Surface electrical charge distribution in white dwarfs

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We investigate the influence of the surface electrical charge in the static equilibrium configuration of white dwarfs, this is possible by solving numerically the hydrostatic equilibrium equation for the charged case. We consider that the fluid in the star is described by a fully degenerate electron gas and that the electric charge is distributed close to the surface of the white dwarf. We found that super-Chandrasekhar mass white dwarfs are found for a large surface electrical charge.

Keywords: White dwarfs; stellar structure.

1. Introduction

1.1. *Super-Chandrasekhar white dwarfs*

In the last few years have been disclosed observations concerning the existence of a particular event in the universe, the super-luminous type Ia supernovae (SNIa)[1–4]. These events are very interesting because their explosion are the more foreseeables and frequently the brightest incidents in the sky. Some authors argue that the possible progenitor of the event SNIa is the super-Chandrasekhar white dwarf, star which ultrapass the standard Chandrasekhar mass limit, $1.44\,M_\odot$ [5,6]. It is estimated that the super-Chandrasekhar white dwarfs' masses are in the range $2.1-2.8\,M_\odot$ [7,8].

Several authors hint models to explain super-Chandrasekhar white dwarfs. In literature, for example, we found white dwarf models where are considered a strong magnetic field[9,10], in rotation and with different topologies for magnetic field[11–14] and with a electric charge distribution[15].

It is found that white dwarfs with an uniform and very strong magnetic field can attain masses of $\sim 2.9\,M_\odot$. Although these objects exceeds significantly the Chandrasekhar limit, they suffer from severe stability[16–20]. The effect of the rotation and different topology for the magnetic field help to reach white dwarfs' masses around $5M_\odot$ [11–14]. A similar effect to the ones aforementioned are produced by the electric charge, which produces a force which helps to the one generated by the fluid pressure to counteract more mass and thus avoid gravitational collapse[15].

1.2. *This work*

Such as is realized in Ref. 15, in this article we also studied the influence of the electric charge in the structure of white dwarfs. The main difference between these two works is the profile of the electric charge. Instead of consider the electric charge density proportional to the energy density, $\rho_e = \alpha\rho$, with α being a dimensionless constant, such as is considered in Ref. 15, here we consider that the electric charge is distributed at the star's surface[21] of the form:

$$\rho_e = k\, exp\left[-\frac{(r-R)^2}{b^2}\right], \tag{1}$$

where r and R represent the radial coordinate and the total radius of the uncharged star. Moreover, b represents the width of the electric charge distribution, here we consider that $b = 10\,[\text{km}]$. The charge profile (1) is taking into account since electrons and ions at white dwarfs could important producing surface's strong electric fields.

With the aim of found k, we use the equality:

$$\sigma = \int_0^\infty 4\pi r^2 \rho_e dr, \tag{2}$$

where σ bears the magnitude directly proportional to the electric charge distribution. Considering Eq. (1) in Eq. (2), it is found:

$$k = \frac{\sigma}{4\pi}\left(\frac{\sqrt{\pi}\, b\, R^2}{2} + \frac{\sqrt{\pi}\, b^3}{4}\right)^{-1}. \tag{3}$$

As can be seen, Eq. (3) connects k and σ.

For the fluid, we consider that the fluid pressure and fluid energy density within the object are given by the relations:

$$p(k_F) = \frac{1}{3\pi^2\hbar^3}\int_0^{k_F}\frac{k^4}{\sqrt{k^2+m_e^2}}dk, \tag{4}$$

$$\rho(k_F) = \frac{1}{\pi^2\hbar^3}\int_0^{k_F}\sqrt{k^2+m_e^2}k^2 dk + \frac{m_N\mu_e}{3\pi^2\hbar^3}k_F^3, \tag{5}$$

with m_e, m_N, $\hbar$, μ_e and k_F being respectively the electron mass, the nucleon mass, the reduced Planck constant, the ratio between the nucleon number and atomic number for ions and the Fermi momentum of the electron[22]. It is important to say that we consider $\mu_e = 2$.

2. Stellar structure equations

We shall study the stellar equilibrium configuration of charged white dwarfs using the profile of the electric charge and the equation of state showed in the previous section. We consider that within the star the charged fluid is described by the energy-momentum tensor (EMT):

$$T_{\mu\nu} = (\rho+p)u_\mu u_\nu + p g_{\mu\nu} + \frac{1}{4\pi}\left(F^{\mu\gamma}F_{\varphi\gamma} - \frac{1}{4}g_{\mu\nu}F_{\gamma\beta}F^{\gamma\beta}\right), \tag{6}$$

where ρ and p represent respectively the energy density and fluid pressure. In addition, u_μ beings the fluid's four velocity, $g_{\mu\nu}$ stands the metric tensor and $F^{\mu\gamma}$ represents the Faraday-Maxwell tensor.

The interior space-time of the charged star is described, in Schwarzschild coordinates, by the following line element:

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{7}$$

as indicated, the functions $\nu(r)$ and $\lambda(r)$ dependent on the radial coordinate r only.

For the line element (7), the nonzero components of the Maxwell-Einstein equation are given by the equalities:

$$q'(r) = 4\pi\rho_e(r)r^2e^{\lambda(r)/2}, \tag{8}$$

$$m'(r) = 4\pi r^2\rho(r) + \frac{q(r)q'(r)}{r}, \tag{9}$$

$$p'(r) = -(p(r)+\rho(r))\left(4\pi rp(r) + \frac{m(r)}{r^2} - \frac{q^2(r)}{r^3}\right)e^{\lambda(r)} + \frac{q(r)q'(r)}{4\pi r^4}, \tag{10}$$

with the potential metric of the form:

$$e^{-\lambda(r)} = 1 - \frac{2m(r)}{r} + \frac{q^2(r)}{r^2}, \tag{11}$$

primes indicate the derivations with respect to r. The functions q and m represent the electric charge and the mass inside the sphere of radial coordinate r. Moreover, Eq. (10) represents the hydrostatic equilibrium equation, known also as the Tolman-Oppenheimer-Volkoff equation[23,24]. This equation is modified from its original version to include the electrical part[25].

3. Results and conclusions

The static equilibrium configurations of charged white dwarfs are presented in Fig. 1. On the left and on the right hand side are shown the behavior of total mass $M/M_\odot$

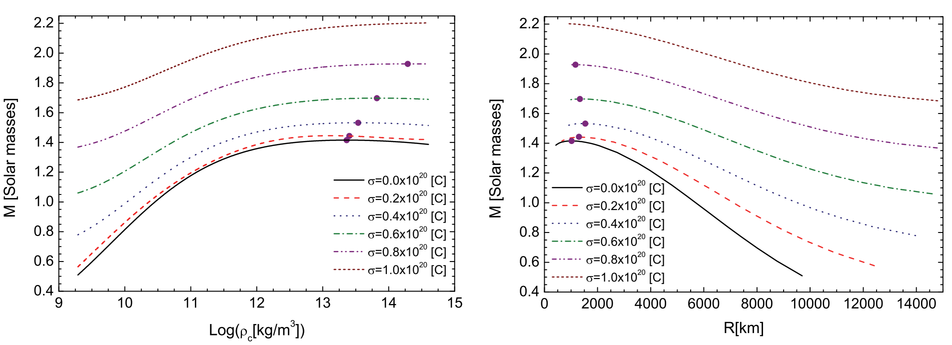

Fig. 1. The mass as a function of the central energy density and against the radius are plotted on the left and right hand side, respectively. The full circles represent the maximum mass points. In both figures are used different values of σ.

versus with central energy density ρ_c and with the radius R, respectively, for different values of σ. For σ lower than 0.8×10^{20} [C], it can be observed that $M/M_{\odot}$ grows with ρ_c until to attain $M_{\max}/M_{\odot}$ (marked with a full circles in purple), after that point, $M/M_{\odot}$ begins to decreases with the increment of ρ_c. At the same time, it is important to say that for $\sigma = 1.0 \times 10^{20}$ [C], $M/M_{\odot}$ increases monotonically with ρ_c, in this way, not maximum mass point is found. In addition, we highlight that the maximum mass found in this model is $2.199 M_{\odot}$, which is obtained using $\sigma = 1.0 \times 10^{20}$[C]. This total mass in the interval masses estimated for the super-Chandrasekhar white dwarfs, 2.1-2.8$M_{\odot}$, check Refs. 8, 7.

In Fig. 1 we can also note that in the uncharged case we obtain a maximum mass is found in a central energy density ρ_c^*, in turn, in the charged cases this points are attained for $\rho_c > \rho_c^*$. Additionally, such as happen for the total mass, the radius change with the total charge.

From the results aforementioned, we can conclude that the electric charge affects notably the static equilibrium configuration of white dwarfs. For a larger σ, white dwarfs with larger masses are found. This can be understand since σ is directly related with the total charge in the star. An increment of the electric charge produce an increment of a force which helps to the fluid pressure to support more mass, thus avoiding the gravitational collapse. More details are presented in Ref. 26.

Acknowledgments

Authors thanks to Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-CAPES and Fundação de Amparo à Pesquisa do Estado de São Paulo-FAPESP, thematic project 2013/26258-4, for grants.

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