



Excited states of spin-(3/2) doubly-heavy baryons within the QCD sum rules method

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Received: 22 February 2022 / Accepted: 10 May 2022
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Abstract Within the framework of the QCD sum rules method, the masses and pole residues of the excited states $2S$ and $1P$ of the doubly-heavy spin-(3/2) baryons Ξ_{cc}^* , Ξ_{bb}^* , Ξ_{bc}^* , Ω_{cc}^* , Ω_{bb}^* , and Ω_{bc}^* are investigated. The excited state masses and pole residues are predicted with reasonable accuracy. We expect that soon, the current predictions can be tested against future experimental data.

1 Introduction

Over the past few years, LHCb has discovered multiple heavily flavored baryons and many resonances containing one heavy bottom quark, as well as, additional baryons having one charm quark. The LHCb collaboration, for example, discovered five new narrow Ω_c^0 states, $\Sigma_b(6097)$ excited state, $\Xi_b(6227)$ bottom-strange state, $\Lambda_b(6146)$, and $\Lambda_b(6152)$ resonances [1–8]. Additionally, the LHCb Collaboration has also identified four narrow peaks of Ω_b^- excited states [9], and the CMS Collaboration has recently discovered a new state $\Xi_b(6100)$ [10]. Moreover, many spin-(1/2) b-baryons (Λ_b^0 , $\Sigma_b^{\pm,0}$, $\Xi_b^{0,-}$, $\Xi_b^{'-}$, Ω_b^0), and spin-(3/2) b-baryons (Λ_b^0 , $\Sigma_b^{*,\pm,0}$, $\Xi_b^{*,0,-}$) were also discovered and their masses are precisely measured [11–15].

The SELEX Collaboration, on the other hand, has observed the doubly-heavy charmed baryon state Ξ_{cc}^+ with spin-(1/2) and mass (3519 ± 1) MeV in the pD^+K^- decay channel [16], and then later was confirmed by the same experiment in 2005 [17]. Whereas, the LHCb Collaboration have recently reported that the ground state mass of Ξ_{cc}^{++} is $(3624.40 \pm 0.72 \pm 0.14)$ MeV [2]. These findings stimulated the search for a new class of long-lived baryons called doubly-heavy baryons.

The doubly-heavy baryons are hadrons containing two heavy quarks of (c and/or b), and one light quark of (d or u or s). The existence of such doubly-heavy baryons is considered a natural consequence of the quark model of hadrons.

Meanwhile, the LHCb has already begun the searching for the doubly-heavy bottom charmed baryon Ξ_{cb} and is still working to improve its signal sensitivity [18].

The existence of such experiments has prompted theorists and phenomenologists to investigate the structural features of the doubly-heavy flavor baryons. The charmed and bottom baryons, which are composed of one (two) heavy quark(s), are of particular interest for studying the dynamics of light quarks in the presence of heavy quarks and may provide an exceptional platform for testing quark model estimations and heavy quark symmetries.

Motivated by the possibility of observing these new doubly-heavy baryons shortly, the theoretical study of such baryons could shed light on the dynamics of the heavy quarks at the hadronic scale, and hence might be represented as a truthful investigation of the interaction between perturbative and non-perturbative QCD pictures. In the theory, the study of doubly-heavy baryons could provide crucial inputs for future experimental discoveries, as well as possibly clarifying and revealing secrets about the nature of other heavy baryons.

One of the most important physical properties of the doubly-heavy baryons is their masses and residues, which can be accessed experimentally. Based on this, many authors have created and established various phenomenology methods and models to estimate the mass spectrum of doubly-heavy baryons, such as the non-relativistic and relativistic quark models [19,20], the three-body Faddeev method [21], the QCD MIT bag model [22], effective field theory with the non-relativistic QCD potential method [23], the heavy quark spin symmetry method [24], the variational approach method [25], the full QCD sum rules method [26], and the lattice QCD method [27,28]. In addition to that, many other

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theoretical works inspired by the above mentioned discoveries also have been carried out to determine the characteristic properties like the masses, lifetimes, and the strong coupling constants of such baryons within various models [29–36].

Thus far, the QCD sum rules method is widely recognized as one of the most essential nonperturbative analytical methods for estimating the ground state masses of the heavy hadrons, and the numerical estimation of this method is extremely accurate. The success of this method made it as one of the most popular theoretical methods for testing the various properties of doubly-heavy baryons. The masses of spin-(1/2) doubly-heavy baryons, for example, were computed using the QCD sum rules approach in [37–41], whereas the ground state masses of spin-(3/2) doubly-heavy baryons were computed in [38–45]. The masses and pole residues of the ground state of spin-(3/2) doubly-heavy baryons Ξ_{QQq}^* , $\Xi_{QQ'q}^*$, Ω_{QQq}^* , and $\Omega_{QQ'q}^*$, on the other hand, have been the focus of previous research [44]. Therefore, there is a lot of work to be done to have a full spectrum of spin-(3/2) doubly-heavy baryons.

Due to its numerical performance, we were tempted to extend the calculations of the QCD sum rules in [44]. However, this time to understand how the QCD sum rules method can be applied to derive the excited state masses and the excited state pole residues of spin-(3/2) doubly-heavy baryons. To do so, in the present work, we'll use the most common type of interpolation current for spin-(3/2) doubly-heavy baryons.

With this in mind, the form of this paper is arranged as follows: Sect. 2 presents the formulation of the QCD sum rules for spin-(3/2) doubly-heavy baryons. The analytical formulations of the QCD sum rules are written in such a way that the reader can use them without proceeding into their derivation. Section 3, focuses on the numerical analysis of the QCD sum rules for excited state masses and excited state pole residues of spin-(3/2) doubly-heavy baryons; 2S and 1P of Ξ_{cc}^* , Ξ_{bb}^* , Ξ_{bc}^* , Ω_{cc}^* , Ω_{bb}^* , and Ω_{bc}^* . Finally, in the last section of this paper, we presented a summary of our findings and some concluding remarks.

2 QCD sum rules for doubly-heavy spin-(3/2) baryon states

One of the most important methods for non-perturbative QCD is the QCD sum rules method. In this method, hadrons are represented by their interpolating quark currents, and the major object of this method is the so-called correlation function $\Pi(p^2)$, which can be expressed in terms of the interpolating quark currents. The correlation function $\Pi(p^2)$ is an analytic function of p^2 that can be defined at both negative (space-like) and positive (time-like) values of p^2 . When p^2 is shifted from large negative values towards positive values,

the correlation function starts receiving contributions from the long-distance quark-gluon interactions. In this region, quarks start to form hadrons, and the phenomenological (or hadronic) side of the correlation function is investigated, and can be expressed in terms of hadronic degrees of freedom by inserting a complete set of intermediate hadronic states with the same quantum numbers as the current operators. In this representation, the correlation function leads to a mass-sum rule, which is in turn related to the physically observed hadrons.

On the other hand, when $p^2 \ll 0$ (space-like), the correlation function is studied in terms of the quarks and gluons and their interactions with the QCD vacuum. This is referred to as theoretical (or QCD) representation. In this representation, the Wilson operator product expansion (OPE) method is usually used to separate the short-and long-distance quark-gluon interactions, by expressing the correlation function in terms of the QCD degrees of freedom, such as quark condensate, gluon condensate, etc.

Finally, equating the two sides of these representations and applying the double Borel transformations to the momentum of the state to suppress the contribution of the higher states and continuum, the QCD sum rules for the physical observables such as masses and residues can be estimated by matching the coefficients of the same structure in both representations of the correlation function.

Consequently, we start the analysis of spin-(3/2) doubly-heavy baryon states by taking into account the two-point correlation function $\Pi_{\mu\nu}(p^2)$ between vacuum states, with no initial and final baryons:

$$\Pi_{\mu\nu}(p^2) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T}\{\eta_\mu(x)\bar{\eta}_\nu(0)\} | 0 \rangle, \quad (1)$$

where \mathcal{T} represents the time ordering operator, η_μ is the interpolating current of the doubly-heavy baryon with spin-(3/2) that injects quarks into the QCD vacuum at point x , and p is the four-momentum of the doubly-heavy baryon.

The conventional expression of the interpolating current, which may be coupled to both the positive parity as well as to negative parity spin-(3/2) doubly-heavy baryons is chosen to be of the following form:

$$\eta_\mu = \frac{1}{\sqrt{3}} \epsilon^{abc} \left\{ (q^{aT} C \gamma_\mu Q^b) Q'^c + (q^{aT} C \gamma_\mu Q'^b) Q^c + (Q^{aT} C \gamma_\mu Q'^b) q^c \right\}, \quad (2)$$

where q denotes the light quark, Q and Q' are the two heavy quarks. T stands for transposition, C for charge conjugation, a , b , and c are for color indices.

The systematic procedure of the QCD sum rules method is started by saturating the above correlation function with full sets of the baryon states with equal quantum numbers as the interpolating current, and by summing on for the ground state and associated excited states, we obtain the physical

part or phenomenological representation of the correlation function as:

$$\Pi^{HAD}(p^2) = \sum_{i=1S, 2S, \text{ and } 1P} \frac{\langle 0 | \eta_\mu(0) | B_i(p) \rangle \langle B_i(p) | \bar{\eta}_\mu(0) | 0 \rangle}{p^2 - m_{B_i}^2} + \dots, \quad (3)$$

where the summation here only includes the required hadronic states produced by the quark current η_μ , and... includes contributions from higher states as well as the continuum.

The matrix elements of the interpolating current evolved into Eq. (3), on the other hand, are parameterized in terms of the three pole residues for the three different baryonic states, $1S$, $2S$, and $1P$,

$$\begin{aligned} \langle 0 | \eta_\mu(0) | B_{1S}(p) \rangle &= \lambda_g u_{\mu(1S)}(p, r), \\ \langle 0 | \eta_\mu(0) | B_{2S}(p) \rangle &= \lambda u_{\mu(2S)}(p, r), \\ \langle 0 | \eta_\mu(0) | B_{1P}(p) \rangle &= \tilde{\lambda} \gamma_5 u_{\mu(1P)}(p, r), \end{aligned} \quad (4)$$

where the λ 's are the pole residues of spin-(3/2) doubly-heavy baryon states, and $u(p, r)$ is the Rarita–Schwinger spinor satisfying the identity,

$$\sum_r u_\mu(p, r) \bar{u}_\nu(p, r) = \sum_i (\not{p} + m_{B_i}) \times \left(g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2 p_\mu p_\nu}{3 m_{B_i}^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3 m_{B_i}} \right), \quad (5)$$

where, $m_{B1S} \equiv m_g$, $m_{B2S} \equiv m$, and $m_{B1P} \equiv \tilde{m}$ are to be the masses of spin-(3/2) doubly-heavy baryon states, as well as, $\lambda_{B1S} \equiv \lambda_g$, $\lambda_{B2S} \equiv \lambda$, and $\lambda_{B1P} \equiv \tilde{\lambda}$ are to be the pole residues of spin-(3/2) doubly-heavy baryon states.

Before going further, a word of caution is in order. The above-mentioned interpolating current η_μ of Eq. (2) couples to both the spin-(1/2) and spin-(3/2) baryons with positive and negative parities. However, because we are only interested in studying spin-(3/2) doubly-heavy baryon states of both positive and negative parities in this work. Therefore, the unwanted spin-(1/2) contributions of both parities should be removed from this study. It is clear from Eq. (5) that the unwanted spin-(1/2) structures are proportional to either $\gamma_\mu \gamma_\nu$, $p_\mu p_\nu$, and $(p_\mu \gamma_\nu - p_\nu \gamma_\mu)$.

Overall, Eq. (5) shows that there are only two independent Lorentz structures left, $\not{p} g_{\mu\nu}$ and $g_{\mu\nu}$, which are coupled only to spin-(3/2) doubly-heavy baryons with both positive and negative parities. On the whole, the hadronic side (HAD) therefore contains the $(1S + 2S + 1P)$ spin-(3/2) doubly-heavy baryonic states, and the excited states are considered as $(2S)$ and $(1P)$ states,

$$\begin{aligned} \Pi^{HAD}(p^2) &= \lambda_g^2 \left(\frac{\not{p} + m_g}{m_g^2 - p^2} \right) g_{\mu\nu} + \lambda^2 \left(\frac{\not{p} + m}{m^2 - p^2} \right) g_{\mu\nu} \\ &\quad + \tilde{\lambda}^2 \left(\frac{\not{p} - \tilde{m}}{\tilde{m}^2 - p^2} \right) g_{\mu\nu} + \dots. \end{aligned} \quad (6)$$

In this regard, the operator product expansion approach (OPE) in the deep Euclidean region $-p^2 \ll m_Q^2$ on the other hand is used to determine the QCD side of the correlation function. The relevant correlation function in terms of the heavy (light) quark propagator may be obtained by putting the explicit expressions of the interpolating currents into the correlation function of Eq. (1) and then, by doing some contractions of all quark fields through the Wicks theorem, we obtain:

$$\begin{aligned} \Pi_{\mu\nu}^{QCD}(p^2) &= \frac{1}{3} \epsilon_{abc} \epsilon_{a'b'c'} \int d^4 x e^{i p \cdot x} \langle 0 | \\ &\quad \times \left\{ -S_Q^{cb'} \gamma_\nu \tilde{S}_{Q'}^{aa'} \gamma_\mu S_q^{bc'} - S_Q^{ca'} \gamma_\nu \tilde{S}_q^{bb'} \gamma_\mu S_Q^{ac'} \right. \\ &\quad - S_{Q'}^{ca'} \gamma_\nu \tilde{S}_Q^{bb'} \gamma_\mu S_q^{ac'} \\ &\quad - S_{Q'}^{ca'} \gamma_\nu \tilde{S}_Q^{bb'} \gamma_\mu S_q^{ac'} - S_q^{ca'} \gamma_\nu \tilde{S}_{Q'}^{bb'} \gamma_\mu S_Q^{ac'} \\ &\quad - S_q^{cb'} \gamma_\nu \tilde{S}_Q^{aa'} \gamma_\mu S_Q^{bc'} - S_{Q'}^{cc'} \text{Tr} \left[S_Q^{ba'} \gamma_\nu \tilde{S}_q^{ab'} \gamma_\mu \right] \\ &\quad - S_q^{cc'} \text{Tr} \left[S_{Q'}^{ba'} \gamma_\nu \tilde{S}_Q^{ab'} \gamma_\mu \right] \\ &\quad \left. - S_Q^{cc'} \text{Tr} \left[S_q^{ba'} \gamma_\nu \tilde{S}_{Q'}^{ab'} \gamma_\mu \right] \right\} |0\rangle, \end{aligned} \quad (7)$$

where $\tilde{S}_{Q(q)}^{ij} \equiv C S_{Q(q)}^{ijT} C$, $S_{Q(q)}^{ij}$ is the heavy (light) quark propagator in the coordinate space, and the explicit mathematical expressions of the heavy (light) quark propagators are,

$$\begin{aligned} S_Q(x) &= \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} - \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q \sqrt{-x^2}), \\ S_q(x) &= i \frac{\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i \frac{m_q \not{x}}{4} \right) \\ &\quad - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i \frac{m_q \not{x}}{6} \right), \end{aligned} \quad (8)$$

where K_1 and K_2 are the modified Bessel functions of the second kind, $\langle \bar{q}q \rangle$ is the quark condensates, and m_0^2 is the conventional parametrization constant in the quark-gluon condensates.

At this point, we make the necessary substitutions of the heavy (light) quark propagators into Eq. (7), the two-point correlation function can be then re-expressed in terms of some spectral densities $\rho(s)$'s with the two independent structures $\not{p} g_{\mu\nu}$ and $g_{\mu\nu}$, respectively,

$$\Pi^{QCD}(p^2) = \Pi_1^{QCD}(p^2) \not{p} g_{\mu\nu} + \Pi_2^{QCD}(p^2) g_{\mu\nu}, \quad (9)$$

where,

$$\begin{aligned} \Pi_1^{QCD}(p^2) &= \int_{s_{min}}^{s_0} \frac{\rho_1(s)}{s - p^2} ds, \\ \Pi_2^{QCD}(p^2) &= \int_{s_{min}}^{s_0} \frac{\rho_2(s)}{s - p^2} ds. \end{aligned} \quad (10)$$

The spectral densities $\rho_1(s)$ and $\rho_2(s)$ in Eq. (10) are obtained from the imaginary part of the correlator function; $\rho_i(s) =$

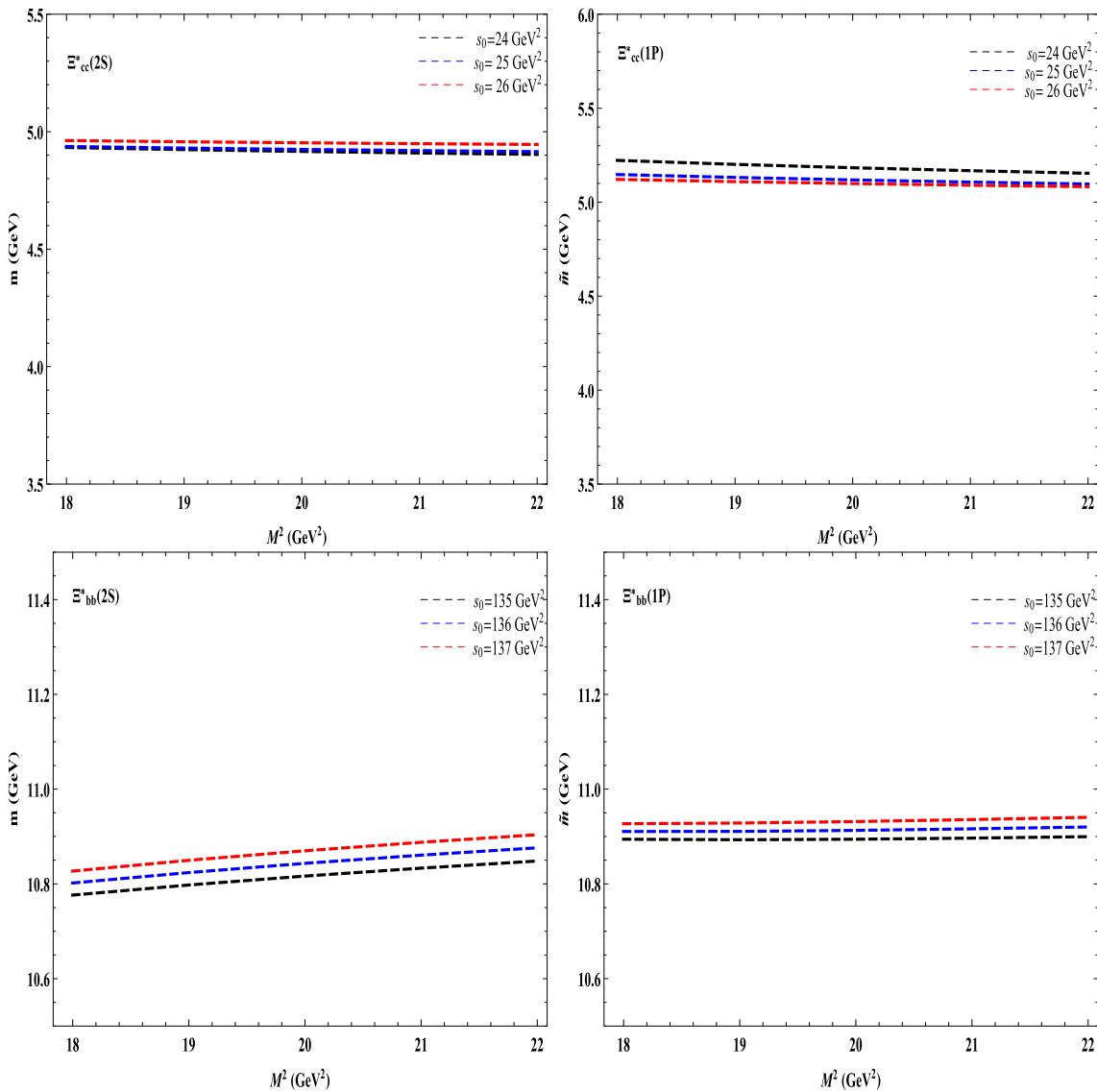


Fig. 1 The dependency of the excited states $2S$, $1P$ masses of the doubly-heavy spin-(3/2) Ξ_{cc}^* , and Ξ_{bb}^* baryons on the Borel mass M^2 at different values of the continuum threshold s_0

$\frac{1}{\pi} \text{Im} \Pi_i(s)$, where $i = 1(2)$ is related to the structures $\not{p} g_{\mu\nu}$ and $g_{\mu\nu}$. Further, s_0 is the continuum threshold, which is not fully arbitrary and it is associated with the energy of the excited states, $s_{min} = (m_Q + m_{Q'})^2$, and finally, the spectral densities $\rho_1(s)$ and $\rho_2(s)$ are written as (see also [44]):

$$\begin{aligned} \rho_1(s) = & \frac{1}{32\pi^4} \int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \left\{ \alpha \left[3xy(x+y)\alpha \right. \right. \\ & \left. \left. - 4(1-x-y)m_{Q'}m_Q - 4m_q(xm_{Q'} + ym_Q) \right] \right\} \\ & - \frac{\langle \bar{q}q \rangle}{24\pi^2} \int_{x_{min}}^{x_{max}} dx \left[(1-x)(3xm_q - 4m_Q) - 4xm_{Q'} \right], \end{aligned} \quad (11)$$

$$\rho_2(s) = \frac{1}{16\pi^4} \int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \left\{ \alpha \left[(x+y)\alpha(xm_{Q'} + ym_Q) \right. \right.$$

$$\begin{aligned} & \left. \left. - m_q(xy\alpha + 3m_Qm_{Q'}) \right] \right\} + \frac{\langle \bar{q}q \rangle}{12\pi^2} \int_{x_{min}}^{x_{max}} dx \\ & \times \left\{ (1-x) \left[2x(m_0^2 + \beta - s) - m_Qm_q \right] \right. \\ & \left. + (xm_q + 3m_Q)m_{Q'} \right\}, \end{aligned} \quad (12)$$

where,

$$\alpha = \frac{m_Q^2}{x} + \frac{m_{Q'}^2}{y} - s,$$

$$\beta = \frac{m_Q^2}{x} + \frac{m_{Q'}^2}{(1-x)} - s,$$

$$y_{min} = \frac{xm_{Q'}^2}{sx - m_Q^2},$$

$$y_{max} = 1 - x,$$

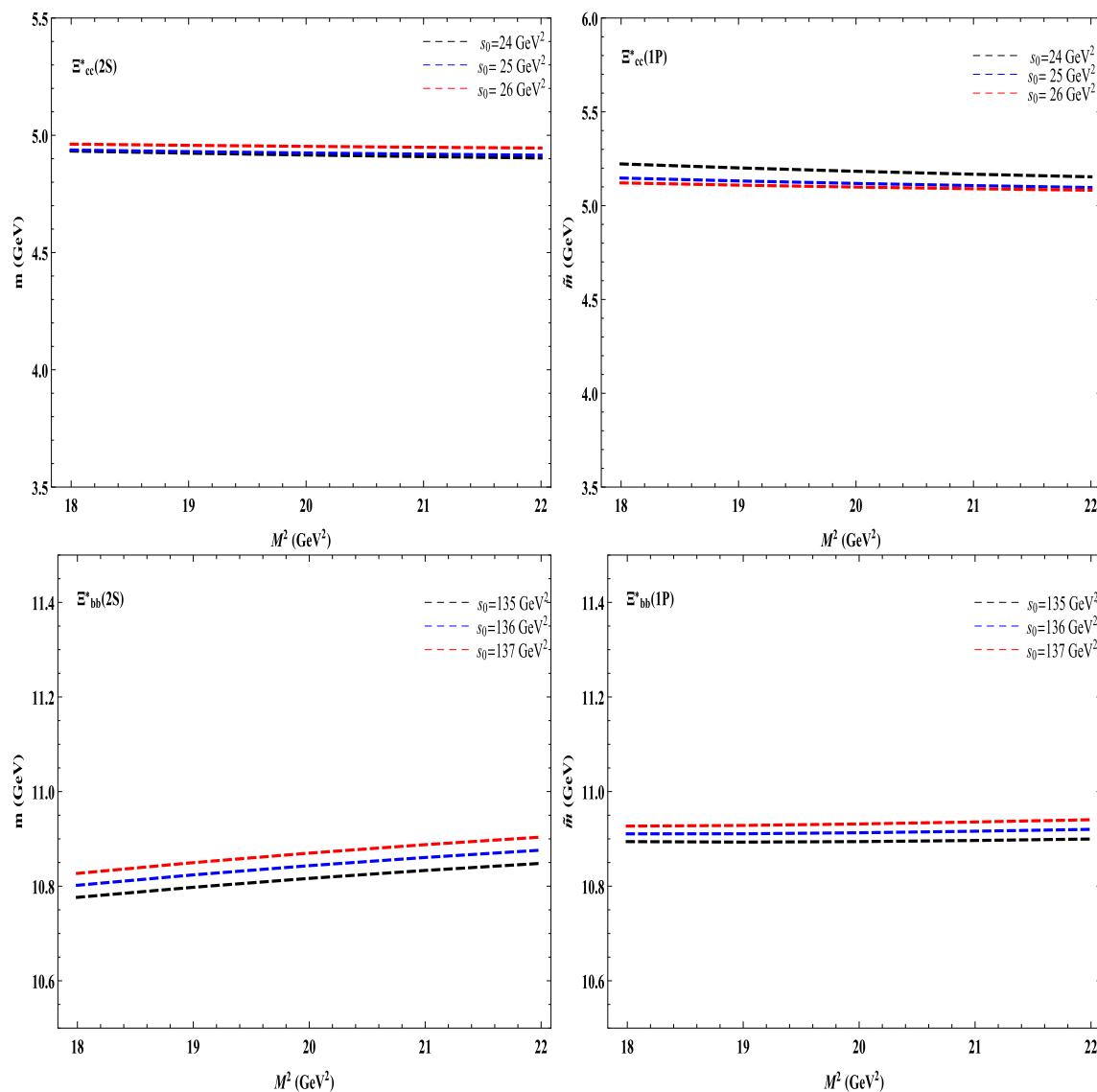


Fig. 2 The dependency of the excited states $2S$, $1P$ masses, and pole residues of the doubly-heavy of spin-(3/2) Ξ_{cc}^* baryon on the Borel mass M^2 at different values of the continuum threshold s_0

$$x_{min} = \frac{1}{2s} \left[s + m_Q^2 - m_{Q'}^2 - \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right],$$

$$x_{max} = \frac{1}{2s} \left[s + m_Q^2 - m_{Q'}^2 + \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right].$$

After calculating both the HAD and QCD sides of the correlation function, we matched the coefficients of the same structures from both sides of Eqs. (6) and (9), and finally applying the Borel transformation to the variable p^2 to eliminate the subtraction terms and suppress the contributions from higher states and the continuum. As a result, the desired sum rules

for the states $1S + 2S + 1P$ are obtained:

$$\begin{aligned} \lambda_g^2 e^{-m_g^2/M^2} + \lambda^2 e^{-m^2/M^2} + \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} &= \Pi_1^{QCD}(M^2), \\ m_g \lambda_g^2 e^{-m_g^2/M^2} + m \lambda^2 e^{-m^2/M^2} - \tilde{m} \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} &= \Pi_2^{QCD}(M^2), \end{aligned} \quad (13)$$

where M^2 is the Borel mass parameter, and $\Pi_1^{QCD}(M^2)$ and $\Pi_2^{QCD}(M^2)$ are the invariant functions of $/pg_{\mu\nu}$ and $g_{\mu\nu}$ structures, respectively.

The derived sum rules in Eq. (13) contain six unknown parameters for the ground state and excited state baryons. However, Eq. (13) is broken down into two sum rule equations, each containing six unknowns: three masses and three pole residues. To solve this system of equations linearly,

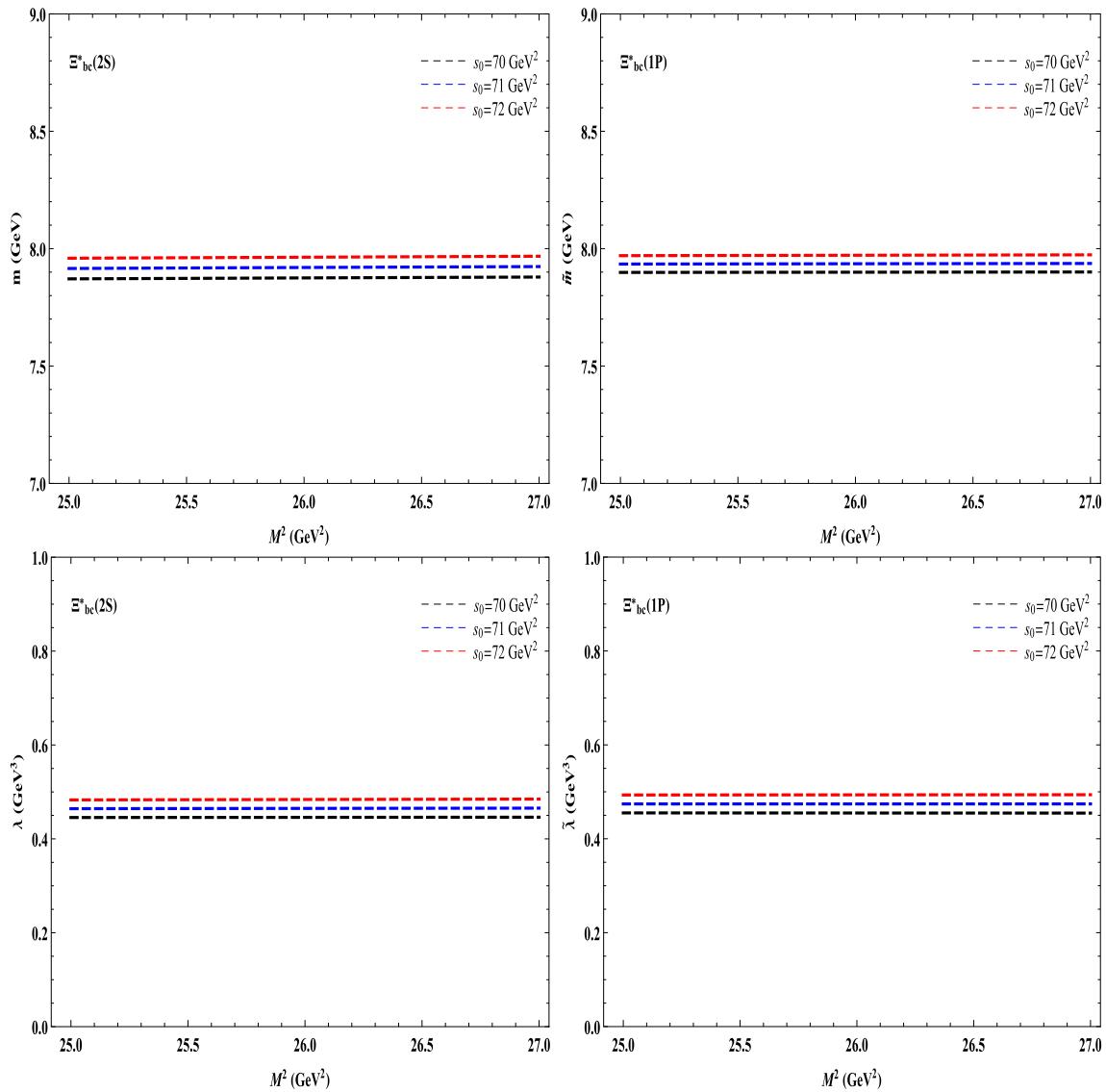


Fig. 3 The dependency of the excited states $2S$, $1P$ masses, and pole residues of the doubly-heavy of spin-(3/2) Ξ_{bc}^* baryon on the Borel mass M^2 at different values of the continuum threshold s_0

we'll need four more equations. The remaining four equations can be found by differentiating all sides of the sum rules in Eq. (13) with respect to $\frac{d}{d(-1/M^2)}$, $\frac{d^2}{d^2(-1/M^2)}$, and thus the six equations with the six unknowns are obtained simultaneously,

$$\begin{aligned}
 m_g^2 \lambda_g^2 e^{-m_g^2/M^2} + m^2 \lambda^2 e^{-m^2/M^2} + \tilde{m}^2 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 = \Pi_1'^{QCD}(M^2), \\
 m_g^3 \lambda_g^2 e^{-m_g^2/M^2} + m^3 \lambda^2 e^{-m^2/M^2} - \tilde{m}^3 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 = \Pi_2'^{QCD}(M^2), \\
 m_g^4 \lambda_g^2 e^{-m_g^2/M^2} + m^4 \lambda^2 e^{-m^2/M^2} + \tilde{m}^4 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 = \Pi_1''^{QCD}(M^2),
 \end{aligned}$$

$$\begin{aligned}
 m_g^5 \lambda_g^2 e^{-m_g^2/M^2} + m^5 \lambda^2 e^{-m^2/M^2} - \tilde{m}^5 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 = \Pi_2''^{QCD}(M^2),
 \end{aligned} \tag{14}$$

$$\text{where, } \Pi_i'^{QCD}(M^2) = \frac{d \Pi_i^{QCD}(M^2)}{d(-1/M^2)}, \text{ and } \Pi_i''^{QCD}(M^2) = \frac{d^2 \Pi_i^{QCD}(M^2)}{d^2(-1/M^2)}.$$

The whole set of six equations above makes the system's solution extremely difficult. Rather, to solve this system of equations, we picked up an appropriate value of s_0 at which the physical part of the correlation function is saturating only the ground state, and the other states, $2S$ and $1P$ are still remain in the continuum zoom. To put it differently, the initial guess of s_0 solved the system of equations and reproduced

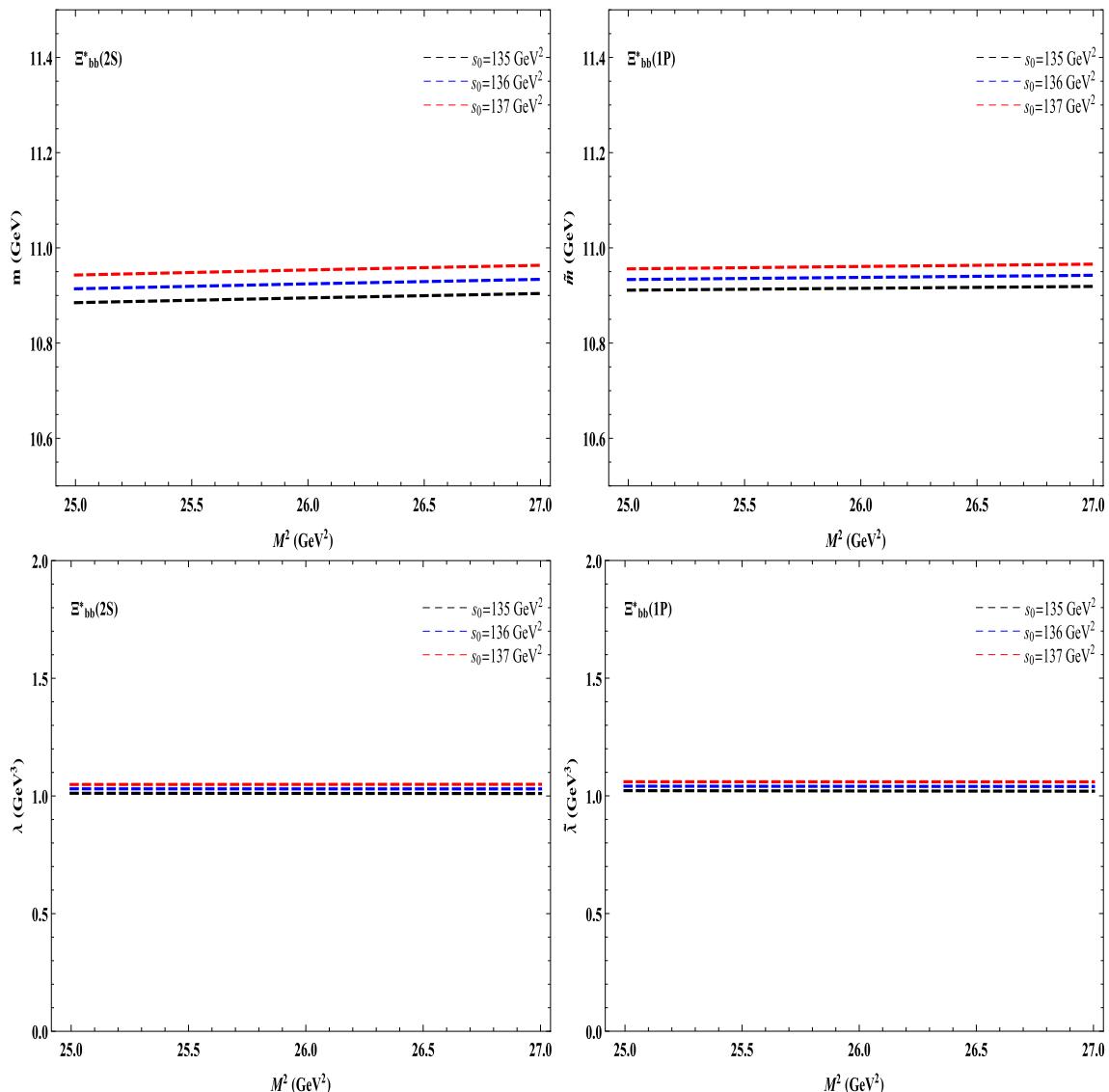


Fig. 4 The dependency of the excited states $2S$, $1P$ masses, and pole residues of the doubly-heavy of spin-(3/2) Ξ_{bb}^* baryon on the Borel mass M^2 at different values of the continuum threshold s_0

only the ground state masses and the pole residues of the spin-(3/2) doubly-heavy baryons m_g and λ_g .

In this regard, we chose the initial s_0 values: $s_0 = (22-26) \text{ GeV}^2$ for the cc , $s_0 = (68-72) \text{ GeV}^2$ for bc , and $s_0 = (133-137) \text{ GeV}^2$ for bb doubly-heavy baryons. At the same time, we allowed the Borel mass parameter M^2 to be varied from 5 GeV^2 up to 30 GeV^2 . From there, we can see that the values of the ground state masses (m_g) and the corresponding pole residues (λ_g) of spin-(3/2) doubly-heavy baryons Ξ_{cc}^* , Ξ_{bb}^* , Ξ_{bc}^* , Ω_{cc}^* , Ω_{bb}^* , and Ω_{bc}^* have begun to appear and converge quickly within $5 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$. The obtained results for (m_g) and (λ_g) with the corresponding s_0 values are also given in [44].

By doing so, we reduced the number of equations and the number of unknowns to four: two excited state masses m , \tilde{m} ,

and two excited state pole residues λ , $\tilde{\lambda}$, while we are still keeping the states $2S$ and $1P$ in the continuum zoom. To put it differently, we will solve the resulting four equations algebraically in terms (m_g) and (λ_g).

While solving the remaining four equations algebraically, we gradually increased the values of continuum zoom s_0 , and we are still keeping the Borel mass parameter M^2 running within the interval, $5 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$. Bearing in mind, the results of the excited state masses and the excited state pole residues of the spin-(3/2) doubly-heavy baryons in Eq. (14) should also not depend on the Borel mass parameter M^2 . In fact, within the interval, $5 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ we observed that there are different working regions where the results are supposedly acceptable and depend only moderately on the Borel mass parameter. For example, in Fig. 1,

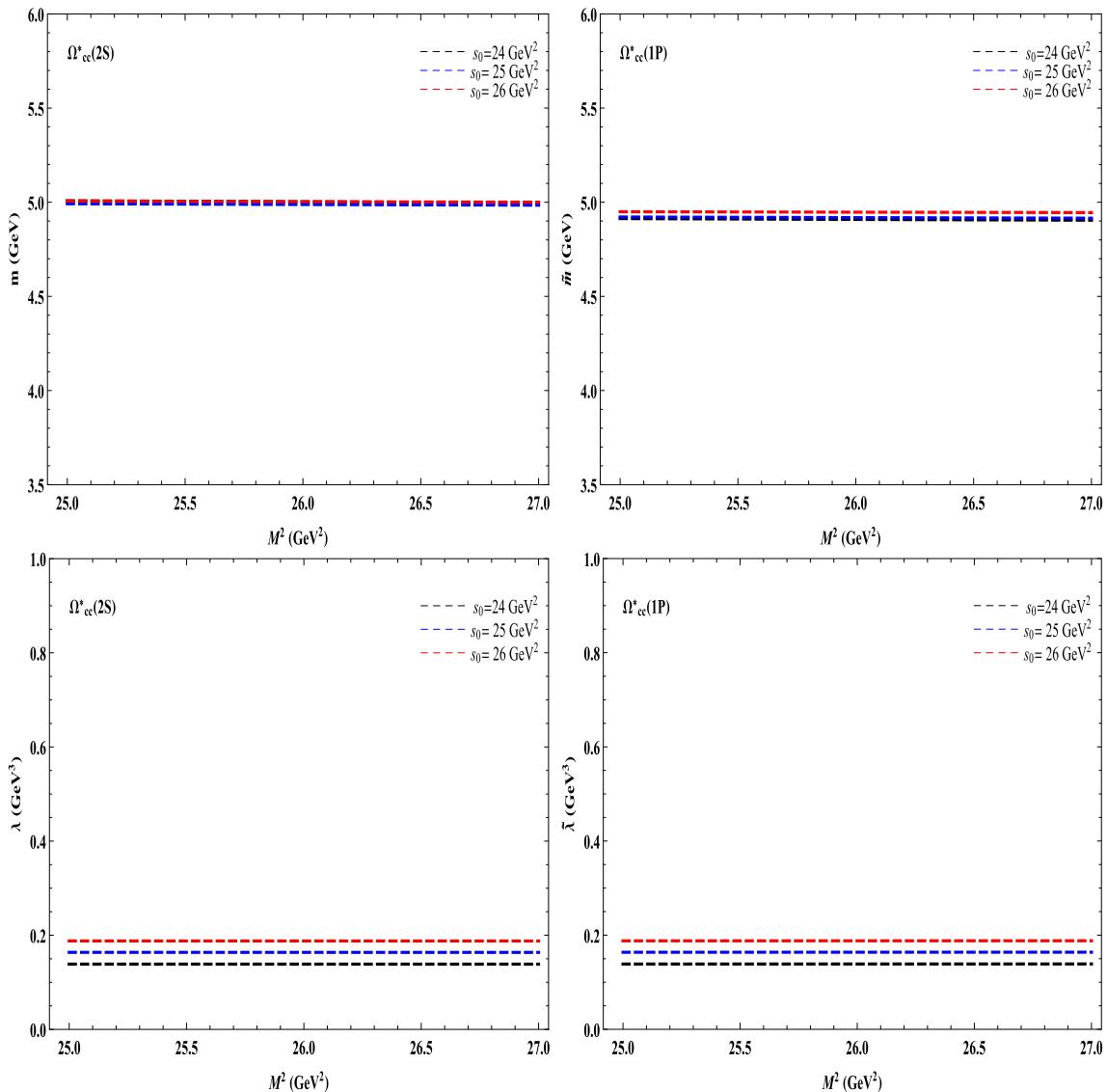


Fig. 5 The dependency of the excited states $2S$, $1P$ masses, and pole residues of the doubly-heavy of spin-(3/2) Ω_{cc}^* baryon on the Borel mass M^2 at different values of the continuum threshold s_0

if we set the working region to be varied in the interval $18 \text{ GeV}^2 \leq M^2 \leq 22 \text{ GeV}^2$, clearly from this figure, we can still see that the excited state masses are rather stable with the variations of the Borel mass parameter.

However, we noticed that the excited state masses and the corresponding pole residues of the spin-(3/2) doubly-heavy baryons began to appear and are rapidly converge within the interval $25 \text{ GeV}^2 \leq M^2 \leq 27 \text{ GeV}^2$. The obtained results for the excited states $2S$ and $1P$ are presented in a series of Figs. 2, 3, 4, 5, 6 and 7 with respect to M^2 , along with the appropriate s_0 values, so that the reader might, if so motivated, repeat our calculations.

3 Numerical analysis

Using the QCD sum rules Eqs. (13) and (14), the masses and pole residues of the excited states of spin-(3/2) doubly-heavy baryons Ξ_{cc}^* , Ξ_{bb}^* , Ξ_{bc}^* , Ω_{cc}^* , Ω_{bb}^* , and Ω_{bc}^* are calculated. Usually, in numerical calculations, the QCD sum rules approach involves a set of fixed input parameters. We used \overline{MS} values for the quark masses: $\bar{m}_b(\bar{m}_b) = (4.16 \pm 0.03) \text{ GeV}$, $\bar{m}_c(\bar{m}_c) = (1.28 \pm 0.03) \text{ GeV}$, and $m_s(2 \text{ GeV}) = (102 \pm 8) \text{ MeV}$ [46, 47]. On the other hand, the values of the quark condensates are taken as $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(246_{-19}^{+28} \text{ MeV})^3$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$, and $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [48]. Moreover, the QCD sum rules technique also includes two auxiliary parameters whose

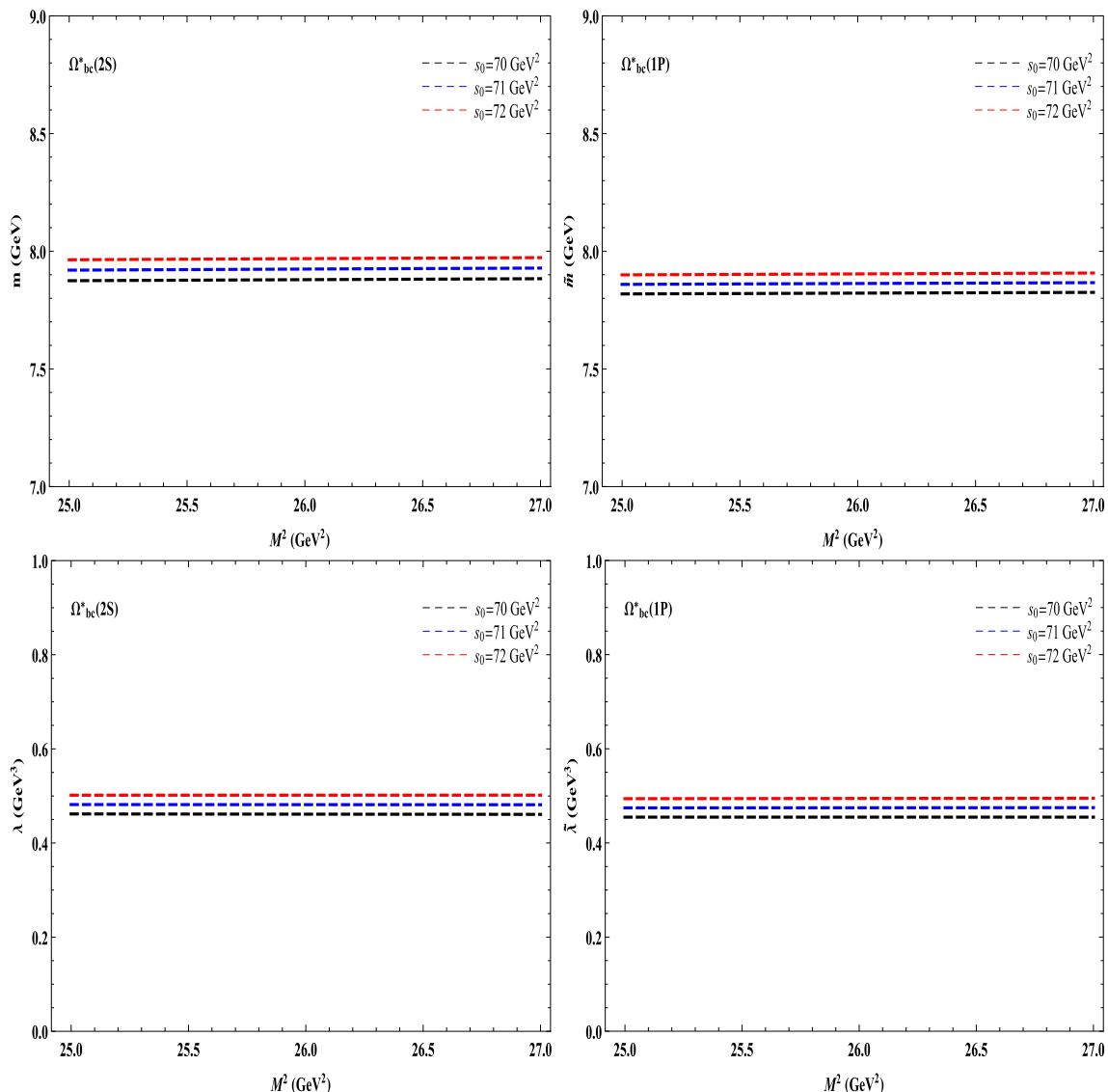


Fig. 6 The dependency of the excited states $2S$, $1P$ masses, and pole residues of the doubly-heavy of spin-(3/2) Ω_{bc}^* baryon on the Borel mass M^2 at different values of the continuum threshold s_0

working regions must be determined, specifically, the continuum threshold s_0 , and the Borel parameter M^2 .

The physical working regions for these parameters should, however, be managed in such a way that the involved physical quantities are roughly independent of these parameters. The working regions of s_0 and M^2 are obtained by the sum rules analysis using the conventional criteria of the QCD sum rules approach, means that, the perturbative contributions must be greater than the condensate contributions, and the pole contributions must be greater than the continuum contributions. As a result, the bounds of M^2 are obtained such that M^2 must be large enough to satisfy the convergence criterion of OPE. Therefore, the working regions of the continuum threshold s_0 and the Borel mass M^2 must be found with the condition

that the masses and pole residues are stable with respect to variations of these parameters.

In Figs. 2, 3, 4, 5, 6 and 7, we showed our numerical findings for the masses and the pole residues of spin-(3/2) doubly-heavy excited state baryons. The masses and the pole residues of the excited states $2S$ and $1P$ of spin-(3/2) doubly-heavy baryons are represented as a function of M^2 for different values of s_0 . The expected masses and the pole residues of the excited states $2S$ and $1P$ showed good stability with respect to the obtained physical working regions of M^2 , and s^0 .

As discussed in the previous section, according to our findings, these requirements are met when M^2 is changed from $25 \text{ GeV}^2 \leq M^2 \leq 27 \text{ GeV}^2$ for all the considered spin-(3/2) doubly-heavy baryons, while for example in Fig. 1, s_0 is modified in the range: $s_0 = (22 - 26) \text{ GeV}^2$ to saturate

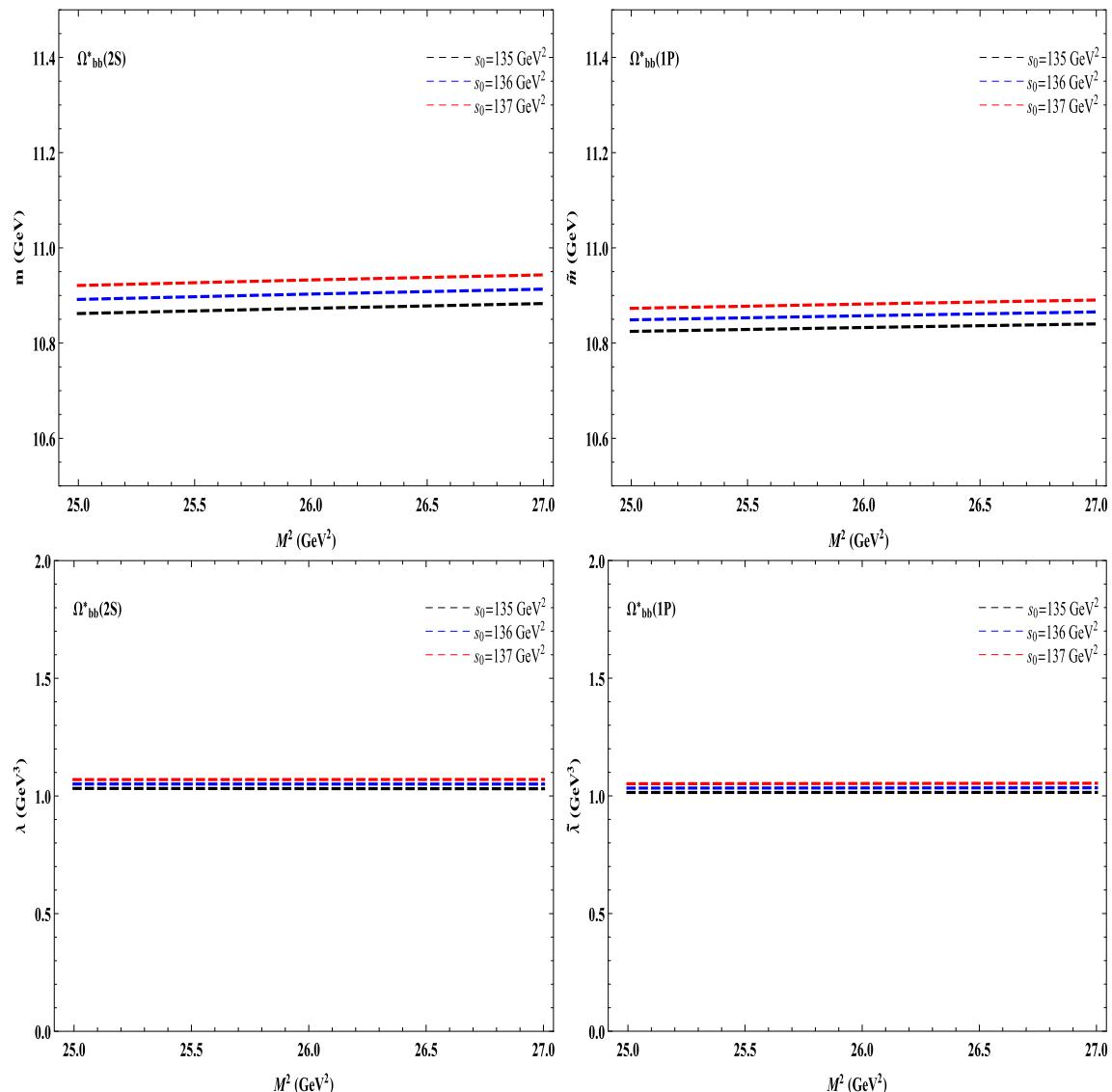


Fig. 7 The dependency of the excited states $2S$, $1P$ masses, and pole residues of the doubly-heavy of spin-(3/2) Ω_{bb}^* baryon on the Borel mass M^2 at different values of the continuum threshold s_0

Table 1 The masses m and \tilde{m} of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons Ξ_{cc}^* , Ξ_{bc}^* , and Ξ_{bb}^* (in units of GeV)

Baryons	Ξ_{cc}^*	Ξ_{bc}^*	Ξ_{bb}^*			
J^P	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)
Our work	4.95 ± 0.05	5.15 ± 0.05	7.91 ± 0.05	7.95 ± 0.05	10.92 ± 0.025	10.95 ± 0.025
Ref. [49]	3.99	3.85	7.27	7.15	10.62	10.51

Table 2 The masses m and \tilde{m} of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons Ω_{cc}^* , Ω_{bc}^* , and Ω_{bb}^* (in units of GeV)

Baryons	Ω_{cc}^*	Ω_{bc}^*	Ω_{bb}^*			
J^P	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)
Our work	5.00 ± 0.05	4.95 ± 0.05	7.90 ± 0.05	7.85 ± 0.05	10.90 ± 0.025	10.85 ± 0.025
Ref. [50]	4.10	3.97	7.48	7.37	10.74	10.64

Table 3 The pole residues λ and $\tilde{\lambda}$ of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons Ξ_{cc}^* , Ξ_{bc}^* , and Ξ_{bb}^* (in units of GeV^3)

Baryons	Ξ_{cc}^*	Ξ_{bc}^*	Ξ_{bb}^*
J^P	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)
Our work	0.175 ± 0.025	0.185 ± 0.025	0.475 ± 0.025

Table 4 The pole residues λ and $\tilde{\lambda}$ of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons Ω_{cc}^* , Ω_{bc}^* , and Ω_{bb}^* (in units of GeV^3)

Baryons	Ω_{cc}^*	Ω_{bc}^*	Ω_{bb}^*
J^P	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)
Our work	0.160 ± 0.025	0.165 ± 0.025	0.475 ± 0.025

the masses and the pole residues of Ξ_{cc}^* (2S), and Ξ_{cc}^* (1P) baryon excited states.

Also, for example in Figs. 2 and 3, considering the heaviest baryons, namely the doubly-heavy baryons including the heavy quark (b), the values of the continuum zoom s_0 should be increased to a sufficiently value in order to saturate again the excited states 2S and 1P.

Finally, in Tables 1, 2, 3 and 4 we displayed our predictions along with the other theoretical approaches for comparison. Our findings are in line with the findings of other methodologies.

4 Conclusion

In the context of the QCD sum rules method, we numerically calculated the excited state masses and the pole residues 2S and 1P of spin-(3/2) doubly-heavy Ξ_{cc}^* , Ξ_{bb}^* , Ξ_{bc}^* , Ω_{cc}^* , Ω_{bb}^* , and Ω_{bc}^* baryons.

The originality of the present work is that the contributions of 1S, 2S, and 1P states in Eqs. (13) and (14) are considered simultaneously.

On the whole, there should be a total of six unknowns: three masses and three pole residues. However, we only have two equations for two independent Lorentz structures $\not{p}g_{\mu\nu}$ and $g_{\mu\nu}$. So, we get the remaining four equations by doing a differentiation for all sides of the sum rule equations with respect to $\frac{d}{d(-1/M^2)}$, $\frac{d^2}{d^2(-1/M^2)}$, and then we solve these equations simultaneously.

We have used a slightly different method to solve this system of equations. We set a suitable value of s_0 and then we saturated the physical part of the correlation function with ground state contributions only. The spectroscopic parameters were then used as input parameters for the remaining set of equations. As a result, we have worked out a system of four equations as an alternative to six equations. Then, we have calculated the masses and the pole residues of the excited states 2S and 1P.

From figures, we have noticed that the masses of the 1P states is slightly exceeding the masses of the 2S states in the doubly-heavy baryons that are containing one light (u or d) quarks. Whereas, the masses of the 1P states is slightly less than the masses of the 2S states in the doubly-heavy baryons that are containing the strange quark. On the other hand, from figures, if one look at the values of the pole residues of the 2S states and that of the 1P states in the same region of s_0 values, one can notice that the values of the pole residues are of the same order to each other, and this is due to the reason that we have mentioned in Sect. 2. That is, the interpolating current in this work have been chosen in such a way that this current can couple to doubly-heavy baryons of both positive and negative parities.

By comparing the QCD sum rules results in this work with the results of the hyper-central constituent quark models [49, 50], we have noticed that the 2S and 1P mass states of the doubly-heavy baryons are exceeding the mass states of the doubly-heavy baryons obtained within the potential models. Finally, we hope that future LHC experimental results can be compared to the current estimates.

Acknowledgements This research was funded by Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2022R106), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data underlying the results are available as part of the article and no additional source data are required.]

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Funded by SCOAP³.

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