AN EXTREMUM RELATIONSHIP FOR LEPTON MASSES*

R. A. SHAFFER

Department of Physics, Lehigh University,
Bethlehem, Pennsylvania

(*Received 28 November 1966*)

Abstract

It is suggested that the muon and electron have mass values which maximize the dimensionless quantity Γ/Q (decay rate/Q-value) for classes of meson decays and β decays, respectively. As a consequence, $M_{\Xi^-} - M_{\Xi^0}$ is predicted to be 5.2 ± 0.3 MeV.

A RECENT paper [1] suggested a new approach to the problem of the anomalous mass of the muon. The idea is based on the dominance of the $\mu + \nu_\mu$ decay mode of the pion, and kaon, in spite of the large muon-electron mass ratio. Given a $V - A$ weak interaction [2], the muon has, within 0.13 per cent, the mass which maximizes the quantity $\Gamma_{\pi\mu}/Q_{\pi\mu} + \Gamma_{K\mu}/Q_{K\mu}$, where Γ is the decay rate and Q the Q-value for the indicated two-body decay modes.

The above idea has also been applied to the mass of the electron and the corresponding β-decays $\pi^- \to \pi^0 + e^- + \overline{\nu_e}$ and $K^0 \to K^+ + e^- + \overline{\nu_e}$ [3]. (The latter decay has not yet been observed. However, this is as expected because of the dominance of strangeness-changing decay modes for the K-mesons which follows from the much greater energies available in those cases.) With the assumption of a universal, $V - A$, weak interaction, and a conserved-vector current, it was found that the value of the electron's mass which maximizes the quantity $\Gamma_{\pi\beta}/Q_{\pi\beta} + \Gamma_{K\beta}/Q_{K\beta}$ is $0.517\,^{+0.007}_{-0.005}$ MeV. This is to be compared with the electron's mass which is 0.511 MeV.

Whereas $\pi_{\mu 2}$ and $K_{\mu 2}$ are the only two-body decays involving muons, it is interesting to conjecture that the electron's mass maximizes Γ/Q for a larger class of β-decays. We consider this possibility here.

A β-decay is defined as a decay of the type $A \to B + e^- + \overline{\nu_e}$, where A and B are hadrons which are members of the same isotopic multiplet (particles whose mass differences are believed to be due to electromagnetic effects). With this definition there are the meson β-decays indicated above in the pseudoscalar-meson octet and the following decays in the baryon octet:

$$\Xi^- \to \Xi^0,\ \Sigma^- \to \Sigma^0,\ \Sigma^0 \to \Sigma^+ \ \text{and}\ N \to P\ [4].$$

* Research sponsored in part by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation (at Oak Ridge National Laboratory).

To obtain explicit expressions for the matrix elements of the above decays, we use the hadron weak currents suggested by Cabibbo [5] and the couplings determined by Brene, Hellesen and Roos [6]. E.g. for the baryons we have for $A \to B$,

$$< B \left| J^i_{V\lambda} + J^i_{A\lambda} \right| A > = \cos \theta_B \times \bar{u} \left[f^{iBA} \gamma_\lambda \right.$$

$$\left. + \left(\alpha_A d^{iBA} + (1 - \alpha_A) f^{iBA} \right) \beta \gamma_\lambda \gamma_5 \right] u,$$

with $\beta = 1.15$ and $\alpha_A = 0.67 \pm 0.03$. With the assumption that the vector Cabibbo angle, θ_V, equals the baryon angle, θ_B, $\cos \theta$ can be factored from J_λ for all of the decays.

Using the hadron current given by Cabibbo's theory and the $V - A$ lepton current, we obtain for a given decay

$$\Gamma_i / Q_i = C_i E_{0i}{}^4 \left[(1 - x_i{}^2)^{1/2} (2 - 9x_i{}^2 - 8x_i{}^4) \right.$$

$$\left. + 15 x_i{}^4 \ln \left((1 + (1 - x_i{}^2)^{1/2}) / x_i \right) \right] / (1 - x_i),$$

where $E_{0i} = M_A - M_B$, $x_i = M_e / E_{0i}$, and C_i is independent of mass values. We wish to find the value of M_e which, given all other mass values and couplings, maximizes $F = \sum_{i=1}^{6} \Gamma_i / Q_i$. Introducing $R_i = E_{0i} / E_{0\pi}$ and calling $x = M_e / E_{0\pi}$, we have $F = F(x, E_{0\pi}, C_i, R_i)$, and we require x so that $\partial F / \partial x = 0$. Using the tables of Rosenfeld *et al.*, [7] we obtain [8]

$$M_e = 0.57 {}^{+0.08}_{-0.06} \text{ MeV}.$$

The result is not particularly sensitive to the $d - f$ ratio for the axial currents (α_A above). In fact, most of the uncertainty in the above result is provided by the uncertainty in $E_{0\Xi}$. Thus, if $E_{0\Xi} = 6.5 \pm 0.0$ MeV with all other data as above, we would have

$$M_e = 0.57 \pm 0.01 \text{ MeV}.$$

We see that agreement is obtained with the conjecture that the electron's mass maximizes Γ/Q for all β-decays provided that $E_{0\Xi}$ is somewhat less than 6.5 MeV. (It is perhaps interesting to point out that the relatively sparse data on leptonic Ξ-decays was not used in obtaining a value for d/f in Ref. [6]).

Momentum-transfer dependent form-factors, and recoil and radiative corrections were ignored in the above calculations. They could provide a correction of the order of one percent [9]. We suggest, therefore, that $M_{\Xi^-} - M_{\Xi^0} \simeq 5.2 \pm 0.3$ MeV. This is consistent with the experimental data. However, it disagrees slightly with the value predicted by $SU(3)$ [10].

As pointed out [3], the effect of varying a lepton's mass in the case of the β-decays is not as dramatic as that produced in the two-body leptonic decays. Nevertheless, for the β-decays, the non-zero value of the electron's mass enhances Γ/Q by 5 per cent.

Possible leptonic decays of resonances, both meson and baryon, have not been included in the above considerations. There is as yet no information on mass splittings within isotopic multiplets of resonances. But more importantly, it may be necessary to make a fundamental distinction between resonances and long-lived states with regard to weak interactions [11].

The speculative nature of the above considerations is obvious. In considering further this approach to lepton dynamics one might keep the following in mind: The dimensionless character of Γ/Q. The fact that maximizing this quantity for classes of decay processes yields the masses of both charged leptons. (This suggests that there might be a relationship between the quantum numbers associated with the muon and electron and the association of particular decay processes with each lepton.)

Further development of the extremum relationship could make an "explanation" of the muon mass (and the electron mass) in terms of self-energies generated through field interactions unnecessary, and perhaps even meaningless.

To summarize, we suggest that given the Cabibbo theory of weak interactions and the hadron masses, both the muon and electron have masses which maximize the dimensionless quantity Γ/Q for classes of decay processes associated with these particles. This conjecture requires that $M_{\Xi^-} - M_{\Xi^0} \leqslant 5.5$ MeV.

Acknowledgements

The author would like to thank Professor W.G. Holladay, Professor M.A. Melvin, and Dr. T.A. Welton for helpful discussions. Part of this research was done while the author was at the Oak Ridge National Laboratory during the summer of 1966. He would like to thank Dr. J.L. Fowler for the hospitality extended to him at the Laboratory.

References

1. R.A. SHAFFER, *Phys. Rev.* **142**, 990 (1966).

2. R.P. FEYNMAN and M. GELL-MANN, *Phys. Rev.* **109**, 193 (1958); R.E. MARSHAK and E.C.G. SUDARSHAN, *Phys. Rev.* **109**, 1860 (1958). We assume throughout that neutrinos have mass zero.

3. R.A. SHAFFER, *Phys. Rev.* **150**, 1298 (1966).

4. See the first paper under Ref. [2].

5. N. CABIBBO, *Phys. Rev. Letters* **10**, 531 (1963); see also R.E. MARSHAK, University of Rochester Rep. No. UR-875-102 (1965).

6. N. BRENE, B. HELLESEN, and M. ROOS, *Phys. Rev. Letters* **11**, 344 (1964). See also W. WILLIS *et al.*, *Phys. Rev. Letters* **13**, 291 (1964). The first paper lists estimated errors for the *d-f* ratio.

7. A.H. ROSENFELD *et al.*, *Rev. mod. Phys.* **37**, 633 (1965).

8. We use an expansion of Γ/Q correct to $O(x^5 \ln x)$. This results in a possible error of no mo than 0.1 per cent.

9. See, e.g. D.R. HARRINGTON, *Phys. Rev.* **120**, 1482 (1960), and references cited therein.

10. See, e.g. *Proceedings of the International Conference on Weak Interactions*, p. 196. Argon National Laboratory, October 25-27, 1965, ANL-7130.

11. K. HALLER, L.F. LANDOVITZ and I. GOLDBERG, *Nuovo Cim.* **48A**, 303 (1967).