SINGLE, DOUBLE, AND CENTRAL DIFFRACTIVE DISSOCIATION*

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Recent results on proton diffractive dissociation are updated and summarized with emphasis on the upgraded LHC kinematics. The high missing mass background is modified to meet recent developments in theory and experiment. In the differential cross section of single diffractive dissociation, a structure (dip-bump) is predicted around $t = -1.1$ GeV^2.

1. Single, double, and central diffraction dissociation

We consider diffraction dissociation with the configurations listed below:

$$\text{SD}: \ pp \to \ pX(pY)\,, \tag{1}$$

$$\text{DD}: \ pp \to \ XY\,, \tag{2}$$

$$\text{CD (DPE)}: \ pp \to \ pZp\,, \tag{3}$$

$$\text{CD}_\text{S}: \ pp \to \ XZp\,, \tag{4}$$

$$\text{CD}_\text{D}: \ pp \to \ XZY\,, \tag{5}$$

where X and Y represent diffraction dissociated protons (nucleon resonances), and Z are diffraction produced mesons in the central system. Note that SD (2) implies two symmetric reactions, *i.e.* $p + p' \to X + p'$ and $p + p' \to p + X$.

Single diffraction (SD) and double diffraction (DD) were studied in Refs. [1, 2], where an original approach to the parametrization of the inelastic vertices, based on their similarity with the structure function in deep inelastic

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(DIS) photon–proton scattering, was used. Central diffraction (CD), see *e.g.* [3], is different, both conceptually and technically. In this paper, similarly to [4], we treat SD, DD, and CD on equal footing.

Apart from the update, we present two new results, namely the modified background and the prediction of the dip-bump structure in SD.

2. Compilation of the basic formulae

In this section, we update and compile the main formulas relevant to single (SD)-, double (DD)-, and central (CED) diffractive dissociation.

The basic variables used below are: $M^2 - - - M_1^2$ and M_2^2 are the squared missing masses in SD and DD, respectively, $\alpha(t)$ is the leading Regge trajectory (the contribution from secondary trajectories can be neglected at the LHC), $\beta(t) \approx e^{b\alpha(t)}$ is the Regge residua function, where b is an adjustable parameter; s and t are the standard Mandelstam variables.

The fraction ξ of the momentum of the proton carried by exchanged Pomeron is related to the rapidity gap width (Δy) by $\xi = e^{-\Delta y}$. The variable ξ, in turn, defined as $\xi_{\mathrm{SD}} = M^2/s$ (for SD), $\xi_{\mathrm{DD}} = M_1^2 M_2^2/(s\,s_0)$ (for DD, here s_0 is a universal normalization parameter), and $\xi_{\mathrm{CD}} = \xi_1 \xi_2 = M^2/s$ (for CD). Furthermore ξ is related to the Feynman scaling variable x_{F}: $\xi = 1 - x_{\mathrm{F}}$. The relations between the kinematic variables Δy, ξ, and M^2 are important.

More details concerning the variables and their interrelations can be found in Refs. [1–4].

The SD dissociation differential cross section is given by

$$\frac{\mathrm{d}^2\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}\xi} = \frac{1}{N(s)} \left[\frac{\beta^2(t)}{16\pi}\,(\xi)^{1-2\alpha(t)} \right] \sigma_{\mathrm{T}}^{Pp}(s\xi, t)\,, \tag{6}$$

$$\frac{\mathrm{d}^2\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}M^2} = \frac{1}{N(s)} \left[\frac{1}{M^2}\frac{\beta^2(t)}{16\pi}\left(\frac{M^2}{s}\right)^{2-2\alpha(t)} \right] \sigma_{\mathrm{T}}^{Pp}\left(M^2, t\right)\,, \tag{7}$$

where P stand for the Pomeron, and p for proton.

The DD dissociation cross section reads

$$\frac{\mathrm{d}^3\sigma_{\mathrm{DD}}}{\mathrm{d}t\mathrm{d}M_1^2\mathrm{d}M_2^2} = \frac{1}{N(s)} \left[\frac{1}{16\pi M_1^2 M_2^2}\left(\frac{M_1^2 M_2^2}{s s_0}\right)^{2-2\alpha(t)} \right] \sigma_{\mathrm{T}}^{Pp}(M_1^2, t)\,\sigma_{\mathrm{T}}^{Pp}(M_2^2, t)\,. \tag{8}$$

For the CD dissociation differential cross section, one has

$$\frac{\mathrm{d}^4\sigma_{\mathrm{CD}}}{\mathrm{d}t_1\mathrm{d}t_2\mathrm{d}\xi_1\mathrm{d}\xi_2} = \frac{1}{N(s)\beta^2(0)(s/s_0)^{\alpha(0)-1}} \left[\frac{\beta^2(t_1)}{16\pi}\xi_1^{1-2\alpha(t_1)} \right]$$
$$\times \left[\frac{\beta^2(t_2)}{16\pi}\xi_2^{1-2\alpha(t_2)} \right] \kappa\,\sigma_{\mathrm{T}}^{PP}(s\xi_1\xi_2, t)\,. \tag{9}$$

The cross sections integrated in M^2 and t are

$$\frac{\mathrm{d}\sigma_{\mathrm{SD}}}{\mathrm{d}M^2} = \int_{-1}^{0} \frac{\mathrm{d}^2\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}M^2}\mathrm{d}t \,, \tag{10}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{SD}}}{\mathrm{d}t} = \int_{1.4}^{0.05s} \frac{\mathrm{d}^2\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}M^2}\mathrm{d}M^2 \,, \tag{11}$$

for the case of SD, and

$$\frac{\mathrm{d}^2\sigma_{\mathrm{DD}}}{\mathrm{d}M_1^2\mathrm{d}M_2^2} = \int_{-1}^{0} \frac{\mathrm{d}^3\sigma_{\mathrm{DD}}}{\mathrm{d}t\mathrm{d}M_1^2\mathrm{d}M_2^2}\mathrm{d}t \,, \tag{12}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{DD}}}{\mathrm{d}t} = \int_{1.4}^{0.05s/1.4} \mathrm{d}M_1^2 \int_{1.4}^{0.05s/M_1^2} \frac{\mathrm{d}^3\sigma_{\mathrm{DD}}}{\mathrm{d}t\mathrm{d}M_1^2\mathrm{d}M_2^2}\mathrm{d}M_2^2 \,, \tag{13}$$

for the case of DD.

The integrated cross section in $\log_{10}\xi$ of SD is

$$\frac{\mathrm{d}\sigma_{\mathrm{SD}}}{\mathrm{d}\log_{10}\xi} = \int_{-1}^{0} \frac{\mathrm{d}^2\sigma_{\mathrm{SD}}}{\mathrm{d}t\mathrm{d}\log_{10}\xi}\mathrm{d}t \,. \tag{14}$$

The fully integrated cross sections are

$$\sigma_{\mathrm{SD}} = \int_{1.4}^{0.05s} \mathrm{d}M^2 \int_{-1}^{0} \mathrm{d}t \frac{\mathrm{d}^2\sigma_{\mathrm{SD}}}{\mathrm{d}M^2\mathrm{d}t} \,, \tag{15}$$

$$\sigma_{\mathrm{DD}} = \int_{1.4}^{0.05s} \mathrm{d}M_1^2 \int_{1.4}^{0.05s/M_1^2} \mathrm{d}M_2^2 \int_{-1}^{0} \frac{\mathrm{d}^3\sigma_{\mathrm{DD}}}{\mathrm{d}t\mathrm{d}M_1^2\mathrm{d}M_2^2}\mathrm{d}t \,, \tag{16}$$

$$\sigma_{\mathrm{CD}} = \int_{1.4/s}^{0.05} \mathrm{d}\xi_1 \int_{1.4/s}^{0.05/\xi_1} \mathrm{d}\xi_2 \int_{-1}^{0} \mathrm{d}t_1 \int_{-1}^{0} \mathrm{d}t_2 \frac{\mathrm{d}\sigma_{\mathrm{CD}}^4}{\mathrm{d}t_1\mathrm{d}t_2\mathrm{d}\xi_1\mathrm{d}\xi_2} \,. \tag{17}$$

The explicit form of the nucleon trajectory is given in Refs. [1, 2, 5]. Resonances on this trajectory appear with spins $J = 5/2, 9/2, 13/2, \ldots$

Our basic idea [1] is the dominance of direct-channel resonances

$$\Im A_{\text{res}}^{PP}\left(M_Z^2,\tilde{t}\right) = \sum_{i=f,P}\sum_{J}\frac{\left[f_i(\tilde{t})\right]^{J+2}\Im\alpha_i\left(M_Z^2\right)}{\left(J-\Re\alpha_i\left(M_Z^2\right)\right)^2+\left(\Im\alpha_i\left(M_Z^2\right)\right)^2}$$
$$+F_{\text{BG}}\left(M^2,\tilde{t}\right),\tag{18}$$

where F_{BG} is the high-M^2 smooth background, see the next section.

For CD, it is convenient to use the variables $\Delta\eta=\ln\frac{s}{M_Z^2}$ (rapidity gap) and η_c (the center of the centrally-produced system in η)

$$\frac{\mathrm{d}^4\sigma_{\text{CD}}}{\mathrm{d}t_1\mathrm{d}t_2\mathrm{d}\Delta\eta\mathrm{d}\eta_c} = A_{\text{CD}}\beta^2(t_1)\beta^2(t_2)\tilde{W}_2^{PP}\left(se^{-\Delta\eta},t_1,t_2\right)\tag{19}$$
$$\times e^{\frac{1}{2}[\alpha_P(t_1)-1][\Delta\eta+\eta_c]}e^{\frac{1}{2}[\alpha_P(t_2)-1][\Delta\eta-\eta_c]}.$$

The integrated cross section for CD is given by

$$\sigma_{\text{CD}}=\int\limits_{t_{1\min}}^{t_{1\max}}\mathrm{d}t_1\int\limits_{t_{2\min}}^{t_{2\max}}\mathrm{d}t_2\int\limits_{\Delta\eta_{\min}}^{\Delta\eta_{\max}}\int\limits_{\eta_{c\min}}^{\eta_{c\max}}\frac{\mathrm{d}^4\sigma_{\text{CD}}}{\mathrm{d}t_1\mathrm{d}t_2\mathrm{d}\Delta\eta\mathrm{d}\eta_c},\tag{20}$$

where $t_{1\min}=t_{2\min}=-\infty$, $t_{1\max}=t_{2\max}=0$ GeV^2, $\Delta\eta_{\min}=3$, $\Delta\eta_{\max}=\ln\left(\frac{s}{s_0}\right)$, $s_0=1$ GeV^2, $\eta_{c\min}=-\frac{1}{2}\left(\Delta\eta-\Delta\eta_{\min}\right)$, and $\eta_{c\max}=\frac{1}{2}\left(\Delta\eta-\Delta\eta_{\min}\right)$ [3].

2.1. Dip-bump in SD

A particularly interesting prediction concerns the t dependence of single diffractive dissociation cross section. According to the available data, the differential cross section in the measured range, $t<-1$ GeV^2, is a structureless exponential. SD has much in common with elastic proton–proton scattering. In particular, a diffractive structure in the differential cross section is expected.

This diffractive structure is not seen in the measured range, $t<-1$ GeV^2, see Fig. 1, but its position and shape can be predicted from a model [6], successful in describing high-energy elastic proton–proton scattering. It was tested and summarized in many papers (see, *e.g.*, [5] and references therein). Due to its simplicity and efficiency, the model was used to predict the dip-bump structure in elastic K^+p scattering, where the dip-bump structure was predicted [6] around $t\approx-2$ GeV^2. The prediction depends merely on two parameters, namely b entering the forward slope $B(s)=b+\ln(s/s_0)$, where s_0 is a universal normalization parameter, and λ appearing in the expression of the (logarithmically rising) total cross sections, $\sigma_{\text{t}}=\sigma_0+\lambda\ln(s/s_0)$.
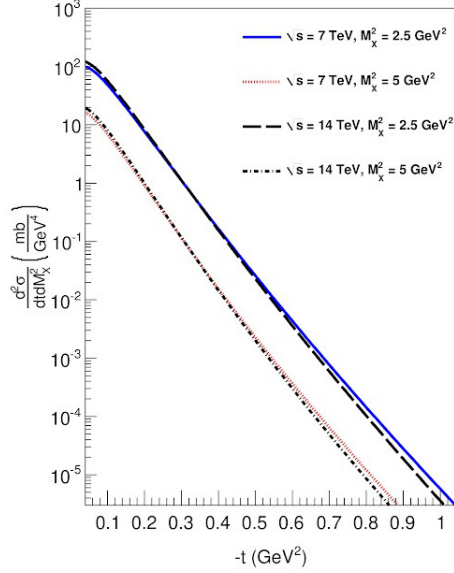
Fig. 1. SD differential cross section at the LHC, from [1].

The invariant spinless high-energy scattering amplitude reads

$$A(s,t) = \frac{is\sigma_0}{\alpha'b} \sum_{i=1}^{2} c_i R_i^2 e^{R_i^2 t} ,$$ (21)

where $c_1 = 1$, $c_2 = \lambda b - \epsilon$, $R_1^2 = \alpha'(b + L - i\pi/2)$, $R_2^2 = \alpha'(L - i\pi/2)$, $L \equiv \ln(s/s_0)$.

As shown in Ref. [6], the position of the dip for linear trajectories is at

$$t_{\text{dip}} = \frac{1}{\alpha'b} \ln \frac{b\gamma L}{b + L}$$ (22)

and the second maximum is at

$$t_{\text{max}} = \frac{1}{\alpha'b} \ln \frac{b\gamma L}{b + L} .$$ (23)

Here, γ is a parameter defined in Ref. [6]. The ratio of the differential cross section at the maximum to that at the minimum is

$$\left(\frac{d\sigma}{dt}\right)_{\text{max}} \bigg/ \left(\frac{d\sigma}{dt}\right)_{\text{min}} \approx L^2 .$$ (24)

The values of the free parameters are fitted to the data on proton–proton scattering. This was done in many papers, the result depending on the

required precision, kinematic range involved, number and quality of the data point, *etc.* It is clear that better fits can be obtained if low energy data and secondary trajectories, as well as proton–antiproton scattering, are involved. Recall that for our purposes (prediction of the dip-bump in SD), only two parameters, namely b and λ (or $\epsilon = 1 - \lambda b$) are important. They can be estimated from fits to the data and/or from model calculations [1]. Note that the value of b in SD and DD, as well as in CD, depends on the masses of excited states, $b \to b(M_i^2)$. As a simple test here, we set the missing mass at its lowest value: $M^2 = (m_p + m_\pi)^2$.

From fits, we have $b = 16.7$ and $\lambda = 0.0583$ (or $\epsilon = 0.0264$), while, according to our models predictions, we have $b = 9.38$ and $\epsilon = 0.027$. Taking the "average" between these two sets of values, we predict the dip in SD to appear near $t = -1.1 \pm 0.1$ GeV^2, followed by a maximum (bump) at $t = -1.3 \pm 0.1$ GeV^2, both within the range of the future experiments at the upgraded LHC[1].

3. Conclusions

We have revised and updated our earlier results on single (SD), double (DD), and central (CED) diffractive dissociation in the kinematic region of the upgraded LHC. We predict a structure in the SD differential cross section, around $t = (-1.1 \pm 0.1)$ GeV^2, similar to the familiar dip-bump in elastic scattering. Future measurements at the upgraded LHC energy may find it.

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