universe

IMPACT FACTOR 2.9

CITESCORE 3.6

Article

Nothing into Something and Vice Versa: A Cosmological Scenario

Leonid Marochnik

Special Issue
Universe: Feature Papers 2023 – Cosmology

Edited by
Dr. Kazuharu Bamba

MDPI

https://doi.org/10.3390/universe9100445

universe

MDPI

Article

Nothing into Something and Vice Versa: A Cosmological Scenario

Leonid Marochnik

Physics Department, University of Maryland, College Park, MD 20742, USA; lmarochnik@gmail.com

Abstract: In the almost empty universe (with almost no matter in it), stochastic gravitational waves (SGW) of finite amplitude produce a de Sitter regime as a solution, which is invariant with respect to the Wick rotation. Asymptotically, super horizon SGWs do not "feel" difference between Lorentzian and Euclidean spacetime and belong simultaneously to both of them. The universe is finishing its evolution in Euclidean spacetime, i.e., it disappears into nothing. Quantum fluctuations of the gravitational field (gravitons) produce a de Sitter regime again in Euclidean spacetime where the current universe finished its existence, and due to the invariance of the de Sitter regime with respect to Wick rotation, the next universe starts its life with de Sitter inflation in Lorentzian spacetime. Such a scenario assumes that a permanent process of birth, death and rebirth of an infinite sequence of universes takes place on an infinite time axis.

Keywords: universes; birth; death; rebirth; permanently

1. Introduction

The contribution of visible and invisible matter to the dynamics of the modern universe is approximately 30% [1,2], i.e., very small. In the process of expansion of the universe, the energy density of matter in it decreases as $a(t)^{-3}$, where $a(t)$ is a scale factor. Therefore, when the universe expands, for example, three times, then the energy density of matter in it will be 1%, i.e., the influence of matter on the dynamics of the universe can be neglected. The universe will turn out to be composed of gravitational waves of finite amplitude, which will have to determine its asymptotic behavior. Of course, some contribution to the overall energy balance will be made by the already-discovered gravitational waves caused by the collisions of black holes and other similar processes [3–6]. However, these facts only confirm the reality of the existence of gravitational waves, which were hypothetical until confirmed by the LIGO and Virgo detectors. When we are dealing with small amplitude fluctuations of a metric tensor, as Lifshitz once showed [7], they can be divided into three types: scalar, vector and tensor (gravitational waves). Such a separation is not possible for fluctuations of finite amplitude due to the non-linearity of the Einstein equations. However, in the absence of matter, any fluctuations of finite amplitude are gravitational waves of finite amplitude, and since only they remain asymptotically, they must also determine the evolution of the Universe. The present work is devoted precisely to this issue.

The exact equations for the stochastic gravitational waves (SGW) in spacetime with fluctuations of the metrical tensor g_{ik} of arbitrary form were obtained by the late Grigory Vereshkov [[8], Appendix A.1.][1]. The general theory of such waves with arbitrary tensor g_{ik} covers three full journal pages. Thus, we are unable to reproduce them here for the reader's convenience. However, in Appendix A.2., the exact equations for SGWs in the FLRW metric are also presented. For the convenience of the reader, we present these equations again in Section 2. In Section 3, we present the exact solution to the set of these equations. The discussion of the obtained solution is the content of Section 4. In this work we assumed from the very beginning that the universe is empty in the sense that it is already at that stage of expansion when the contribution of matter to the overall dynamics is already negligibly small.

Citation: Marochnik, L. Nothing into Something and Vice Versa: A Cosmological Scenario. *Universe* **2023**, *9*, 445. https://doi.org/10.3390/universe9100445

Academic Editor: Kazuharu Bamba

Received: 29 August 2023
Revised: 24 September 2023
Accepted: 27 September 2023
Published: 9 October 2023

2. Grigori Vereshkov' s Equations for Stochastic Gravitational Waves of Finite Amplitude

The general approach to the problem is described in Appendix A.1. of [8]. The situation was considered when the geometric characteristics of spacetime metric $\hat{g}_{ik}$, the connection $\hat{\Gamma}^l_{ik}$ and the curvature $\hat{R}_{ik}$ are fluctuating functions for some physical reasons. It is assumed, however, that there exist regularly determined components of these functions g_{ik}, ${\Gamma^l}_{ik}$ and R_{ik}. It was assumed also that the standard relations of Riemannian geometry are satisfied. Extracting the background geometry from the fluctuating geometry is a non-trivial problem because of the nonlinearity of Einstein's equations. However, the functional integration method allows us to do that. Also, it is considered that the mean value of the random function $< {\psi_i}^k >$ in the statistical ensemble is zero, i.e., $< {\psi_i}^k >= 0$ by definition. Referring the reader for physical and mathematical details to Section A.1. of Appendix A of work [8], I will proceed to Section A.2. "Stochastic Nonlinear Gravitational Waves over the FLRW Background", where the equations of stochastic gravitational waves in the FLRW metric are written explicitly.

We take the FLRW metric of the flat universe as the background metric. In this case, we have

$$\begin{aligned} &ds^2 = dt^2 - a(t)^2\gamma_{\alpha\beta}dx^\alpha dx^\beta = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2) \\ &{R_0}^0 = -3\tfrac{\ddot{a}}{a} {R_\alpha}^\beta = -{\delta_\alpha}^\beta\left(\tfrac{\ddot{a}}{a} + 2\tfrac{\dot{a}^2}{a^2}\right) \end{aligned} \tag{1}$$

In the synchronous gauge we have

$$ {\psi_0}^0 = 0 \quad {\psi_0}^\alpha = 0 \tag{2}$$

Einstein's equations in the explicit form read (before averaging, [4], Appendix A)

$$\sqrt{\frac{\hat{g}}{g}}\hat{g}^{0l}\hat{R}_{0l} \equiv -3\frac{\ddot{a}}{a} - \frac{1}{4}\left\{\ddot{\psi} - \frac{\dot{a}}{a}\dot{\psi} - \frac{1}{a^2}\left({X_\mu}^\nu\psi^{,\mu}\right)_{,\nu}\right\} - \frac{1}{4}{\dot{\psi}_\mu}{}^\nu{\dot{\psi}_\nu}{}^\mu + \frac{1}{8}\dot{\psi}^2 = 0 \tag{3}$$

$$\begin{aligned} \sqrt{\tfrac{\hat{g}}{g}}\hat{g}^{\beta l}\hat{R}_{\alpha l} \equiv \quad & -{\delta_\alpha}^\beta\left(\tfrac{\ddot{a}}{a} + 2\tfrac{\dot{a}^2}{a^2}\right) + \tfrac{1}{2}\Big\{-\tfrac{1}{2}{\delta_\alpha}^\beta\left[\ddot{\psi} + 3\tfrac{\dot{a}}{a}\dot{\psi} - \tfrac{1}{a^2}\left({X_\mu}^\nu\psi^{,\mu}\right)_{,\nu}\right] \\ & -\tfrac{1}{a^2}\left({X_\mu}^\nu{\dot{\psi}_\alpha}{}^{\beta,\mu} - {X_\mu}^\beta{\psi_\alpha}^{\nu,\mu} - {X_\mu}^\nu{\psi_\alpha}^{\mu,\beta}\right)_{,\nu}\Big\} \\ & +\tfrac{1}{4a^2}\left[{X_\lambda}^\beta{\psi_{\mu,\alpha}}^\nu{\psi_\nu}^{\mu,\lambda} - \tfrac{1}{2}{X_\lambda}^\beta\psi_{,\alpha}\psi^{,\lambda} - 2{X_\alpha}^\nu{\psi_{\nu,\mu}}^\lambda{\psi^{\mu,\beta}}_\lambda\right] \end{aligned} \tag{4}$$

$$\sqrt{\frac{\hat{g}}{g}}\hat{g}^{0l}\hat{R}_{\alpha l} \equiv \frac{1}{2}\left(-{\dot{\psi}_{\alpha,\mu}}{}^\mu + \frac{\dot{a}}{a}\psi_{,\alpha}\right) - \frac{1}{4}\left({\psi_{\nu,\alpha}}^\mu{\dot{\psi}_\mu}{}^\nu\right) - \frac{1}{2}\psi_{,\alpha}\dot{\psi} - 2X_\alpha \tag{5}$$

$$X^k_l \equiv (\exp\psi)^k_l = \delta^k_l + \hat{\psi}^k_l + \frac{1}{2!}\hat{\psi}^m_l\hat{\psi}^k_m + \frac{1}{3!}\hat{\psi}^m_l\hat{\psi}^n_m\hat{\psi}^k_n + \ldots \tag{6}$$

In this approach, the exponential parameterization Equation (6) has been used, which automatically provides conservation of the energy–momentum tensor (EMT) of gravitational waves [9][2]. In these equations, dots are derivatives over physical time t. All operations with the spatial indexes are conducted with Euclidean metric. Averaging of Equations (3) and (4)[3] leads to the fact that all terms in the curly brackets are zeroed (because the mean of the random variable ψ is zero) and we get

$$-3\frac{\ddot{a}}{a} = \frac{1}{4} < {\dot{\psi}_\mu}{}^\nu{\dot{\psi}_\nu}{}^\mu - \frac{1}{2}\dot{\psi}^2 > \tag{7}$$

$$3\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2} = \frac{1}{4a^2} < {X_\lambda}^\alpha{\psi_{\mu,\alpha}}^\nu{\psi_\nu}^{\mu,\lambda} - \frac{1}{2}{X_\lambda}^\alpha\psi_{,\alpha}\psi^{,\lambda} - 2{X_\alpha}^\nu{\psi_{\nu,\mu}}^\lambda{\psi_\lambda}^{\mu,\alpha} > \tag{8}$$

We can define the energy density and pressure of nonlinear gravitational wave medium as follows

$$3\frac{\dot{a}^2}{a^2} = \kappa\rho_{GW} \tag{9}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho_{GW} + 3p_{GW}) \tag{10}$$

where $\kappa = 8\pi G$, speed of light $c = 1$ and G is the gravitational constant. Thus, the energy density of such a nonlinear gravitational wave medium reads

$$\kappa\rho_{GW} = \frac{1}{8} < \dot{\psi}_\mu{}^\nu \dot{\psi}_\nu{}^\mu - \frac{1}{2}\dot{\psi}^2 + \frac{1}{a^2}\left(X_\mu{}^\nu \psi_\lambda{}^{\sigma,\mu}\psi_{\sigma,\nu}{}^\lambda - 2X_\mu{}^\nu \psi_\lambda{}^{\sigma,\mu}\psi_{\nu,\sigma}{}^\lambda - \frac{1}{2}X_\mu{}^\nu \psi_{,\nu}\psi^{,\mu}\right) > \tag{11}$$

The pressure of such a nonlinear gravitational wave medium can be found from Equations (7), (10) and (11). Finally, we turn to the equations for gravitational waves. They need to be divided into equations of constraints and equations of proper dynamics. The constraint equations can be obtained from (3)–(8). The equations of proper dynamics follow from (4). They read

$$\begin{aligned} &\ddot{\psi}_\alpha{}^\beta + 3\tfrac{\dot{a}}{a}\dot{\psi}_\alpha{}^\beta - \tfrac{1}{a^2}\left(X_\nu{}^\mu \psi_\alpha{}^{\beta,\mu} - X_\mu{}^\beta \psi_\alpha{}^{\nu,\mu} - X_\mu{}^\nu \psi_\alpha{}^{\mu,\beta}\right)_{,\nu} \\ &-\tfrac{1}{2}\delta_\alpha{}^\beta\left[\ddot{\psi} + 3\tfrac{\dot{a}}{a}\dot{\psi} - \tfrac{1}{a^2}(X_\mu{}^\nu \psi^{,\mu})_{,\nu}\right] = \\ &+\tfrac{1}{2a^2}\left[X_\lambda{}^\beta \psi^\nu{}_{\mu,\alpha}\psi_\nu{}^{\mu,\lambda} - \tfrac{1}{2}X_\lambda{}^\beta \psi_{,\alpha}\psi^{,\lambda} - 2X_\alpha{}^\nu \psi^\lambda{}_{\nu,\mu}\psi_\lambda{}^{\mu,\beta}\right] - \\ &\tfrac{1}{2a^2} < \left[X_\lambda{}^\beta \psi^\nu{}_{\mu,\alpha}\psi_\nu{}^{\mu,\lambda} - \tfrac{1}{2}X_\lambda{}^\beta \psi_{,\alpha}\psi^{,\lambda} - 2X_\alpha{}^\nu \psi^\lambda{}_{\nu,\mu}\psi_\lambda{}^{\mu,\beta}\right] > \end{aligned} \tag{12}$$

Thus, Equations (7)–(12) describe the backreaction of non-linear SGWs of the arbitrary amplitude $\psi_\alpha{}^\beta$ on the background $a(t), p(t), \rho(t)$, which after averaging depends on t only. Equations (8), (11) and (12) can be rewritten in the following convenient form

$$3\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2} = \frac{1}{4} < EMT > \tag{13}$$

$$\kappa\rho_{GW} = \frac{1}{8} < \dot{\psi}_\mu{}^\nu \dot{\psi}_\nu{}^\mu - \frac{1}{2}\dot{\psi}^2 + EMT\Big) > \tag{14}$$

$$\kappa p_{GW} = \frac{1}{8} < \dot{\psi}_\mu{}^\nu \dot{\psi}_\nu{}^\mu - \frac{1}{2}\dot{\psi}^2 - \frac{1}{3}EMT > \tag{15}$$

$$\begin{aligned} &\ddot{\psi}_\alpha{}^\beta + 3\tfrac{\dot{a}}{a}\dot{\psi}_\alpha{}^\beta - \tfrac{1}{a^2}\left(X_\nu{}^\mu \psi_\alpha{}^{\beta,\mu} - X_\mu{}^\beta \psi_\alpha{}^{\nu,\mu} - X_\mu{}^\nu \psi_\alpha{}^{\mu,\beta}\right)_{,\nu} \\ &-\tfrac{1}{2}\delta_\alpha{}^\beta\left[\ddot{\psi} + 3\tfrac{\dot{a}}{a}\dot{\psi} - \tfrac{1}{a^2}(X_\mu{}^\nu \psi^{,\mu})_{,\nu}\right] = \tfrac{1}{2}(EMT - < EMT >) \end{aligned} \tag{16}$$

where the energy–momentum tensor (EMT) is

$$EMT = \frac{1}{a^2}\left[X_\lambda{}^\beta \psi^\nu{}_{\mu,\alpha}\psi_\nu{}^{\mu,\lambda} - \frac{1}{2}X_\lambda{}^\beta \psi_{,\alpha}\psi^{,\lambda} - 2X_\alpha{}^\nu \psi^\lambda{}_{\nu,\mu}\psi_\lambda{}^{\mu,\beta}\right] \tag{17}$$

3. Solution for (13)–(16) Is the de Sitter Regime

In this section, we show that the de Sitter regime is a solution to Equations (13)–(16). One can check this statement by substitution of $a(t) = exp(Ht)$ into the LHSs of (13)–(16). From the LHS of (13) one can find the average value of EMT, which is

$$< EMT >= 36H^2 \tag{18}$$

After that, we get from the LHSs of (14) and (15)

$$\kappa\rho_{GW} = 3H^2 \quad \kappa p_{GW} = -3H^2 \tag{19}$$

Thus, (19) confirms our hypothesis that SGWs of finite amplitude produce de Sitter expansion (20)

$$a(t) = exp(Ht) \tag{20}$$

To figure out the behavior of SGW themselves, we have to solve (12) over the background (20), which looks impossible even numerically. As a result of averaging, all background functions $a(t), \rho(t), p(t)$ are functions of time only; meanwhile, the SGW tensor $\psi_\alpha{}^\beta$ is a function of coordinates and time, which can be found using some approximations. In paper [8], it was shown that gravitational waves of small amplitude are unable to produce a de Sitter expansion; meanwhile, Equations (19) and (20) tell us that SGW of arbitrary amplitude can do that. There is no contradiction here. The gravitational waves of small amplitude are unable to build a de Sitter expansion if their wavelengths are sub horizon (shorter than the distance to the horizon) because formally speaking integrals (6) and (7) in work [8] are divergent. Meanwhile, super horizon wavelengths can produce a de Sitter regime because of changing the limits of integrations from [0, ∞} for sub horizon waves to interval [0, 1] for super horizon waves in Equations (6) and (7) of work [8]. Thus, one can think that the solutions to Equations (19) and (20) describe waves of finite amplitude with the super horizon wavelengths. This fact testifies in favor of super horizon wavelengths as solutions to Equations (19) and (20).

4. Invariance of de Sitter Regime with Respect to Wick Rotation

In a Euclidean spacetime interval between events, (1) reads

$$ds^2 = d\tau^2 + a\left(\tau\right)^2\gamma_{\alpha\beta}dx^\alpha dx^\beta = d\tau^2 + a\left(\tau\right)^2\left(dx^2 + dy^2 + dz^2\right) \tag{21}$$

where $t = i\tau$. Recall that $H = \frac{1}{a}\frac{da}{dt}$ by definition. Thus, the Hubble constant in Euclidean spacetime τ $H_E = \frac{1}{a}\frac{da}{d\tau}$ is

$$H_E = iH \tag{22}$$

Thus, the de Sitter regime is invariant with respect to the transfer from Lorentzian spacetime to Euclidean spacetime and vice versa, which follows from (24)

$$exp(Ht) = exp(H_E \cdot \tau) \tag{23}$$

5. What Kind of SGWs are Capable to Provide the Invariance of the de Sitter Regime with Respect to Lorentzian and Euclidean Spacetime?

To answer this question, we have to go back to (16) and (17). Note that (17) does not contain time derivatives and only the two first terms contain the time derivatives in (16) for $\alpha \neq \beta$. This means that such invariance can be provided by super horizon wavelengths $\lambda >> ct$. In such a case, spatial derivatives can be neglected compared to time derivatives, and (16) takes the following form ($\alpha \neq \beta$)

$$\ddot{\psi}_\alpha{}^\beta + 3\frac{\dot{a}}{a}\dot{\psi}_\alpha{}^\beta = 0 \tag{24}$$

Obviously, (24) is invariant with respect to change of variables $t = i\tau$. This fact means that such waves belong to both Lorentzian and Euclidean spacetimes simultaneously. This means, in turn, there is no topologically impenetrable barrier between Lorentzian and Euclidean spacetimes for such waves and the de Sitter regime. Note also that such waves provide an overwhelming input to the effect because of the infinite length of the interval which they occupied $ct < \lambda \leq \infty$.

One more argument in favor of the idea of this work follows from the consideration of the back reaction of SGWs of small amplitude but of arbitrary wavelengths on the background metrics. Such waves are described by Equations (A26)–(A31) of Appendix A.3 of work [8]. As it is shown in work [8], Section 1.2, they are unable to form a de Sitter regime in Lorentzian spacetime because of divergency of integrals. However, they can easily do so in Euclidean spacetime ([8], Section 1.2)[4]. Thus, the invariance of the de Sitter regime with respect to the transfer from Lorentzian to Euclidean spacetime and vice versa, the absence of a topologically impenetrable barrier between Lorentzian and Euclidean spacetimes for super horizon wavelengths and the ability of waves of arbitrary wavelengths but small amplitude to form a de Sitter regime only in Euclidean spacetime allows us to suppose that the final stage of the evolution of the universe is "tunneling" (using instanton terminology) [12] from our Lorentzian spacetime to Euclidean spacetime. In other words, our Lorentzian universe is going to disappear into "Nothing"[5]. In the frame of such a scenario, the modern de Sitter expansion of the universe (dark energy effect) looks like the last stage of evolution of our universe before its "dives" into nothing.

6. Cosmological Scenario

One can suppose that a new universe should start from quantum fluctuations in Euclidean spacetime, i.e., from the place where the previous universe had completed its existence. In one-loop approximation, the equation of state of gravitons in the Euclidean spacetime reads [13]

$$\rho_g = \frac{3\hbar N_g \cdot H^4}{8\pi^2} = -p_g \tag{25}$$

where $\hbar$ is Planck constant, N_g is number of gravitons in the universe and ρ_g and p_g are the energy density and pressure of gravitons, respectively. Note the important fact that (25) is invariant with respect to Wick rotation $t = i\tau$ because (25) contains H^4 (recalling that $i^4 = 1$). Thus, this de Sitter expansion is invariant with respect to the transfer from Euclidean to Lorentzian spacetime and vice versa. As it follows from (25), in one loop approximation, gravitons produce de Sitter expansion in Lorentzian space-time. Thus, we can consider (25) as an initial condition for the formation of a new universe that starts with a de Sitter expansion, which eventually should lead to a new Big Bang in a new universe. This process probably should start near the Planck time $t_p \approx 5.4 \cdot 10^{-44}$ s. This number should be close to the initial conditions used by inflation theories [14–18] for the present universe. The modern inflation theories applicable to the existing universe usually start approximately at $10^8 t_p$ and last about $\left(10^{11} - 10^{12}\right)t_p$. However, we need to remember the words of Steven Weinberg [19]: "So far, the details of inflation are unknown, and the whole idea of inflation remains a speculation, though one that is increasingly plausible". The scenario proposed assumes that a permanent process of birth, death and rebirth of an infinite sequence of universes takes place on an infinite time axis.

7. Conclusions

I am deeply grateful to Zeev Dashevsky, who drew my attention to a book written 100 years ago by Alexander Friedmann [20], the man who predicted the expansion of the universe [21] a few years before its discovery by Edwin Hubble, making him the founding father of modern cosmology. In his book, Friedmann quotes the famous phrase from the book "Ecclesiastes": "What was, will be, and what has been done is what will be done- and there is nothing new under the Sun," meaning that even with all the progress in our ideas about the world, our approach fundamentally remains the same as it was millennia ago for Aristotle.

Funding: This research received no external funding.

Data Availability Statement: Data sharing is not applicable.

Acknowledgments: I am deeply grateful to John Payne for stimulating discussion that allowed me to state the content of this work more clearly.

Conflicts of Interest: The author declares no conflict of interest.

Notes

1 The material constituting the content of the Appendix A of work [8] was prepared by the late Grigory Vereshkov. It was to be included in our joint paper [9]. We did not include it in the final version of [9], since [9] was entirely devoted to quantum effects. After Grigory passed away, I decided to publish his work, including it as Appendix A to my paper [8] with a reference to its author.

2 There are other types of parameterizations as well, e.g., Fierz–Pauli parameterization [10] and a linear parametrization used in most of the works known to us (see [9] and references therein). The linear parameterization does not provide conservation of EMT, which can be conducted "by hand" [11].

3 In this paper, averaging is conducted over a three-space (see, e.g., [11]).

4 Note that in Section 1.2 of work [8] the incorrect normalization was used. In the right hand sides (RHS) of Formulas (16)–(20), (23) and (25) of [8], the Hubble constant H must be changed for 1. This correction does not change the main result of this section, which proves that the back reaction of SGWs of small amplitude can produce a de Sitter expansion in Euclidean spacetime only.

5 To the best of my knowledge, the first time such a scenario was proposed in paper [12], although the problem of tunneling gravitational instantons from Euclidean spacetime into Lorentzian spacetime of the universe has a large literature (see, e.g., [12] and references therein].

References

1. Bennett, C.L.; Larson, D.; Weiland, J.L.; Jarosik, N.; Hinshaw, G.; Odegard, N.; Smith, K.M.; Hill, R.S.; Gold, B.; Halpern, M.; et al. Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Final maps and results. *Astrophys J.* **2012**, *208*, 20. [CrossRef]
2. Abbott, T.M.; Aguena, M.; Alarcon, A.; Allam, S.; Alves, O.; Amon, A.; Andrade-Oliveira, F.; Annis, J.; Avila, S.; Bacon, D.; et al. Dark Energy Survey Year 3 results: Cosmological constraints from galaxy clustering and weak lensing. *Phys. Rev. D* **2022**, *105*, 023520. [CrossRef]
3. Demorest, P.B.; Ferdman, R.D.; Gonzalez, M.E.; Nice, D.; Ransom, S.; Stairs, I.H.; Arzoumanian, Z.; Brazier, A.; Burke-Spolaor, S.; Chamberlin, S.J.; et al. Limits on the stochastic gravitational wave background from the North American nanohertz observatory for gravitational waves. *Astrophys. J.* **2013**, *762*, 94. [CrossRef]
4. Nakamal, T.; Silk, J.; Kamionkowski, M. Stochastic gravitational waves associated with the formation of primordial black holes. *Phys. Rev. D* **2017**, *95*, 043511. [CrossRef]
5. Christensen, N. Stochastic gravitational wave backgrounds. *Rep. Prog. Phys.* **2019**, *82*, 016903. [CrossRef] [PubMed]
6. Reardon, D.J.; Zic, A.; Shannon, R.M.; Hobbs, G.B.; Bailes, M.; Di Marco, V.; Kapur, A.; Rogers, A.F.; Thrane, E.; Askew, J.; et al. Search for an isotropic gravitational-wave background with the Parkes Pulsar Timing Array. *Astrophys. J. Lett.* **2023**, *951*, L6. [CrossRef]
7. Lifshitz, E.M. On the gravitational stability of the expanding universe. *JETP* **1946**, *16*, 587.
8. Marochnik, L. Dark Energy and Inflation from Gravitational Waves. *Universe* **2017**, *3*, 72. [CrossRef]
9. Vereshkov, G.; Marochnik, L. Quantum gravity in Heisenberg representation and self-consistent theory of gravitons in macroscopic spacetime. *J. Mod. Phys.* **2013**, *4*, 2. [CrossRef]
10. Fierz, M.; Pauli, W.E. On relativistic wave equations for particles of arbitrary spin in an electromagnetic field. *Proc. R. Soc. Lond.* **1939**, *173*, 211–232.
11. Abramo, L.R.; Brandenberger, R.H.; Mukhanov, V.F. Energy-momentum tensor for cosmological perturbations. *Phys. Rev. D* **1997**, *56*, 3248. [CrossRef]
12. Marochnik, L. Dark energy from instantons. *Gravit. Cosmol.* **2013**, *19*, 178–187. [CrossRef]
13. Marochnik, L.; Usikov, D.; Vereshkov, G. Cosmological acceleration from virtual gravitons. *Found. Phys.* **2008**, *38*, 546–555. [CrossRef]
14. Starobinsky, A.A. A new type of isotropic cosmological models without singularity. *Phys. Lett. B* **1980**, *91*, 99–102. [CrossRef]
15. Guth, A.H. Inflationary universe: A possible solution to the horizon and flatness problems. *Phys. Rev. D* **1981**, *23*, 347. [CrossRef]
16. Linde, A.D. A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. *Phys. Lett. B* **1982**, *108*, 389. [CrossRef]
17. Mukhanov, V.; Chibisov, G. Vacuum energy and large-scale structure of the Universe. *Sov. Phys. JETP* **1982**, *56*, 258–265.
18. Albrecht, A.; Steinhardt, P.J. Cosmology for grand unified theories with radiatively induced symmetry breaking. *Phys. Rev. Lett.* **1982**, *48*, 1220. [CrossRef]
19. Weinberg, S. *Cosmology*; Oxford University Press: Oxford, UK, 2008.

20. Friedmann, A. *The World as Space and Time*, 1st ed.; Мир как Пространство и Время (English translation: The World as Space and Time); First Russian Edition, 1923; Second Russian Edition, 1965; Minkowski Institute Press: Montreal, QC, Canada, 2014; p. 62.
21. Friedmann, A. Über die krümmung des raumes. *Z. Für Physik.* **1922**, *10*, 377–386. [CrossRef]