



Strength of pairing interaction for hyperons in multistrangeness hypernuclei

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ABSTRACT

Pairing correlations play a very important role in atomic nuclei. Although several effective pairing interactions have been used in mean field calculations for nucleons, little is known about effective pairing interactions for hyperons. Based on the quark model, we propose a relationship between effective pairing interactions for hyperons and for nucleons; e.g., for Λ s, the strength of the pairing interaction is $4/9$ of that for nucleons. A separable pairing force of finite range which has been widely applied to describing pairing correlations in normal nuclei is used to investigate pairing effects in multi- Λ Ca, Sn and Pb hypernuclei.

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Since the discovery of the first Λ hypernucleus from cosmic rays [1], the study of hypernuclei has been one of very interesting topics in nuclear physics [2–9]. Most of the observed hypernuclei are of single- Λ . So far there is only one confirmed double- Λ hypernucleus ${}^6_{\Lambda\Lambda}\text{He}$ [10] and two candidates ${}^{13}_{\Lambda\Lambda}\text{B}$ [11] and ${}^{10}_{\Lambda\Lambda}\text{Be}$ [12]. Nevertheless, many theoretical efforts have been devoted to investigating the structure of more double- Λ and even multistrangeness ($-S \geq 3$) hypernuclei [13–20].

Hypernuclei are unique quantum many-body systems for the investigation of hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions which are, in turn, crucial for understanding the hypernuclear structure as well as the hypernuclear matter and properties of neutron stars. With the strangeness degree of freedom, a hyperon can move deep inside the nucleus and serve as an impurity for probing nuclear properties that are not accessible by conventional methods developed for normal nuclei. Lots of many-body techniques for normal nuclei, including various mean-field models, have been extended to hypernuclei [18–39]. In particular, the relativistic mean-field (RMF) models [40–49] which have been very successful in describing normal nuclei in the whole nuclear chart are also used extensively to study hypernuclei.

In the RMF models, one needs effective interactions both for the particle-hole (ph) and particle-particle (pp) channels. For normal nuclei, a large amount of effective interactions for the ph channel have been proposed, see, e.g., Refs. [50–58]. Meanwhile, both zero-range and finite-range pairing forces have been used in the pp channel [59–62]. For hypernuclei, YN and YY interactions in the ph channel can be either obtained by fitting experimental data or estimated with the naive quark model [29,63–68]. However, effective interactions for hyperons in the pp channel are much less known. In this Letter, we propose a way to estimate the strength of pairing interactions for hyperons based on the meson exchange picture and the naive quark model.

From the quark model we know that nucleons consist of three u/d quarks and hyperons consist of, besides u/d quarks, one or more s quarks. Next we use $n_{u/d}$ to label the number of u/d quarks in a baryon (a nucleon or a hyperon) and define g_{BM} to be the coupling constant of a non-strangeness meson M (σ , ω , ρ , ...) to a baryon B (N , Λ , ...). According to the OZI rule only u/d quarks are involved in the coupling of a non-strangeness meson to a baryon at the tree level. Therefore the following relation holds: $g_{YM} = n_{u/d}/3 \cdot g_{NM}$. Similar discussions have been made with the quark-meson coupling model [69]. If g_{NM} is known, one can readily get g_{YM} . For example, since $n_{u/d} = 2$ in Λ s, $g_{\Lambda M} = 2/3 \cdot g_{NM}$; this has been proposed and used in the study of Λ -hypernuclei [63,64,70–72].

The exchange of the meson M between two baryons B_1 and B_2 results in an interaction with the strength proportional to

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$g_{B_1M}g_{B_2M}$ [73]. For single- Λ hypernuclei, the central potential for the Λ in the mean field generated by nucleons is proportional to $g_{NM}g_{\Lambda M}$ while that for nucleons is proportional to g_{NM}^2 , leading to the well known observation that the depth of the potential for the Λ is roughly 2/3 of that for nucleons [74–77].

The strength for the YY interaction is proportional to g_{YM}^2 , i.e., $n_{u/d}^2/9$ ($=g_{YM}^2/g_{NM}^2$) of that for the NN interaction. Since the pairing force is the residual of the two-body BB interaction, the ratio $n_{u/d}^2/9$ holds also between the strength of the pairing interaction for hyperons and that for nucleons. Note that mesons consisting of strange quarks may be exchanged between hyperons and result in possible deviations of this ratio from $n_{u/d}^2/9$. In this work we restrict our discussions in the framework of conventional RMF models with non-strangeness mesons.

As an illustration, we use the relativistic Hartree-Bogoliubov (RHB) theory to study the effects of the $\Lambda\Lambda$ pairing in multi-strangeness hypernuclei. The RHB theory provides a unified description of the relativistic mean field and the pairing correlations via the Bogoliubov transformation. The RHB equation consists of the single particle Hamiltonian h_B and the pairing field Δ [42,78]

$$\int d^3\mathbf{r}' \begin{pmatrix} h_B - \lambda & \Delta \\ -\Delta^* & -h_B + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}, \quad (1)$$

where λ is the Fermi energy and E_k and $(U_k, V_k)^T$ are the quasi-particle energy and wave function, respectively.

For the ph channel, the Dirac Hamiltonian for nucleons has been established [40–49] and that for Λ s reads [72]

$$h_\Lambda(\mathbf{r}) = \boldsymbol{\alpha} \cdot \mathbf{p} + V_\Lambda(\mathbf{r}) + \beta(m_\Lambda + S_\Lambda(\mathbf{r})) + T_\Lambda(\mathbf{r}), \quad (2)$$

where the scalar, vector and tensor potentials are

$$\begin{aligned} S_\Lambda(\mathbf{r}) &= g_{\sigma\Lambda}\sigma(\mathbf{r}), \\ V_\Lambda(\mathbf{r}) &= g_{\omega\Lambda}\omega_0(\mathbf{r}), \\ T_\Lambda(\mathbf{r}) &= -\frac{f_{\omega\Lambda\Lambda}}{2m_\Lambda}\beta(\boldsymbol{\alpha} \cdot \mathbf{p})\omega_0(\mathbf{r}). \end{aligned} \quad (3)$$

The tensor potential $T_\Lambda(\mathbf{r})$ is included to achieve the small spin-orbit splitting for the Λ [79–81]. We adopt two effective interactions NLSH-A [51,64] and PK1-Y1 [54,66] which have been extensively used in the study of Λ hypernuclei.

In the pp channel, the pairing potential reads

$$\begin{aligned} \Delta(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2) \\ = \int d^3\mathbf{r}_1' d^3\mathbf{r}_2' \sum_{\sigma_1'\sigma_2'} V(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2, \mathbf{r}_1'\sigma_1', \mathbf{r}_2'\sigma_2') \kappa(\mathbf{r}_1'\sigma_1', \mathbf{r}_2'\sigma_2'), \end{aligned} \quad (4)$$

where V is the effective pairing interaction and κ is the pairing tensor

$$\kappa(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2) = \sum_{k>0} V_k^*(\mathbf{r}_1\sigma_1) U_k(\mathbf{r}_2\sigma_2). \quad (5)$$

We use the separable pairing force of finite range proposed by Tian et al. [60,82–84]

$$V(\mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2, \mathbf{r}_1'\sigma_1', \mathbf{r}_2'\sigma_2') = -G\delta(\mathbf{R} - \mathbf{R}')P(\mathbf{r})P(\mathbf{r}')\frac{1 - P_\sigma}{2}, \quad (6)$$

where G is the pairing strength and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ are the center of mass and relative coordinates, respectively. $P(\mathbf{r})$ denotes the Gaussian function,

$$P(\mathbf{r}) = (4\pi a^2)^{-3/2} e^{-r^2/4a^2}, \quad (7)$$

where a is the effective range of the pairing force. For nucleons, the pairing strength $G_N = 728 \text{ MeV}\cdot\text{fm}^3$ and the effective range $a = 0.644 \text{ fm}$ have been obtained by fitting the momentum dependence of the pairing gap in the nuclear matter calculated from the Gogny force. According to our proposal discussed before, the pairing strength for Λ s is taken to be $G_\Lambda = 4/9 \cdot G_N$.

We have carried out calculations with the multidimensionally-constrained (MDC) RHB theory [85], one of the recently developed MDC covariant density functional theories (MDC-CDFTs) [49,85–88]. For simplicity, we choose doubly-magic ^{40}Ca , ^{132}Sn and ^{208}Pb as the core nuclei and study even-even-even hypernuclei $^{40-S}_{-\Lambda}\text{Ca}$ ($-S = 0-20$), $^{132-S}_{-\Lambda}\text{Sn}$ ($-S = 0-40$) and $^{208-S}_{-\Lambda}\text{Pb}$ ($-S = 0-70$). All these hypernuclei are spherical and have vanishing neutron and proton pairing gaps according to our MDC-RHB calculations. In Fig. 1, the two-Lambda separation energies $S_{2\Lambda}$ are shown for them. One can find that $S_{2\Lambda}$ decreases monotonically with the number of Λ s increasing. As far as the two-Lambda separation energy is concerned, at least 20, 40 and 70 Λ s can be bound to the core nuclei ^{40}Ca , ^{132}Sn and ^{208}Pb , respectively. There are sudden drops in $S_{2\Lambda}$ when $-S = 2, 8, 20, 34, 40$ and 58 . These numbers are magic or semi-magic numbers for Λ s. Since the spin-orbit splitting is very small in the single Λ spectrum, these numbers actually correspond to shell closures or sub-closures in the single particle level scheme of a harmonic oscillator potential.

The pairing gap is one of the typical quantities to characterize pairing effects. We have calculated the average pairing gap as [89]

$$\Delta_\Lambda = \sum_k \langle u_k v_k \Delta_k \rangle / \sum_k \langle u_k v_k \rangle, \quad (8)$$

where Δ_k is the pairing gap corresponding to a single Λ state k in the canonical basis and u_k^2 and v_k^2 give the empty and occupation probabilities, respectively. Fig. 2 shows the $\Lambda\Lambda$ pairing gaps of $^{40-S}_{-\Lambda}\text{Ca}$, $^{132-S}_{-\Lambda}\text{Sn}$ and $^{208-S}_{-\Lambda}\text{Pb}$. It can be seen that for almost every hypernucleus, the pairing gaps of Λ s obtained from PK1-Y1 and NLSH-A are very similar. One can also find that the $\Lambda\Lambda$ pairing gaps are zero when the strangeness number is 2, 8, 20, 34, 40, 58 and 70, consistent with the conclusion drawn from the two-Lambda separation energies that they are magic or semi-magic numbers for Λ s. There is a clear dependence of Δ_Λ on the mass number of the core nucleus. For $^{40-S}_{-\Lambda}\text{Ca}$, $^{132-S}_{-\Lambda}\text{Sn}$ and $^{208-S}_{-\Lambda}\text{Pb}$, the maximal values of Δ_Λ are a bit smaller than 0.8 MeV, around 0.6 MeV and smaller than 0.6 MeV, respectively. That is, the heavier the core, the smaller the pairing gap of Λ s. This dependence is consistent with the observation in normal open shell nuclei that the pairing gap for nucleons decreases with the mass number. We will discuss more about this dependence later. Meanwhile, when comparing the maximal values of Δ_Λ with Δ_N , one may also notice that the pairing effects of Λ s are weaker than nucleons; e.g., $\Delta_\Lambda < 0.8 \text{ MeV}$ for $^{44}_{\Lambda}\text{Ca}$, while for an open shell nucleus with $A = 44$, Δ_N is about 1.8 MeV according to the empirical formula $\Delta_N \approx 12A^{-1/2} \text{ MeV}$ [90]. Since the pairing strength for Λ s has been taken as 4/9 of that for nucleons, it is not unexpected that the pairing effects of Λ s are weaker compared to nucleons.

There have not been much work on the pairing effects of hyperons in finite nuclei, though some efforts were made to the study of double- Λ and multistrangeness ($-S \geq 3$) hypernuclei [13–20]. In Ref. [19], Güven et al. have investigated multistrangeness hypernuclei with the Hartree-Fock-Bogoliubov (HFB) theory and obtained interesting results concerning pairing effects of Λ s. Next we make a brief comparison of our results with Ref. [19]. In Fig. 3(a), the pairing gaps for Λ s in three typical multistrangeness hypernuclei $^{46}_{6\Lambda}\text{Ca}$, $^{160}_{28\Lambda}\text{Sn}$ and $^{272}_{64\Lambda}\text{Pb}$ are compared with the HFB results [19]. It can be seen that $\Lambda\Lambda$ pairing gaps of these three nuclei in the present work are smaller than those given in Ref. [19]. Furthermore, the Δ_Λ from the MDC-RHB theory decreases faster with A

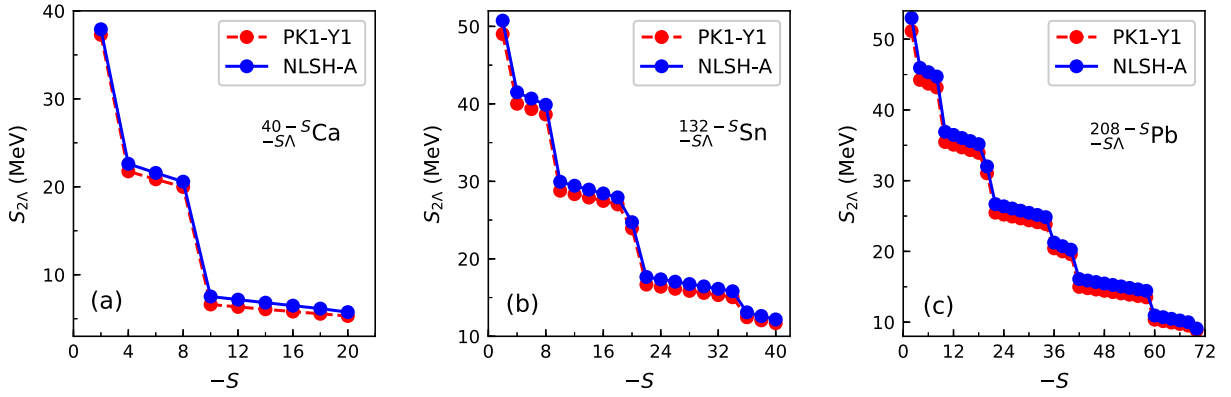


Fig. 1. (Color online) The two-Lambda separation energy as a function of the strangeness number $-S$ for (a) $^{40}_{-S\Lambda}\text{Ca}$ ($-S = 0-20$), (b) $^{132}_{-S\Lambda}\text{Sn}$ ($-S = 0-40$) and (c) $^{208}_{-S\Lambda}\text{Pb}$ ($-S = 0-70$) obtained in the MDC-RHB calculations. The effective interactions PK1-Y1 and NLSH-A are used for the ph channel and the separable pairing force of finite range with the pairing strength $G_\Lambda = 4/9 \cdot G_N = 323.56 \text{ MeV}\cdot\text{fm}^3$ is used for pp channel.

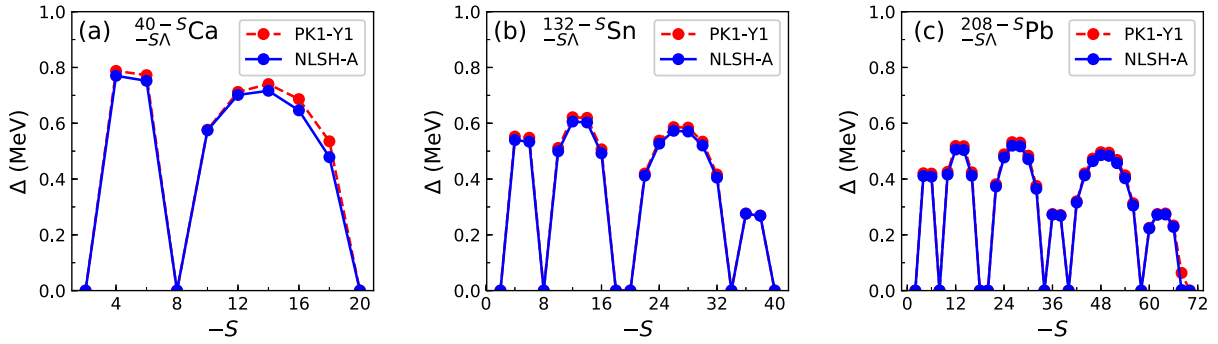


Fig. 2. (Color online) The pairing gap of Λ s as a function of the strangeness number $-S$ for (a) $^{40}_{-S\Lambda}\text{Ca}$ ($-S = 0-20$), (b) $^{132}_{-S\Lambda}\text{Sn}$ ($-S = 0-40$) and (c) $^{208}_{-S\Lambda}\text{Pb}$ ($-S = 0-70$) obtained in the MDC-RHB calculations. The effective interactions PK1-Y1 and NLSH-A are used for the ph channel and the separable pairing force of finite range with the pairing strength $G_\Lambda = 4/9 \cdot G_N = 323.56 \text{ MeV}\cdot\text{fm}^3$ is used for pp channel.

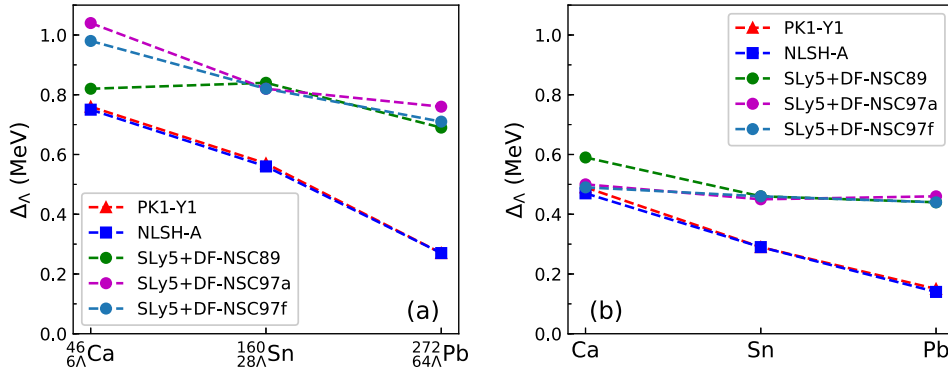


Fig. 3. (Color online) (a) $\Lambda\Lambda$ pairing gaps for $^{46}_{6\Lambda}\text{Ca}$, $^{160}_{28\Lambda}\text{Sn}$ and $^{272}_{64\Lambda}\text{Pb}$ and (b) the average $\Lambda\Lambda$ pairing gaps as defined in Eq. (9) for $^{40}_{-S\Lambda}\text{Ca}$ ($-S = 6-20$), $^{132}_{-S\Lambda}\text{Sn}$ ($-S = 18-40$) and $^{208}_{-S\Lambda}\text{Pb}$ ($-S = 58-70$) compared with the HFB results [19] with SLy5 for the NN interaction and DF-NSC89, DF-NSC97a and DF-NSC97f for the ΛN interaction in the ph channel. In the MDC-RHB calculations, the effective interactions PK1-Y1 and NLSH-A are used for the ph channel and the separable pairing force of finite range with the pairing strength $G_\Lambda = 4/9 \cdot G_N = 323.56 \text{ MeV}\cdot\text{fm}^3$ is used for pp channel.

than the HFB predictions. This conclusion holds also for the average pairing gap

$$\bar{\Delta}_\Lambda \equiv \frac{1}{m} \sum_{-S} \Delta_\Lambda(A_{-S\Lambda}^X), \quad X = \text{Ca, Sn and Pb}, \quad (9)$$

for $^{40}_{-S\Lambda}\text{Ca}$ ($-S = 6-20$ and $m = 8$), $^{132}_{-S\Lambda}\text{Sn}$ ($-S = 18-40$ and $m = 12$) and $^{208}_{-S\Lambda}\text{Pb}$ ($-S = 58-70$ and $m = 12$), as seen in Fig. 3(b). In Ref. [19], a zero-range δ force is adopted for the $\Lambda\Lambda$ pairing and its strength for $^{40}_{-S\Lambda}\text{Ca}$, $^{132}_{-S\Lambda}\text{Sn}$ and $^{208}_{-S\Lambda}\text{Pb}$ has been adjusted separately within a sharply truncated pairing window. The adjustment was made by fitting the average pairing gap (9) to the maximal pairing gap in uniform hypernuclear matter given in Ref. [91] at

a certain density corresponding to the averaged density of $^{40}_{-S\Lambda}\text{Ca}$ ($-S = 6-20$), $^{132}_{-S\Lambda}\text{Sn}$ ($-S = 18-40$) and $^{208}_{-S\Lambda}\text{Pb}$ ($-S = 58-70$), respectively. Thus for Ca, Sn and Pb isotopes, the pairing strengths are different and the pairing interaction is always the strongest for Pb, as seen in TABLE IV of Ref. [19]. In the present work, however, a global finite-range pairing force [Eq. (6)] is adopted and, thus, there is no hard cut-off for the pairing window.

In normal nuclei, it has been well known that the pairing gap for nucleons Δ_N declines more or less with $\sqrt{A}$ [90]. This decreasing tendency is roughly consistent with the dependence of $\Lambda\Lambda$ pairing gaps with respect to the number of Λ s obtained in the present work. Nevertheless, in Ref. [19], the decrease of the $\Lambda\Lambda$

pairing gaps is much gentler with the number of Λ s. This is quite interesting and should be investigated further.

To summarize, we have proposed a relationship between effective pairing interactions for hyperons and for nucleons based on the quark model. Namely, the ratio between the strength of the effective pairing interaction for the hyperon Y consisting of $n_{u/d}$ u/d quarks and that for the nucleon is $n_{u/d}^2/9$. For Λ s, this ratio is simply 4/9. A separable pairing force of finite range has been implemented in the MDC-RHB theory to investigate $\Lambda\Lambda$ pairing effects in multi- Λ Ca, Sn and Pb hypernuclei. By examining the two-Lambda separation energy $S_{2\Lambda}$ and the pairing gap Δ_Λ , it is revealed that $-S = 2, 8, 20, 34, 40$ and 58 are magic or semi-magic numbers for Λ s. The Δ_Λ decreases with the mass number of the core nucleus increasing. It is also found that the pairing effects of Λ s are weaker than nucleons due to the suppression of the pairing strength by a factor of 4/9.

Finally, let us make two further remarks. First, one may notice that the ratio $n_{u/d}^2/9$ is probably very rough. Other factors such as the violation of the OZI rule [92,93], medium effects [69], possible different couplings of ρ to baryons [94] and mass splittings for baryons [80] may alternate this ratio or make the relation more complex between pairing interactions of Λ s and nucleons. Second, although in the present work the ratio $n_{u/d}^2/9$ between the strength of the pairing interaction for hyperons and that for nucleons has been used in the framework of the RMF models, we expect it is also applicable to non-relativistic mean field models.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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