# LIGHT HADRON SPECTRUM FROM LATTICE QCD

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We present a full ab-initio calculation of the masses of the nucleon and other light hadrons, using lattice calculations. In our 2+1 flavor analysis, pion masses down to 190 MeV are used to extrapolate to the physical point with lattice sizes of approximately four times the inverse pion mass. Three lattice spacings are used for a continuum extrapolation. All systematics are controlled. Our results completely agree with experimental observations.

QCD is asymptotically free, at high energies the interaction gets weaker and weaker[1] enabling perturbative calculations based on a small coupling parameter (e.g. for at least ten a hundred times higher energies than the mass of the proton). Much less is known about t other side: when the coupling gets large, and the physics becomes non-perturbative. The ma generation of hadrons belongs to these non-perturbative phenomena. This presentation is bas on the detailed hadron spectrum study of the Budapest-Marseille-Wuppertal Collaboration[3].

Before we study the question in more detail it is illustrative to summarize the most impo tant qualitative features of the hadron mass generation. In the early universe the temperatu (T) was very high. There was a smooth transition[4] between a high T phase dominated quarks and gluons and a low T phase dominated by hadrons. In the high T phase the hi temperature was manifested by motion. The motion was diluted by the expansion of the ea universe. Nevertheless a small fraction of this motion remained with us confined in protons. a consequence the kinetic energy inside the proton is observed as the mass of the particle.

After this illustration let us discuss the systematic field theoretical approach.

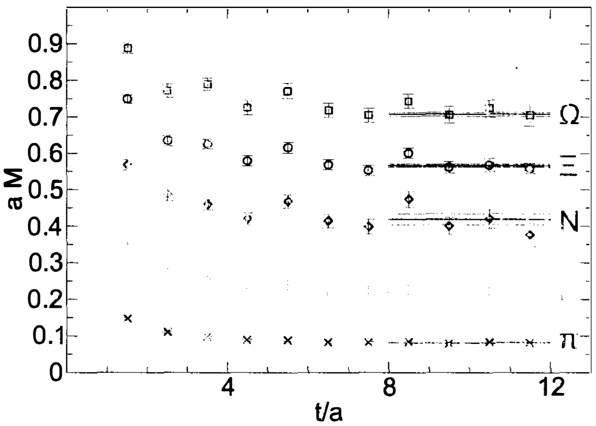
To explore QCD in the non-perturbative regime, the most systematic technique is to discret the QCD Lagrangian on a hypercubic space-time lattice with spacing $a$, to evaluate its Gree functions numerically and to extrapolate the resulting observables to the continuum ($a \to 0$). convenient way to carry out this discretization is to place the fermionic variables on the sites the lattice, whereas the gauge fields are treated as $3 \times 3$ matrices connecting these sites. In t sense, lattice QCD is a classical four-dimensional statistical physics system.

In order to be able to resolve the structure of the proton the lattice has to be fine enoug Typically 0.1 fm is used for that purpose. The lattices have about 50 points in each directio Since at each lattice points we have dozens of variables the numerical treatment of such a syst is quite difficult, mathematically it corresponds to a one billion dimensional integral.

The first step is to generate vacuum configurations. For the classical theory the vacu is just a trivial configuration with vanishing field strengths. In the quantum theory, howev the vacuum is fluctuating around this classical configuration. Typically a few hundred of th configurations are enough to calculate various observables with a few percent accuracy.

For many years calculations were performed using the quenched approximation, which as-
umes that the fermion determinant (obtained after integrating over the quark fields) is inde-
endent of the gauge field. Although this approach omits the most CPU-demanding part of
. full QCD calculation, a thorough determination of the quenched spectrum took almost 20
ears. It was shown[6] that the quenched theory agreed with the experimental spectrum to ap-
roximately 10 percent for typical hadron masses and demonstrated that systematic differences
vere observed between quenched and two flavor QCD beyond that level of precision[6,7].

Including the effects of the light sea-quarks has dramatically improved the agreement between
xperiment and QCD results. Several works appeared in the literature, which included these
ea-quark effects also in the light hadron spectrum. Efforts are being made to calculate (part of)
he QCD spectrum with 2+1 flavor staggered quarks[8], with non-perturbatively $O(a)$-improved
Vilson quarks[9,10] with almost physically light quark masses (albeit in small volumes at one
ingle lattice spacing [11]), with domain wall fermions[12] or domain wall fermions on staggered
onfigurations[13]. There are also two flavor calculations using unimproved, $O(a)$-improved[14] and
wisted-mass Wilson fermions[15] and overlap quarks[16].




igure 1: Effective masses $aM=\log[C(t/a)/C(t/a+1)]$, where $C(t/a)$ is the correlator at time $t$, for $\pi$, $K$, $N$, $\Xi$
ıd $\Omega$ at our lightest simulation point with $M_\pi \approx 190$ MeV ($a \approx 0.085$ fm with physical strage quark mass). The
orizontal lines indicate the masses extracted form the correlators by using single mass correlated cosh/sinh fits.

However, all of these studies have neglected one or more of the ingredients required for a
ıll and controlled calculation. The five most important of those are:

*a.* The inclusion of the $u$, $d$ and $s$ quarks to the fermion determinant with an exact algorithm
ıd with an action, whose universality class is QCD. For the light-hadron spectrum, the effects
the heavier $c$, $b$ and $t$ quarks are included in the coupling constant and light quark masses.

*b.* A complete determination of the masses of the light ground state mesons, octet and
ecuplet baryons. Three of these are used to fix the masses of the isospin averaged light ($m_{ud}$)
ıd strange ($m_s$) quark masses and the overall scale in physical units.

*c.* Large volumes to guarantee small finite-size effects and at least one simulation at a
gnificantly larger volume to confirm the smallness of these effects. In large volumes, finite-
ze corrections to the spectrum are exponentially small[17,18]. As a conservative rule of thumb
$_\pi L \gtrsim 4$, with $M_\pi$ the pion mass and $L$ the lattice size, guarantees that finite-volume errors in
.e spectrum are around or below the percent level. Resonances require special care. Their
ııte volume behavior is more involved [19,20].

*d.* Controlled interpolations and extrapolations of the results to physical $m_{ud}$ and $m_s$ (or
entually simulating directly at these masses). While interpolations in $m_s$, corresponding to
$_K \simeq 495$ MeV, are straightforward, the extrapolations for $m_{ud}$, corresponding to $M_\pi \simeq 135$ MeV,

are difficult. They need CPU-intensive calculations with $M_\pi$ reaching down to 200 MeV or les

*e.* Controlled extrapolations to the continuum limit, requiring that the calculations b performed at no less than three values of the lattice spacing, in order to guarantie that th scaling region is reached.

The analysis presented in this paper includes all five ingredients listed above, thus providin a calculation of the light hadron spectrum with fully controlled systematics.

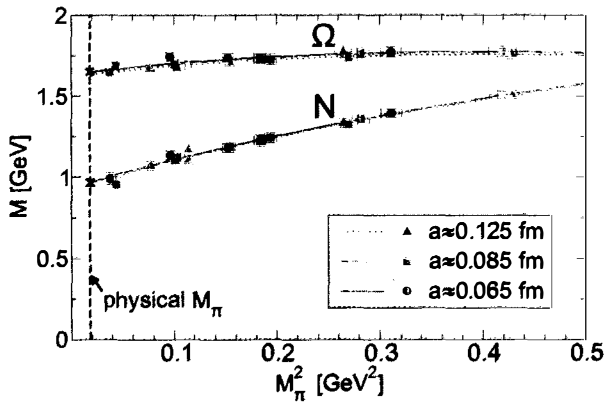



Figure 2: Pion mass dependence of the $N$ and $\Omega$ for all three values of the lattice spacing. The scale in th case is set by $M_\Xi$ at the physical point. Triangles/dotted lines correspond to $a\approx0.125$ fm, squares/dashed lin to $a\approx0.085$ fm and circles/solid lines to $a\approx0.065$ fm. The curves are the corresponding fits. The crosses are t continuum extrapolated values in the physical pion mass limit. The lattice-spacing dependence of the results barely significant statistically despite the factor of 3.7 separating the squares of the largest ($a\approx0.125$ fm) an smallest ($a\approx0.065$ fm) lattice spacings.

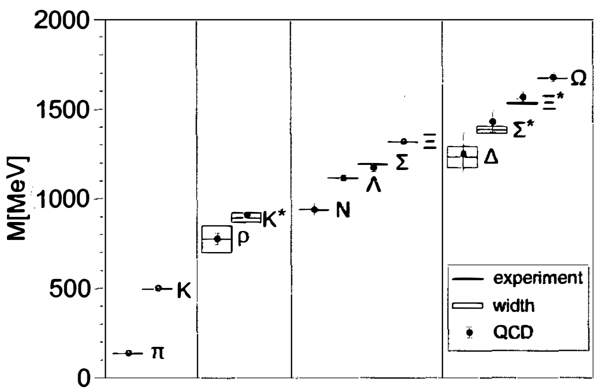
We choose a tree-level, $O(a^2)$-improved Symanzik gauge action[21] and work with tree-leve clover-improved Wilson fermions, coupled to links which have undergone six levels of stout lin averaging[22]. (The precise form of the action is presented in ref.[23].) We perform a series of 2+ flavor simulations, that is we include degenerate $u$ and $d$ sea quarks and an additional $s$ se quark. We fix $m_s$ to its approximate physical value. We vary $m_{ud}$ in a range which extend down to $M_\pi \approx 190$ MeV.

To set the overall physical scale, any dimensionful observable can be used. Since both th $\Omega$ and $\Xi$ are reasonable choices, we carry out two analyses, one with $M_\Omega$ ($\Omega$ set) and one wit $M_\Xi$ ($\Xi$ set). We find that for all three lattice spacings both quantities give consistent results. determine the masses of the baryon octet ($N$, $\Sigma$, $\Lambda$, $\Xi$) and decuplet ($\Delta$, $\Sigma^*$, $\Xi^*$, $\Omega$) and thos members of the light pseudoscalar ($\pi$, $K$) and vector meson ($\rho$, $K^*$) octets which do not requir the calculation of disconnected propagators. Typical effective masses are shown in Figure 1.

Shifts in hadron masses due to the finite size of the lattice are systematic effects. There a two different effects. The first type of volume dependence is related to virtual pion exchang The second type of volume dependence exists only for resonances. The coupling between th resonance state and its decay products leads to a non-trivial level structure in finite volum The literature provides a conceptually satisfactory framework for these effects [19,20]. We too both effects into account.

Our three flavor scaling study[23] showed that hadron masses deviate from their continuu values by less than approximately 1 percent for a lattice spacing upto $a\approx0.125$ fm. Since th statistical errors of the hadron masses calculated in this paper are similar in size, we do n expect significant scaling violations here. This is confirmed by Figure 2.

As indicated, we performed two separate analyses, setting the scale with $M_\Xi$ and $M_\Omega$. Th $\Xi$ set is shown on Figure 3. With both scale setting procedures we find that the masses agr vith the hadron spectrum observed in nature[24].




'igure 3: The light hadron spectrum of QCD. Horizontal lines and bands are the experimental values with their ecay widths. Our results are shown by filled circles. Vertical error bars represent our combined statistical and ystematic error estimates. The $\pi$, $K$ and $\Xi$ have no error bars, since they are used to set the light quark mass, the strange quark mass and the overall scale, respectively.

Thus, our study strongly suggests that QCD is the theory of the strong interaction, also at ›w energies, and furthermore that lattice studies have reached the stage where all systematic rrors can be fully controlled.

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