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Radiative corrections to Lorentz-invariance violation with higher-order operators: Fine-tuning problem revisited

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Abstract

We study the possible effects of large Lorentz violations that can appear in the effective models in which the Lorentz symmetry breakdown is performed with higher-order operators. For this we consider the Myers and Pospelov extension of QED with dimension-five operators in the photon sector and standard fermions. We focus on the fermion self-energy at one-loop order and find small and finite radiative corrections in the even CPT sector. In the odd CPT sector a lower dimensional operator is generated which contains unsuppressed effects of Lorentz violation leading to a possible fine-tuning. For the calculation of divergent diagrams we use dimensional regularization and consider an arbitrary background four-vector.

1. Introduction

New physics standing in the form of Lorentz symmetry violation has been a starting point for several effective models beyond the standard model [1]. A low energy remnant of this type is strongly motivated by the idea that spacetime changes drastically due to the appearance of some level or discreteness or spacetime foam at high energies. The effective approach has been shown to be extremely successful in order to contrast the possible Lorentz and CPT symmetry violations with experiments. A great part of these searches have been given within the framework of the standard model extension with several bounds on Lorentz symmetry violation provided [2, 3, 4]. In general most of the studies on Lorentz symmetry violation have been performed with operators of mass dimension $d \leq 4$, [5]. In part because the higher-order theories present some problems in their quantization [6]. However, in the last years these operators have received more attention and several bounds have been put forward [7, 8, 9, 10, 11]. Moreover, a generalization has been constructed to include non-minimal terms in the effective framework of the standard model extension [12].

Many years ago Lee-Wick [13] and Cutkosky [14] studied the unitarity of higher-order theories using the formalism of indefinite metrics in Hilbert space. They succeeded to prove that unitarity can be conserved in some higher-order models by restricting the space of asymptotic states. This has stimulated the construction of several higher-order models beyond the standard model [15]. One example is the Myers and Pospelov model based on dimension-five operators describing possible effects of quantum gravity [16, 17]. In the model the Lorentz symmetry violation is characterized by a preferred four-vector n [18, 19]. The preferred four-vector may be thought to come from a spontaneous symmetry breaking in an underlying fundamental theory. One of the original motivations to incorporate such terms was to produce cubic modifications in the dispersion relation, although an exact calculation yields a more complicated structure usually with the

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gramian of the two vectors k and n involved. The Myers and Pospelov model has become an important arena to study higher-order effects of Lorentz-invariance violation [8, 20, 21, 22].

This work aims to contribute to the discussion on the fine-tuning problem due to Lorentz symmetry violation [23], in particular when higher-order operators are present. There are different approaches to the subject, for example using the ingredient of discreteness [24] or supersymmetry [25]. For renormalizable operators, including higher space derivatives, large Lorentz violations can or cannot appear depending on the model and regularization scheme [26]. However, higher-order operators are good candidates to produce strong Lorentz violations via induced lower dimensional operators [27]. Some attempts to deal with the fine tuning problem considers modifications in the tensor contraction with a given Feynman diagram [16] or just restrict attention to higher-order corrections [28]. However in both cases the problem comes back at higher-order loops [29]. Here we analyze higher-order Lorentz violation by explicitly computing the radiative corrections in the Myers and Pospelov extension of QED. We use dimensional regularization which eventually preserves unitarity, thus extending some early treatments [18, 20].

2. Lorentz fine-tuning

Consider the Yukawa model [23, 30]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 + \bar{\psi}(i\not{\partial} - m)\psi + g_Y\bar{\psi}\phi\psi \ , \quad (1)$$

where the Lorentz violation was implemented by modifying the propagators. In particular, the fermion propagator changes as

$$\frac{i}{\not{k} - m + i\epsilon} \longrightarrow \frac{i f\left(|\vec{k}|/\Lambda\right)}{\not{k} - m + i\epsilon} \ , \quad (2)$$

where Λ is an explicit cutoff and $f\left(|\vec{k}|/\Lambda\right)$ obeys $f(0) = 1$ and $f(\infty) = 0$. In this way the scalar self-energy $\Pi(p)$ is given by

$$\Pi(p) = -ig_Y^2 \int \frac{d^4k}{(2\pi)^4} f\left(|\vec{k}|/\Lambda\right) f\left(|\vec{k} + \vec{p}|/\Lambda\right) \times \mathrm{Tr}\left[\frac{1}{(\not{k} - m + i\epsilon)}\frac{1}{(\not{k} + \not{p} - m + i\epsilon)}\right] . \quad (3)$$

The above integral has been ultraviolet regularized and is therefore convergent. We expect to recover the usual divergencies in the limit $\Lambda \to 0$ of the first terms of the expansion around $p = 0$

$$\Pi(p) = \Pi(0) + \frac{p_0 p_0}{2!}\left(\frac{\partial^2\Pi(p)}{\partial p_0 \partial p_0}\right)_{p=0} + \frac{p_i p_i}{2!}\left(\frac{\partial^2\Pi(p)}{\partial p_i \partial p_i}\right)_{p=0} + \mathrm{conv} \ . \quad (4)$$

Above we have considered that the mixed terms

$$\frac{\partial\Pi(0)}{\partial p_\mu}, \quad \frac{\partial^2\Pi(0)}{\partial p_0 \partial p_i}, \quad \frac{\partial^2\Pi(0)}{\partial p_i \partial p_j} \ , \quad (5)$$

vanish. We have from rotational invariance

$$\Pi(p) = \Pi(0) + \frac{1}{2!}(p_0^2 - \vec{p}^2)\left(\frac{\partial^2\Pi(p)}{\partial p_1 \partial p^1}\right)_{p=0} + p_0^2\, \eta_{LV} + \Pi_\Lambda \ , \quad (6)$$

where the Lorentz violation is parametrized by the quantity

$$\eta_{LV} = \frac{1}{2!}\left(\frac{\partial^2\Pi(p)}{\partial p_0 \partial p_0} + \frac{\partial^2\Pi(p)}{\partial p_1 \partial p_1}\right)_{p=0} . \quad (7)$$

After some calculation, both contributions give

$$\left(\frac{\partial^2\Pi(p)}{\partial p_0 \partial p_0} + \frac{\partial^2\Pi(p)}{\partial p_1 \partial p_1}\right)_{p=0} = -ig_Y^2 \int \frac{d^4k}{(2\pi)^4} \times f(|\vec{k}|/\Lambda)\left(-2\left(\frac{\partial f\left(|\vec{k} + \vec{p}|/\Lambda\right)}{\partial p_1}\right)_{p=0} \times \mathrm{Tr}\left[\frac{1}{(\not{k} - m)}\gamma^1\frac{1}{(\not{k} - m)^2}\right] + \left(\frac{\partial^2 f\left(|\vec{k} + \vec{p}|/\Lambda\right)}{\partial p_1^2}\right)_{p=0} \mathrm{Tr}\left[\frac{1}{(\not{k} - m)^2}\right]\right) . \quad (8)$$

The dominant term can be obtained by setting $m = 0$

$$\eta_{LV} = -2ig_Y^2 \int \frac{d^4k}{(2\pi)^4}\frac{f(|\vec{k}|/\Lambda)}{k^2} \times \left(\frac{\partial^2 f(|\vec{k} + \vec{p}|/\Lambda)}{\partial p_1^2}\right)_{p=0} . \quad (9)$$

Integrating in k_0, and changing $\left(\frac{\partial^2 f\left(\frac{|\vec{k}+\vec{p}|}{\Lambda}\right)}{\partial p_1^2}\right)_{p=0} = \frac{\partial^2 f\left(\frac{|\vec{k}|}{\Lambda}\right)}{\partial k_1^2}$, we have

$$\eta_{LV} = -\frac{2g_Y^2}{16\pi^3}\int \frac{d^3k}{(2\pi)^4}\frac{1}{|\vec{k}|} f\left(|\vec{k}|/\Lambda\right)\frac{\partial^2 f(|\vec{k}|/\Lambda)}{\partial k_1^2} \ , \quad (10)$$

and using $\frac{\partial^2 f\left(|\vec{k}|/\Lambda\right)}{\partial k_1^2} = \frac{f''(x)}{3\Lambda^2} + \frac{2f'(x)}{3x\Lambda^2}$ inside the integral where the derivatives are with respect to $x = |\vec{k}|/\Lambda$, we arrive at

$$\eta_{LV} = \frac{g_Y^2}{12\pi^2}\left(1 + 2\int_0^\infty dx\, x\, (f'(x))^2\right) . \quad (11)$$

We see that the Lorentz violation is not suppressed by the ultraviolet scale. The term η_{LV} can be interpreted as a modification of the velocity of light as can be seen from the correction of the dispersion relation $E = \sqrt{\vec{p}^2c^2 + m^2c^4}$ in the form $E^2 - \vec{p}^2 - m^2 - \Pi(p) = 0$. By using the value of the standard model couplings one can set the value $\eta_{LV} \geq 10^{-3}$. On the other hand, from the well tested bound on c one has $\eta_{LV} \leq 10^{-20}$, which is clearly in conflict with the above result. This leads to a fine-tuning of the wave function in order to produce an acceptable size of the radiative correction.

3. The QED extension with dimension-5 operators

The Myers-Pospelov Lagrangian extension of QED with modifications in the photon sector can be written as [16]

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\xi}{2m_{\rm Pl}} \\ &\times \; n_\mu \epsilon^{\mu\nu\lambda\sigma} A_\nu (n\cdot\partial)^2 F_{\lambda\sigma} \,, \qquad (12) \end{aligned}$$

where $m_{\rm Pl}$ is the Planck mass, ξ a dimensionless coupling parameter and n is a four-vector defining a preferred reference frame. In addition we introduce the gauge fixing Lagrangian term, $\mathcal{L}_{G.F} = -B(x)(n\cdot A)$, where $B(x)$ is an auxiliary field.

The field equations for A_μ and B derived from the Lagrangian $\mathcal{L} + \mathcal{L}_{G.F}$ read,

$$\partial_\mu F^{\mu\nu} + g\epsilon^{\nu\alpha\lambda\sigma} n_\alpha (n\cdot\partial)^2 F_{\lambda\sigma} = Bn^\nu \,, \qquad (13)$$

$$n\cdot A = 0 \,, \qquad (14)$$

where $g = \frac{\xi}{m_{\rm Pl}}$. Contracting Eq. (13) with ∂_ν produces $(\partial\cdot n)B = 0$, which allows us to set $B = 0$. In the same way, the contraction of Eq. (13) with n_ν in momentum space, leads to $k\cdot A = 0$.

We can choose the polarization vectors $e^{(a)}_\mu$ with $a = 1, 2$ to lie on the orthogonal hyperplane defined by k and n [31], satisfying $e^{(a)}\cdot e^{(b)} = -\delta^{ab}$ and

$$\begin{aligned} -\sum_a (e^{(a)}\otimes e^{(a)})_{\mu\nu} &= -(e^{(1)}_\mu e^{(1)}_\nu + e^{(2)}_\mu e^{(2)}_\nu) \\ &\equiv e_{\mu\nu} \,, \qquad (15) \\ \sum_a (e^{(a)}\wedge e^{(a)})_{\mu\nu} &= e^{(1)}_\mu e^{(2)}_\nu - e^{(2)}_\mu e^{(1)}_\nu \\ &\equiv \epsilon_{\mu\nu} \,. \qquad (16) \end{aligned}$$

In particular, one can set

$$\begin{aligned} e^{\mu\nu} &= \eta^{\mu\nu} - \frac{(n\cdot k)}{D}(n^\mu k^\nu + n^\nu k^\mu) + \frac{k^2}{D}n^\mu n^\nu \\ &\quad + \frac{n^2}{D}k^\mu k^\nu \,, \qquad (17) \\ \epsilon^{\mu\nu} &= \frac{1}{\sqrt{D}}\epsilon^{\mu\alpha\rho\nu} n_\alpha k_\rho \,, \qquad (18) \end{aligned}$$

with $D = (n\cdot k)^2 - n^2k^2$. The photon propagator can be written as

$$\Delta_{\mu\nu}(k) = -\sum_{\lambda=\pm 1} \frac{P^{(\lambda)}_{\mu\nu}(k)}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}} \,, \qquad (19)$$

where

$$P^{(\lambda)}_{\mu\nu} = \frac{1}{2}(e_{\mu\nu} + i\lambda\epsilon_{\mu\nu}) \,. \qquad (20)$$

is the orthogonal projector

4. The fermion self-energy

We compute the fermion self-energy with the modifications introduced only via the Lorentz violating photon propagator (19). The one loop-order approximation to the fermion self-energy is

$$\Sigma(p) = ie^2\int \frac{d^4k}{(2\pi)^4}\gamma^\mu \left(\frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2}\right)\gamma^\nu \Delta_{\mu\nu}(k) \,, \qquad (21)$$

which can be decomposed into its CPT even part

$$\begin{aligned} \Sigma^{(+)}(p) &= -\frac{ie^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\gamma^\mu \left(\frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2}\right) \\ &\quad \times \frac{\gamma^\nu e_{\mu\nu}}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}} \,, \qquad (22) \end{aligned}$$

and CPT odd part

$$\begin{aligned} \Sigma^{(-)}(p) &= -\frac{ie^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\gamma^\mu \left(\frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2}\right) \\ &\quad \times \frac{\gamma^\nu i\lambda\epsilon_{\mu\nu}}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}} \,. \qquad (23) \end{aligned}$$

Next we expand in powers of external momenta obtaining

$$\Sigma(p) = \Sigma(0) + p_\alpha \left(\frac{\partial\Sigma(p)}{\partial p_\alpha}\right)_{p=0} + \Sigma_g \,, \qquad (24)$$

where Σ_g are convergent terms in the limit $g \to 0$ depending on quadratic and higher powers of p.

The strategy to compute the next integrals is:

i) Perform a Wick rotation to $k_E = (ik_0, \vec{k})$, and extend analytically the preferred four vector to $n_E = (in_0, \vec{n})$.

ii) Use dimensional regularization in spherical coordinates for divergent integrals.

To begin, we are interested on the first two even contributions in Eqs. (22) and (24), which are

$$\Sigma^{(+)}(0) = -\frac{ime^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)} \times \frac{\gamma^\mu e_{\mu\nu}\gamma^\nu}{k^2+2g\lambda(k\cdot n)^2\sqrt{D}}, \quad (25)$$

$$\frac{\partial\Sigma^{(+)}(0)}{\partial p_\alpha} = -\frac{ie^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2-m^2)} - \frac{2k_\alpha^2}{(k^2-m^2)^2}\right]\frac{\gamma^\mu\gamma^\alpha\gamma^\nu e_{\mu\nu}}{k^2+2g\lambda(k\cdot n)^2\sqrt{D}}. \quad (26)$$

Following our strategy leads to

$$\Sigma^{(+)}(0) = e^2 m\sum_\lambda \int \frac{d^4k_E}{(2\pi)^4} \times \frac{1}{(k_E^2+m^2)(k_E^2-2g\lambda(k_E\cdot n_E)^2\sqrt{D_E})}, \quad (27)$$

$$\frac{\partial\Sigma^{(+)}(0)}{\partial p_\alpha} = -\frac{e^2}{2}(n_\nu n^\alpha - \frac{n_E^2}{2}\eta_\nu^\alpha)\gamma^\nu\sum_\lambda \int \frac{d^4k_E}{(2\pi)^4} \times \left[\frac{1}{(k_E^2+m^2)} - \frac{k_E^2}{2(k_E^2+m^2)^2}\right] \times \frac{k_E^2}{D_E}\frac{1}{(k_E^2-2g\lambda(k_E\cdot n_E)^2\sqrt{D_E})}, \quad (28)$$

where we have used $\gamma^\mu e_{\mu\nu}\gamma^\nu = 2$ and $D_E = (n_E\cdot k_E)^2 - k_E^2 n_E^2$. The calculation produces

$$\Sigma^{(+)}(0) = \frac{e^2 m}{8\pi^2}\left(1-\ln\left(\frac{g^2m^2(n_E^2)^3}{16}\right)\right), \quad (29)$$

$$p_\alpha\frac{\partial\Sigma^{(+)}(0)}{\partial p_\alpha} = -\frac{e^2}{16\pi^2}\left(\frac{1}{2}\not{p} - \frac{\not{n}(n\cdot p)}{n_E^2}\right) \times \left(1+\ln\left(\frac{g^2m^2(n_E^2)^3}{16}\right)\right).$$

Let us emphasize that the renormalization in the even sector involves small corrections without any possible fine-tuning. Also, the radiative corrections to the mass and wave function are finite and have the usual logarithmic divergence in the limit $g \to 0$.

Now we compute the lower dimensional operator $\bar{\psi}\not{n}\gamma_5\psi$, which arises in the radiative correction of the odd sector. According to Eqs. (23) and (24) it comes from

$$\Sigma^{(-)}(0) = -2ge^2(\epsilon_{\mu\alpha\beta\nu}n^\alpha\gamma^\mu\gamma^\sigma\gamma^\nu)\int\frac{d^4k}{(2\pi)^4} \times \frac{k^\beta k_\sigma}{(k^2-m^2)}\frac{(n\cdot k)^2}{((k^2)^2-4g^2(k\cdot n)^4D)}. \quad (30)$$

We can extract the correction from the most general form of the above integral $F\,\delta^\beta_\sigma + R\,n^\beta n_\sigma$, considering $\epsilon_{\mu\alpha\beta\nu}n^\alpha\gamma^\mu\gamma^\beta\gamma^\nu = 3!i\not{n}\gamma^5$, which requires to find

$$F = -\frac{2ge^2}{3n^2}\int\frac{d^4k}{(2\pi)^4} \times \frac{D(n\cdot k)^2}{(k^2-m^2)((k^2)^2-4g^2(k\cdot n)^4D)}. \quad (31)$$

For this divergent element we have in d dimensions

$$F = \frac{2ige^2n_E^2}{(d-1)}\mu^{4-d}\int\frac{d\Omega}{(2\pi)^d}\sin^2\theta\cos^2\theta \times \int_0^\infty\frac{d|k_E|\,|k_E|^{d-1}M^2}{(|k_E|^2+m^2)(|k_E|^2+M^2)}, \quad (32)$$

where $M^2 = \frac{1}{4g^2n_E^6\sin^2\theta\cos^4\theta}$, and θ is the angle between n_E and k_E and $|k_E| = \sqrt{k_E^2}$. Next considering the solid angle element in d dimensions,

$$\int d\Omega = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}\int_0^\pi d\theta(\sin\theta)^{d-2}, \quad (33)$$

and using the approximation $M^2 \gg m^2$ with $d = 4-\varepsilon$ the dominant contribution is

$$F = -\frac{ie^2}{\varepsilon\pi^2}\left(\frac{1}{24g(n^2)^2} + \frac{gm^2n^2}{96}\right) + \frac{ie^2}{1152g\pi^2(n^2)^2}\Big[-24\ln(g^2\pi\mu^2(n^2)^3)) - 160 + 24\gamma_E + 8\ln(64)\Big]. \quad (34)$$

We see that the presence of the high scale g is to produce small and finite corrections in the CPT even sector (29) and to fine-tune the parameters in the CPT odd sector of the theory. To deal with the large Lorentz corrections in the odd sector we can take a step further in the renormalization program. We consider the usual subtraction for the mass by choosing the physical mass m_P to be the pole of the renormalized 2-point function, which in our Lorentz violating theory also depends on $(n\cdot p)$. Let us write accordingly

$$iG^R(\not{p},(n\cdot p)) = \frac{i}{\not{p} - m_R + \Sigma_R(\not{p},(n\cdot p))}. \quad (35)$$

In order to have a good limit of the usual theory when taking the limit $g \to 0$ we must choose the renormalization points around $\not{p}$ and $(n \cdot p)$ [32]. Only after this procedure is implemented we may complete the study on the possible fine-tuning aspects of the theory.

5. Conclusions

Effective field theory provides a very powerful tool in order to check for consistent Lorentz symmetry violation at low energies. This is especially true for effective theories with higher-order operators where operators generated via radiative corrections may be unprotected against fine-tuning. However, in the Myers and Pospelov model we have shown that the same symmetries that allow an operator to be induced will also dictate the size of the correction.

We have considered the even and odd CPT parts coming from modifications in the photon propagator. We have shown that the radiative corrections to the even CPT sector are given by small contributions to the usual parameters of the standard model couplings. On the contrary, in the odd CPT sector we have found large Lorentz violations in the induced axial operator of mass dimension-3. For the calculation we have used dimensional regularization in order to preserve unitarity and considered a general background which incorporates the effects of higher-order time derivatives.

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