may be the first gluon emitted, the first term on the right-hand side of (25), or that it may be found in either of the Z's in the second term on the right-hand side of (20). The factor of 2 in front of the second term on the right-hand side of (24) comes from the choice of which of the two Z's the measured gluon comes from.

In order to solve (24), it is convenient to define $Y = \ell n\; z_1/z, y = \ell n\; z_2/z$ and to note that

$$T(x_{10}, Z_1; Q, z) = T(Y, Qx_{10}), \tag{26}$$

at least when ρ is very small. Write

$$T(Y, Qx_{10}) = \int \frac{d\omega}{2\pi i} e^{\omega Y} T_\omega(Qx_{10}) \tag{27}$$

where the ω-integral goes parallel to the imaginary axis and to the right of any singularities in T_ω. Substituting (27) into (24) one finds

$$T_\omega(Qx_{10}) = \frac{2\alpha\ C_F}{\pi} \frac{v(Qx_{10})}{\omega + \frac{4\ \alpha\ C_F}{\pi} \ell n(x_{10}/\rho)} + \frac{4\ \alpha\ C_F}{\pi} \int dx_{12} \frac{\tilde{K}(x_{10}, x_{12}) T_\omega(x_{12}Q)}{\omega + \frac{4\ \alpha\ C_F}{\pi} \ell n(x_{10}/\rho)}. \tag{28}$$

Equation (28) can be rewritten as

$$T_\omega(Qx_{10}) = \frac{2\ \alpha\ C_F}{\pi\ \omega} v(Qx_{10}) + \frac{4\ \alpha\ C_F}{\pi\ \omega} \int dx_{12} K(x_{10}, x_{12}) T_\omega(x_{12}Q) \tag{29}$$

where

$$K(x_{10}, x_{12}) = \tilde{K}(x_{10}, x_{12})\ -\ \delta(x_{10} - x_{12})\ \ell n(x_{10}/\rho). \tag{30}$$

In fact, when $\rho = 0$ K is the BFKL kernel. To see this use the identity[12]

$$\frac{\pi}{2} \int_0^\infty bdb\ J_0(b\ x_{01})\ J_0(b\ x_{02})\ J_0(b\ x_{12}) = \frac{1}{\sqrt{[(x_{21} + x_{20})^2 - x_{10}^2][x_{10}^2 - (x_{21} - x_{20})^2]}}.$$

Then, (12), (25), (29) and (30) give

$$K(x_{10}, x_{12}) = \frac{x_{10}^2}{x_{12}} \int bdb[\psi(1) + \ell n 2 - \ell n\ bx_{10}]\ J_0(bx_{10}) J_0(bx_{12}) \tag{31}$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$. In arriving at (31) we have used

$$\int_\rho^\infty \frac{dx_{20}}{x_{20}}\ J_0(bx_{20}) = \lim_{\lambda \to 0} \{ \int_0^\infty\ dx_{20}\ x_{20}^{\lambda-1}\ J_0(bx_{20}) - \int_0^\rho\ dx_{20}\ x_{20}^{\lambda-1}\ J_0(bx_{20}) \} \tag{32}$$

and

$$\int_0^\infty\ dx_{20}\ x_{20}^{\lambda-1}\ J_0(bx_{20}) = 2^{-1+\lambda} b^{-\lambda}\ \frac{\Gamma(\lambda/2)}{\Gamma(1 - \lambda/2)}. \tag{33}$$

We have also set $J_0(bx_{20}) = 1$ in the second term on the right-hand side of (32) an approximation which neglects terms linear in ρ.

From (31) and (33), it is straightforward to show that

$$\int\ dx_{12}\ K(x_{10},\ x_{12}) x_{12}^\lambda\ = \chi(\lambda)\ x_{10}^\lambda \tag{34}$$

with

$$\chi(\lambda) = \psi(1) - \frac{1}{2}\psi(1 - \lambda/2) - \frac{1}{2}\psi(\lambda/2), \tag{35}$$

which equations show the equivalence of K with the usual BFKL kernel. Thus in order to solve (29) one writes

$$T_\omega(x_{10}\, Q) \;=\; \int \frac{d\lambda}{2\pi i}(Q\, x_{10})^\lambda\, T_{\lambda\omega} \tag{36}$$

where the λ-integral runs parallel to the imaginary axis. Substituting (36) into (29) and using (34) gives

$$T_{\lambda\omega} \;=\; \frac{2\,\alpha\, C_F}{\pi}\,\frac{v_\lambda}{\omega - \frac{4\,\alpha\, C_F}{\pi}\,\chi(\lambda)}. \tag{37}$$

From (27), (36) and (37), we find

$$T(Y, Qx_{10}) = \int \frac{d\lambda}{2\pi i}\,\frac{2\,\alpha\, C_F}{\pi}\, v_\lambda (Qx_{10})^\lambda\, e^{\frac{4\,\alpha\, C_F}{\pi}\chi(\lambda)Y} \tag{38}$$

which integral is easily evaluated in the saddle point approximation about $\lambda = 1$,

$$T = \frac{\alpha\, C_F\, v_1}{\pi}\, Qx_{10}\,\frac{e^{(\alpha_P - 1)Y}}{\sqrt{7\,\alpha\, C_F\,\zeta(3)Y}} \tag{39}$$

With $\alpha_P = \frac{8\,\alpha\, C_F}{\pi}\,\ell n 2$. Substituting (39) into (23) gives

$$zF(z, Q^2) \;=\; \frac{2\,\alpha\, C_F v_1}{\pi}\,(QR)\,\frac{e^{(\alpha_P - 1)Y}}{\sqrt{7\,\alpha\, C_F \zeta(3)Y}}, \tag{40}$$

where

$$R = \frac{1}{2}\int d^2\underline{x}_{10}\, dz_1\, x_{10}\, \Phi^{(0)}(\underline{x}_{10},\; z_1) \tag{41}$$

is the average transverse radius of the onium state, and now $Y = \ell n 1/z$.

Comparing (40) and (21) we see that the effect of the soft gluons in the onium wavefunction leads to an enhancement of F by a factor proportional to

$$\frac{Q\; R\; e^{(\alpha_P - 1)Y}}{\sqrt{\frac{7}{2}\alpha\; N_c\;\zeta(3)Y}} \tag{42}$$

a factor normally associated with the zero momentum transfer BFKL pomeron.

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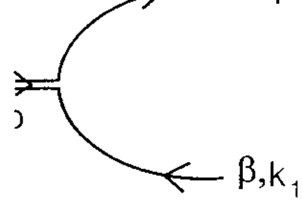


Fig 1

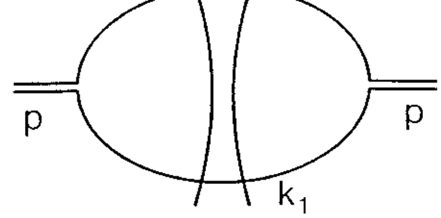
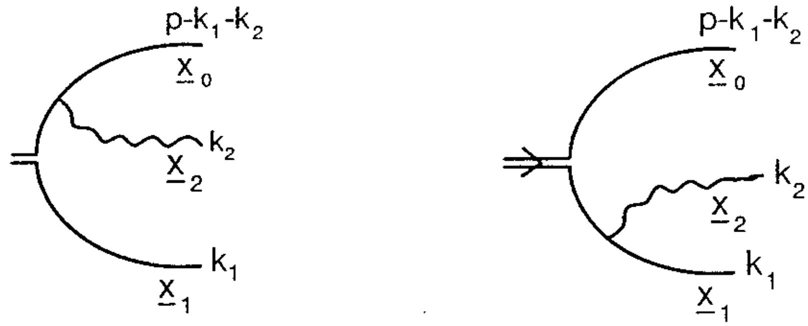
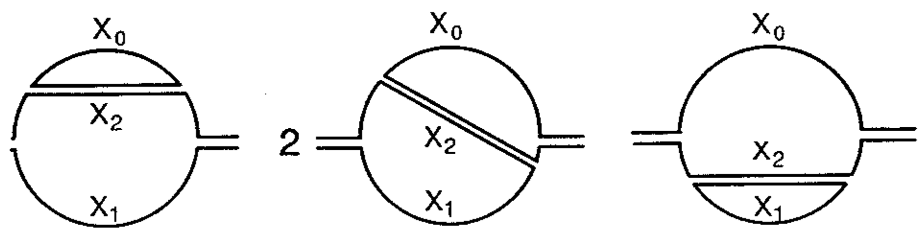


Fig 2



A B

Fig 3



A B C

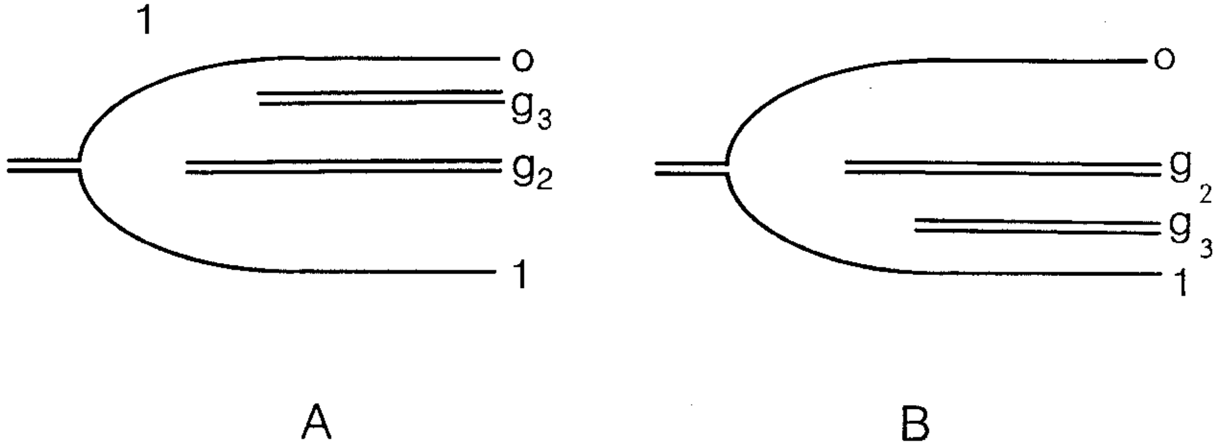


Fig 5