Measurements of quantum fields and the Unruh effect: a particle detector perspective

by

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Statement of Contributions

Jose de Ramon Rivera was the sole author this thesis, although portions of this thesis are direct adaptations of collaborative publications. Chapters 1, 2, 3, 6 and 9 are new content produced for this thesis by Jose de Ramon Rivera alone.

Chapter 4 is adapted for this thesis from the publication [1] which was prepared in collaboration with Jose's supervisor Eduardo Martin-Martinez and Luis Garay. Jose de Ramon Rivera conducted a majority of the research on this project with some guidance/input from his supervisors and collaborators. Jose de Ramon Rivera wrote the draft manuscript alone which was then collaboratively edited.

Chapter 5 is adapted for this thesis from the publication [2] which was prepared in collaboration with Jose's supervisor Eduardo Martin-Martinez and Maria Papageorgiou. Jose de Ramon Rivera conducted a majority of the research on this project with some guidance/input from his supervisors and collaborators. Jose de Ramon Rivera wrote the draft manuscript alone which was then collaboratively edited.

Chapter 7 is adapted for this thesis from the publication [3] which was prepared in collaboration with Jose's supervisor Eduardo Martin-Martinez and Luis Garay. Jose de Ramon Rivera conducted a majority of the research on these projects with some guidance/input from his supervisor and collaborators. Jose de Ramon Rivera wrote the draft manuscript alone which was then collaboratively edited.

Chapter 8 is an original extension of the contents published [4] which was prepared in collaboration with Jose's supervisor Eduardo Martin-Martinez, Luis Garay and Raul Carballo-Rubio.

Abstract

This thesis is devoted to the study of detector models in quantum field theory. A detector model aims to describe a hypothetical physical situation, realistic or not, in which a measurement is performed on a quantum field by means of a quantum mechanical apparatus. The goal is to design a procedure by which one can analyze local features of the field theory with a strong physical base. Within this context, we will analyze the so-called Unruh effect, which states that accelerated observers will experience the ordinary field's vacuum fluctuations as thermal fluctuations at a temperature proportional to their acceleration.

The text covers an adapted introduction to quantum field theory and the Unruh effect, which contextualizes and links the original contents developed by the author. These original works discuss the following topics:

- A particle detector-based measurement of the two-point function in quantum fields.
- Causality issues and impossible measurements in general detector models in QFT.
- The non-monotonic behavior of thermodynamic features of accelerated detectors, also known as Anti-Unruh effect.
- Robustness of the Unruh effect against ultra-violet deformations of the correlations.

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Table of Contents

List of Figures x

List of Tables xiii

1 Introduction 1

2 Quantum field theory 8

2.1 Canonical quantization . . . 9

2.1.1 Issues with canonical quantization . . . 12

2.2 Covariant quantization of the Klein-Gordon field . . . 15

2.3 Measurement theory of quantum fields . . . 18

3 Particle detectors 22

3.1 Particle detectors and interaction Hamiltonians . . . 22

3.2 Detectors, statistics and responses . . . 28

3.2.1 The point-like UDW model . . . 34

4 From detecting particles to measuring fluctuations 39

4.1 Sequences of interactions . . . 40

4.1.1 Stationary correlations . . . 42

4.2 Strong and fast couplings . . . 43

4.2.1 Comb of interactions . . . 47

4.3 Probing the two-point correlator with fast kicks 49
4.4 Discussion 52

5 The price to pay: Violations of causality in particle detector physics 55
5.1 Faster-than-light signaling in detector models 56
5.2 "Impossible measurements" and superluminal propagation of initial data 62
5.3 Impossible measurements with weakly coupled detectors 67
5.4 Discussion 69

6 The Unruh effect 71
6.1 The Unruh effect without detectors 72
6.1.1 The KMS condition 80
6.1.2 Some comments on the algebraic derivation of the Unruh effect 85
6.2 The Unruh effect and the response of particle detectors 86

7 The Anti-Unruh effect 91
7.1 The Anti-Unruh effect and the KMS condition 92
7.1.1 $\tilde{C}$ does not depend on the KMS parameter 94
7.1.2 $\tilde{C}$ depends on the KMS parameter 95
7.2 Parameter space dependence of the Anti-Unruh phenomena 98
7.3 Discussion 100

8 The Unruh effect without thermality 103
8.1 Motivation 103
8.1.1 Deformations of the two-point correlator 104
8.2 Infinitely adiabatic limit for uniformly accelerated detectors 105
8.2.1 Accelerated trajectories 107
8.3 Preservation of the Unruh effect 110
8.4 Examples 112

8.4.1 No deformation 112
8.4.2 Lorentz-invariant deformations 113
8.4.3 Lorentz-violating deformations 115
8.4.4 Regarding the KMS condition 117

9 Conclusions **119**

References **123**

List of Figures

5.1 Causal relations between simply connected, non intersecting sets (grey and white) in two spacetime dimensions. Black lines represent the future or past lightcone of the sets or points between them. $a), b)$: Examples of non causally orderable sets. $c), d)$: Examples of sets that are causally orderable, but that do not causally precede each other according to our definition. $e)$: Spacelike separated sets. $f)$: Example of a set causally preceding another set. 58

5.2 Two detectors A and B are coupled to the field over regions $\mathcal{O}_{\text{A,B}}$. The initial data are encoded in the field state through a unitary intervention over region $\mathcal{O}_{\text{C}}$. Notice that region $\mathcal{O}_{\text{A}}$ is partially invading the past and future lightcones of regions $\mathcal{O}_{\text{B}}$ and $\mathcal{O}_{\text{C}}$ respectively. 63

6.1 Here we represent the Minkowski diagram. Regions I and II are characterized by $t \leq |x_1|$ and are commonly known as Rindler wedges. The blue lines represent surfaces of constant Rindler coordinate η, whereas the red lines represent surfaces of constant Rindler coordinate ξ. For positive values of η the surfaces are straight lines of positive slope, whereas positive values of ξ represent the red curves that lay in region I. 75

7.1 *Weak Anti-Unruh effect*: Derivative of the response function with respect to the KMS temperature $T_{\text{KMS}} = 1/\beta$ for 1+1D, $m = 1$. The different lines correspond to values of $\Omega = 15$ (inverted purple triangles), $\Omega = 10$ (red triangles), $\Omega = 5$ (green rhombi), $\Omega = 2$ (orange squares), $\Omega = 0.5$ (blue circles). The two plots represent the short and the long time regimes. Namely **Right.** $\sigma = 1$ with a Gaussian switching $\chi(\tau/\sigma) = \pi^{-1/4}e^{-\tau^2/(2\sigma^2)}$, and **Left.** $\sigma \to \infty$, independently of the switching. We see that for a broad range of the parameters this derivative is negative (i.e., the response function decreases as the KMS temperature increases), even for adiabatic (eternal) switching. 96

7.2 *Strong Anti-Unruh effect*: Figure *a)* shows the EDR temperature $T_{\text{KMS}} = 1/\beta$ as a function of the KMS temperature for 1+1D, $m = 1$, $\sigma = 0.04$, and $\Omega = 1$. For large KMS temperatures, $T_{\text{EDR}} \simeq T_{\text{KMS}}$, while for small ones the EDR temperature actually decreases, as seen in the zoomed Fig. *b)*. Figure *c)* displays (for $m = 1$, $\sigma = 0.04$, and $T_{\text{KMS}} = 8$) the apparent linearity of Ω/T_{EDR} with Ω and hence the almost independence of the EDR temperature with Ω. Figure *d)* shows (for the same parameters as Fig. *c)*) that this dependence is actually present although it is extremely weak. 97

7.3 *Anti-Unruh effect dependence on the mass*: For any switching function shape, in the limit of infinite interaction time $\sigma \to \infty$, we show the derivative of the response function with respect to the KMS temperature $T_{\text{KMS}} = 1/\beta$ for 1+1D and for $m = 0.1$ (Left) and $m = 10$ (Right). The different lines correspond again to values of $\Omega = 15$ (inverted purple triangles), $\Omega = 10$ (red triangles), $\Omega = 5$ (green rhombi), $\Omega = 2$ (orange squares), $\Omega = 0.5$ (blue circles). The right figure shows how for $m \ll T_{\text{KMS}}$ the oscillations of the derivative generate Anti-Unruh effect in the low temperature zone, whereas the left shows that when the KMS temperature drops below tha mass scale (in this case $m = 10$), the AntiUnruh effect disappears. We see in both figures that the Anti-Unruh effect can exist for values of Ω below and above the mass scale m. 99

8.1 The contour γ_i encloses all the poles $\{\bar{z}_j, \bar{z}_k, \bar{z}_l\}$ that are originated from the isolated pole z_i^0 of the undeformed Wightman function. . . . 111

8.2 Complex poles of the undeformed Wightman function. 112

8.3 Displacement of the poles for the deformation in equation (8.36). . . . 114

8.4 Displacement of the poles for the deformation that is obtained by changing the sign in front of ℓ^2 in equation (8.36). 115

List of Tables

8.1 Comparison of the KMS condition and the sufficient conditions for the preservation of the Unruh effect in the adiabatic limit. Note that "Imaginary periodicity" refers to the property of the Wightman function, not its pull-back 118

Chapter 1

Introduction

This thesis is devoted to the study of particle detector models, and the role they have played and may have yet to play in our understanding of a variety of results in quantum field theory, such as the Unruh effect. Roughly speaking, a particle detector model aims to describe a situation in which a quantum mechanical device, the “detector”, interacts locally with a quantum field. After the interaction, the statistics of the detector will generally depend on the state of the quantum field prior to the interaction “around” the spacetime region in which the interaction has taken place. Hence, particle detector models provide a conceptual tool to study local aspects of QFT.

When exposed to this prescription of what a detector model is and what it is expected to achieve, the reader’s mind will possibly follow one of these streams of thought:

- The reader that is, in some sense, more philosophically methodical will start questioning the very concept, necessity or usefulness of such models for studying local aspects of QFT.

- The reader with a more pragmatic attitude will wonder what realistic physical systems could play such a role.

Indeed, the first kind of reader would request a model-independent justification for the use of detector models. Conversely, the second kind reader could be interested in detector protocols by themselves as physical processes, rather than as a tool to conceptualize QFT quantities. After all, one could argue that the experimental procedures that induce actual measurements of quantum fields necessarily pass through some kind of detector system, e.g a photodetector.

It is legitimate to wonder why a particular quantum field theory would require the introduction of “external” objects that do not belong to the field theory itself in order to extract local information from the quantum field. The rough idea is that the mathematical objects available from the field theory are difficult to interpret physically. Whereas at the level of the whole Hilbert space one can link the eigenstates of, the number operator to the phenomenology associated with particles, as is customary in high energy physics, it is not immediate to link the mathematical objects to physical quantities when considering local observables and local operations. In contrast, a device playing the role of a detector can be analyzed in more mundane terms (at least as mundane as quantum mechanics allows). A typical question one may find difficult to answer, e.g., is whether a field’s state resembles a thermal reservoir when restricted to a spacetime wedge and in what sense. This is the central question posed by the so-called Unruh effect, and constitutes an example of which an answer is given by inquiring whether a detector undergoing an accelerated trajectory reaches thermal equilibrium with the field, as it would with a thermal state.

What are the challenges in interpreting QFT quantities? We will tackle this in due time, but first, let us recall some commonly accepted facts for finite-dimensional quantum theory. According to quantum theory, physical systems can be represented by Hilbert spaces. Mutually exclusive physical properties are then associated with orthogonal closed subspaces of this Hilbert space, which can be represented by projectors. One-dimensional projectors represent properties that are elementary, and these can be represented by rays of vectors in the Hilbert space. A set of subspaces in which each subspace is given by the span of an element of an orthogonal basis represents therefore an exclusive and exhaustive set of possibilities [5, 6].

More generally, one can have continuous properties, in which one has a countably additive function that associates subsets of some Borel set to subspaces of the Hilbert space, in such a way that whenever two sets are disjoint the associated projectors are orthogonal, and that the projector associated with the whole set is the identity in the Hilbert space. Such a function is known in mathematics as a projector-valued measure (PVM). There is a one-to-one correspondence between self-adjoint operators in the Hilbert space and PVM’s [7].

States of the system are represented by positive linear functionals over the space of bounded operators in this Hilbert space, in such a way that for each exhaustive and exclusive set of properties we have a probability distribution. It follows that states can also be represented by operators, at least in dimensions greater than two. This result, known as Gleason's theorem [8], states that any probability measure over the closed subspaces of a Hilbert space of dimension greater than two can be written as

$$\langle \cdot \rangle = \mathrm{tr}[\cdot \hat{\rho}] \tag{1.1}$$

for some trace class, unit-trace, positive operator $\hat{\rho}$, the so-called density matrix. Note then that Gleason's result can be understood as a first principles derivation of the so-called Born's rule. States that cannot be written as a convex combinations of other states are called pure states. Importantly, pure states are represented by rank-one projectors, therefore establishing a duality between states and properties.

Regarding quantum measurements, a pragmatic definition of measurement is just some procedure that grants empirical access to the probabilities as calculated by Born's rule for a given preparation of the system. In this thesis we will remain agnostic about the nature of the state of the system as either ontic or epistemic, and we will not commit to particular interpretation of what happens to the system whenever a property is measured. We will consider, however, the notion of non-selective measurement, which does not require to introduce a state update rule in the theory. Indeed, let $\hat{\rho}_0$ be the initial state before a measurement. Consider the spectral decomposition of an observable $\hat{A}$, i.e., $\hat{A} = \sum_{\mathrm{A}} a \hat{P}_{\mathrm{A}}$. After a non-selective measurement of $\hat{A}$ the state of the system, denoted as $\hat{\rho}|_{\mathrm{A}}$, is given by

$$\hat{\rho}|_{\mathrm{A}} = \sum_{\mathrm{A}} \hat{P}_{\mathrm{A}} \hat{\rho}_0 \hat{P}_{\mathrm{A}}. \tag{1.2}$$

This operation is considered a measurement because it transforms a state into a probabilistic mixture of states $\hat{P}_{\mathrm{A}}$ associated with each outcome, with a weight given by $\mathrm{tr}[\hat{P}_{\mathrm{A}} \hat{\rho}_0]$, i.e. Born's rule. This map does not require the introduction of new dynamics, since it can be implemented through environmental decoherence [9].

An important concept for our purposes is statistical independence of measurements. Consider for example two commuting observables $[\hat{A}, \hat{B}] = 0$ of a closed quantum system. A non-selective measurement of observable $\hat{A}$ does not affect the expectation value of $\hat{B}$. Since $[\hat{A}, \hat{B}] = 0$, it holds that $[\hat{P}_{\mathrm{A}}, \hat{B}] = 0 \ \forall a$. Making use of this, we can easily see that

$$\mathrm{tr}(\hat{B} \hat{\rho}|_{\mathrm{A}}) = \mathrm{tr}(\hat{B} \hat{\rho}_0), \tag{1.3}$$

which means that the expectation value of $\hat{B}$ does not depend on a non-selective measurement of observable $\hat{A}$.

Finally, we mention that one can gain some insights with respect to measurements by considering a more detailed account of how experiments are really carried out. For instance, in quantum optics and high energy physics experiments measurements are usually thought of as destructive. The system under consideration, say a photon, is measured and after is discarded, and therefore the state of the system after the measurement does not need to be specified. Although this line of thought in quantum optics and high energy physics requires the conflicting notion of particle, it has had a portentous success. More generally, one may have more confidence in how to measure some systems than others, i.e., one may want to use an external device to measure a target system. If one has two systems that share a common state, it is obvious that one can gain information about the one by measuring the other, given that both share some correlations. Moreover, these correlations can be thought of as created dynamically through a controlled interaction between the detector and the system.

The general formalism that accounts for this type of measurement protocols, also called Von Neumann measurements [6], is the one of the positive-operator valued measures (POVM), for every PVM in the system plus detector Hilbert space induces a countably additive, normalized function that takes values on positive operators acting over the target system's space. The opposite is also true, given such POVM one can always find a PVM on an enlarged system that reproduces the POVM, although this one is not unique, so interpreting a POVM in terms of physical properties generally requires the actual PVM used to implement the measurement in the system plus detector Hilbert space [10].

As we will discuss, the formalism of von Neumann measurements plays a relevant role in relativistic quantum physics. The issues surrounding the notion of measurement and the interpretation of quantum probabilities are most commonly discussed in the context of non-relativistic quantum mechanics, and particularizing these notions to relativistic systems, however, faces unique challenges [11].

A relativistic theory could be defined as a theory that makes explicit reference to spacetime, and that respects its symmetries in such a way that its predictions are not dependent on the reference frame. It can be concluded from the postulates of special relativity [12] that nothing can travel faster than light (technically, nothing can travel faster than light if at some point it travelled slower). This in turn induces a causal structure, which determines whether two events in spacetime can be causally related, either as a cause or an effect, and those that cannot. Events that can be related by a causal relation are said to be timelike separated, whereas events that cannot be related by causal relations are said to be spacelike separated. Given a spacetime event, denoted by x, it causal future and its causal past are given by $\mathcal{J}^+(\mathsf{x})$

and $\mathcal{J}^-(\mathsf{x})$ respectively.

Regarding quantum theory, these assertions have crystallized into two independent sets of assumptions about relativistic quantum theory. On the one hand, one would like to enforce, given that the theory makes explicit reference to spacetime regions, that properties associated with spacelike separated events are fully independent, in the sense that the projectors associated with them commute. This general idea is referred to as the microcausality condition, and it is a kinematic condition in the sense that it does not depend on the underlying dynamics of the theory. The second set of conditions make reference to the covariance of the theory, its symmetries and the spectrum of the operators associated with those symmetries. This is somehow a dynamical condition in the sense that it imposes conditions over timelike related events, i.e. over the time evolution of the system, propagation of initial conditions etc.

It turns out that these two independent requirements, when applied together to quantum theory, lead to a series of common-theme results. An example of such type of result is the non-existence of a relativistic position operator. Indeed, one common place in relativistic quantum physics is that a relativistic quantum theory requires the use of fields. Perhaps the first indication that this should be the case stems from the original efforts of Dirac for building a relativistic version of the Schrodinger equation for the wavefunction [13], namely the Dirac equation, only to discover that its solutions must include negative energy solutions. Negative energy solutions are not acceptable for reasons that do not have to do with causality, namely that a system that can take arbitrarily low energies will be unavoidably unstable. An exposition more in-tune with measurement theory takes the form of a no-go theorem, due to Malament [14], which deals with the existence of a projector valued measure from subregions of a spacelike surface. Indeed, Schrodinger's equation is nothing but the position representation of the unitary evolution of the state of a particle, and therefore specifiying a PVM for the position in a spacelike surface, or a position operator, is equivalent to specifying a Schrodinger-type equation.

Such a PVM should be compatible with the causal structure of the theory by imposing that the statistics of the particle are independent in spacelike separation i.e. the microcausality condition. This implies that the PVM could be split in spacelike separated regions, and the sub-PVMs associated with these subregions should commute [15]. Also, if the PVM is covariant, then it should carry a representation of the Lorentz group, which is implemented unitarily. This unitary implementation should fulfill the condition that the Hamiltonian that generates it has a spectrum bounded from below, as explained before. In a few words, the no-go theorem expresses the impossibility of having such PVM, which implies that relativistic quantum physics cannot rely on a quantum mechanical description.

This problem can be overcome when one resorts to a field theoretic description. As we will see, the particle detector approach allows one to perform measurement of local quantities which otherwise seem difficult to approach. Indeed, the usual approaches in QFT, such as in high-energy physics, do not rely on local properties of the field theory, but on the global notion of particle [16]. We will see that such objects cannot exist locally, and further that the projectors that do exist locally are too complicated to treat or to interpret. Moreover, we will see that phenomena predicted by QFT, such as the Unruh effect, again rely on either particles or complicated mathematical constructs, and that particle detectors again provide a nice balance between both approaches.

In this thesis we present some of our results in the topic of particle detectors in QFT and the Unruh effect. The organization of the chapters is as follows:

- Chapter 2 introduces the formalism of the Klein-Gordon field from the point of view of both canonical quantization and covariant quantization, as well as a discussion of some aspects of local measurement theory in QFT.
- Chapter 3 introduces a variety of (linear) particle detector models, most importantly the point-like Unruh DeWitt model. It includes a detailed derivation of the statistics of general detector models at the first orders in perturbation theory.
- Chapter 4 is based on our work in [1]. There, a simple procedure to measure the local correlations of a quantum field using detector models is discussed.
- Chapter 5 is based on our work in [2]. In there we analyze causality issues, such as Sorkin's impossible measurements, in the context of general detector models.
- Chapter 6 focuses on the Unruh effect, and its traditional description through different methods. It contains a detailed, yet simplified description of the Kubo-Martin-Schwinger (KMS) condition, as well as some discussion on thermometry within the framework of perturbation theory.
- Chapter 7 is based on our work in [3], in which we investigated aspects of the so-called Anti-Unruh effect, which describes an unusual, non-monotonic behavior of accelerated particle detectors with the acceleration.

- Chapter 8 is based on our work in [4]. There, we discuss the robustness of the Unruh effect in the presence of ultraviolet deformations of the field theory.

- Finally, we conclude and summarize in chapter 9.

Chapters 2, 3 and 6 include mostly standard contents that have not been developed by the author of this thesis, but that help to contextualize and link the original results together. Some of the contents of these chapters, however, settle notation and even contain partial original results. Regarding the original contents included in this thesis, chapter 2 is not specially relevant aside from settling some notation regarding QFT. Chapter 3 contains some important derivations regarding the point-like UDW model, e.g. the relevant transition probabilities, but the reader that may take such derivations for granted may safely skip most of it. However, it also presents some original generalizations of particle detector models that are relevant for chapter 5, and some unpublished derivations of the statistics of detector models in this general context. Finally, chapter 6 describes the Unruh effect from different formalisms in order to stress the relevance of the original work, but a full understanding of these descriptions is not necessary in order to understand the results discussed in chapters 7 and 8. Nonetheless, it also contains some results regarding the KMS condition that will be used in these chapters, but the relevant derivations are gathered within its own subsection.

Chapter 2

Quantum field theory

The framework of quantum field theory provides us with a conceptual machinery and a mathematical formalism that accounts for quantum as well as relativistic phenomena. It is considered to be the most complete theory of nature, since in principle it can describe all physical phenomena in which quantum gravitational effects are not relevant.

The most elementary example of a quantum field is perhaps given by the real, scalar bosonic field in flat spacetime. For such theory, one wishes to describe a quantum version of the so-called Klein-Gordon field, which is a classical field that fulfills the Klein Gordon equation

$$(\partial_\mu \partial^\mu - m^2)\phi(\mathsf{x}) = 0 \tag{2.1}$$

where $\partial_\mu \partial^\mu$ stands for the D'Alambertian $-\partial_t^2 + \Delta$, where Δ is the Laplacian in the spatial coordinates, and m is the mass of the field. There are several ways of describing a quantum version of this field theory. The most successful ones are canonical quantization, the path integral quantization and the algebraic quantization. While canonical quantization is the most straightforward method, and keeps a close resemblance the usual quantization methods of quantum mechanics, the path integral method is better suited to study gauge theories. The algebraic approach is more sophisticated mathematically [17], and has the goals of describing field theory axiomatically, but also to make sense of local observables [18], and to study QFT in curved spacetimes [19].

2.1 Canonical quantization

The most standard formulation of the Klein-Gordon field would resort to the so-called canonical quantization [20]. The starting point is to acknowledge that the classical Klein-Gordon field can be equivalently described by applying the principle of stationary action to the functional

$$S = \int \mathrm{d}V \mathcal{L}(\mathsf{x}) \tag{2.2}$$

where $\mathrm{d}V = \mathrm{d}t\mathrm{d}^3\boldsymbol{x}$, and the Lagrangian density $\mathcal{L}$ is given by

$$\mathcal{L}(\mathsf{x}) = -\frac{1}{2}\partial_\mu\phi(\mathsf{x})\partial^\mu\phi(\mathsf{x}) - \frac{m^2}{2}\phi^2(\mathsf{x}). \tag{2.3}$$

Since the process of extremizing the action does not make explicit reference to the inertial reference frame, and the Lagrangian density is written in term of Lorentz invariant quantities, this theory is explicitly Lorentz invariant.

Following the recipe for canonical quantization, one chooses a time parameter representing the coordinate time for some inertial reference frame, and defines the conjugate momentum field as

$$\pi(t,\boldsymbol{x}) = \frac{\partial}{\partial\partial_t\phi}\mathcal{L} = \partial_t\phi(t,\boldsymbol{x}), \tag{2.4}$$

and defines the Hamiltonian density as the Legendre transform

$$\mathcal{H}(t,\boldsymbol{x}) = \partial_t\phi(t,\boldsymbol{x})\pi(t,\boldsymbol{x}) - \mathcal{L}(t,\boldsymbol{x}) = \frac{1}{2}\pi^2(t,\boldsymbol{x}) + \frac{1}{2}\partial_i\phi(t,\boldsymbol{x})\partial^i\phi(t,\boldsymbol{x}) + \frac{m^2}{2}\phi^2(t,\boldsymbol{x}), \tag{2.5}$$

where i ranges over the spatial coordinates, and the total Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{x}\mathcal{H}(t,\boldsymbol{x}). \tag{2.6}$$

In canonical quantization, the fields ϕ and π are operators fulfilling the equal-time canonical commutation relations

$$[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{x}')] = \mathrm{i}\delta(\boldsymbol{x}-\boldsymbol{x}')\mathbb{I}, \tag{2.7}$$

where $\mathbb{I}$ denoted the identity operator.

The Hilbert space is constructed by defining implicitly the operators of creation and annihilation as

$$\hat{\phi}(\mathsf{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}}(\hat{a}_{\boldsymbol{k}}e^{\mathsf{ikx}} + \hat{a}^{\dagger}_{\boldsymbol{k}}e^{-\mathsf{ikx}}), \tag{2.8}$$

where $\omega_k = \sqrt{m^2 + \boldsymbol{k}^2}$ and $\mathsf{k} = (\omega_k, \boldsymbol{k})$. This is an expansion in modes of the quantum field. Each of the creation and annihilation operators is labeled by an index $\boldsymbol{k}$ that relates them to a solution of the Klein-Gordon equation with well-defined wave number. The equal-time canonical commutation relations imply the following commutation relations for the creation and annihilation operators

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}] = 0, \tag{2.9}$$

and

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}'}] = \delta(\boldsymbol{k} - \boldsymbol{k}')\mathbb{I}. \tag{2.10}$$

These canonical commutation relations are then used to define the Hilbert space upon which this operators act. First, one defines the vacuum vector $|0\rangle$ and the so-called one-particle Hilbert space $\mathcal{H}$. Then, one constructs the second-quantization, or Fock space of the Hilbert space $\mathcal{H}$ as

$$F = \bigoplus_{n=0}^{\infty} \mathcal{H}_n \tag{2.11}$$

where

$$\mathcal{H}_0 = \mathrm{span}\{|0\rangle\}, \tag{2.12}$$

$$\mathcal{H}_1 = \mathcal{H} \tag{2.13}$$

and $\mathcal{H}_n$ is given by the n-times symmetrized tensor product, i.e.

$$\mathcal{H}_n = \mathcal{H}\underbrace{\otimes_S \cdots \otimes_S}_{n}\mathcal{H}. \tag{2.14}$$

The one particle Hilbert space is identified with vectors of the form

$$\hat{a}^{\dagger}_{\boldsymbol{k}} |0\rangle, \tag{2.15}$$

which implies, through the canonical commutation relations, that

$$\hat{a}_{\boldsymbol{k}} |0\rangle = 0. \tag{2.16}$$

For our purposes, it is relevant to study the correlations of the field operator evaluated in two spacetime points, that is, the so-called Wightman function

$$W(\mathsf{x},\mathsf{x}') = \langle 0|\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')|0\rangle . \tag{2.17}$$

Given the expansion (2.8), the field operator acting over the vacuum state can be written as

$$\hat{\phi}(\mathsf{x}) |0\rangle = \int \frac{\mathrm{d}^3\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}}(\hat{a}_{\boldsymbol{k}} e^{\mathrm{i}\mathsf{kx}} + \hat{a}^\dagger_{\boldsymbol{k}} e^{-\mathrm{i}\mathsf{kx}}) |0\rangle = \int \frac{\mathrm{d}^3\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}} e^{-\mathrm{i}\mathsf{kx}} \hat{a}^\dagger_{\boldsymbol{k}} |0\rangle , \tag{2.18}$$

which allows to calculate

$$\langle 0|\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')|0\rangle = \int \frac{\mathrm{d}^3\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}} e^{\mathrm{i}\mathsf{kx}} \int \frac{\mathrm{d}^3\boldsymbol{p}}{\sqrt{2\omega_{\boldsymbol{p}}}} e^{-\mathrm{i}\mathsf{px}'} \langle 0|\hat{a}_{\boldsymbol{k}}\hat{a}^\dagger_{\boldsymbol{p}}|0\rangle = \int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} e^{\mathrm{i}\mathsf{k}(\mathsf{x}-\mathsf{x}')}, \tag{2.19}$$

where we have used the canonical commutation relations.

Moreover, one can write the Wightman function in terms of Lorentz invariant scalars, for instance, consider the case in which $\mathsf{x}-\mathsf{x}'$ is a timelike vector (e.g. in [21]).

Since $\mathsf{x}-\mathsf{x}'$ is a timelike vector, there is a reference frame in which $\mathsf{x}-\mathsf{x}'$ is at rest, that is, its only non zero component is the time component. Moreover, since the interval of a four vector is Lorentz invariant, the value of this component is

$$\Delta = \mathrm{sgn}(t-t')\sqrt{-(\mathsf{x}-\mathsf{x}')^2}. \tag{2.20}$$

Let Λ be the Lorentz transformation that takes us to that reference frame for each τ, τ':

$$\int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} e^{\mathrm{i}\mathsf{k}(\mathsf{x}-\mathsf{x})} = \int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} e^{\mathrm{i}\mathsf{k}\Lambda(\Delta,\boldsymbol{0})} = \int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} e^{\mathrm{i}(\Lambda^t\mathsf{k})(\Delta,\boldsymbol{0})}. \tag{2.21}$$

Now, since $\frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}}$ is a Lorentz invariant measure, we can change variables from k to $\mathsf{k}' = \Lambda^t\mathsf{k}$ leaving the measure untouched, i.e. $\frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} = \frac{\mathrm{d}^3\boldsymbol{k}'}{2\omega_{\boldsymbol{k}'}}$. Thus,

$$\int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} e^{\mathrm{i}(\Lambda^t\mathsf{k})(\Delta,\boldsymbol{0})} = \int \frac{\mathrm{d}^3\boldsymbol{k}'}{2\omega_{\boldsymbol{k}'}} e^{\mathrm{i}\mathsf{k}'(\Delta,\boldsymbol{0})} = \int \frac{\mathrm{d}^3\boldsymbol{k}'}{2\omega_{\boldsymbol{k}'}} e^{-\mathrm{i}\omega_{\boldsymbol{k}'}\Delta}. \tag{2.22}$$

Now, since $\omega_{\boldsymbol{k}} = \sqrt{m^2 + \boldsymbol{k}^2}$, one can simplify the integral in spherical coordinates, integrating the angular variables

$$\int \frac{\mathrm{d}^3\boldsymbol{k}'}{2\omega_{\boldsymbol{k}'}} e^{-\mathrm{i}\omega_{\boldsymbol{k}'}\Delta} = 2\pi \int_0^\infty \frac{k^2 \mathrm{d}k}{\sqrt{m^2+k^2}} e^{-\mathrm{i}\sqrt{m^2+k^2}\Delta}. \tag{2.23}$$

This integral can be performed in terms of modified Bessel functions of the second kind

$$2\pi \int_0^\infty \frac{k^2 \mathrm{d}k}{\sqrt{m^2+k^2}} e^{-\mathrm{i}\sqrt{m^2+k^2}\Delta} = \frac{m}{2\pi\mathrm{i}\Delta} K_1(\mathrm{i}m\Delta). \tag{2.24}$$

In particular, when $m = 0$ the integral simplifies to

$$2\pi \int_0^\infty \mathrm{d}k k\, e^{-\mathrm{i}k\Delta} = -\frac{1}{\Delta^2}. \tag{2.25}$$

The fact that the Wightman function depends on the points x, x' through the interval Δ is not surprising, since the vacuum is Lorentz invariant. Note that the integral expression is not absolutely convergent, which means that the expression has to be understood in the sense of distributions.

2.1.1 Issues with canonical quantization

There are (at least) two features of quantum field theory that are not totally apparent in this quantization scheme. One is the lack of definition of field operator itself, the other is the arbitrariness of the mode expansion (2.8). The first problem is most pertinent in the context of chapter 4 of this thesis, whereas the second becomes relevant for the analysis of the Unruh effect, as we will see in chapter 6.

Regarding the first point, it is indeed part of the lore of quantum field theory that, in contrast with classical field theory, the value of the field at a point is not well defined. It is not difficult to illustrate this behavior from the point of view of standard canonical quantization. Consider the norm of the vector $\hat{\phi}(\mathsf{x})\,|0\rangle$:

$$\|\hat{\phi}(\mathsf{x})\,|0\rangle\|^2 = \int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}}, \tag{2.26}$$

which does not converge. A way to interpret this fact in physical grounds is that the value of the field's amplitude at a point has infinite variance in the vacuum.

In order to get operators with finite variance one has to "average" the field operator by smearing it with a spacetime function, say $f(\mathsf{x})$, such that

$$\hat{\phi}(f) = \int \mathrm{d}V \hat{\phi}(\mathsf{x}) f(\mathsf{x}) \tag{2.27}$$

This leads to the conclusion that one should think of the field operator as an "operator valued distribution", that takes sufficiently well-behaved functions in space and time to operators in the Hilbert space. Indeed, in that case it can be checked that

$$\|\hat{\phi}(f)\,|0\rangle\|^2 = \int \frac{\mathrm{d}^3\boldsymbol{k}}{2\omega_{\boldsymbol{k}}} |\tilde{f}(\omega_k, \boldsymbol{k})|^2, \tag{2.28}$$

where the symbol $\tilde{\cdot}$ denotes the Fourier transform.

Regarding the second point, note that in proceeding with canonical quantization one makes a choice of mode expansion. One could, in principle, have chosen a different set of modes to describe quantize the system, and wonder if the underlying physics is any different.

A Bogoliubov transformation for a system that fulfills the canonical commutation relations (CCR) is just a linear transformation acting over the creation and annihilation operators that preserves the canonical commutation relations [22]. Consider a generic expansion of the quantum field

$$\hat{\phi} = \sum_k \xi_k \hat{a}_k + \xi_k^* \hat{a}_k^\dagger, \tag{2.29}$$

where we have substituted the integrals by sums in order to represent the calculation schematically, since the pass to the continuum does not play a role for the moment. The elements ξ_k live in the space of solutions of the Klein Gordon equation. The creation and annihilation operators fulfill the canonical commutation relations $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'}$. Now, consider a second expansion

$$\hat{\phi} = \sum_k \zeta_k \hat{b}_k + \zeta_k^* \hat{b}_k^\dagger. \tag{2.30}$$

where $\{\zeta_k, \zeta_k^*\}$ is a different basis. Since they are both basis of the same vector space, they can be related by a linear transformation

$$\zeta_k = \sum \alpha_{kl} \xi_l + \beta_{kl} \xi_l^*, \tag{2.31}$$

where α and β are two matrices. The preservation of the canonical commutation relations implies the following relations

$$\alpha\alpha^\dagger - \beta\beta^\dagger = \mathbb{I} \tag{2.32}$$

and

$$\alpha^*\beta^\dagger - \beta^*\alpha^\dagger = 0. \tag{2.33}$$

Note that the creation and annihilation operators $\hat{a}_i$ and $\hat{b}_k$ are then linearly related. Indeed,

$$\hat{b}_k = \alpha_{kl}\hat{a}_i + \beta_{kl}\hat{a}_i^\dagger. \tag{2.34}$$

This implies that the state annihilated by a set of annihilation operators does not need to be annihilated by the second set, concretely if the coefficients in β are not zero. This further implies that the vacuum in one quantization contains particles with respect to the other quantization.

Bogoliubov tranformations are just canonical transformations in the phase space that describes the field theory at the classical level, which we shall describe more thoroughly in next section.

At this point, it seems that the choice in (2.8) was arbitrary, and there is no reason to not consider other choices as legitimate. Is this arbitrariness a problem? In finite dimensional systems, von Neumann's theorem [23] states that all representations of the canonical commutation relations are unitarily equivalent. In our case, this means that the vacuua associated with different decompositions are related through unitary transformations. It can be argued that when describing two systems that are unitarily equivalent, one is actually describing the same physical system, since the unitary transformation is just a relabelling of the relevant physical quantities.

For infinite dimensional systems, such as quantum field theories, this is not always the case. There are many instances in which two quantizations are not unitarily equivalent. This is the case, for instance, when describing the Unruh effect from the point of view of Bogoliubov transaformations, as we will discuss thoroughly in chapter 6. In flat spacetime, despite phenomena like the Unruh effect, it is reasonable to expect the ordinary canonical quantization to be the representation of quantum fields, for it is the one that respects the symmetries of flat spacetime. In other situations, such as quantum fields defined on curved spacetime backgrounds, the symmetries do not single out a privileged quantization scheme.

2.2 Covariant quantization of the Klein-Gordon field

Canonical quantization makes use of the field amplitude and its canonically conjugate momentum defined over a spacelike surface. Since theory is relativistic, the physics should not depend on the choice of spacelike surface, and it would be desirable for the quantization scheme to not make explicit reference to it.

Actually, it is possible to quantize a field theory using covariant relations that do not make explicit reference to the canonical momentum or the Hamiltonian. Moreover, these relations are straightforwardly extendable to Lorentzian, globally hyperbolic curved backgrounds for linear fields fulfilling hyperbolic equation of motion. In any representation, the field operator $\hat{\phi}$ should have the following properties as a spacetime distribution [24]:

- $\hat{\phi}^\dagger(f) = \hat{\phi}(f^\star)$.
- The weak Klein-Gordon equation

$$\hat{\phi}\left((\nabla_a \nabla^a + m^2) f\right) = 0, \tag{2.35}$$

  for any compactly supported f.
- The covariant commutation relations

$$[\hat{\phi}(f), \hat{\phi}(h)] = \mathrm{i} \int \mathrm{d}V \, f(\mathsf{x}) G[h](\mathsf{x}) \mathbb{I}, \tag{2.36}$$

  where G denotes the difference between the advanced and retarded propagators of the classical equations of motion, i.e.

$$G = G_A - G_R. \tag{2.37}$$

  The advanced and retarded propagators are the unique propagators of the Klein-Gordon equation such that the solutions are past and future compact respectively [25]. Finally $\mathrm{d}V = \mathrm{d}^{n+1}\mathsf{x}\sqrt{-|g|}(\mathsf{x})$ is the canonical element of volume.

In addition, note that these properties do not depend on the quantization scheme chosen; they constitute a set of algebraic relations for the field amplitude. Therefore, they constitute the starting point for most rigorous treatments of the Klein-Gordon equation in curved spacetimes.

Before proceeding further with the quantization of the field, let us point out that the space of solutions of the Klein Gordon equation

$$(\nabla_a \nabla^a - m^2)\varphi = 0 \tag{2.38}$$

is in one to one correspondence with the span of all the field operators $\hat{\phi}(f)$. Indeed, for any compactly supported spacetime function f, $G[f]$ is a solution of the Klein-Gordon equation. G has a non-trivial kernel as a map, so there is not a one-to one correspondence between spacetime functions and solutions. This is obvious because not every spacetime function is a solution of the Klein Gordon equation. However, the kernel of G is precisely the set of functions of the form $(\nabla_a \nabla^a + m^2)h$ for some compactly supported function h, because for such functions the advanced and retarded propagators act equally.

This means that if for two spacetime smearings f and g it holds that $G[f] = G[g]$, that is, f and g are associated with the same solution of the Klein Gordon equation, then $G[f - g] = (\nabla_a \nabla^a + m^2)h$, and hence $\hat{\phi}(f) = \hat{\phi}(g)$. Therefore, the vector space given by the span of spacetime smeared field operators is in one-to one correspondence with the space of solutions of the Klein Gordon equation. Let us denote by $\boldsymbol{\xi}$ the elements of the space.

In addition, the space of solutions is a symplectic space, since it possesses an antisymmetric, non-degenerate bilinear form given by the commutator. Let $\boldsymbol{\xi}$ and $\boldsymbol{\xi}'$ be two solutions and f, f' two smearings associated with them. Then we define the symplectic form as

$$\boldsymbol{\Omega}(\boldsymbol{\xi}, \boldsymbol{\xi}') = \int \mathrm{d}V f(\mathsf{x}) G[f'](\mathsf{x}). \tag{2.39}$$

To proceed further with Fock quantization, one again has to make a choice of creation and annihilation operators. The task in Fock quantization is to find the relevant Hilbert space describing the one-particle states, and then to upgrade this Hilbert space to a Fock space by performing direct sums and tensor products. The creation and annihilation operators are then defined to implement transitions between the different n-particle states. This can always be accomplished with a mathematical object called a *complex structure* [26].

We define a linear complex structure $\mathscr{J}$ compatible with $\boldsymbol{\Omega}$ as a linear map that fulfills

- $\mathscr{J}^2 = -\mathbb{I}$.
- $(\boldsymbol{\xi}, \boldsymbol{\xi}') = \boldsymbol{\Omega}(\mathscr{J}\boldsymbol{\xi}, \boldsymbol{\xi}')$ defines a positive, real inner product.

It can be shown that these imply that complex structures are canonical transformations, i.e. they preserve the

$$\boldsymbol{\Omega}(\mathscr{J}\boldsymbol{\xi}, \mathscr{J}\boldsymbol{\xi}') = \boldsymbol{\Omega}(\boldsymbol{\xi}, \boldsymbol{\xi}'). \tag{2.40}$$

Complex structures have only two distinct eigenvalues, i and −i, with two orthogonal subspaces associated with those eigenvalues, π_{i} and $\pi_{-\mathrm{i}}$:

$$\pi_{\pm\mathrm{i}} = \frac{\mathbb{I} \mp \mathrm{i}\mathscr{J}}{2}. \tag{2.41}$$

The ranges of π_{i} and $\pi_{-\mathrm{i}}$ are called the spaces of positive and negative frequency respectively.

With an abuse of notation, let us define $\hat{\phi}(\boldsymbol{\xi}) = \hat{\phi}(f)$ for any f associated with the solution $\boldsymbol{\xi}$. One can define the annihilation operators associated with the complex structure as

$$\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}) = -\frac{\mathrm{i}}{\hbar}\hat{\phi}(\pi_{-\mathrm{i}}\boldsymbol{\xi}) \tag{2.42}$$

in such a way that

$$\hat{\phi}(\boldsymbol{\xi}) = \mathrm{i}\hbar\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}) + (\mathrm{i}\hbar\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}'))^{\dagger}. \tag{2.43}$$

The covariant commutaton relations then imply that

$$[\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}), \hat{a}_{\mathscr{J}}(\boldsymbol{\xi}')] = 0 \tag{2.44}$$

and

$$[\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}), (\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}'))^{\dagger}] = \frac{1}{2\hbar}((\boldsymbol{\xi}, \boldsymbol{\xi}') - \mathrm{i}\boldsymbol{\Omega}(\boldsymbol{\xi}, \boldsymbol{\xi}'))\mathbb{I}. \tag{2.45}$$

Since $\frac{1}{2\hbar}((\boldsymbol{\xi}, \boldsymbol{\xi}') - \mathrm{i}\boldsymbol{\Omega}(\boldsymbol{\xi}, \boldsymbol{\xi}'))$ defines a positive inner product, it can be used to define the Hilbert space $\mathcal{H}$ of the one particle states , in which the state vectors are given by linear combinations and limits in the $(\cdot, \cdot)$-norm of the solutions $\boldsymbol{\xi}$.

According to Fock quantization, we define $|0\rangle_{\mathscr{J}}$ as the vacuum state, and the elements of the one-particle Hilbert space are then labelled by $|\boldsymbol{\xi}\rangle = (\hat{a}_{\mathscr{J}}(\boldsymbol{\xi}))^{\dagger}\,|0\rangle_{\mathscr{J}}$.

Therefore, different Fock quantizations can be encoded in the choice of complex structure $\mathscr{J}$. Interestingly, consider the two-point function of a vacuum state is determined by the choice of $\mathscr{J}$

$$\begin{aligned}\langle 0_{\mathscr{J}}|\hat{\phi}(f)\hat{\phi}(f')|0\rangle_{\mathscr{J}} &= \hbar^2 \langle 0_{\mathscr{J}}|[\hat{a}_{\mathscr{J}}(G[f]), (\hat{a}_{\mathscr{J}}(G[f]'))^\dagger]|0\rangle_{\mathscr{J}} \\ &= \frac{\hbar}{2}(\boldsymbol{\Omega}(\mathscr{J}G[f], Gf') - \mathrm{i}\boldsymbol{\Omega}(G[f], G[f'])). \end{aligned} \tag{2.46}$$

Finally, given this state, one can associate to every local region a mathematical object called von Neumann algebra [18]. Given a bounded region $\mathcal{O}$, the local algebra is the weak closure of the algebra of (bounded functions of) field operators such that the smearing f has support in $\mathcal{O}$ [17]. These algebras are broadly regarded as representing the set of possible local observables, or alternatively local operations. Although we will not enter in much detail, we mention that the local algebras have features that are intrinsic to infinite dimensional systems and that have striking physical consequences. We discuss these characteristics below.

2.3 Measurement theory of quantum fields

Formulating a measurement theory that is consistent with relativity is not devoid of problems. First, there are some issues regarding the description of local properties in QFT. It turns out that under some mild assumptions [17], the local algebras are type III von Neumann algebras. These algebras are *not* isomorphic to any algebra of bounded operators acting over a Hilbert space. In particular, this implies that the set of local observables/operations does not contain minimal projectors [27].

To get a grasp of the consequences of this, consider a bipartite system consisting on two harmonic oscillators. The local algebra of all bounded operators acting only over one of the oscillators does not contain any finite rank projectors, but is still isomorphic to the space of bounded operators acting over the Hilbert space associated with a single harmonic oscillator. Namely, we can identify elements of the form $|\psi\rangle\langle\psi| \otimes \mathbb{I}$ which will play the role of minimal projectors.

This means, roughly speaking, that we can specify the subsystem independently of the whole Hilbert space. This is the case assumed in quantum mechanics, as well as in probability theory, when one first specifies the subsystems one one is interested in and then "glues" them together to form composite systems. In particular, the duality between states and properties alluded to in the introduction still holds. A von

Neumann algebra (with trivial center) that is isomorphic to the algebra of bounded operators is a type I von Neumann algebra. The fact that the local algebras in QFT are type III has the consequence that the duality between states and properties of the system does not hold locally [11].

Is this so dramatic? There is no immediate answer to this, for the question is full of subtleties. In type III algebras, states can hold properties locally in some sense [28, 17]. Projectors in type III have the property of being always equivalent to each other, in the sense that they are all related through a partial isometry. Since the identity is a projector, one gets that for any state of the form $\langle \cdot \rangle_W = \langle \hat{W}^\dagger \cdot \hat{W} \rangle$, where $\hat{W}$ relates the projector $\hat{P}$ to the identity, $\langle \hat{P} \rangle_W = \langle \hat{W}^\dagger \hat{P} \hat{W} \rangle = \langle \mathbb{I} \rangle = 1$, and any projector $\hat{P}'$ orthogonal to $\hat{P}$ fulfills $\langle \hat{P}' \rangle_W = 0$.

The difference between the local algebras in QFT and the ones encountered in quantum mechanics is that in QFT one cannot think of the properties associated with the projectors in the local regions as properties of a subsystem independent of the surroundings; one has to always consider the subsystem in the context of its environment. But even this is a subtle statement, if the quantum field possesses the so-called split property, or funnel property [29, 28]. If this is the case, then whenever two local algebras associated with two regions are such that the closure of one is contained in the other (consider, e.g. two concentric spacetime bubbles with different radii), then there is a type I algebra that contains the algebra associated with the small region and is contained in the algebra associated with the larger region.

It is easy to see that then any algebra in the commutant of the large region, particularly any local algebra in the causal complement of the large region, is contained in the commutant of the type I algebra, which will be again type I. Finally, since the algebra generated by two commuting type I algebras is isomorphic to their tensor product, the algebra associated with two spacelike separated regions is isomorphic to the tensor product of the algebras, as far as the regions are not the causal complement of each other. This means that, while we cannot think of the subsystems associated with local regions independently of the global Hilbert space, we can think of the algebras associated with two spacelike separated regions independently as far as there is a *safety region* that separates them. Yet, many authors have pointed out interpretational shortcomings when the local algebras are involved [30, 11].

Beyond the definition of local properties, there is also the problem of the instantaneous state update. This, obviously, becomes problematic since joint probability distributions will depend on the order in which the measurements are performed, which generally depends on the reference frame. Therefore, it becomes necessary to modify the prescription of the state update for the extraction of frame-independent probabilities of successive measurements. A covariant version of the state update rule that gives rise to frame independent probability assignments was developed by Hellwig and Kraus [31]. For more modern discussions of this topic, see [32, 33].

Finally, and most importantly for the purposes of this thesis, there is the possibility of superluminal signaling. Beyond the possibility of transmission of EPR correlations (which are dependent on the interpretation of the result of the measurement) an important challenge was posed by Sorkin's work in [34] on the impossibility of (idealized) measurements in QFT, based solely on relativistic considerations. This poses the question, e.g., of whether one could measure non-local QFT quantities such as Wilson loops [35]. Sorkin's work demonstrates that signaling between two spacelike separated regions A and B can be 'mediated' by an operation on a third region C that is partially in the causal future of A and partially in the causal past of B.

Since superluminal signaling is not compatible with the axioms of relativity, Sorkin's result proves that a naive 'quantum-mechanical' set of idealized measurement rules would fail in relativistic QFT. Furthermore, the result raises the issue of the consistent description of *successive* measurements when more than two measurements in different spacetime regions are involved. The issue stems from the fact that a partial causal order can be defined between pairs of extended (bounded) regions, but cannot be naturally extended to multiple regions (unless they are pointlike). Sorkin suggests that a resolution can be given in the more 'spacetime oriented' formulation of sum-over histories approach (See, e.g., [36]).

Sorkin's result was further analyzed in [37, 38], and more recently in [39], where they studied what conditions can be imposed as requirements for local, field-valued POVM measurements to avoid superluminal signalling. Finally, a formal resolution of the Sorkin problem has been proposed recently in the context of Fewster and Verch framework for measurements in algebraic QFT [40, 29]. This resolution is given, precisely, in the context of (explicitly local) von Neumann measurements.

In summary, it is still an open problem to find a consistent description of measurements in QFT, since it is unclear what are the consequences of measuring a quantum field on a region. In this context, particle detector models provide us with an alternative framework for the description of local measurements in QFT. On the one hand, considering a finite dimensional system avoids many of the subtleties exposed before in terms of properties and states. While the observables induced by projective measurements in an external device still cannot be traced back to local properties of the field in a clear way, one can apply all the principles of standard measurement theory. On the other hand, as we will see, the violations of causality can be interpreted dynamically in terms of the non-relativistic nature of the detector, thereby bringing the Sorkin-type issues to a new stage in which one can identify the situations in which these violations of causality are relevant as being outside the regime validity of the detector model under consideration.

Chapter 3

Particle detectors

3.1 Particle detectors and interaction Hamiltonians

Most analyses that use detector models as a conceptual tool are restricted to the study of real scalar field theories, possibly in curved spacetimes. It is generally argued that the study of local aspects of scalar bosonic fields captures the essence of many aspects that apply to more general QFT's.

It is customary to specify the detector coupling through an interaction Hamiltonian in the interaction picture. In most detector models the spacetime localization of the detector is not specified in the detector's free degrees of freedom, but precisely in this interaction Hamiltonian. This allows one to keep a non-relativistic description of the detector, which admits a wider set of observables like, e.g. a position representation.

Before giving an historical account of the detector models used in the past, we present the most popular form of detector model, namely the point-like Unruh-DeWitt model, which is given by a two level system equipped with a free Hamiltonian and whose two eigenstates referred to as $|g\rangle$ and $|e\rangle$, the ground and the excited state respectively. This two-level system interacts with the quantum field with the following interaction Hamiltonian (in the interaction picture)

$$\hat{H}_{\text{pt}} = \lambda\chi(\tau)\hat{D}(\tau) \otimes \hat{\phi}(\mathsf{x}(\tau)). \tag{3.1}$$

This interaction Hamiltonian contains the basic ingredients that are common to all detector models. These are, in order of appearance

- The coupling constant λ, which is necessary to justify the weak coupling of the interaction, which ultimately allows to perform calculations.
- The switching function χ, which plays the role of effectively describing the process of turning the interaction on and off.
- The proper time of the detector τ. The free evolution of the detector is assumed to be given by the time experienced by an observer fiduciary to the detector.
- $\hat{D}(\tau)$ is an observable of the detector in the interaction picture. In the UDW model this is the operator that transitions between the two eigensates of the Hamiltonian, i.e.

$$\hat{D}(\tau) = e^{\mathrm{i}\Omega\tau} |e\rangle\langle g| + e^{-\mathrm{i}\Omega\tau} |g\rangle\langle e| \tag{3.2}$$

- Finally, $\hat{\phi}(\mathsf{x}(\tau))$ is the field's amplitude evaluated along the spacetime trajectory of the detector $\mathsf{x}(\tau)$.

The UDW model in this form is often dubbed the "point-like" UDW model, for it describes a detector that interacts with the quantum field along a one-dimensional curve. It is expected that in any reasonable detector model, that aims to portray a localized device, there will be a regime of validity in which the point-like limit captures the readings of the detector. Intuitively, whenever the relevant observables are calculated, the dependence on the internal structure of the detector, as well as effects related to its finite size, will be negligible if the relevant wave lenghts are much larger than the effective interaction region between field and detector.

However, as it was noted in chapter 2, the field's amplitude evaluated at a point is not a real observable. This makes the Hamiltonian given by (3.1) also ill-defined as a function of time. However, given that the field's amplitude can be thought of as an operator-valued distribution, one would like to think of the Hamiltonian (3.1) as an operator-valued distribution when smeared over time with a sufficiently smooth function. This is partially the motivation for the introduction of the switching function χ. However, the existence of the dynamics for the point-like Unruh-DeWitt model is not been shown rigorously as far as we are aware.

Of course, one can check that a Dyson expansion of the unitary dynamics generates well defined evolution maps order by order. One finds then that the unitary evolution at the first two orders in perturbation theory is well defined whenever the function χ fulfills some smoothness requirements, but higher orders become increasingly cumbersome to deal with given the distributional nature of the Hamiltonian.

The ill-definiteness of the Hamiltonian for, say, discontinuous switching functions manifests itself with the appearance of divergences in relevant quantities, such as the probability of the detector to transition from its ground state to its excited state. Note that, remarkably, this implies that one can only describe scattering processes since keeping active track of the evolution of the system will be forbidden.

It is possible to generalize the Unruh-DeWitt Hamiltonian in order to get a well-defined interaction Hamiltonian. An effective description of a detector model that couples to a smeared field operator is given by

$$\hat{H}_{\mathrm{sm}} = \lambda\chi(t)\hat{\mu}(t) \otimes \int \mathrm{d}^n\boldsymbol{x}\, F(\boldsymbol{x})\hat{\phi}(\boldsymbol{x}, t). \tag{3.3}$$

This model, the smeared Unruh-DeWitt model, describes a detector that couples to the field in a space region with an intensity regulated by the function F. Note that in the present we have not specified the number of spatial dimensions, which we denote by n. This type of interaction Hamiltonian has become the starting point for many regularization schemes for the point-like model [41, 42].

However, as we will thoroughly study in chapter 5, this way of proceeding comes at the price of generating friction with relativistic causality, and therefore it requires further justification to use smeared couplings.

Historically, the notion of particle detector was suggested by Unruh in his seminal paper, [43], where the detector was already meant to play a clarifying role for the thermality of what would come to be called the Unruh effect, a role that we will explore in detail in chapter 6. In this work, the detector is envisioned in two alternative forms, one is with a quantum particle in box and the other with another quantum field.

Both have advantages and disadvantages. The main disadvantage of using a non-relativistic detector model is that one comes across the aforementioned friction with causality, whereas the main problem with using a quantum field as a detector is the interpretation of the calculated quantities (understood as local observables), as hinted in chapter 2.

The particle in a box-type detector was described through the Hamiltonian

$$\hat{H}_{\mathrm{UW}}(\tau) = \lambda\chi(\tau) \int \mathrm{d}^n\boldsymbol{x}\, \hat{\phi}(\boldsymbol{x}, \tau) \otimes \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_\tau). \tag{3.4}$$

In this case the spectrum of the operator $\hat{\boldsymbol{x}}_t$ corresponds to the spatial manifold over which the field interacts with the detector, so in the case of a particle in a box the spatial integrals run through the extension of the box. Indeed, formally, one can think of this Hamiltonian as

$$\hat{H}_{\text{UW}}(\tau) = \lambda\chi(\tau)\hat{\phi}(\hat{\boldsymbol{x}}_\tau, \tau). \tag{3.5}$$

Although Hamiltonian (3.4) is not explicitly written down in the original paper, it was clearly formulated and analyzed in [44].

Later, in [45], DeWitt introduced the pointlike model previously discussed, which since then became the most popular. It is unclear, to the best of my knowledge, if DeWitt's model was inspired by similar toy models in the context of the light-matter interaction, but one can guess from the source that the point-like model was formulated seeking a simpler description of the phenomenon first predicted by Unruh.

Crucially, one can recover a smeared version of the UDW model when considering only transitions between the detector's energy levels. Indeed, by introducing a decomposition of the identity in the eigenbasis of the detector's free Hamiltonian, say $|E_i\rangle$, on both sides of the operator $\delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_\tau)$ in equation (3.4) we get

$$\hat{H}_{\text{UW}}(\tau) = \lambda\chi(\tau)\int \mathrm{d}^n\boldsymbol{x}\, \hat{\phi}(\boldsymbol{x},\tau) \otimes \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_\tau) \tag{3.6}$$

$$= \sum_{ij} \lambda\chi(\tau)\int d^n\boldsymbol{x}\, e^{-\mathrm{i}(E_i - E_j)\tau}\psi_i^*(\boldsymbol{x})\psi_j(\boldsymbol{x})\hat{\phi}(\boldsymbol{x},\tau) \otimes |E_i\rangle\langle E_j|\,, \tag{3.7}$$

where ψ_i is the ith eigenstate of the free Hamiltonian, which then acts as a complex smearing. When the wave functions associated with a particular pair of eigenstates i, j are localized in a small region compared to the other scales, the transitions between them can be approximated with the pointlike UDW model.

This ihas provided material for much thought in the recent literature. For instance, it has been shown that the interaction of a Hydrogen-like atom and the electromagnetic field can be approximated by the UDW model when the exchange of angular momentum between atom and the field is not relevant [46]. Indeed, the dipole interaction Hamiltonian of an electron in an atom and the electric field is of the form $\hat{\boldsymbol{x}}\cdot\boldsymbol{E}(t,\boldsymbol{x})$. Moreover, the light matter interaction can be used to extend the UDW model in meaningful ways, for instance the light-matter interaction contains many terms that are related to the quantum character of the center of mass of the atom [47].

In addition, a rather ecumenical way to treat detector models was proposed in the context of the quantum temporal probabilities program [36], in which the interaction

Hamiltonian is generally given by a current $\hat{J}$

$$\hat{H}_{\rm curr}(\tau) = \int \mathrm{d}^n \boldsymbol{x} \hat{J}(\boldsymbol{x}, \tau) \otimes \hat{O}(\boldsymbol{x}, \tau) \tag{3.8}$$

and where $\hat{O}$ is a general composite operator for the field.

We conclude this historical review with a series of even more novel generalizations related to particle detectors in curved spacetimes. The point-like model indeed has a straightforward generalization to curved backgrounds [48], and it is been used as such in many scenarios such as quantum field theory in black holes [49, 50], cosmology [51, 52, 53], and Vaidya's spacetime [54].

The extension to curved spacetimes of generalizations of the pointlike model, however, is less trivial and continues to be an area of active research. In [55], the authors discuss the covariance of smeared detector models in flat spacetime. More concretely, they discuss if the transition probabilities predicted by the model vary depending on the reference frame in which the detector model is defined. Their method describes the detector in terms of a Hamiltonian weight

$$\hat{h}(\mathsf{x}) = \lambda \Lambda(\mathsf{x}) \hat{\mu}(\tau(\mathsf{x})) \otimes \hat{\phi}(\mathsf{x}), \tag{3.9}$$

where $\tau(\mathsf{x})$ is a function whose level curves represent the surfaces of the simultaneity of the center of mass of the detector. This formalism was further analysed in the context of curved backgrounds in [56], and problems with relativistic covariance were reported in [57]. From this scalar one can indeed construct a one-parameter family of operators representing the time-dependent interaction Hamiltonian:

$$\hat{H}(\tau) = \int_{\mathcal{E}(\tau)} \mathrm{d}\mathcal{E}\ \hat{h}(\mathsf{x}). \tag{3.10}$$

where $\mathcal{E}(\tau)$ is the family of space-like surfaces associated with $\tau(\mathsf{x})$.

Our contribution to this story, reported in [2], is a generalization of the former, in which we consider scalars of the form

$$\hat{h}(\mathsf{x}) = \lambda \Lambda(\mathsf{x}) \hat{J}(\mathsf{x}) \otimes \hat{\phi}(\mathsf{x}). \tag{3.11}$$

Despite the notation, $\hat{J}(\mathsf{x})$ does not need to be a current, in the sense that the variation on space and time does not have to be given by the adjoint action of operators on the Hilbert space of the detector. Here, Λ is a spacetime function of compact support.

As before,

$$\hat{H}(\tau) = \int_{\mathcal{E}(\tau)} \mathrm{d}\mathcal{E}\ \hat{h}(\mathsf{x}). \tag{3.12}$$

but in this case there is no explicit reference to the detector's proper time, so given any global time function $\mathscr{T}(\mathsf{x})$

$$d\mathcal{E}(\tau) := \mathrm{d}\mathsf{x}^{n+1}\delta(\mathscr{T}(\mathsf{x}) - \tau)\sqrt{|g|}(\mathsf{x}) \tag{3.13}$$

where then the parameter τ is associated with a time parameter that may or may not have anything to do with observers fiduciary to the detector.

One advantage that the Hamiltonian scalar (3.11) possesses is that it allows one to treat field-type, as well as nonrelativistic-type detectors in a unified language. Indeed, in Unruh's seminal paper the field-type detector is described as a complex field pair $\hat{\psi}_M$ and $\hat{\varphi}_m$ with large, yet slightly different masses, and the Hamiltonian weight was just given by

$$\hat{h}(\mathsf{x}) = \lambda\Lambda(\mathsf{x}) \left(\hat{\psi}_M^\dagger(\mathsf{x})\hat{\varphi}_m(\mathsf{x}) + \hat{\psi}_M(\mathsf{x})\hat{\varphi}_m(\mathsf{x})^\dagger\right) \hat{\phi}(\mathsf{x}). \tag{3.14}$$

An even simpler case is given in [29, 58], where the detector is just another real scalar field $\hat{\psi}$,

$$\hat{h}(\mathsf{x}) = \lambda\Lambda(\mathsf{x})\hat{\psi}(\mathsf{x})\hat{\phi}(\mathsf{x}), \tag{3.15}$$

although of course the simplicity there was well motivated since the intention was to give an example within a more general framework.

Note that, given a detector model that can be written in the form (3.11), i.e. embedded in the background spacetime, one can unambiguously determine whether the detector is relativistic or not through the microcausality of the dynamics, that is

$$[\hat{J}(\mathsf{x}), \hat{J}(\mathsf{y})] = 0, \tag{3.16}$$

if x and y are spacelike separated. Indeed, as we will see in chapter 5, this will play a major role when discussing the causality of the measurements performed with the detector.

For the rest of topics covered in this thesis, we will focus primarily on the point-like model. We will use the full version of (3.11) in chapter 5, where we will discuss friction with causality in particle detector models in a rather general fashion.

3.2 Detectors, statistics and responses

This section is devoted to analyzing the statistics of general detector models in the framework of perturbation theory. Other frameworks are possible, and in the literature of particle detectors it has been common to consider the formalism of open quantum systems in the weak coupling regime [59, 60, 61, 62], or more sophisticated considerations within the algebraic approach such in [63], or the Fewster-Verch framework [29].

In the work presented in this thesis the calculations always describe some form of scattering process. Namely, the interaction is switched on and off and is different from zero only for a finite time (we may relax the assumption of compact support when appropriate). The state of the detector is prepared before the interaction and only the state of the detector is measured after, in such a way the it is never assumed that that the there is any interplay between the measurement process and the interaction theory between the detector and the field.

If the detector plus field system is prepared at $\tau = \tau_0$ in a state described by the density matrix $\hat{\rho}$, then the state of the whole system at a later time τ is given by the unitary evolution

$$\hat{\rho}(\tau) = \hat{U}(\tau)\hat{\rho}\hat{U}^{\dagger}(\tau), \tag{3.17}$$

where the unitary operator $\hat{U}$ is given by

$$\hat{U}(\tau) = \mathcal{T}e^{-\mathrm{i}\int_0^{\tau}\mathrm{d}\tau \hat{H}(\tau)} = \sum_{n=0}^{\infty}\frac{(-\mathrm{i})^n}{n!}\int_{\tau_0}^{\tau}...\int_{\tau_0}^{\tau_2}\mathrm{d}^n\tau\ \hat{H}(\tau_n)...\hat{H}(\tau), \tag{3.18}$$

the measure $\mathrm{d}^n\tau$ denoting $\mathrm{d}\tau_1\ldots\mathrm{d}\tau_n$. Now, the dynamics is best described in the interaction picture, which implements deviations of the interacting dynamics from the free dynamics:

$$\hat{\rho}(\tau) = \hat{U}_{\text{free}}(\tau)\hat{U}_I(\tau)\hat{\rho}\hat{U}_I^{\dagger}(\tau)\hat{U}_{\text{free}}^{\dagger}(\tau). \tag{3.19}$$

We assume that the function has compact support and we also assume that the interaction starts at some time $\tau \geq \tau_0$. In other words, we say that there exists a τ_f such that

$$\text{supp}\{\chi(\tau)\} \subset [\tau_0, \tau_f]. \tag{3.20}$$

Therefore, for all $\tau > \tau_f$, the integrals can be extended from their domain to the whole real line without changing their value. Then it can be shown that the evolution in the interaction picture takes the form

$$\hat{S} = \hat{U}_I(\tau > \tau) = \mathcal{T}e^{-\mathrm{i}\int \mathrm{d}\tau \hat{H}_I(\tau)} = \sum_{n=0}^{\infty} \frac{(-\mathrm{i})^n}{n!} \int_{\mathbb{R}^n} \mathrm{d}^n\tau \; \mathcal{T}\left[\hat{H}_I(\tau_1)...\hat{H}_I(\tau_n)\right], \tag{3.21}$$

where $\mathcal{T}[\cdot]$ denotes the time-ordered product and $\mathrm{d}^n\tau = \mathrm{d}\tau_1 \dots \mathrm{d}\tau_n$. Henceforth, we will always assume that the time τ at which we evaluate the state is larger than the time the interaction ends: $\tau > \tau_f$.

To compute the partial state of the detector, $\hat{\rho}_{\mathrm{D}}$, we trace over the field degrees of freedom:

$$\hat{\rho}_{\mathrm{D}} = \mathrm{tr}_\phi \left[\hat{S}\hat{\rho}\hat{S}^\dagger\right]. \tag{3.22}$$

Finally, we assume that prior to $\tau = \tau_0$ the detector and the field are completely uncorrelated, i.e., the initial state has the form of the tensor product

$$\hat{\rho} = \hat{\rho}_{\mathrm{D}}^0 \otimes \hat{\rho}_\phi. \tag{3.23}$$

This last assumption allows one to write the evolution in the interaction picture in terms of a completely positive trace-preserving (CPTP) map, given by the quantum channel

$$\mathcal{E}[\hat{\rho}_{\mathrm{D}}^0] = \mathrm{tr}_\phi \left[\hat{S}\hat{\rho}_{\mathrm{D}}^0 \otimes \hat{\rho}_\phi \hat{S}^\dagger\right]. \tag{3.24}$$

We shall consider the effect of the interaction up to some order in perturbation theory. The perturbation parameter is encoded in the coupling constant λ. The way to proceed is then to expand the unitary evolution in this parameter, i.e., $\hat{S} = \mathbb{I} + \hat{S}^{(1)} + \hat{S}^{(2)} + \dots$ with $\hat{S}^{(k)}$ the k-term of the Dyson expansion of S in the coupling strength λ, which is defined as

$$\begin{aligned}\hat{S}^{(k)} &= (-\mathrm{i})^k \int_{-\infty}^{\infty} \int_{-\infty}^{\tau_k} ... \int_{-\infty}^{\tau_2} \mathrm{d}\tau_k ... \mathrm{d}\tau_1 \hat{H}_I(\tau_k)...\hat{H}_I(\tau_1) \\ &= \frac{(-\mathrm{i})^k}{k!} \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} \mathrm{d}\tau_k ... \mathrm{d}\tau_1 \mathcal{T}\hat{H}_I(\tau_k)...\hat{H}_I(\tau_1). \end{aligned} \tag{3.25}$$

Therefore, the channel that describes the interaction with the detector at leading and next to leading order in perturbation theory is given by

$$\begin{aligned}\mathcal{E}[\hat{\rho}_{\mathrm{D}}] = {} & \hat{\rho}_{\mathrm{D}} + \lambda \mathrm{tr}_\phi \left(\hat{S}^{(1)}\hat{\rho}_{\mathrm{D}} \otimes \hat{\rho}_\phi + \hat{\rho}_{\mathrm{D}} \otimes \hat{\rho}_\phi \hat{S}^{\dagger(1)}\right) \\ & + \lambda^2 \mathrm{tr}_\phi \left(\hat{S}^{(1)}\hat{\rho}_{\mathrm{D}} \otimes \hat{\rho}_\phi \hat{S}^{\dagger(1)} + \hat{S}^{(2)}\hat{\rho}_{\mathrm{D}} \otimes \hat{\rho}_\phi + \hat{\rho}_{\mathrm{D}} \otimes \hat{\rho}_\phi \hat{S}^{\dagger(2)}\right) + \mathcal{O}(\lambda^3). \end{aligned} \tag{3.26}$$

As explained before, the most general detector model, which linearly couples to a real scalar field, can be specified through the Hamiltonian weight

$$\hat{h}(\mathsf{x}) = \lambda\Lambda(\mathsf{x})\hat{J}(\mathsf{x}) \otimes \hat{\phi}(\mathsf{x}), \tag{3.27}$$

together with a choice of folitation $\mathscr{T}$ such that

$$\hat{H}(\tau) = \int_{\mathcal{E}(\tau)} \mathrm{d}\mathcal{E}\ \hat{h}(\mathsf{x}). \tag{3.28}$$

For a general linear detector model, the map (3.26) can be written in terms of spacetime integrals

$$\begin{aligned}\mathcal{E}[\hat{\rho}_{\text{D}}] = \hat{\rho}_{\text{D}} &- \mathrm{i}\lambda \int \mathrm{d}V \Lambda(\mathsf{x})[\hat{J}(\mathsf{x}), \hat{\rho}_{\text{D}}] \langle\hat{\phi}(\mathsf{x})\rangle \\ &+ \lambda^2 \iint \mathrm{d}V\mathrm{d}V'\Lambda(\mathsf{x})\Lambda(\mathsf{x}')\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}}\hat{J}(\mathsf{x}) \langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle \\ &- \frac{\lambda^2}{2} \iint \mathrm{d}V\mathrm{d}V'\Lambda(\mathsf{x})\Lambda(\mathsf{x}')\mathcal{T}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}} \langle\mathcal{T}\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle \\ &- \frac{\lambda^2}{2} \iint \mathrm{d}V\mathrm{d}V'\Lambda(\mathsf{x})\Lambda(\mathsf{x}') \left[\mathcal{T}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}}\right]^\dagger \langle\mathcal{T}\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle^* + \mathcal{O}(\lambda^3),\end{aligned} \tag{3.29}$$

where we have defined, for every operator $\hat{F}$ acting over the field

$$\langle\hat{F}\rangle = \mathrm{tr}_\phi\left[\hat{F}\hat{\rho}_\phi\right] \tag{3.30}$$

and we have used the fact that given two commuting, time-dependent observables $\hat{A}(\tau)$ and $\hat{B}(\tau)$,

$$\mathcal{T}[\hat{A}(\tau)\hat{A}(\tau')\hat{B}(\tau)\hat{B}(\tau')] = \mathcal{T}[\hat{A}(\tau)\hat{A}(\tau')]\mathcal{T}[\hat{B}(\tau)\hat{B}(\tau')]. \tag{3.31}$$

Also, in the following we will slightly abuse the notation by denoting the spacetime Heavy side function $\theta(\mathscr{T}-\mathscr{T}')$, and related functions, also by $\theta(\tau-\tau')$ when integrated against the spacetime volume.

The form of equation (3.29) suggests writing the action of the field over the detector degrees of freedom as

$$\mathcal{E}[\hat{\rho}_{\text{D}}] = \hat{\rho}_{\text{D}} + \lambda\Xi[\hat{\rho}_{\text{D}}] + \lambda^2\Theta[\hat{\rho}_{\text{D}}] + \mathcal{O}(\lambda^3), \tag{3.32}$$

where Ξ, Θ are linear maps defined as

$$\Xi[\hat{\rho}_{\text{D}}] = -\mathrm{i}\left[\int \mathrm{d}V \Lambda(\mathsf{x}) \langle\hat{\phi}(\mathsf{x})\rangle \hat{J}(\mathsf{x}), \hat{\rho}_{\text{D}}\right] \tag{3.33}$$

and

$$\begin{aligned}\Theta[\hat{\rho}_{\text{D}}] = &\iint \mathrm{d}V\mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}}\hat{J}(\mathsf{x}) \langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle \\ &- \frac{1}{2}\iint \mathrm{d}V\mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\mathcal{T}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}} \langle\mathcal{T}\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle \\ &- \frac{1}{2}\iint \mathrm{d}V\mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}') \left[\mathcal{T}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}}\right]^\dagger \langle\mathcal{T}\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle^* .\end{aligned} \tag{3.34}$$

We will often refer to the two-point function of the field as

$$W(\mathsf{x}, \mathsf{x}') = \langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle \tag{3.35}$$

regardless of the two-point function being the vacuum two-point function or in a different state.

Finally, one can use the following property of the time ordered product:

$$\iint \mathrm{d}\tau\mathrm{d}\tau' \mathcal{T}[\hat{A}(\tau)\hat{A}(\tau')] = 2\iint \mathrm{d}\tau\mathrm{d}\tau' \theta(\tau-\tau')\hat{A}(\tau)\hat{A}(\tau') \tag{3.36}$$

$$= \iint \mathrm{d}\tau\mathrm{d}\tau' \hat{A}(\tau)\hat{A}(\tau') + \iint \mathrm{d}\tau\mathrm{d}\tau' \mathrm{sign}(\tau-\tau')\hat{A}(\tau)\hat{A}(\tau'), \tag{3.37}$$

to write the second integral in (3.34) as

$$\begin{aligned}&\frac{1}{2}\iint \mathrm{d}V\mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\mathcal{T}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}} \langle\mathcal{T}\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle \\ &= \iint \mathrm{d}V\mathrm{d}V' \frac{1+\mathrm{sgn}(\tau-\tau')}{2}\Lambda(\mathsf{x})\Lambda(\mathsf{x}')\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}} \langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle\end{aligned} \tag{3.38}$$

and the third as

$$\begin{aligned}&\frac{1}{2}\iint \mathrm{d}V\mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}') \left[\mathcal{T}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}')\hat{\rho}_{\text{D}}\right]^\dagger \langle\mathcal{T}\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle^* \\ &= \iint \mathrm{d}V\mathrm{d}V' \frac{1-\mathrm{sgn}(\tau-\tau')}{2}\Lambda(\mathsf{x})\Lambda(\mathsf{x}')\hat{\rho}_{\text{D}}\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}') \langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle .\end{aligned} \tag{3.39}$$

This allows one to decompose the map Θ in (3.34) as

$$\Theta[\hat\rho_{\rm D}] = \iint \mathrm{d}V \mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}') \left(\hat J(\mathsf{x}')\hat\rho_{\rm D}\hat J(\mathsf{x}) - \frac{1}{2}\{\hat J(\mathsf{x})\hat J(\mathsf{x}'),\hat\rho_{\rm D}\}\right) \langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle$$
$$+ \left[-\frac{1}{2}\iint \mathrm{d}V \mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\mathrm{sgn}(\tau-\tau')\hat J(\mathsf{x})\hat J(\mathsf{x}') \langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle , \hat\rho_{\rm D}\right]. \tag{3.40}$$

This splitting allows the identification of the dissipative part of the dynamics, given by the first integral in (3.40), and a Hamiltonian part given by the second. Indeed, the second integral can be written again as a nested integral

$$-\frac{1}{2}\iint \mathrm{d}V \mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\mathrm{sgn}(\tau-\tau')\hat J(\mathsf{x})\hat J(\mathsf{x}') \langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle$$
$$= -\frac{\mathrm{i}}{2}\iint \mathrm{d}V \mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\theta(\tau-\tau')(-\mathrm{i}) \left(\hat J(\mathsf{x})\hat J(\mathsf{x}') \langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle - \hat J(\mathsf{x}')\hat J(\mathsf{x}) \langle\hat\phi(\mathsf{x}')\hat\phi(\mathsf{x})\rangle \right) \tag{3.41}$$

which further indicates that it can be written as a single integral of an operator

$$-\frac{\mathrm{i}}{2}\iint \mathrm{d}V \mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\theta(\tau-\tau')(-\mathrm{i}) \left(\hat J(\mathsf{x})\hat J(\mathsf{x}') \langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle - \hat J(\mathsf{x}')\hat J(\mathsf{x}) \langle\hat\phi(\mathsf{x}')\hat\phi(\mathsf{x})\rangle \right)$$
$$= -\mathrm{i}\int \mathrm{d}V \hat V(\mathsf{x}) \tag{3.42}$$

where we have defined

$$\hat V(\mathsf{x}) = -\frac{\mathrm{i}}{2}\int \mathrm{d}V' \Lambda(\mathsf{x})\Lambda(\mathsf{x}')\theta(\tau-\tau') \left(\hat J(\mathsf{x})\hat J(\mathsf{x}') \langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle - \hat J(\mathsf{x}')\hat J(\mathsf{x}) \langle\hat\phi(\mathsf{x}')\hat\phi(\mathsf{x})\rangle \right). \tag{3.43}$$

Interestingly, one can further split this effective potential into two terms by noticing that

$$\langle\hat\phi(\mathsf{x})\hat\phi(\mathsf{x}')\rangle = \frac{1}{2}\langle[\hat\phi(\mathsf{x}),\hat\phi(\mathsf{x}')]\rangle + \frac{1}{2}\langle\{\hat\phi(\mathsf{x}),\hat\phi(\mathsf{x}')\}\rangle$$
$$= K(\mathsf{x},\mathsf{x}') + \mathrm{i}G_R(\mathsf{x},\mathsf{x}') - \mathrm{i}G_A(\mathsf{x},\mathsf{x}') \tag{3.44}$$

where we have defined

$$K(\mathsf{x},\mathsf{x}') = \frac{1}{2}\langle\{\hat\phi(\mathsf{x}),\hat\phi(\mathsf{x}')\}\rangle \tag{3.45}$$

and decomposed the commutator of the field according to the covariant commutation relations. If we multiply expression (3.44) by the step function $\theta(\tau-\tau')$, that is

$$\theta(\tau-\tau')\,\langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle=\theta(\tau-\tau')K(\mathsf{x},\mathsf{x}')+\mathrm{i}\theta(\tau-\tau')G_R(\mathsf{x},\mathsf{x}')-\mathrm{i}\theta(\tau-\tau')G_A(\mathsf{x},\mathsf{x}')$$

we realize that the last two terms do not actually depend on the choice of time function $\mathscr{T}$, since they have support in the past and future lightcone. Indeed, if $\mathsf{x}\in J^+(\mathsf{x}')$, then $\mathscr{T}(\mathsf{x})\geq\mathscr{T}(\mathsf{x}')$ since $\mathscr{T}$ is a timelike folliation of the spacetime under consideration. Similarly, if $\mathsf{x}\in J^-(\mathsf{x}')$ then $\mathscr{T}(\mathsf{x})\leq\mathscr{T}(\mathsf{x}')$. But microcausality enforces that $\langle[\hat{\phi}(\mathsf{x}'),\hat{\phi}(\mathsf{x}')]\rangle$ is different form zero (is the distributional sense) if $\mathsf{x}\in J^+(\mathsf{x})\cup J^-(\mathsf{x}')$, so the choice of $\mathscr{T}$ is irrelevant. Moreover, since G_A has support only in the past lightcone $\mathrm{i}\theta(\mathscr{T}-\mathscr{T}')G_A(\mathsf{x},\mathsf{x}')=0$, and since G_R has only support in the past lightcone $\mathrm{i}\theta(\mathscr{T}-\mathscr{T}')G_R(\mathsf{x},\mathsf{x}')=\mathrm{i}G_R(\mathsf{x},\mathsf{x}')$.

In summary, the effective potential induced by the interaction with the field takes the form

$$\begin{aligned}\hat{V}(\mathsf{x})=&\frac{1}{4}\{\Lambda(\mathsf{x})\hat{J}(\mathsf{x}),\int\mathrm{d}V'G_R(\mathsf{x},\mathsf{x}')\Lambda(\mathsf{x}')\hat{J}(\mathsf{x}')\}\\&-\frac{\mathrm{i}}{4}\int\mathrm{d}V'\Lambda(\mathsf{x})\Lambda(\mathsf{x}')\theta(\tau-\tau')[\hat{J}(\mathsf{x}),\hat{J}(\mathsf{x}')]K(\mathsf{x},\mathsf{x}'),\end{aligned}\tag{3.46}$$

and the map Θ takes the form

$$\begin{aligned}\Theta[\hat{\rho}_{\mathrm{D}}]=&\iint\mathrm{d}V\mathrm{d}V'\Lambda(\mathsf{x})\Lambda(\mathsf{x}')\left(\hat{J}(\mathsf{x}')\hat{\rho}_{\mathrm{D}}\hat{J}(\mathsf{x})-\frac{1}{2}\{\hat{J}(\mathsf{x})\hat{J}(\mathsf{x}'),\hat{\rho}_{\mathrm{D}}\}\right)\langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\rangle\\&-\mathrm{i}\left[\int\mathrm{d}V\hat{V}(\mathsf{x}),\hat{\rho}_{\mathrm{D}}\right].\end{aligned}\tag{3.47}$$

Remarkably, we have isolated the contribution to second order in perturbation theory that depends explicitly on the choice of proper time $\mathscr{T}$. The dissipation term of the map (3.47) does not depend of this choice, because it can be written in terms of covariant integrals. The effective potential $\hat{V}$ contains a contribution that potentially depends on this choice, and another that does not. The figure of merit in this case seems to be the strength of the field's correlations evaluated in the interaction region, that is, the support of Λ.

All the information that one can gather from any detector model at next to leading order in perturbation theory is encoded in the maps Ξ, Θ. We notice that the parameters that depend on the field theory are all given in terms of the mean value of the field amplitude, $\langle\hat{\phi}(\mathsf{x})\rangle$ and the two-point correlator, or Wightman function $\langle\hat{\phi}(\mathsf{x})\rangle$

evaluated in the support of the interaction of the field with the detector. Therefore, assuming some level of control in the parameters of the interaction leads to measuring different, possibly complex smearings of the one and two-point functions of the field in the initial state $\hat{\rho}_\phi$.

The basic quantities predicted by the detector model are the transition probabilities between different states. Indeed, one can calculate the probability of transitioning from a state $|i\rangle$ to a state $|f\rangle$, after the detector has interacted with the quantum field, with Born's rule

$$P_{i\to f} = \langle f|\mathcal{E}[|i\rangle\langle i|]|f\rangle \,. \tag{3.48}$$

Expanding this expression in perturbation theory, one can identify the contributions coming from the maps Ξ and Θ:

$$P_{i\to f} = |\,\langle f|i\rangle\,|^2 + \lambda\,\langle f|\Xi[|i\rangle\langle i|]|f\rangle + \lambda^2\,\langle f|\Theta[|i\rangle\langle i|]|f\rangle + \mathcal{O}(\lambda^3). \tag{3.49}$$

3.2.1 The point-like UDW model

The point-like UDW model is specially relevant for this thesis, since it is the model of choice for most of the contents. Therefore, it is useful to particularize the quantities defining the maps Ξ and Θ in the more general context, and define new quantities that are particular to the UDW model. In this chapter, we will refer to the maps Ξ and Θ and the current $\hat{J}$ particularized to the point-like UDW model as Ξ_{pt} and Θ_{pt}. This notation will be dropped in later chapters whenever the model of choice is obvious by context.

The Hamiltonian weight can be particularized to the point-like UDW model by introducing the following distribution current:

$$\hat{J}_{\text{pt}}(\mathsf{x}) = \int \mathrm{d}s \frac{1}{\sqrt{-|g(s)|}} \delta(\mathsf{x} - \mathsf{x}(s))\chi(s)\hat{\mu}(s) \tag{3.50}$$

where

$$\hat{\mu}(\tau) = e^{\mathrm{i}\Omega\tau}\,|e\rangle\langle g| + e^{-\mathrm{i}\Omega\tau}\,|g\rangle\langle e|\,. \tag{3.51}$$

Particularizing this expression for the leading order contribution in equation (3.33) to the point-like current (3.50), it is immediate to check that

$$\Xi_{\text{pt}}[\hat{\rho}_{\text{D}}] = -\mathrm{i}[\mathcal{Q}(\Omega)\,|e\rangle\langle g| + \mathcal{Q}(-\Omega)\,|g\rangle\langle e|\,, \hat{\rho}_{\text{D}}], \tag{3.52}$$

where we have defined

$$\mathcal{Q}(\Omega,\chi)=\int \mathrm{d}\tau\chi(\tau)\,\langle\hat{\phi}(\mathsf{x}(\tau))\rangle\,e^{\mathrm{i}\Omega\tau}. \tag{3.53}$$

On the other hand, the map Θ_{pt} takes the form

$$\Theta_{\mathrm{pt}}[\hat{\rho}_{\mathrm{D}}]=-\mathrm{i}[\int \mathrm{d}\tau\hat{V}_{\mathrm{pt}}(\tau),\hat{\rho}_{\mathrm{D}}]+\sum\gamma_{ab}(\hat{A}_a\hat{\rho}_{\mathrm{D}}\hat{A}_b^\dagger-\frac{1}{2}\{\hat{A}_b^\dagger\hat{A}_a,\hat{\rho}_{\mathrm{D}}\}) \tag{3.54}$$

where the operators $\hat{A}$ are given by

$$\hat{A}_{1,2}=|e\rangle\langle g|\,,|g\rangle\langle e|\,. \tag{3.55}$$

The Hamiltonian part of the map Θ_{pt} is given by

$$\hat{V}_{\mathrm{pt}}(\tau)=\mathcal{L}(\Omega,\chi,\tau)\,|e\rangle\langle e|-\mathcal{L}(-\Omega,\chi,\tau)\,|g\rangle\langle g| \tag{3.56}$$

where we have defined

$$\mathcal{L}(\Omega,\chi,\tau)=-\mathrm{i}\chi(\tau)\int^{\tau}\mathrm{d}\tau'\chi(\tau')\left(e^{\mathrm{i}\Omega(\tau-\tau')}\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\rangle-e^{-\mathrm{i}\Omega(\tau-\tau')}\langle\hat{\phi}(\mathsf{x}(\tau'))\hat{\phi}(\mathsf{x}(\tau))\rangle\right). \tag{3.57}$$

We find that the two-level system has the particular property that, for all χ

$$[\hat{V},\hat{H}_{\mathrm{free}}]=0. \tag{3.58}$$

This property is only an approximation for more general detectors, which works in the long adiabatic limit [64].

Finally, the diffusion part of Θ_{pt} is specified by the matrix γ, which is given by

$$\gamma_{ab}=\begin{pmatrix}\mathcal{F}_{\mathrm{pt}}(\Omega,\chi) & \mathcal{R}_{\mathrm{pt}}(\Omega,\chi)\\ \mathcal{R}^*_{\mathrm{pt}}(\Omega,\chi) & \mathcal{F}_{\mathrm{pt}}(-\Omega,\chi)\end{pmatrix} \tag{3.59}$$

where we have defined the following functionals

$$\mathcal{F}_{\mathrm{pt}}(\Omega,\chi)=\iint \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau)\chi(\tau')\,\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\rangle\,e^{-\mathrm{i}\Omega(\tau-\tau')} \tag{3.60}$$

$$\mathcal{R}_{\mathrm{pt}}(\Omega,\chi)=\iint \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau)\chi(\tau')\,\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\rangle\,e^{\mathrm{i}\Omega(\tau+\tau')}. \tag{3.61}$$

During the thesis we will often abuse the notation and denote the pull-back of the two-point function along the trajectory of the detector by

$$\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\rangle = W(\tau,\tau'). \tag{3.62}$$

Moreover, we will often find that the pull-back of the correlations of certain two-point functions are stationary, which means that

$$\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\rangle = \langle\hat{\phi}(\mathsf{x}(\tau-\tau'))\hat{\phi}(\mathsf{x}(0))\rangle\,, \tag{3.63}$$

or, again with a slight abusing notation

$$W(\tau,\tau') = W(\tau-\tau'). \tag{3.64}$$

The case of stationary correlations will play a major role in what follows, concretely when calculating quantities in the context of the Unruh effect. As we will see, the functionals involved in the detector's next-to-leading order, dissipative statistics, $\mathcal{F},\mathcal{R}$ are the figures of merit in that case, an therefore it is useful to further simplify them as they take the simpler forms:

$$\mathcal{F}_{\text{pt}}(\Omega,\chi) = \int \mathrm{d}\tau\bar{\chi}\star\chi(\tau)W(\tau)e^{-\mathrm{i}\Omega\tau} \tag{3.65}$$

$$\mathcal{R}_{\text{pt}}(\Omega,\chi) = \int \mathrm{d}\tau(e^{-\mathrm{i}\bar{\Omega}\tau}\chi)\star(e^{\mathrm{i}\Omega\tau}\chi)(\tau)W(\tau), \tag{3.66}$$

where we have defined the convolution in such a way that

$$\bar{f}\star g(\tau) = \int f^*(\tau'-\tau)g(\tau')\mathrm{d}\tau'. \tag{3.67}$$

These expressions suggest that these quantities can be written in terms of the Fourier transform of $W(\tau)$, and indeed

$$\mathcal{F}_{\text{pt}}(\Omega,\chi) = \int \mathrm{d}\omega|\tilde{\chi}|^2(\omega)\tilde{W}(\omega+\Omega) \tag{3.68}$$

$$\mathcal{R}_{\text{pt}}(\Omega,\chi) = \int \mathrm{d}\omega\tilde{\chi}(\omega+\Omega)\tilde{\chi}(\omega-\Omega)\tilde{W}(\omega), \tag{3.69}$$

For the point-like UDW model, the quantity that has played a major role in the lliterature is the excitation probability of the detector of transitioning from its ground state to its excited state, which by direct substitution is given by

$$P_{g\to e} = \lambda^2\,\langle e|\Theta[|g\rangle\langle g|]|e\rangle = \lambda^2\mathcal{F}_{\text{pt}}(\Omega,\chi) + \mathcal{O}(\lambda^3). \tag{3.70}$$

where we have used that the leading order map and the Hamiltonian part of the next to leading order map vanish when applied to the state $|g\rangle\langle g|$.

The case most often studied in the literature is the case of a detector that interacts for a long time with the field. The long time limit is achieved by choosing switching function χ whose support is wide, in such a way that for all practical purposes the statistics of the detector are indistinguishable from an always switched detector. One could be tempted to simply substitute the switching function by a constant function, e.g. $\chi(\tau) = 1$ for all τ, but this often leads to divergences in the statistics. Indeed, the correlations of the field are not expected to be $\mathbb{R}^2$ integrable in τ, τ', since this would imply that the field's correlations vanish for pairs of events in the asymptotic future, regardless of how close those events may be. This is specially true in situations in which the correlations are stationary.

There are multiple ways of addressing this situation, but in this thesis we will make use of asymptotic expansions of the relevant quantities in some time scale. More concretely, it will be relevant the so-called adiabatic, long time limit. By adiabatic, we mean that we take a limit in which the switching function reaches a constant value and such that the derivatives of the switching function tend to zero with the same scale, which we shall refer to as σ. If $\chi(s)$ is a smooth, compactly supported function of s, then a function that fulfils the conditions exposed above can be constructed as

$$\chi_\sigma(\tau) = \chi\left(\frac{\tau}{\sigma}\right). \tag{3.71}$$

Indeed, the pointwise limit of χ_σ when $\sigma \to \infty$ is given by

$$\lim_{\sigma\to\infty} \chi_\sigma(\tau) = \chi(0), \tag{3.72}$$

a constant value, whereas

$$\lim_{\sigma\to\infty} \frac{\mathrm{d}}{\mathrm{d}\tau}\chi_\sigma(\tau) = 0. \tag{3.73}$$

The statistics of the (dissipative part of the) UDW model in the presence of stationary correlations, with adiabatic switching, are then given by

$$\mathcal{F}_{\text{pt}}(\Omega, \chi_\sigma) = \int \mathrm{d}\omega |\tilde{\chi}_\sigma|^2(\omega)\tilde{W}(\omega + \Omega) \tag{3.74}$$

$$\mathcal{R}_{\text{pt}}(\Omega, \chi_\sigma) = \int \mathrm{d}\omega \tilde{\chi}_\sigma(\omega + \Omega)\tilde{\chi}_\sigma(\omega - \Omega)\tilde{W}(\omega), \tag{3.75}$$

but using the properties of the Fourier transform

$$\tilde{\chi}_\sigma(\omega) = \sigma\tilde{\chi}(\sigma\omega), \tag{3.76}$$

and with a change of variable of the form $\omega' = \sigma\omega$, the expressions take the form

$$\mathcal{F}_{\text{pt}}(\Omega, \chi_\sigma) = \sigma \int \mathrm{d}\omega' |\tilde{\chi}|^2(\omega')\tilde{W}(\omega'/\sigma + \Omega) \tag{3.77}$$

$$\mathcal{R}_{\text{pt}}(\Omega, \chi_\sigma) = \sigma \int \mathrm{d}\omega' \tilde{\chi}(\omega' + \Omega\sigma)\tilde{\chi}(\omega' - \Omega\sigma)\tilde{W}(\omega). \tag{3.78}$$

Finally, if we assume that we can take the limit inside the integrals, which will depend on whether the Fourier transform decays faster than $\tilde{W}(\omega)$ grows, $\mathcal{F}$ and $\mathcal{R}$ take the following asymptotic expressions for large σ:

$$\mathcal{F}_{\text{pt}}(\Omega, \chi_\sigma) \sim \sigma\tilde{W}(\Omega) \int \mathrm{d}\omega' |\tilde{\chi}|^2(\omega') \tag{3.79}$$

$$\mathcal{R}_{\text{pt}}(\Omega, \chi_\sigma) \sim 0, \tag{3.80}$$

where we have assume that, given that $\tilde{\chi}$ is integrable, the limit of $\tilde{\chi}(\omega' + \Omega\sigma)\tilde{\chi}(\omega' - \Omega\sigma)$ for $\sigma \to \infty$ is zero.

Therefore, for stationary correlations, the asymptotic behavior of the excitation probability for long times (and at leading order in the coupling constant λ) takes the form

$$P_{g\to e} \sim \sigma \times \lambda^2 \tilde{W}(\Omega). \tag{3.81}$$

where we have reabsorbed the value of the integral in (3.79) in the coupling constant without loss of generality.

Note that a similar calculation for the de-excitation probability leads to

$$P_{e\to g} \sim \sigma \times \lambda^2 \tilde{W}(-\Omega). \tag{3.82}$$

Similarly to the celebrated weak-coupling limit used in open quantum systems, the dissipative part of the statistics is well defined in the long-time limit when keeping scale $\lambda^2\sigma$ is finite.

Chapter 4

From detecting particles to measuring fluctuations

In this chapter we review the results from [1], in which some aspects of measurement theory of quantum field with detectors models were investigated. In this chapter we assume the point-like UDW model. This work was concerned with the possibility of measuring the Wightman function of a quantum field between two spacetime events by analyzing the statistics of a particle detector.

Most analyses in the UDW particle detector literature focus on long-time, adiabatic response of the detector to the quantum field, because of the ties of these processes to the notion of "particle". In this work we pointed out that, conceptually, the finite time response can answer complementary questions that drastically depart form the notion of particle, such as the direct measurement of the Wightman function between two spacetime points.

Waiting for equilibration formally requires taking the adiabatic interaction limit, which makes the nature of the predictions non local. The response of the detector will omit the details of the local structure of the field. Some questions, as how different effects dominate at different time scales cannot be addressed with this setup. For instance, an accelerated detector interacting with a broad class of states will finish in a thermal state at the Unruh temperature [63]. To a large extent, everlasting accelerated detectors can erase any non-Lorentz invariant effects in the theory, as shown in [4], a fact that we will thoroughly discuss in 8 . In addition, the whole world line of the detector is integrated in order to calculate the excitation probability, and the response of the detector becomes highly dependent on the particular trajectory of the detector, therefore yielding different predictions for different trajectories when the detector crosses the same spacetime points. Whereas this fact is an advantage for the analysis of the Unruh effect, as we will see in chapter 6, it poses a challenge to direct measurements of the correlation functions of the field, since for a given detector phenomenology, we cannot separate its dependence on the detector trajectory from the dependence on the field state.

Analyzing the response of the detector for finite time interactions can address the limitations of the analyses mentioned above. We will outline a plausible procedure that in theory could lead to direct measure of the correlations of the quantum field between two spacetime events, thus avoiding the ambiguity of whether the pull-back on a particular trajectory or the actual Wightman function is being probed.

In order to do so, it will prove essential to study the time-correlations of a quantum field from a detector-model perspective. This may be achieved by analyzing aspects of measurement theory involving repeatedly switching the detector on and off, and studying the deviations from uncorrelated statistics, as we illustrate in the following sections.

In this chapter, we will introduce the following notation. If $W(\tau,\tau')$ is the pull back of the two-point correlator of the field, we denote the quadratic form

$$\mathcal{W}^{\Omega}[f,g] = \iint \mathrm{d}\tau \mathrm{d}\tau' f(\tau) g(\tau') e^{-\mathrm{i}\Omega(\tau-\tau')} W(\tau,\tau') \tag{4.1}$$

that is, the pullback in the sense of distributions of the Wightman function of the field along the field's trajectory.

4.1 Sequences of interactions

Consider a switching function composed of a sequence of N repetitions of a particular time dependent function $\xi(\tau)$. Specifically,

$$\chi(\tau) = \sum_{l=0}^{N} \xi_l^{\tau_0}(\tau). \tag{4.2}$$

Here the notation $\xi_l^{\tau_0}(\tau)$ has to be understood as

$$\xi_l^{\tau_0}(\tau) = \xi(\tau - \tau_0 - l\zeta), \tag{4.3}$$

where ξ is a certain function, which we call "tooth" function, τ_0 is the time at which the first tooth is centered, and the rest of the teeth will be centred at $\tau_0 + l\zeta$

for $l = 1, 2, \ldots, N$. Hence, ζ is the constant lapse between each two consecutive interactions. The excitation probability (up to leading order) in these conditions is readily obtained by direct substitution the comb function χ in (3.70) by means of expression (3.60):

$$\mathcal{F}(\Omega, \chi) = \sum_{l=0}^{N} \sum_{s=0}^{N} \mathcal{W}^{\Omega}[\xi_l^{\tau_0}, \xi_s^{\tau_0}]. \tag{4.4}$$

It is useful to separate this sum in two parts:

1. The sum of terms where the integrals only involve tooth functions centered at the same time (local terms). These terms are proportional to $\mathcal{W}[\xi_l^{\tau_0}, \xi_l^{\tau_0}]$, and therefore each one of them independently can be interpreted as the excitation probability of the detector when only one switching process is implemented.

2. The sum of terms where the integrals involve tooth functions evaluated at different times (non-local terms).

Now for the non-local terms we see that the sum runs for l, s from 1 to N, with $l \neq s$. Equivalently, we can write this sum with the index l running from 1 to N and separate the sum for $s < l$ and for with $s > l$. After making this separation explicit, we can write (4.4) as

$$\sum_{\{s \neq l\}}^{N} \mathcal{W}^{\Omega}[\xi_l^{\tau_0}, \xi_s^{\tau_0}] = \sum_{l=0}^{N} \sum_{s=l+1}^{N} \mathcal{W}^{\Omega}[\xi_l^{\tau_0}, \xi_s^{\tau_0}] + \sum_{l=0}^{N} \sum_{s=0}^{l-1} \mathcal{W}^{\Omega}[\xi_l^{\tau_0}, \xi_s^{\tau_0}]. \tag{4.5}$$

Here the first sum can be rearranged by changing the name of the indices. We define $n = s$ and $m = l - s$, in such a way that we have

$$\sum_{l=0}^{N} \sum_{s=l+1}^{N} \mathcal{W}^{\Omega}[\xi_l^{\tau_0}, \xi_s^{\tau_0}] = \sum_{m=1}^{N} \sum_{n=0}^{N-m} \mathcal{W}^{\Omega}[\xi_{n+m}^{\tau_0}, \xi_n^{\tau_0}]. \tag{4.6}$$

Similarly, we can write the second sum as

$$\sum_{l=0}^{N} \sum_{s=0}^{l-1} \mathcal{W}^{\Omega}[\xi_l^{\tau_0}, \xi_s^{\tau_0}] = \sum_{m=1}^{N} \sum_{n=0}^{N-m} \mathcal{W}^{\Omega}[\xi_n^{\tau_0}, \xi_{n+m}^{\tau_0}]. \tag{4.7}$$

It is clear in this form that both sums in the r.h.s. of (4.5) are complex conjugate of each other. Gathering these results together, we write the excitation probability (4.4) as

$$\mathcal{F}(\chi,\Omega) = \sum_{n=0}^{N} \mathcal{F}(\xi_n,\Omega) + 2\mathrm{Re}\left[C(\Omega,\tau_0)\right]. \tag{4.8}$$

Here we have notated the local terms as

$$\mathcal{F}(\xi_n,\Omega) = \mathcal{W}^{\Omega}[\xi_n^{\tau_0},\xi_n^{\tau_0}] \tag{4.9}$$

and the non-local terms are collected as

$$C(\Omega,\tau_0) = \sum_{m=1}^{N}\sum_{n=0}^{N-m} \mathcal{W}^{\Omega}[\xi_{n+m}^{\tau_0},\xi_n^{\tau_0}]. \tag{4.10}$$

Note that if instead of a succession of N teeth the switching function were composed of a single tooth ξ, the transition probability would be given by just $\mathcal{F}(\xi_0,\Omega)$. In this form it can be readily seen that the local contribution to the total excitation probability consists of the sum of the probability of excitation switched on only for the duration of every individual tooth.

On the other hand $C(\Omega,\tau_0)$ gathers the contributions that come from the existence of time-correlations in the state $\hat{\rho}_\phi$.

Note that equation (4.8) works for any general comb interactions even if the two-point function is not stationary (i.e., even if it does not depend only on the difference of times). We conclude this section with some remarks specific to the situation when we impose stationarity.

4.1.1 Stationary correlations

We can consider situations in which the two-point function is stationary. We remind the reader that with this we mean that

$$W(\tau,\tau') = W(\tau-\tau'), \tag{4.11}$$

i.e., the correlations between two events depend only on the difference of times between them. It is interesting to consider the stationarity condition as it is fulfilled in paradigmatic scenarios in physics of detectors interacting with quantum fields, which is our main goal here. Such scenarios comprise, for instance, the interaction of inertial detectors with vacuum and thermal baths, as well as the interaction of uniformly accelerated detectors with vacuum, as we will discuss in more detail chapters 6,7 and 8.

When the situation is stationary it is very easy to see how the information about the target correlations is imprinted in the excitation probability. This allows us to conceive a simple theoretical protocol to extract this information.

For stationary correlations we can see that neither (4.9) nor the different terms in (4.10) depend on n or τ_0. Indeed,

$$\mathcal{W}^{\Omega}[\xi^{\tau_0}_{n+m},\xi^{\tau_0}_{n}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{d}\tau'\xi(\tau-\tau_0-(n+m)\zeta)\xi(\tau'-\tau_0-n\zeta)e^{-\mathrm{i}\Omega(\tau-\tau')}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{d}\tau'\xi(\tau-m\zeta)\xi(\tau')e^{-\mathrm{i}\Omega(\tau-\tau')}W(\tau-\tau') = \mathcal{W}^{\Omega}[\xi^{0}_{m},\xi^{0}_{0}].$$

Using the definition of stationarity we can see that (4.8) simplifies to

$$P^{+} = NP^{+}_{\xi,0} + 2\lambda^2\mathrm{Re}\left[C(\Omega,0)\right], \tag{4.12}$$

where

$$P^{+}_{\xi,0} = \lambda^2\mathcal{W}[\xi^0_0,\xi^0_0] \tag{4.13}$$

is again the excitation probability associated with one single interaction and

$$C(\Omega,0) = \sum_{m=1}^{N}(N-m)\,\mathcal{W}^{\Omega}[\xi^0_m,\xi^0_0]. \tag{4.14}$$

is the time-correlation term.

4.2 Strong and fast couplings

In this section we will consider that the comb of interactions consists of short and intense interactions. We require the interactions to be intense in order to not vanish in the limit where the interaction time is very short, which is the requirements needed to study quantities related to the value of the correlations in a single point. Note first that, as far as the quantities involved are all finite, the validity of the perturbative regime is not in jeopardy if the coupling constant is smaller than any other scale of the problem. First, we will investigate the asymptotics of the detector's transition probabilities in the case of ultra-local interactions. We shall calculate the excitation probability when the switching function is short and intense but with a finite width, which we will call η. More concretely we are interested in the behavior of the excitation probability when this parameter η is small. We can parametrize a switching function with those characteristics in a sufficiently general form as:

$$\chi = \varphi_\eta = \frac{1}{\eta}\varphi(\tau/\eta), \tag{4.15}$$

where φ is an integrable, positive and normalized function. Notice that in order to avoid the introduction of additional timescales we must endow the switching functions with dimensions of inverse time. The one-parameter characterization (4.15) leads to families of nascent delta functions. If we smear a function $f(\tau)$ with any member of this family and we take the limit $\eta \to 0$, we have

$$\lim_{\eta\to 0}\int \frac{\mathrm{d}\tau}{\eta}\varphi(\tau/\eta)f(\tau) = \lim_{\eta\to 0}\int \mathrm{d}\bar{\tau}\varphi(\bar{\tau})f(\eta\bar{\tau}) = f(0)\int \mathrm{d}\bar{\tau}\varphi(\bar{\tau}) = f(0), \tag{4.16}$$

where we have performed the change of variables $\bar{\tau} = \tau/\eta$ and we have assumed that $\varphi(\tau)f(\eta\tau)$ is dominated by an integrable function for all η, in such a way that we can take the limit inside the integral [65].

If we calculate the action of bi-functionals of the form (4.1) over members of these families we get:

$$\mathcal{W}^{\Omega}[\varphi_\eta, \varphi'_\eta] = \int_{-\infty}^{\infty}\mathrm{d}\bar{\tau}\int_{-\infty}^{\infty}\mathrm{d}\bar{\tau}'\varphi(\bar{\tau})\varphi'(\bar{\tau}')e^{-\mathrm{i}\Omega\eta(\bar{\tau}-\bar{\tau}')}W(\eta\bar{\tau}, \eta\bar{\tau}'), \tag{4.17}$$

but in this case one has to be careful when taking the limit $\eta \to 0$, as W is not a function (but a distribution) in general.

We analyze the excitation probability of a detector coupled to a scalar Klein-Gordon field with a switching function of the form (4.15). In this context the excitation probability will not be well defined in the limit $\eta \to 0$. This will be caused, as we will see, by the well-known ultra-violet divergent character of relativistic quantum field theories [19].

It is clear that if the interaction time goes to zero without varying the intensity of the interaction, then the excitation probability will vanish in the instantaneous interaction limit. However, it is a very different case if we increase the intensity of the interactions as they get shorter in a delta switching limit.

It is well known [41, 42] that taking arbitrarily sudden interactions causes UV divergences in the model. Regarding the ratio between the intensity of the interaction and the interaction time we have three options. First as we mentioned above, the interaction time decreases as the intensity increases, but at a slower rate, so we remain in the case where the excitation probability vanishes as the interaction time vanishes. Second, the intensity increases faster than the interaction time decreases. In this case a divergence in the excitation probability is expected regardless of the UV structure of the theory. We are interested in the third option: If the interaction intensity increases at the same rate as the interaction time decreases, as in (4.15), then the excitation probability has an UV divergence.

Indeed if we use $\chi = \delta_{\tau_0}$ as switching function in the excitation probability, we obtain

$$P^+ = \lambda^2 \mathcal{W}(\delta_{\tau_0}, \delta_{\tau_0}) = \lambda^2 W(\tau_0, \tau_0), \tag{4.18}$$

which turns out to be divergent in the limit of coincidence, as we know from chapter 2. From the point of view of the detector, this means that the detector is interacting with all frequencies of the field, as the change in the interaction happens in an arbitrarily short time.

It is well known, however, that for any state $\hat{\rho}_{\mathrm{T}}$ fulfilling the so-called Hadamard condition [19] we can write

$$\langle \hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})\rangle_{\hat{\rho}_{\mathrm{T}}} = F_{\hat{\rho}_{\mathrm{T}}}(\mathsf{x},\mathsf{y}) + \langle 0|\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})|0\rangle\,, \tag{4.19}$$

where $F_{\hat{\rho}_{\mathrm{T}}}(\mathsf{x},\mathsf{y})$ is a regular function and $\langle 0|\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})|0\rangle$ is the Wightman function of the scalar field in the Minkowski vacuum. As $F_{\hat{\rho}_{\mathrm{T}}}(\mathsf{x},\mathsf{y})$ is regular, the second term as $\eta \to 0$ is going to dominate. Having clarified the importance of the vacuum Wightman function in more general grounds, we will analyze the excitation probability of an UDW detector coupled to the Minkowski vacuum through a nascent delta switching.

In those conditions, the Wightman function takes the form (see, for instance, [21])

$$\langle 0|\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})|0\rangle = \int_0^\infty \frac{\mathrm{d}\omega}{(2\pi)^d}\frac{D_d(\omega,m)}{2\omega}e^{-\mathrm{i}\omega\Delta(\mathsf{x}-\mathsf{y})}, \tag{4.20}$$

where

$$\Delta(\mathsf{x}-\mathsf{y}) = \sqrt{-(\mathsf{x}-\mathsf{y})^2}\,\mathrm{sgn}(\mathsf{x}-\mathsf{y})^0$$

is the spacetime interval between x and y, d is the spatial dimension of the theory and $D_d(\omega, m)$ is the density of states of the field for a certain value of the energy ω

in that dimension. The latter can be written in a closed form [21]:

$$D_d(\omega, m) = \frac{2^{1-d}\pi^{d/2}}{\Gamma(d/2)}|\omega|(\sqrt{\omega^2 - m^2})^{d-2}\Theta(\omega - m). \tag{4.21}$$

We see that the Wightman function diverges for Klein-Gordon fields in the coincidence limit, that is for $\mathsf{x} = \mathsf{y}$, as their density of states is unbounded.

Hence, we have that the functional (4.1) is

$$\begin{aligned} \mathcal{W}_{0,d}[\varphi_\eta, \varphi_\eta] = & \int_0^\infty \frac{\mathrm{d}\omega}{(2\pi)^d} \frac{D_d(\omega)}{2\omega} \int_{-\infty}^\infty \int_{-\infty}^\infty \mathrm{d}\tau \mathrm{d}\tau' \\ & \times \frac{1}{\eta^2}\varphi(\tau/\eta)\varphi(\tau'/\eta) e^{-\mathrm{i}\left(\omega\Delta\left(\mathsf{x}(\tau)-\mathsf{y}(\tau')\right)+\Omega(\tau-\tau')\right)}. \end{aligned} \tag{4.22}$$

In order to analyze the asymptotic behavior as $\eta \to 0$, we perform the following changes of variables:

$$\bar{\omega} = \omega\eta, \qquad \bar{\tau} = \tau/\eta, \qquad \bar{\tau}' = \tau'/\eta, \tag{4.23}$$

so (4.22) takes the form

$$\begin{aligned} \mathcal{W}_{0,d}^{\Omega}[\varphi_\eta, \varphi_\eta] = & \int_0^\infty \frac{\mathrm{d}\bar{\omega}}{(2\pi)^d} \frac{D_d(\bar{\omega}/\eta, m)}{2\bar{\omega}} \int_{-\infty}^\infty \int_{-\infty}^\infty \mathrm{d}\bar{\tau} \mathrm{d}\bar{\tau}' \\ & \times \varphi(\bar{\tau})\varphi(\bar{\tau}') e^{-\mathrm{i}\left(\bar{\omega}\Delta\left(\mathsf{x}(\eta\bar{\tau})-\mathsf{y}(\eta\bar{\tau}')\right)/\eta+\Omega\eta(\bar{\tau}-\bar{\tau}')\right)}. \end{aligned} \tag{4.24}$$

Now, it can be seen that the density of states from (4.21) fulfills the following:

$$D_d(\bar{\omega}/\eta, m) = \eta^{1-d} D_d(\bar{\omega}, m\eta). \tag{4.25}$$

For $d \geq 1$ we write

$$\begin{aligned} \mathcal{W}_{0,d}^{\Omega}[\varphi_\eta, \varphi_\eta] = \eta^{1-d} & \int_0^\infty \frac{\mathrm{d}\bar{\omega}}{(2\pi)^d} \frac{D_d(\bar{\omega}, m\eta)}{2\bar{\omega}} \int_{-\infty}^\infty \int_{-\infty}^\infty \mathrm{d}\bar{\tau} \mathrm{d}\bar{\tau}' \\ & \times \varphi(\bar{\tau})\varphi(\bar{\tau}') e^{-\mathrm{i}\left(\bar{\omega}\Delta\left(\mathsf{x}(\eta\bar{\tau})-\mathsf{y}(\eta\bar{\tau}')\right)/\eta+\Omega\eta(\bar{\tau}-\bar{\tau}')\right)}. \end{aligned} \tag{4.26}$$

Finally, assuming that the conditions of the dominated convergence theorem [65] are fulfilled in the integrals we can take the limit of the quantity $\eta^{d-1}\mathcal{W}_{0,d}[\varphi_\eta, \varphi_\eta]$ (for

$d > 1$) when η goes to zero inside the integral. This limit leads to the asymptotic behavior of the excitation probability:

$$\begin{aligned}\lim_{\eta\to 0}\eta^{d-1}\mathcal{W}_{0,d}[\varphi_\eta,\varphi_\eta] &= \int_0^\infty \frac{\mathrm{d}\bar{\omega}}{(2\pi)^d}\frac{D_d(\bar{\omega},0)}{2\bar{\omega}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\bar{\tau}\mathrm{d}\bar{\tau}'\varphi(\bar{\tau})\varphi(\bar{\tau}')e^{-\mathrm{i}\bar{\omega}(\tau-\tau')}\\ &= \int_0^\infty \frac{\mathrm{d}\bar{\omega}}{(2\pi)^d}\frac{D_d(\bar{\omega},0)}{2\bar{\omega}}|\tilde{\varphi}(\bar{\omega})|^2,\end{aligned} \tag{4.27}$$

where $\tilde{\varphi}$ is the Fourier transform of the function φ, which we assume that decays faster than any polynomial. Also, we have taken into account that

$$\begin{aligned}\lim_{\eta\to 0}\frac{\Delta(\eta\bar{\tau},\eta\bar{\tau}')}{\eta} &= \lim_{\eta\to 0}\frac{1}{\eta}\sqrt{-(\mathsf{x}(\eta\bar{\tau})-\mathsf{x}(\eta\bar{\tau}'))^2}\,\mathrm{sgn}\,(\eta(\bar{\tau}-\bar{\tau}'))\\ &= |\dot{\mathsf{x}}^2(0)|(\bar{\tau}-\bar{\tau}') = \bar{\tau}-\bar{\tau}',\end{aligned} \tag{4.28}$$

where we have expanded the numerator at first order in η and we have taken into account that any timelike trajectory fulfills $\dot{\mathsf{x}}^2 = -1$. Thus, we can write the leading order behavior of the excitation probability as $\eta \to 0$ as

$$P^+ \sim \lambda^2\eta^{1-d}\int_0^\infty \frac{\mathrm{d}\bar{\omega}}{(2\pi)^d}\frac{D_d(\bar{\omega},0)}{2\bar{\omega}}|\tilde{\varphi}(\bar{\omega})|^2, \tag{4.29}$$

with $d > 1$. The case $d = 1$ is special because the divergence is not only ultraviolet, but also infrared, that is for $m = 0$ the spectral density $D_1(\omega,0)/2\omega$ is not bounded from below. We expect, however, a logarithmic divergence as m goes to zero (see, for instance, [21]). We see from equation (4.25), that the massless limit and our delta switching limit are the same in $1+1$ dimensions.

We see from (4.29) that the response of the detector ignores both the mass of the field and the energy gap of the detector for $d > 1$. Therefore the detector's response is asymptotically independent of its state of motion as the interaction time goes to zero. We observe that the detector interacts with the field as if it were following an inertial trajectory. Note that as the excitation probability has to be Lorentz invariant [55], it does not depend on the velocity of the detector.

We have studied the structure of the divergences of an UDW detector when it is coupled to a Klein-Gordon field for arbitrary mass and arbitrary dimensions, and for arbitrary Hadamard states.

4.2.1 Comb of interactions

We will see that under those assumptions the behavior of the non-local terms and of the individual excitation probabilities in (4.8) are, in principle, qualitatively different as the latter can diverge. Also the non-local terms are well defined in the limit $\eta \to 0$ under some assumptions and, as we argue in section 4.3, can be used for probing the two-point function.

In this subsection we apply a comb of interactions of the form (4.2), where this time the teeth are nascent delta functions of the form (4.15). We restrict our study here to the time-correlations involving the non-local terms given in equation (4.10). Then the total switching function acquires the form:

$$\chi(\tau) = \sum_{l=0}^{N} \varphi_{l,\eta}^{\tau_0}(\tau), \tag{4.30}$$

where

$$\varphi_{l,\eta}^{\tau_0}(\tau) = \frac{1}{\eta}\varphi\left(\frac{\tau - l\zeta - \tau_0}{\eta}\right). \tag{4.31}$$

Substituting in (4.10) we get

$$C(\Omega,\tau_0) = \sum_{m=1}^{N}\sum_{n=0}^{N-m} \mathcal{W}^{\Omega}[\varphi_{n+m,\eta}^{\tau_0}, \varphi_{n,\eta}^{\tau_0}]. \tag{4.32}$$

The expression of the action of the functionals $\mathcal{W}^{\Omega}$ over the nascent delta switchings within the non-local terms is

$$\begin{aligned} &\mathcal{W}^{\Omega}[\varphi_{n+m,\eta}^{\tau_0}, \varphi_{n,\eta}^{\tau_0}] \\ &= e^{-\mathrm{i}\Omega\zeta m}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{d}\tau'\varphi(\bar{\tau})\varphi(\bar{\tau}')e^{-\mathrm{i}\Omega\eta(\tau-\tau')} \\ &\quad\times W\left(\eta\bar{\tau} + \tau_0 + (n+m)\zeta, \eta\bar{\tau}' + \tau_0 + n\zeta\right), \end{aligned} \tag{4.33}$$

where we have performed the following changes of the variables:

$$\bar{\tau} = \frac{\tau - (n+m)\zeta - \tau_0}{\eta}, \quad \bar{\tau}' = \frac{\tau' - n\zeta - \tau_0}{\eta}. \tag{4.34}$$

In conclusion, we can take the limit $\eta \to 0$ inside the integral sign, and write

$$\lim_{\eta\to 0}\mathcal{W}^{\Omega}[\varphi_{n+m,\eta}^{\tau_0}, \varphi_{n,\eta}^{\tau_0}] = W\left(\tau_0 + (n+m)\zeta, \tau_0 + n\zeta\right). \tag{4.35}$$

Therefore we conclude that the non-local contribution to the excitation probability has a well defined behavior in the limit $\eta \to 0$ and we write

$$\begin{aligned}&\lim_{\eta\to 0} C(\Omega,\tau_0)\\ &= \sum_{m=1}^{N} e^{-\mathrm{i}\Omega\zeta m} \sum_{n=0}^{N-m} W\left(\tau_0 + (n+m)\zeta, \tau_0 + n\zeta\right). \end{aligned} \tag{4.36}$$

This behavior with η is crucial. It means that, for series of fast and intense kicks the deviations from statistical independence in the excitation probability of the detector only depend on the correlations in the initial state $\hat{\rho}_\phi$ between the events when they take place.

This result, which seems trivial, is what will allow us to reconstruct the Wightman function from the excitation probability of the detector, as we will see in section 4.3.

To conclude this subsection we remark that, for simplicity in the expressions, we have applied evenly distributed interactions in time. Nonetheless, it is easy to check that the former statement remains true even when the interactions are not evenly distributed.

4.3 Probing the two-point correlator with fast kicks

In this section we finally propose a method for probing the two-point function of the state $\hat{\rho}_\phi$. The method, as it has been advanced in previous sections, will consist in predicting the value of the two-point correlator between two events by designing interactions with the field that are sharp and intense with a nascent delta switching centered in those two events and measuring their excitation probabilities. Repeated probing of quantum fields has already proven useful to, for example, carry out measurements of the generating functional in quantum simulations of interacting QFTs [66].

A first characteristic of this method is that we have to be able to measure the excitation probability with single kicks, and measure the excitation probability with two kicks for the same couple of events. This requires being able to ensure that if we repeat the experiment the two events, located at τ_1, τ_2, are equivalent to other two events located at $(\tau_1 + \alpha, \tau_2 + \alpha)$, where α is an arbitrary lapse of proper time.

Besides, the accurate measurement of an excitation probability requires a huge amount of experiments. So if we want to predict the two-point correlator we need to repeat the experiment as many times as necessary and be sure that the quantities that we are measuring do not change if we perform the experiment at two different times.

All these considerations lead to the necessity of considering almost exclusively stationary situations.

The theoretical protocol that can be used, in principle, for probing the two-point correlator by applying two fast kicks requires the following assumptions:

- We can measure the excitation probability of the detector regardless of the number of interactions we are applying. More concretely, this means that we can measure the excitation probability when both a double-kick and a single-kick interaction are implemented at any time of our choice. In our notation, we have access to the quantities $P^+_{\xi,0}$, $P^+_{\xi,1}$ and P^+. The indices $0, 1$ denote that we apply the same interaction, described by the tooth function ξ, at two different times separated by a lapse ζ.
- We are free to change the parameters of the detector: the energy gap Ω and the lapse between interactions ζ.
- Besides, we assume that we can shrink the tooth functions in the way we studied in section 4.2, that is, we can choose tooth functions ξ in such a way they act like nascent delta functions, φ_η.
- We also assume that we can kick the target with intense interactions but not intense enough for coming out from the grounds of perturbation theory in the coupling constant λ.

Under these conditions, we particularize the expression (4.2) for the switching function for $N = 2$ and apply our results straightforwardly. First, the excitation probability in (4.8) takes the form

$$\begin{aligned} P^+ &= P^+_{\xi_1} + P^+_{\xi_0} + 2\lambda^2 \mathrm{Re}\,[C(\Omega, \zeta, 2, \tau_0)] \\ &= P^+_{\xi_1} + P^+_{\xi_0} + 2\lambda^2 \mathrm{Re}\,\left[\mathcal{W}^\Omega[\xi_1^{\tau_0}, \xi_0^{\tau_0}]\right]. \end{aligned} \tag{4.37}$$

We define now the following function:

$$\mathcal{S}(\Omega, \tau_0) := \frac{1}{2\lambda^2}\left(P^+ - P^+_{\xi_1} - P^+_{\xi_0}\right) = \mathrm{Re}\,\left[\mathcal{W}^\Omega[\xi_1^{\tau_0}, \xi_0^{\tau_0}]\right], \tag{4.38}$$

which only involves measurable quantities as we have claimed. By doing so we realize that we can obtain the value of Re $\left[\mathcal{W}^{\Omega}[\xi_1^{\tau_0}, \xi_0^{\tau_0}]\right]$ just measuring the excitation probability in several processes.

Finally, for tooth functions of the form (4.15) we calculate the delta-switching limit of (4.38):

$$\lim_{\eta\to 0} \mathrm{Re}\left[\mathcal{W}^{\Omega}[\varphi_{1,\eta}^{\tau_0}, \varphi_{0,\eta}^{\tau_0}]\right] = \mathrm{Re}[e^{-\mathrm{i}\Omega\zeta} W(\zeta+\tau_0, \tau_0)]$$
$$= \cos(\Omega\zeta)\mathrm{Re}[W(\zeta+\tau_0,\tau_0)] + \sin(\Omega\zeta)\mathrm{Im}[W(\zeta+\tau_0,\tau_0)]. \tag{4.39}$$

Under these conditions we can build the whole two-point correlator by forcing the quantity $\Omega\zeta$ to take some concrete values, that is, if we synchronize the detector with the lapse between interactions. On the one hand, if $\Omega\zeta = 2\pi k$ where k is an integer, then we have that

$$\lim_{\eta\to 0} \mathcal{S}(2\pi k\zeta^{-1}, \tau_0) = \mathrm{Re}[W(\zeta+\tau_0,\tau_0)]. \tag{4.40}$$

On the other hand, if we synchronize the detector by choosing the energy gap to fulfill $\Omega\zeta = 2\pi k' + \pi/2$, with k' a different integer, we have that

$$\lim_{\eta\to 0} \mathcal{S}\left((2\pi k' + \pi/2)\zeta^{-1}, \tau_0\right) = \mathrm{Im}[W(\zeta+\tau_0,\tau_0)]. \tag{4.41}$$

Thus we can build the whole two-point correlator in terms of measurable quantities:

$$W(\zeta+\tau_0,\tau_0) = \lim_{\eta\to 0}\left[\mathcal{S}(2\pi k\zeta^{-1}, \tau_0) + \mathrm{i}\mathcal{S}\left((2\pi k' + \pi/2)\zeta^{-1}, \tau_0\right)\right]. \tag{4.42}$$

In light of this result, we see that we can approximately measure the two-point correlator between whatever two events by measuring the excitation probability of each interaction separately, and the excitation probability of the joint interaction. Further, as we pointed out at the beginning of this section the expression is completely meaningful. In terms of stationary correlations the statistic $\mathcal{S}$ takes the form

$$\mathcal{S}(\Omega, 0) = \frac{1}{2\lambda^2}\left(P^+ - 2P^+_{\xi_1}\right). \tag{4.43}$$

So, once we know $P^+_{\xi_1}$, which can be measured in an independent experiment, we have full information about the two point function by measuring the excitation probability P^+ with the comb of nascent deltas:

$$W(\zeta) = \lim_{\eta\to 0}\left[\mathcal{S}(2\pi k\zeta^{-1}, 0) + \mathrm{i}\mathcal{S}\left((2\pi k' + \pi/2)\zeta^{-1}, 0\right)\right]. \tag{4.44}$$

4.4 Discussion

In the context of a quantum field theory probed by a particle detector, we have shown that we can use the detector response to time-separated interactions for measuring the two-point correlator of the field. We have analyzed the general form of the excitation probability of the detector from the ground state to any other state at leading order in perturbation theory, focusing in the simplest case of UDW point-like model.

We can write this excitation probability after two fast interactions as the sum of the individual excitation probabilities plus deviations from statistical independence. These deviations encode crucial information about the target two-point correlation function smeared over the switching function that has two peaks centered at different times. We have further analyzed the particularly relevant case where the correlations are stationary, which corresponds to interesting cases such as thermalization or any other equilibration process.

Next, we have taken the formal limit when the interactions are fast and intense, thus resembling delta distributions. In this limit the correlations are only dependent on the two-point correlator evaluated at the times where the interactions take place, and we have discussed the asymptotics and scales associated with this.

We developed a detection scheme in which both the real and imaginary components of the two-point correlator can be directly measured in two different times. This protocol assumes that once the lapse of time between interactions ζ is fixed, we can change the energy gap of the detector Ω. Both the real (imaginary) part of the correlator can be measured if we synchronize the energy gap of the detector to perform an even (odd) number of free cycles between interactions.

Among these, a particularly important technical aspect is that the value predicted cannot be thought as the value of a function overall, but as the outcome of a distribution acting over narrow smearing functions. However, the point-like information about the Wightman function can be made meaningful when it is understood as a limit of regular finite-smearing distributions. However this does not present a problem in this case as the non-local terms in (4.33) do not involve the coincidence limit, so the limit of the switching functions to delta distributions is well defined.

Importantly, we recall that (4.38) involves the value of the difference of individual excitation probabilities. Both the excitation probability of the combined process and the individual excitation probabilities diverge in the delta-switching limit, but these divergences are spurious and point-independent: the different excitation probabilities at different times diverge but their difference remains finite. This allows us to, in principle, suggest possible experimental protocols that obtain the value of the asymptotic expression $W_{\hat{\rho}_{\mathrm{T}}}(\zeta + \tau_0, \tau_0)$ as $\eta \to 0$. We show that, provided that the detector's probability distributions after each interaction separately are known, we can directly extract information from the two-point correlator of the field operator in the state $\hat{\rho}_\phi$. Further, we showed in section 4.2 that if the interactions are fast enough, then they are completely determined by the correlator evaluated at the times the interactions take place. We finished the general discussion in section 4.3, where we proposed a procedure for evaluating the correlator at two different times directly.

Assuming the UDW model, this method allows one to measure the Wightman function between two events that are timelike separated for a wide variety of states and theories. For comparison, in quantum optics, there are well-known techniques such as homodyne detection [67] that can probe the field quadratures of single particular modes. Notice that these results are limited to massless fields. In contrast the methods we propose to probe the full two point function of a relativistic quantum field along an arbitrary relativistic trajectory, and are easily generalizable to curved backgrounds, whereas to the authors' knowledge, all the methods in the literature are restricted to non-relativistic setups for probing a single mode of a quantum field, and not the full two point function along an arbitrary relativistic trajectory.

We have discussed the mathematical subtleties of evaluating an object of distributional nature at a point, then seeing that this can be done as far as the Wightman distribution does not diverge inside the light-cone. Of course, this is the case for the most common theories that can be found in the literature.

We have also compared this method of measuring the Wightman distribution with the more conventional way, that for a stationary state, consists in waiting for the detector to reach equilibrium. This fast-interaction approach avoids taking the long-time limit, leading thus to measurements that do not involve the non-local character of equilibration. In summary, finite time interactions can be used to probe the Wightman function of a quantum field. This has the advantage that we are able to measure the correlation function in a spacetime localized manner at two points, rather than its pullback over a full trajectory, which is the information one gets when waiting for equilibration.

The measurement of the two-point function of a a quantum field between two space-time points can be also motivated from a different point of view, based on quantum field theory in curved space-times. Intuitively, given that the vacuum two-point function of a quantum field depends on the background geometry, it is not surprising that the metric between two events can be written in terms on the vacuum correlations between those two events. This observation was first made in [68], where the concrete dependence was calculated. The fact that our work shows that the correlations between time-like events can be written as the outcome of excitation probabilities opens an interesting avenue to relate the geometry of space-time to the outcome of measurement of particle detectors.

Chapter 5

The price to pay: Violations of causality in particle detector physics

In this chapter we review some of the results from [2], which are concerned with causality issues in the context of general detector models.

The main subject of this chapter is to deepen the analysis of the friction between *smeared* detector models and relativistic causality for general detector models in curved spacetimes, with an emphasis in the problem of the so-called "impossible measurements in QFT" [34, 39, 40]. Crucially, the causality issues we will tackle are introduced by the very fundamental construction of the model per-se, and not by extra approximations that introduce non-locality, such as the rotating-wave approximation [69], or other a-posteriori non-relativistic approximations [70].

A fully relativistic measurement scheme for QFT in which the detector is another quantum field (and the interactions have certain locality preserving properties) is of course devoid from causality issues (e.g., the FV-framework [29],[40]). However it is perhaps still reasonable to approach measurements in QFT from much simpler, effective, non-relativsitic detector models e.g. the Unruh-DeWitt model. In this chapter we will be concerned with structural aspects of UDW-type models related to the interplay between their *non-relativistic* nature and their spacetime *localization* through their smearing. We will pay especial attention to the possibility of superluminal signaling in smeared models, a phenomenon be unacceptable in relativistic physics.

This chapter is divided into the following sections. In section 5.1, we give general (non-peturbative) arguments about signaling between two detectors, and show the absence of faster-than-light signalling, that is, the absence of signalling when the detectors' couplings are constrained to spacelike separated regions.

5.1 Faster-than-light signaling in detector models

This section is devoted to providing a general argument concerning the existence of faster-than-light signalling in measurement schemes with particle detector models. In the underlying quantum field theory superluminal signalling is prevented through the microcausality axiom, which states that the field operators commute in spacelike separation, i.e.

$$[\hat{\phi}(\mathsf{x}), \hat{\phi}(\mathsf{y})] = 0, \tag{5.1}$$

if x and y are spacelike separated.

It is important to clarify that in the case of selective measurements microcausality does not guarantee statistical independence, i.e., the statistics of $\hat{B}$ will generally depend on the outcome of A . It is well known that quantum field theory permits outcome-outcome correlations in spacelike separation [71]. It is also well understood that outcome-outcome correlations do not lead to superluminal signaling. Rather, they are a consequence of the fact that the state of the field (for example, a thermal state, or the vacuum) can display entanglement and classical correlations even between spacelike separated regions [72].

In this chapter we will make use of the most general possible (linear) detector model of the family of models described by the Hamiltonian density (3.11), introduced in chapter 3. Further, if we are to restrict our set of measurements of the field to this kind of detector-based protocol, it is necessary to define signalling in terms of interactions between several detectors.

Let us study then the dynamics of a set of independent detectors interacting with the same quantum field. First, consider two detectors that couple to the same quantum field undergoing an interaction generated by the Hamiltonian weight

$$\begin{aligned}\hat{h}(\mathsf{x}) = \hat{h}_{\text{A}}(\mathsf{x}) + \hat{h}_{\text{B}}(\mathsf{x}) = \lambda_{\text{A}}\Lambda_{\text{A}}(\mathsf{x})\hat{J}_{\text{A}}(\mathsf{x}) \otimes \mathbb{I}_{\text{B}} \otimes \hat{\phi}(\mathsf{x}) \\ + \lambda_{\text{B}}\Lambda_{\text{B}}(\mathsf{x})\mathbb{I}_{\text{A}} \otimes \hat{J}_{\text{B}}(\mathsf{x}) \otimes \hat{\phi}(\mathsf{x}).\end{aligned} \tag{5.2}$$

This scalar generates a joint Hamiltonian for the joint system of the form

$$\hat{H}(t) = \hat{H}_{\text{A}}(t) + \hat{H}_{\text{B}}(t) \tag{5.3}$$

where

$$\hat{H}_{\text{A,B}}(t) = \int_{\mathcal{E}(t)} \mathrm{d}\mathcal{E}\ \hat{h}_{\text{A,B}}(\mathsf{x}). \tag{5.4}$$

Note that this Hamiltonian generates evolution with respect to the same parameter t for both detectors. Although we will not concern ourselves with this in the present chapter, since it has already been studied in [57], it is clear that one needs to properly reparametrize the local Hamiltonians to generate time translations with respect to the same parameter, which in general cannot correspond to the proper time of both detectors.

In order to analyze causal relations between detectors, we need first to define causal relations between (compact) subsets of spacetime. Given a globally hyperbolic spacetime, the future lightcone of a region $\mathcal{O}$, $\mathcal{J}^+(\mathcal{O})$, is the set of all points that lay in the causal future of some point of O. Similarly, $\mathcal{J}^-(\mathcal{O})$, the causal past of a region $\mathcal{O}$, is the set of all points that lay in the causal past of $\mathcal{O}$.

- We say that A and B are causally orderable if $\mathcal{J}^-(\mathcal{O}_{\text{A}}) \cap \mathcal{O}_{\text{B}}$ or $\mathcal{J}^-(\mathcal{O}_{\text{B}}) \cap \mathcal{O}_{\text{A}}$ are empty. If two sets are not orderable, there is not reference frame in which one comes after the other, see fig 5.1.
- We say that A and B are spacelike separated if $(\mathcal{J}^+(\mathcal{O}_{\text{A}}) \cup \mathcal{J}^-(\mathcal{O}_{\text{A}})) \cap \mathcal{O}_{\text{B}}$ or $(\mathcal{J}^+(\mathcal{O}_{\text{B}}) \cup \mathcal{J}^-(\mathcal{O}_{\text{B}})) \cap \mathcal{O}_{\text{A}}$ are empty. Notice that this is a particular case of causally orderable.
- Finally, we have that if $\mathcal{O}_{\text{B}} \subset \mathcal{J}^+(\mathcal{O}_{\text{A}})/O_{\text{A}}$, and $\mathcal{O}_{\text{A}} \subset \mathcal{J}^-(\mathcal{O}_{\text{B}})/O_{\text{B}}$, we say that A causally precedes B. Notice that this is a particular case of causally orderable since although $\mathcal{J}^-(\mathcal{O}_{\text{B}}) \cap \mathcal{O}_{\text{A}} = O_{\text{A}} \neq \emptyset$, it holds that $\mathcal{J}^-(\mathcal{O}_{\text{A}}) \cap \mathcal{O}_{\text{B}} = \emptyset$.

Specifically, we have defined $\mathcal{O}_{\text{A}}$ to precede $\mathcal{O}_{\text{B}}$ if for every observer all the events in $\mathcal{O}_{\text{A}}$ precede any event in $\mathcal{O}_{\text{B}}$, that is, $\mathcal{O}_{\text{A}}$ "comes first" for all observers.

These are covariant statements that are independent of the observer, but one can also define causal relations with respect to a particular foliation $\mathscr{T}(\mathsf{x})$. We say that A precedes B with respect to $\mathscr{T}(\mathsf{x})$ if $\mathscr{T}(\mathsf{x}) < \mathscr{T}(\mathsf{y})$ for all $\mathsf{x} \in \mathcal{O}_{\text{A}}$ and for all $\mathsf{y} \in \mathcal{O}_{\text{B}}$. The two notions are linked by the following facts:

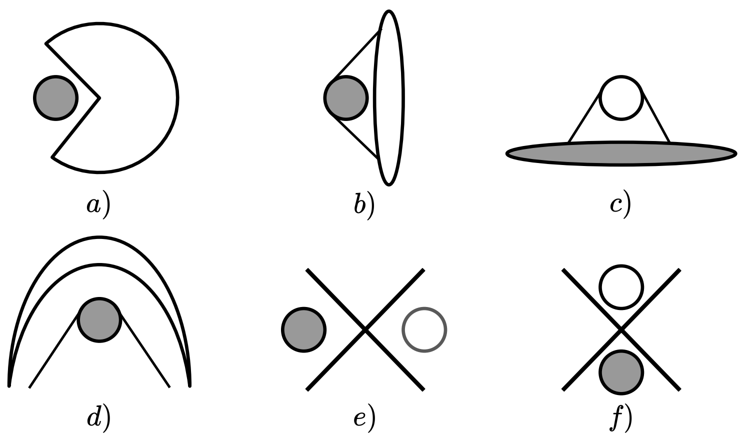


Figure 5.1: Causal relations between simply connected, non intersecting sets (grey and white) in two spacetime dimensions. Black lines represent the future or past lightcone of the sets or points between them. $a), b)$: Examples of non causally orderable sets. $c), d)$: Examples of sets that are causally orderable, but that do not causally precede each other according to our definition. $e)$: Spacelike separated sets. $f)$: Example of a set causally preceding another set.

- If A and B are causally orderable, one precedes the other with respect to some foliation.
- If A and B are spacelike separated, then there are at least two foliations such that A precedes B with respect to one and such that B precedes A with respect to the other.
- If A causally precedes B, then A precedes B with respect to all foliations.

We will say that two detectors obey any of the causal order relations above if the regions $O_{\text{A}} = \text{supp}(\Lambda_{\text{A}})$, $O_{\text{B}} = \text{supp}(\Lambda_{\text{B}})$ obey the respective causal relations described above. See figure 5.1 for examples with simply connected sets.

Given these definitions of causal relations, we can analyze further the implications of the microcausality axiom in detector physics. The Hamiltonians defined by (5.4) are defined respect to some time function $\mathscr{T}(\mathsf{x})$, so the two detectors will naturally have causal relations with respect to the foliation defined by its level curves. If the underlying field theory were not relativistic, we would expect that different foliations give rise to different dynamics for spacelike separated detectors, because in that case the order in which the measurements are done would typically matter. This is exactly what is to be avoided in a relativistic theory, and in the following we will examine this condition in detector models departing from the microcausality condition of the underlying QFT.

Now, recall that the microcausality axiom in curved spacetimes implies that, for two compactly supported spacetime functions $m(\mathsf{x})$ and $l(\mathsf{x})$,

$$\int \text{d}\mathsf{V} \int \text{d}\mathsf{V}'\, l(\mathsf{x}) m(\mathsf{y}) [\hat{\phi}(\mathsf{x}), \hat{\phi}(\mathsf{y})] = 0 \tag{5.5}$$

where $\text{d}\mathsf{V} = \text{d}\mathsf{x}^n \sqrt{|g|}$ and $\text{d}\mathsf{V}' = \text{d}\mathsf{y}^n \sqrt{|g'|}$, if the supports of l and m are spacelike separated. Therefore, the microcausality axiom implies that

$$\left[\hat{h}_{\text{A}}(\mathsf{x}), \hat{h}_{\text{B}}(\mathsf{y})\right] = 0 \tag{5.6}$$

if Λ_{A} and Λ_{B} have spacelike separated supports. This in turn implies that

$$\left[\hat{H}_{\text{A}}(t), \hat{H}_{\text{B}}(t')\right] = 0 \tag{5.7}$$

for all t, t'.

The joint evolution in the of the detectors and the field can be described as a unitary operator acting over the joint state of the system. That is, if $\hat{\rho}_{\text{initial}}$ is the density operator describing the state of the field-detectors system before the interactions are switched-on (respect to the parameter t). The notation A+B indicates that the operator accounts for the interaction of the two detectors, whereas $\hat{S}_{\text{A,B}}$ will denote the scattering matrices associated with the individual interactions generated by the individual interaction Hamiltonians. The total state in the asymptotic future will be given by the transformation

$$\hat{\rho}_{\text{final}} = \hat{S}_{\text{A+B}}\ \hat{\rho}_{\text{initial}}\ \hat{S}^{\dagger}_{\text{A+B}} \tag{5.8}$$

where $\hat{S}_{\text{A+B}}$ is the so-called scattering operator[1]. The scattering operator is unitary and can be formally written as the Dyson series

$$\hat{S}_{\text{A+B}} = \sum_n \left(\frac{-\mathrm{i}}{\hbar}\right)^n \frac{1}{n!} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \mathrm{d}t^n \mathcal{T}(\hat{H}_{\text{A}}(t_1) + \hat{H}_{\text{B}}(t_1) \ldots \hat{H}_{\text{A}}(t_n) + \hat{H}_{\text{B}}(t_n)). \tag{5.9}$$

Intuitively, we would like to ensure that if two detectors A and B are coupled to the field in spacelike separation, one cannot conclude whether the other one is coupled to the field or not. Therefore, a minimum non-signaling requirement would be that if A interacts first with the quantum field in any foliation, i.e. if B is not in the causal future of A, then all expectation values of observables of detector B should not depend on magnitudes of detector A, e.g. the coupling constant λ_{A}. If the expectation values of observables of B depend on λ_{A}, then its value could be used to encode, and then signal information.

It is not a priori obvious why the causal behaviour of the underlying QFT, e.g. the microcausality axiom, would guarantee the causal behaviour of detectors. However, we will see that this is guaranteed under some conditions [73]. As we will show below, in the context of particle detector models, faster-than-light signalling is prevented if the joint scattering matrix factorizes when the detectors are causally orderable. In particular, if B does not intersect with the past of A, we would have

$$\hat{S}_{\text{A+B}} = \hat{S}_{\text{B}}\hat{S}_{\text{A}}. \tag{5.10}$$

We will refer to this property as causal factorization.

[1] It is common in the UDW literature to denote the evolution operator by $\hat{U}$. In this work, however, we prefer to denote it with $\hat{S}$ to emphasize the fact that these maps represent scattering operators and we adopt a notation analog to, e.g., [40] .

To see that causal factorization prevents acausal signalling, consider the local statistics of the detector A, given by the partial trace

$$\hat{\rho}_{\text{A}} = \text{tr}_{\text{B},\phi}(\hat{S}_{\text{A+B}}\ \hat{\rho}_{\text{initial}}\ \hat{S}^{\dagger}_{\text{A+B}}). \tag{5.11}$$

Now, if causal factorization holds, then

$$\hat{\rho}_{\text{A}} = \text{tr}_{\text{B},\phi}(\hat{S}_{\text{B}}\hat{S}_{\text{A}}\ \hat{\rho}_{\text{initial}}\ \hat{S}^{\dagger}_{\text{A}}\hat{S}^{\dagger}_{\text{B}}). \tag{5.12}$$

But $\hat{S}_{\text{B}}$ depends only on operators acting over the subspaces associated with the field and the detector B, therefore it can be permuted within the partial trace:

$$\begin{aligned}\hat{\rho}_{\text{A}} &= \text{tr}_{\text{B},\phi}(\hat{S}_{\text{A}}\ \hat{\rho}_{\text{initial}}\ \hat{S}^{\dagger}_{\text{A}}\hat{S}^{\dagger}_{\text{B}}\hat{S}_{\text{B}}) \\ &= \text{tr}_{\text{B},\phi}(\hat{S}_{\text{A}}\ \hat{\rho}_{\text{initial}}\ \hat{S}^{\dagger}_{\text{A}}).\end{aligned} \tag{5.13}$$

Therefore, we have shown that if causal factorization holds, there is no local (spacetime compact) measurement carried though a detector interaction that can be used to receive signals from another detector outside the causal past of such interaction.

Note that in the particular case where A and B are spacelike separated, then causal factorization implies

$$\hat{S}_{\text{A+B}} = \hat{S}_{\text{B}}\hat{S}_{\text{A}} = \hat{S}_{\text{A}}\hat{S}_{\text{B}}, \tag{5.14}$$

which implies that neither detector A can signal to detector B nor detector B can signal to detector A, that is, it prevents faster than light signaling.

It is rather intuitive why condition (5.10) should hold if, e.g. A precedes B respect to the concrete foliation in which the interaction has been defined, as the unitary evolution factorizes by construction. This can be used to argue that the factorization will be independent of the foliation if A causally precedes B in the sense given at the beginning of this section. The proof is also simple if A and B are spacelike separated, in which case the factorization also holds independently of the foliation. What is less trivial, however, is that the factorization holds if the detectors are causally orderable, which is a covariant statement that does not depend on the foliation either.

In conclusion, causal factorization prevents faster-than-light signalling, as far as only two detectors are involved. The result can be extended to some limited scenarios with many detectors. For example, if one has more than two detectors, say $\text{A}, \text{B}_1, \dots, \text{B}_N$, one can always define the collection of all the detectors that are not

A as a single detector A^c. If all the detectors in A^c and A are causally orderable, with A preceding the rest, then again causal factorization will hold and

$$\hat{S}_{\Sigma B_i + A} = \hat{S}_{\Sigma B_i} \hat{S}_A = \hat{S}_{A^c} \hat{S}_A \tag{5.15}$$

and the measurements on A will not be affected by the other detectors.

It could be tempting to claim that this implies that the signals sent by a detector can only reach other detectors in the causal future of its interaction region. Indeed, causal factorization ensures this as long as we consider schemes involving two detectors. Obviously, if a detector B can only receive signals from its causal past, then another single detector A can only send signals to B if B is in the causal future of A.

However, if more than two detectors are involved, then causal factorization does not solve all the possible frictions that the detector models can have with relativistic causality.

5.2 "Impossible measurements" and superluminal propagation of initial data

We have defined signalling so far as the transmission of information between detectors through their interaction with the field. We have seen that, in a two-detector scenario, a detector localized in some region is irrelevant for another detector localized in its causal complement, which means that the detector only influences, in some sense, its own causal future.

However, as pictured in Sorkin's impossible measurements paper [34], there are subtleties associated with the detectors not being in a definite causal ordering when considering more than two measurements. Namely, even if the response of the detector A cannot be influenced by the detector B, the influence of detector A over B can still carry information about events that happened outside the causal past of B, which is obviously not acceptable.

In order to understand how Sorkin's problem can manifest in measurement's models with particle detectors, we shall first analyze a different kind of signalling in which the information is not encoded in the interaction, but in the initial state of the system.

Indeed, a detector can also be thought of as a repeater, that is, given some initial state of the field (possibly coming from another interaction), the detector can register

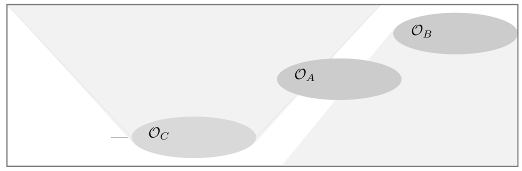


Figure 5.2: Two detectors A and B are coupled to the field over regions $\mathcal{O}_{\text{A,B}}$. The initial data are encoded in the field state through a unitary intervention over region $\mathcal{O}_{\text{C}}$. Notice that region $\mathcal{O}_{\text{A}}$ is partially invading the past and future lightcones of regions $\mathcal{O}_{\text{B}}$ and $\mathcal{O}_{\text{C}}$ respectively.

the initial data and propagate it back to the field. In this case, one may fear that a detector can re-emit information in a non-causal manner. In this subsection we will prove that this is indeed a reasonable concern, since superluminal propagation of initial data is a widespread phenomenon when considering non-relativistic systems.

For instance, one could imagine a scenario in which a detector A partially precedes and is partially spacelike separated from a second detector B (see figure 5.2). Consider that the state of the system is initially given by

$$\hat{\rho}_{\text{initial}} = \hat{\rho}_{\text{A}} \otimes \hat{\rho}_{\text{B}} \otimes e^{\mathrm{i}\lambda_f \hat{\phi}(f)} \hat{\rho}_{\phi} e^{-\mathrm{i}\lambda_f \hat{\phi}(f)} \tag{5.16}$$

where $\hat{\rho}_{\text{A,B},\phi}$ are arbitrary states of the detectors A, B and the quantum field respectively, and $\hat{\phi}(f)$ is a smeared field operator which is compactly supported in region[2] $\mathcal{O}_{\text{C}}$, spacelike separated from B, but not from A. If $\mathcal{O}_{\text{C}}$ is spaceilike separated from B, the local statistics of B should not be affected by the value of the constant λ_f, otherwise detector A would be acting as an agent for superluminal signalling.

More generally, one can consider the case in which the initial state of the detectors plus field has the form

$$\hat{\rho}_{\text{initial}} = \hat{U} \hat{\rho}_0 \hat{U}^\dagger, \tag{5.17}$$

where $\hat{\rho}_0$ is an arbitrary reference state of the joint system, and $\hat{U} = \mathbb{I}_{\text{A}} \otimes \mathbb{I}_{\text{B}} \otimes \hat{U}_\phi$ is an arbitrary unitary acting on the field's Hilbert space, so that $\hat{U}_\phi$ is localized in $\mathcal{O}_{\text{C}}$ (contained in the causal complement of the interaction region $\mathcal{O}_{\text{B}}$). It is clear that

$$[\hat{U}, \hat{S}_{\text{B}}] = 0. \tag{5.18}$$

[2] One can think of this as a third party Charles encoding information in the field in region through a spacetime localized unitary action in region $\mathcal{O}_{\text{C}}$.

$\hat{U}$ can be thought of as encoding a set of initial data[3]. The statistics of detector B can only depend on $\hat{U}$ if detector A's interaction region overlaps with the causal past of B and $\hat{S}_{\text{A}}$ does not commute with $\hat{U}$ (e.g., as shown in Fig. 5.2). To avoid superluminal signalling, it should hold that the local statistics of B do not depend on the choice of $\hat{U}$, i.e.

$$\hat{\rho}_{\text{B}} = \text{tr}_{\text{A},\phi}(\hat{S}_{\text{A+B}}\hat{U}\ \hat{\rho}_0\ \hat{U}^\dagger \hat{S}^\dagger_{\text{A+B}}) = \text{tr}_{\text{A},\phi}(\hat{S}_{\text{A+B}}\ \hat{\rho}_0\ \hat{S}^\dagger_{\text{A+B}}). \tag{5.19}$$

Further, since B is localized (at least partially) in the future of A, and it is spacelike separated from the set of initial data implemented by $\hat{U}$, B cannot be fully contained in the causal past of A. We conclude that A does not causally precede B, in the terminology of the last section.

Imposing condition (5.19) for all initial density operators is equivalent to

$$\text{tr}_{\text{A},\phi}(\hat{V}\ \hat{\sigma}\ \hat{V}^\dagger) = \text{tr}_{\text{A},\phi}(\hat{\sigma}) \tag{5.20}$$

for any arbitrary density operator $\hat{\sigma}$, where $\hat{V}$ is a unitary given by

$$\hat{V} = \hat{S}_{\text{A+B}}\hat{U}\hat{S}^\dagger_{\text{A+B}}. \tag{5.21}$$

This implies that if $\hat{D}_{\text{B}}$ is an operator acting on detector B (i.e. it commutes with the field operators and with the operators acting on detector A) then

$$\text{tr}(\hat{V}^\dagger \hat{D}_{\text{B}} \hat{V} \hat{\sigma}) = \text{tr}(\hat{D}_{\text{B}} \hat{\sigma}) \tag{5.22}$$

for all $\hat{\sigma}$. For our purposes, this implies that

$$\hat{V}^\dagger \hat{D}_{\text{B}} \hat{V} = \hat{D}_{\text{B}}, \tag{5.23}$$

or equivalently

$$[\hat{D}_{\text{B}}, \hat{V}] = 0 \tag{5.24}$$

for all operators acting over detector B. Assuming that A precedes B, the connection with the propagation of initial data is more clear when one uses causal factorization. Then, $\hat{S}_{\text{A+B}} = \hat{S}_{\text{B}}\hat{S}_{\text{A}}$ and condition (5.24) can be written as

$$[\hat{S}^\dagger_{\text{B}} \hat{D}_{\text{B}} \hat{S}_{\text{B}}, \hat{S}_{\text{A}} \hat{U} \hat{S}^\dagger_{\text{A}}] = 0, \tag{5.25}$$

[3] We can always thing without loss of generality that the action of $\hat{U}$ is localized in a subset of a Cauchy surface in the causal past of $\mathcal{O}_{\text{C}}$

for all unitaries in the causal complement of B. If we think of $\hat{S}_{\text{B}}^\dagger \hat{D}_{\text{B}} \hat{S}_{\text{B}}$ as an induced operator acting on the field localized in region B and of $\hat{S}_{\text{A}} \hat{U} \hat{S}_{\text{A}}^\dagger$ as the evolution of the initial data given by interaction A, we can interpret condition (5.24) as meaning that the interaction A does not propagate initial data superluminally, since the propagated data still lays within the causal complement of region B. This condition is related to the unitary restriction of the condition discussed in [39], but more general in the sense that allows for auxiliary degrees of freedom representing the devices used to implement the measurement.

The relevant question now is whether condition (5.24) holds for general detector models. Unfortunately the answer is generally negative. It is easy to corroborate using perturbation theory that the localization region of $\hat{S}_{\text{A}} \hat{U} \hat{S}_{\text{A}}^\dagger$ is not the causal future of $\hat{U}$, but the causal future of A. Indeed, using Dyson's expansion

$$\begin{aligned}\hat{S}_{\text{A}}\, \hat{U}\, \hat{S}_{\text{A}}^\dagger &= \hat{U} - \frac{\mathrm{i}}{\hbar} \int \mathrm{d}V [\hat{h}_{\text{A}}(\mathsf{x}), \hat{U}] \\ &- \frac{1}{2\hbar^2} \int \mathrm{d}V \int \mathrm{d}V' \mathcal{T} \left[\hat{h}_{\text{A}}(\mathsf{x}), [\hat{h}_{\text{A}}(\mathsf{y}), \hat{U}]\right] \\ &+ \mathcal{O}(\lambda_{\text{A}}^3).\end{aligned} \tag{5.26}$$

If we pay attention to the first term, which is given by the density

$$[\hat{h}_{\text{A}}(\mathsf{x}), \hat{U}] = \lambda_{\text{A}} \Lambda_{\text{A}}(\mathsf{x}) \hat{J}_{\text{A}}(\mathsf{x}) \otimes \mathbb{I}_{\text{B}} \otimes [\hat{\phi}(\mathsf{x}), \hat{U}], \tag{5.27}$$

we realize that microcausality ensures that no x outside the lightcone of $\hat{U}$ can contribute to the integral. This means that regardless of the localization of region B, the leading order propagation of initial data is still localized in the lightcone of $\hat{U}$ and the propagation is causal.

Now, at second order, the contribution will be given by the kernel

$$\left[\hat{h}_{\text{A}}(\mathsf{x}), [\hat{h}_{\text{A}}(\mathsf{y}), \hat{U}]\right] \tag{5.28}$$

where the time-ordering is implemented considering that y precedes x respect to the foliation $\mathscr{T}(\mathsf{x})$. Because of microcausality, y will also be constrained to lie within the lightcone of $\mathcal{O}_{\text{C}}$, but x can be anywhere. One can use Jacobi's identity to expand this kernel as follows

$$\begin{aligned}&\left[\hat{h}_{\text{A}}(\mathsf{x}), [\hat{h}_{\text{A}}(\mathsf{y}), \hat{U}]\right] \\ &= \left[[\hat{h}_{\text{A}}(\mathsf{x}), \hat{h}_{\text{A}}(\mathsf{y})], \hat{U}\right] + \left[\hat{h}_{\text{A}}(\mathsf{y}), [\hat{h}_{\text{A}}(\mathsf{x}), \hat{U}]\right],\end{aligned} \tag{5.29}$$

such that x has to lie in the lightcone of the initial data for the second term not to vanish, but the first one will not generally vanish when x is outside the lightcone of $\mathcal{O}_{\text{C}}$.

One can see that in general, unless $[\hat{h}_{\text{A}}(\mathsf{x}), \hat{h}_{\text{A}}(\mathsf{y})] = 0$ when x and y are spacelike separated, the propagation will not be causal anymore. Similar results were found in [57] when addressing violations of relativistic covariance.

Indeed, one can further expand the commutator of the Hamiltonian densities as

$$\begin{aligned}
&[\hat{h}_{\text{A}}(\mathsf{x}), \hat{h}_{\text{A}}(\mathsf{y})] \\
&= \lambda_{\text{A}}^2 \Lambda_{\text{A}}(\mathsf{x}) \Lambda_{\text{A}}(\mathsf{y}) [\hat{J}_{\text{A}}(\mathsf{x}), \hat{J}_{\text{A}}(\mathsf{y})] \otimes \mathbb{I}_{\text{B}} \otimes \hat{\phi}(\mathsf{x}) \hat{\phi}(\mathsf{y}) \\
&+ \lambda_{\text{A}}^2 \Lambda_{\text{A}}(\mathsf{x}) \Lambda_{\text{A}}(\mathsf{y}) \hat{J}_{\text{A}}(\mathsf{x}) \hat{J}_{\text{A}}(\mathsf{y}) \otimes \mathbb{I}_{\text{B}} \otimes [\hat{\phi}(\mathsf{x}), \hat{\phi}(\mathsf{y})].
\end{aligned} \tag{5.30}$$

Again, microcausality ensures that the second term in (5.30) vanishes in spacelike separation, but the first one will not vanish, nor will commute with $\hat{U}$ in general , unless $[\hat{J}_{\text{A}}(\mathsf{x}), \hat{J}_{\text{A}}(\mathsf{y})] = 0$ in spacelike separation. In general it is not difficult to argue (following a similar combinatoric procedure as in [57], together with a recursive use of Jacobi's identity) that if the interaction Hamiltoinan density of A is microcausal (for example for a pointlike detector), the propagation of initial data is causal in all orders in perturbation theory.

If this condition holds, it means that either all points in $\operatorname{supp} \Lambda_{\text{A}}$ are causally connected (which is only possible for a pointlike detector) or that the detector is a relativistic field. Since by assumption the system is non-relativistic and generally smeared, we conclude that the detector's dynamics carry superluminal propagation of initial data at second order in perturbation theory.

Note that since for point-like detectors there is not superluminal propagation, one can disregard this kind of faster-than-light signalling for "small enough" detectors. Whether a detector is small or not will depend, of course, on the parameters of the problem.

The preceding discussion provides a dynamical interpretation of the impossible measurements problem, in the sense that it links superluminal signalling with superluminal propagation within the device that is implementing the measurement. It is clear then, that if the detector is a relativistic quantum field then there is not superluminal propagation of initial data under some assumptions in the dynamics of the coupling, as it is shown in full rigor in [40]. In our case, however, we have to understand this kind of faster-than-light signalling as a fundamental feature of non-relativistic particle detector models that restricts their usage to regimes where these superluminal features are negligible or irrelevant for the results at hand.

5.3 Impossible measurements with weakly coupled detectors

We have seen that faster-than-light signalling is present in smeared non-relativistic particle detector models. However, calculations involving particle detectors are most commonly carried out in perturbation theory. Indeed, not only the justification of the model is jeopardized for strong couplings, but also some of the most interesting phenomenology, such emission and absorption of particles, can be described at quadratic order in the coupling strengths. Not only that, this is also the leading order for most phenomena in relativistic quantum information (e.g., detector's responses [21], communication [74], entanglement harvesting [75] and the Fermi Problem [76, 77, 78, 79], etc..). This section is devoted to analyze the order in perturbation theory at which superluminal propagation of initial data, described in last section, plays a role in measurement schemes involving more than two detectors.

Let us slightly extend the set-up described in section 5.2 by assuming that the unitary $\hat{U}$ in (5.17) is implemented by a weakly coupled detector C, in such a way that we can write $\hat{U} = \hat{S}_{\text{C}}$. We can now determine at which order in perturbation theory the dynamics exhibits superluminal signalling, that is, at which order in perturbation theory condition (5.25) fails to hold.

In order to do so, we first define the operator

$$\hat{K} = [\hat{S}_{\text{B}}^{\dagger}\hat{D}_{\text{B}}\hat{S}_{\text{B}}, \hat{S}_{\text{A}}\hat{S}_{\text{C}}\hat{S}_{\text{A}}^{\dagger}]. \tag{5.31}$$

If this operator vanished there would be no superluminal propagation of initial data. We can determine the first order in the coupling strengths at which $\hat{K}$ does not trivially vanish.

We can expand $\hat{K}$ in the coupling strengths by writing $\hat{K} = \hat{K}^{(0)} + \hat{K}^{(1)} + ...$, where each $\hat{K}^{(j)}$ contains integrals involving j Hamiltonians. Each term $\hat{K}^{(j)}$ will contain contributions from orders in the coupling constants of detectors C+A+B in such a way that all the powers add up to j. It is easy to see that

$$\hat{K}^{(0)} = [\hat{D}_{\text{B}}, \mathbb{I}] = 0, \tag{5.32}$$

and that the linear term will also vanish

$$\hat{K}^{(1)} = \left[\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t[\hat{H}_{\mathrm{B}}(t), \hat{D}_{\mathrm{B}}], \mathbb{I}\right] + [\hat{D}_{\mathrm{B}}, -\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t[\hat{H}_{\mathrm{A}}(t), \mathbb{I}]] + [\hat{D}_{\mathrm{B}}, -\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t\hat{H}_{\mathrm{C}}(t)] = 0. \tag{5.33}$$

The fact that the first two terms in (5.33) vanish is obvious, while the third vanishes because B and C are spacelike separated.

The higher order terms can be calculated similarly, but given the increasing complexity of the calculations it is more practical to reason which terms will vanish based on the following observations:

1. The zeroth order in the coupling constant of detector C cannot contribute to any order in $\hat{K}$, because at that order condition (5.25) is satisfied trivially.
2. The zeroth order in A cannot contribute at any order either, because B and C are spacelike separated.
3. Finally, the zeroth order in B cannot contribute at any order because in that case the induced observable $\hat{S}_{\mathrm{B}}^{\dagger}\hat{D}_{\mathrm{B}}\hat{S}_{\mathrm{B}}$ acts trivially over the field.

Therefore, $\hat{K}$ cannot have any contributions at quadratic order, because any quadratic contribution will involve the zeroth order of at least one of the detectors. Hence,

$$\hat{K}^{(2)} = 0. \tag{5.34}$$

This is not surprising if we take into account the result of section 5.1, because at quadratic order the detectors interact only pairwise. Since the detectors only influence each other pair-wise, the measurements cannot exhibit this type of superluminal signalling that involves necessarily three detectors. It is expected that this argument carries through in general calculations involving quadratic orders in perturbation theory.

Interestingly, and perhaps less intuitively, the third order will also vanish. Indeed, the only term that can contribute at third order in perturbation theory, given the observations made above, is the one that involves the linear order of each in the three detectors.

$$\hat{K}^{(3)} = -\frac{\mathrm{i}}{\hbar^3}\left[\int_{-\infty}^{\infty}\mathrm{d}t[\hat{H}_{\mathrm{B}}(t),\hat{D}_{\mathrm{B}}],[\int_{-\infty}^{\infty}\mathrm{d}t\hat{H}_{\mathrm{A}}(t),\int_{-\infty}^{\infty}\mathrm{d}t\hat{H}_{\mathrm{C}}(t)]\right]. \tag{5.35}$$

The operator in the first entry of the nested commutator acts over the space of detector B and over the quantum field, whereas the operator on the second entry is given by

$$\int \mathrm{d}V\int \mathrm{d}V'[\hat{h}_{\mathrm{A}}(\mathsf{x}),\hat{h}_{\mathrm{C}}(\mathsf{y})]. \tag{5.36}$$

Following the reasoning of section 5.2, the microcausality condition forces this operator to be localized in the causal future of C, and therefore commutes with field operators localized in region B. We then conclude that

$$\hat{K}^{(3)} = 0. \tag{5.37}$$

The interpretation in this case is that the superluminal propagation of initial data happens only at quadratic order in the detector that may act as a repeater. Since having a quadratic contribution in one of the detectors implies that at least one of the others contributes at zeroth order, the arguments given above force $\hat{K}^{(3)} = 0$, and no superluminal propagation can happen.

The moral is that, assuming that all the measurements are weakly performed with detectors, impossible measurements are not present in most calculations done in the literature. One should be careful, however, when handling non-pertubative methods for smeared detectors.

5.4 Discussion

We have analyzed whether generalized Unruh-DeWitt-type detector models fulfill minimum requirements regarding relativistic causality. In other words, we have discussed whether non-relativistic systems coupled to quantum fields can be used to model repeatable measurements on quantum fields without incurring in incompatibilities with relativistic causality.

In particular, we have investigated compatibility with relativistic causality in detector-based measurements by demanding that the signals emitted by each of the detectors should be constrained to lie within their associated future light-cones. Furthermore, we have formulated Sorkin's "impossible measurements" problem in terms of particle detector-based measurements, linking in this context the "impossible measurements" issues to the non-relativistic dynamics of the detector. The physical intuition is that, when a detector is spatially extended, the information propagating inside the detector is not constrained to travel subluminally since the detector is a non-relativistic system.

Chapter 6

The Unruh effect

What has come to be known as the Unruh effect [43] stands for a prediction of quantum field theory involving the vacuum as seen by accelerated observers. Roughly speaking, it asserts that an observer following a relativistic accelerated trajectory will experience Minkowski's vacuum as a thermal bath, with a temperature proportional to their acceleration. The proportionality constant involves a combination of the speed of light, Planck's constant and Boltzman's constant.

The fact that this effect is predicted to be measurable in the intersection of the three most successful branches of physics, namely relativity, quantum mechanics and thermodynamics, is partially what makes its study interesting. Actually, it is one of the few predictions in physics that requires these three conceptual frameworks, other examples including the closely related Hawking radiation, and other gravitational effects. A particularly appealing feature of the Unruh effect is that is expected to occur in flat spacetime backgrounds, and thereby it could be measured, in principle, in an experimental setting on Earth [80]. However, the temperatures predicted for ordinary accelerations achievable in a laboratory are extremely low, and therefore this effect appears to be virtually out of reach for experimental probing.

The Unruh effect, as we have loosely defined above, can be considered uncontroversial. However, the fact that it is so far from experimental testing has led to a whole field of study focused on finding effects that may reveal its signature indirectly. In addition, much research has been dedicated to the study Unruh-like effects, that is, physical phenomena that depart from the usual situation staged in the traditional Unruh effect but that share some of its most defining features. Examples include, for instance, the relation of the Unruh effect with Larmor radiation [81], simulation in classical systems [82], or the relation of the Unruh effect with circular motion [83], and rapid repeated interactions [84], or the effect on the center of mass of atomic systems [85]. These are some examples; it is not our goal to give a complete account of these phenomena here for it would take us far from our of topic.

Another source of controversy involves the correct identification of the figure of merit behind the traditional Unruh effect. Most approaches seem to identify the origin of the Unruh effect with the existence of correlations between the degrees of freedom of two causally separated regions, and the presence of an event horizon, but some counterexamples [86] have led some authors to consider a more local rationale.

The link with Minkowski's vacuum may be derived through several methods, each with its own advantages and flaws, and that differ basically in how they define the concepts involved in the statement. We follow the classification of these methods suggested in [87]:

- The first method is to study the quantum field theory as a chain of harmonic oscillators, and to make use of Bogoliubov transformations to relate inertial and accelerated observers [43].
- Second, there is the fully rigorous approach of algebraic quantum field theory, which makes use of modular theory and local algebras. The accelerated observer in this case is associated with the automorphism group of the subalgebra of operators constrained to a causal wedge [88].
- Finally, the approach taken in this thesis reaches the conclusion by introducing a detector that follows an accelerated trajectory [45].

For the rest of the chapter we will briefly review these approaches, highlighting their strengths and weaknesses, with a special stress in the use of detector models and their relation to the Unruh effect.

6.1 The Unruh effect without detectors

Chronologically, the first attempts to describe the Unruh effect were done using the first method, i.e. comparing the observable field states as perceived in inertial versus accelerated frames. Different non-inertial observers may naturally understand different states (with different particle content) as the vacuum state of the field. More precisely, a comparison is performed between the different notions of "particle" encoded in the creation and annihilation operators that are naturally associated with quanta for each observer.

For general bosonic systems, the creation and annihilation operators associated with different observers are assumed to relate to each other by linear transformations (the Bogoliubov transformations mentioned in chapter 2). Hence, the comparison between different observers relies on being able to calculate such linear maps. When one aims to export these methods to quantum field theory, which can be envisioned as a set of infinitely many bosonic modes, one runs into some difficulties. It can be shown that the Bogoliubov transformations are not well defined when different vacuum states are not unitarily equivalent. Although not devoid of problems, some important lessons may be learned from the Bogoliubov method, as we will illustrate in what follows.

The Unruh effect, as derived from this method, has close analogies to chains of harmonic oscillators. Indeed, consider the Hilbert space of a system of two harmonic oscillators, given by the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$. The Hamiltonian of the system is given by

$$\hat{H}_{12} = \omega(\hat{a}_1^\dagger \hat{a}_1 \otimes \mathbb{I}_1 + \mathbb{I}_2 \otimes \hat{a}_2^\dagger \hat{a}_2 + \mathbb{I}_{12}). \tag{6.1}$$

A two mode squeezing operator is defined as

$$\hat{S}(\zeta) = \exp\left(\zeta \hat{a}_1^\dagger \otimes \hat{a}_2^\dagger - \zeta^* \hat{a}_1 \otimes \hat{a}_2\right), \tag{6.2}$$

where $\zeta = re^{\mathrm{i}\phi}$. The squeezing operator satisfies $\hat{S}(\zeta)^\dagger \hat{S}(\zeta) = \hat{S}(\zeta)\hat{S}(\zeta)^\dagger = \mathbb{I}_{12}$, i.e. it is a unitary operator. Further, it can be shown that the adjoint action of the squeezing operator over the creation and annihilation operators is a Bogoliubov transformation i.e.

$$\hat{b}_i = \hat{S}^\dagger(\zeta)\hat{a}_i\hat{S}(\zeta) = \cosh(r)\hat{a}_1 + e^{\mathrm{i}\phi}\sinh(r)\hat{a}_2^\dagger, \tag{6.3}$$

which obviously fulfills the properties of a Bogoliubov transformation.

A two-mode vacuum squeezed state is given by

$$|\zeta\rangle = \hat{S}(\zeta)\,|0\rangle_1 \otimes |0\rangle_2 = \frac{1}{\cosh(r)} \sum_{n=0}^{\infty} \left(e^{\mathrm{i}\phi}\tanh(r)\right)^n |n\rangle_1 \otimes |n\rangle_2\,, \tag{6.4}$$

which we have expressed in the Fock basis. Clearly, this is an entangled state, since it cannot be written as a product of states. The partial state of the system when tracing out the second oscillator turns out to be a thermal state, i.e.

$$\hat{\rho}(\zeta) = \frac{1}{\cosh^2(r)} \sum_{n=0}^{\infty} (\tanh(r))^{2n} |n\rangle_1 \langle n|_1 = \frac{1}{\cosh^2(r)} \exp\left[2 \log \tanh(r) \hat{a}_1^\dagger \hat{a}_1\right]. \quad (6.5)$$

Therefore the partial state is thermal with temperature

$$\beta(\zeta) = \frac{2}{\omega} \log \tanh(r). \quad (6.6)$$

What is the analogy between this simple system and the Unruh effect? First, for the sake of the argument, let us write the field operator (schematically) as the sum over modes

$$\hat{\phi}(t, x_1, \boldsymbol{x}) = \sum_k \frac{e^{-\mathrm{i}\omega_k t + \mathrm{i}k_1 x_1 + \mathrm{i}\boldsymbol{k}\boldsymbol{x}}}{\sqrt{2\omega_k}} \hat{a}_k + \frac{e^{\mathrm{i}\omega_k t - \mathrm{i}k_1 x_1 - \mathrm{i}\boldsymbol{k}\boldsymbol{x}}}{\sqrt{2\omega_k}} \hat{a}_k^\dagger. \quad (6.7)$$

Here the sum runs through all the possible values of k_1 and $\boldsymbol{k}$ and $\omega_k = \sqrt{m^2 + k_1^2 + |\boldsymbol{k}|^2}$, where k_1 is associated with the coordinate x_1 and $\boldsymbol{k}$ is a two-dimensional vector associated with the components perpendicular to such x_1.

For the full calculation, the sum should be understood as an integral. However, the pass to the continuum in this method requires some technical detail, such expansions in wave packets [21], that does not contribute to the general understanding of the Unruh effect. Moreover, even if done as fairly as possible this method cannot be carried out fully rigorously for reasons we will see. Hence, we restrict ourselves to working with this schematic expression.

Now, the Minkowski vacuum is the state that is annihilated by all the $\hat{a}_k$, i.e.

$$\hat{a}_k |0\rangle = 0. \quad (6.8)$$

Note that time translating the modes generated by ∂_t, as functions over the Minkoswski spacetime, induces a unitary evolution in the Hilbert space, given by the formula

$$e^{-\mathrm{i}\omega_k t} \hat{a}_k = e^{\mathrm{i}\hat{H}_M t} \hat{a}_k e^{-\mathrm{i}\hat{H}_M t}, \quad (6.9)$$

where it is clear that $\hat{H}_M$ has to be given by

$$\hat{H}_M = \sum_k \frac{\omega_k}{2} \left(\hat{a}_k^\dagger \hat{a}_k + \hat{a}_k \hat{a}_k^\dagger\right). \quad (6.10)$$

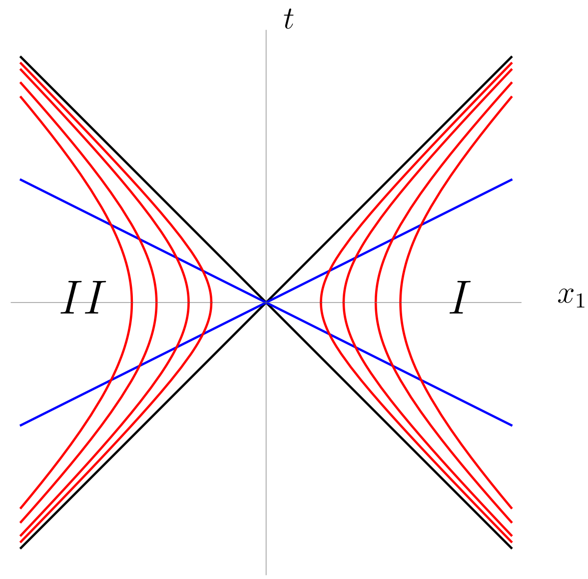


Figure 6.1: Here we represent the Minkowski diagram. Regions I and II are characterized by $t \leq |x_1|$ and are commonly known as Rindler wedges. The blue lines represent surfaces of constant Rindler coordinate η, whereas the red lines represent surfaces of constant Rindler coordinate ξ. For positive values of η the surfaces are straight lines of positive slope, whereas positive values of ξ represent the red curves that lay in region I.

Therefore, time translations in the field are generated by

$$\hat{\phi}(t, \boldsymbol{x}) = e^{\mathrm{i}\hat{H}_M t}\hat{\phi}(0, \boldsymbol{x})e^{-\mathrm{i}\hat{H}_M t}. \tag{6.11}$$

However, as we mentioned in chapter 2, the quantization procedure in quantum field theory allows for different choices. Indeed, the field expansion may be written also in terms of the so-called Rindler coordinates [22], which are the suitable coordinates for accelerated observers. These coordinates are given, implicitly, by

$$t = \xi \sinh(\eta), \quad x_1 = \xi \cosh(\eta), \tag{6.12}$$

see figure 6.1 for a graphical representation of these coordinates.

Indeed, we can write the field operator as

$$\hat{\phi}(\eta, \xi, \boldsymbol{x}) = \hat{\phi}_I(\eta, \xi, \boldsymbol{x}) + \hat{\phi}_{II}(\eta, \xi, \boldsymbol{x}) =$$
$$\sum_{\mathsf{k}} u_{I\mathsf{k}}(\eta, \xi, \boldsymbol{x})\hat{b}_{I\mathsf{k}} + u^*_{I\mathsf{k}}(\eta, \xi, \boldsymbol{x})\hat{b}^\dagger_{I\mathsf{k}} + \sum_{\mathsf{k}} u_{II\mathsf{k}}(\eta, \xi, \boldsymbol{x})\hat{b}_{II\mathsf{k}} + u^*_{II\mathsf{k}}(\eta, \xi, \boldsymbol{x})\hat{b}^\dagger_{II\mathsf{k}}, \tag{6.13}$$

where $u_{I\mathsf{k}}$ and $u_{II\mathsf{k}}$ are a basis of solutions of the Klein Gordon equation in the coordinates, η, ξ and $\boldsymbol{x}$. These functions are indexed by $\mathsf{k} = (\Omega, \boldsymbol{k})$. Also, these functions are supported in regions I and II of the Minkowski diagram respectively and fulfill

$$u_{I\mathsf{k}}(\eta, \xi, \boldsymbol{x}) = u^*_{II\tilde{\mathsf{k}}}(\eta, -\xi, \boldsymbol{x}), \tag{6.14}$$

where $\tilde{\mathsf{k}} = (\Omega, -\boldsymbol{k})$, with $\Omega > 0$.

Further, they fulfill

$$u_{I\mathsf{k}}(\eta, \xi, \boldsymbol{x}) = e^{-\mathrm{i}\Omega\eta} u_{I\mathsf{k}}(0, \xi, \boldsymbol{x}) \tag{6.15}$$

and

$$u_{II\mathsf{k}}(\eta, \xi, \boldsymbol{x}) = e^{\mathrm{i}\Omega\eta} u_{II\mathsf{k}}(0, \xi, \boldsymbol{x}). \tag{6.16}$$

In other words, they are the positive energy solution of the Klein Gordon with respect to the time evolution generated by the vector

$$\partial_\eta = x_1 \partial_t - t\partial_{x_1}, \tag{6.17}$$

which is the generator of Lorentz boosts [20].

Mimicking the technique used to find the Hamiltonian of time translations, we see that ∂_η can be represented as the unitary action of the the operators $\hat{b}_{I\mathsf{k}}$, which will given by

$$e^{\mathrm{i}\hat{K}\eta} \hat{b}_{I\mathsf{k}} e^{-\mathrm{i}\hat{K}\eta} = e^{-\mathrm{i}\Omega\eta} \hat{b}_{I\mathsf{k}}. \tag{6.18}$$

Since the creation and annihilation operators commute for the two regions, we get that the generator of boost can be written for region II as for region I but with an extra minus sign for region II, that is

$$\hat{K} = \sum_{\mathsf{k}} \frac{\Omega}{2} \left(\hat{b}^\dagger_{I\mathsf{k}} \hat{b}_{I\mathsf{k}} + \hat{b}_{I\mathsf{k}} \hat{b}^\dagger_{I\mathsf{k}} \right) - \sum_{\mathsf{k}} \frac{\Omega}{2} \left(\hat{b}^\dagger_{II\mathsf{k}} \hat{b}_{II\mathsf{k}} + \hat{b}_{II\mathsf{k}} \hat{b}^\dagger_{II\mathsf{k}} \right). \tag{6.19}$$

Then, $\hat{K}$, the generator of boosts, generates future oriented evolution in region I respect to the parameter η, whereas in region II, the generator of boosts generates past oriented evolution.

The evolution of the field has to be future oriented. Further, η cannot be associated with a trajectory since it is dimensionless, and ∂_η is not a normalized timelike vector. Accelerated observers (with positive acceleration) will evolve with proper time given by

$$\partial_\tau = a\partial_\eta. \tag{6.20}$$

Thus, the appropriate Hamiltonian is given by the boost times a in region I and minus the boost times a in region II:

$$\hat{H} = \sum_{\mathsf{k}} \frac{a\Omega}{2}\left(\hat{b}^\dagger_{I\mathsf{k}}\hat{b}_{I\mathsf{k}} + \hat{b}_{I\mathsf{k}}\hat{b}^\dagger_{I\mathsf{k}}\right) + \sum_{\mathsf{k}} \frac{a\Omega}{2}\left(\hat{b}^\dagger_{II\mathsf{k}}\hat{b}_{II\mathsf{k}} + \hat{b}_{II\mathsf{k}}\hat{b}^\dagger_{II\mathsf{k}}\right). \tag{6.21}$$

This operator generates quantum evolution

$$e^{\mathrm{i}\hat{H}\tau}\hat{b}_{i\mathsf{k}}e^{-\mathrm{i}\hat{H}\tau} = e^{-\mathrm{i}E_{i\mathsf{k}}\tau}\hat{b}_{i\mathsf{k}}, \tag{6.22}$$

where $i = I, II$ and $E_{i\mathsf{k}}$ is the (positive) energy of each mode. This justifies the fact that these are the modes that are seen by an accelerated observer.

Now, one could calculate the Bogoliubov coefficients that relate Minkowski's plane wave modes with the Rindler modes directly, but there is a much more elegant way of proceeding that makes use of a theorem by Bisognano and Wichmann [89]. In a restricted version, this theorem states the following:

Let $\hat{\phi}$ be a real scalar quantum field in the Minkowski spacetime. Then

$$\left(e^{-\pi\hat{K}}\hat{\phi}e^{\pi\hat{K}} - \mathcal{J}[\hat{\phi}]\right)|0\rangle = 0 \tag{6.23}$$

where $\mathcal{J}$ denotes CPT operation (in the Heisenberg picture) plus a reflection of the 2-dimensional space perpendicular to the Rindler wedges. Its action may be written for a real scalar field in Rindler coordinates as

$$\mathcal{J}[\hat{\phi}(\eta, \xi, \boldsymbol{x})] = \hat{\phi}(\eta, -\xi, \boldsymbol{x}). \tag{6.24}$$

This theorem is actually much more general; it applies to complex fields with tensor and spinor structures and even to interacting fields. From this, one can derive that the CPT operation times a reflection (CRT) acts over the creation and annihilation operators associated with the Rindler modes as

$$\mathcal{J}[\hat{b}_{I\mathsf{k}}] = \hat{b}_{II\tilde{\mathsf{k}}}. \tag{6.25}$$

Note that, for each mode, we assume that we can analytically extend the evolution of the creation and annihilation operators [90] the evolution generated by η:

$$e^{-\pi\hat{K}}\hat{b}_{I\mathsf{k}}e^{\pi\hat{K}} = e^{\pi\Omega}\hat{b}_{I\mathsf{k}} \tag{6.26}$$

and its Hermitian conjugate

$$e^{-\pi\hat{K}}\hat{b}^\dagger_{I\mathsf{k}}e^{\pi\hat{K}} = e^{-\pi\Omega}\hat{b}^\dagger_{I\mathsf{k}}. \tag{6.27}$$

Applying this and substituting in the expression given by Bisognano's theorem, (6.23), we get

$$\left(e^{-\pi\hat{K}}\hat{\phi}e^{\pi\hat{K}} - \mathcal{J}\hat{\phi}\mathcal{J}^\dagger\right)|0\rangle_M = \tag{6.28}$$

$$\left(\sum_{\mathsf{k}} u_{I\mathsf{k}}(\eta,\xi,\boldsymbol{x})[e^{\pi\Omega}\hat{b}_{I\mathsf{k}} - \hat{b}^\dagger_{II\tilde{\mathsf{k}}}] + \sum_{\mathsf{k}} u^*_{I\mathsf{k}}(\eta,\xi,\boldsymbol{x})[e^{-\pi\Omega}\hat{b}^\dagger_{I\mathsf{k}} - \hat{b}_{II\tilde{\mathsf{k}}}] + \right.$$

$$\left.\sum_{\mathsf{k}} u_{II\mathsf{k}}(\eta,\xi,\boldsymbol{x})[e^{\pi\Omega}\hat{b}_{II\mathsf{k}} - \hat{b}^\dagger_{I\tilde{\mathsf{k}}}] + \sum_{\mathsf{k}} u^*_{II\mathsf{k}}(\eta,\xi,\boldsymbol{x})[e^{-\pi\Omega}\hat{b}^\dagger_{II\mathsf{k}} - \hat{b}_{I\tilde{\mathsf{k}}}]\right)|0\rangle_M = 0. \tag{6.29}$$

These four terms are modulated by independent functions, thereby they all have to vanish as Hilbert space-valued functions.

We will show now that realize that the first and the fourth vanishing terms determine annihilation operators for Minkowki's vacuum, and that they are supported in regions I and II respectively. Indeed, if we define the following operators (up to a proportionality constant),

$$\hat{d}_{I\mathsf{k}} \propto e^{\pi\Omega}\hat{b}_{I\mathsf{k}} - \hat{b}^\dagger_{II\tilde{\mathsf{k}}}, \tag{6.30}$$

and

$$\hat{d}_{II\mathsf{k}} \propto e^{\pi\Omega}\hat{b}_{II\mathsf{k}} - \hat{b}^\dagger_{I\tilde{\mathsf{k}}}, \tag{6.31}$$

we find a set operators associated with the functions $u_{I,II\mathsf{k}}$ that annihilate Minkowski's vacuum. These annihilation operators can be written as combination of the usual creation and annihilation operators, but the definition of $\hat{d}_{I\mathsf{k}}$ and $\hat{d}_{II\mathsf{k}}$ does not involve $\hat{a}^\dagger_k$, otherwise their action over Minkowski's vacuum would be nonzero.

It is easy to see that $\hat{d}_{I\mathsf{k}}$ and $\hat{d}_{II\mathsf{k}}$ commute. The proportionality constants are found by imposing the canonical commutation relations, i.e

$$[\hat{d}_{i\mathsf{k}}, \hat{d}^\dagger_{i\mathsf{k}}] = 1, \tag{6.32}$$

which forces the proportionality constant to be

$$\frac{1}{\sqrt{e^{2\pi\Omega}-1}} \tag{6.33}$$

Therefore, we have that these Minkowki's creation and annihilation operators are related to Rindler's creation and annihilation operators by

$$\hat{d}_{I\mathsf{k}} = \frac{1}{\sqrt{e^{2\pi\Omega}-1}} e^{\pi\Omega}\hat{b}_{I\mathsf{k}} - \frac{1}{\sqrt{e^{2\pi\Omega}-1}}\hat{b}^{\dagger}_{II\tilde{\mathsf{k}}}, \tag{6.34}$$

and

$$\hat{d}_{II\mathsf{k}} = \frac{1}{\sqrt{e^{2\pi\Omega}-1}} e^{\pi\Omega}\hat{b}_{II\mathsf{k}} - \frac{1}{\sqrt{e^{2\pi\Omega}-1}}\hat{b}^{\dagger}_{I\tilde{\mathsf{k}}}. \tag{6.35}$$

Schematically, we can write

$$\begin{pmatrix} d_{I\mathsf{k}} \\ d_{II\tilde{\mathsf{k}}} \end{pmatrix} = \begin{pmatrix} \cosh(r) & 0 \\ 0 & \cosh(r) \end{pmatrix} \begin{pmatrix} \hat{b}_{I\mathsf{k}} \\ \hat{b}_{II\tilde{\mathsf{k}}} \end{pmatrix} + \begin{pmatrix} 0 & -\sinh(r) \\ -\sinh(r) & 0 \end{pmatrix} \begin{pmatrix} b^{\dagger}_{I\mathsf{k}} \\ b^{\dagger}_{II\tilde{\mathsf{k}}}, \end{pmatrix} \tag{6.36}$$

where we have defined r as

$$\cosh(r) = \frac{e^{\pi\Omega}}{\sqrt{e^{2\pi\Omega}-1}}. \tag{6.37}$$

We conclude that, mode by mode, Minkowski's vacuum behaves like a two-mode squeezed state with respect to the Rindler modes. The temperature associated with the restriction to one of the regions is proportional to a geometrical factor over the energy of each mode.

$$\beta \propto -\frac{1}{E_{\mathsf{k}}}\log\tanh^2(r) = 2\pi\frac{\Omega}{E_{\mathsf{k}}}. \tag{6.38}$$

But the energy of each mode is observer dependent. For accelerated observers, the energy of each mode is $E_{\mathsf{k}} = a\Omega$. Then for such observers the associated temperature is

$$\beta = \frac{2\pi}{a}. \tag{6.39}$$

The conclusion is therefore reached, since an accelerated observer will interact only with the modes associated with region I, and their notion of particle will stem from the one associated with the operators $\hat{b}_{I\mathbf{k}}$. Therefore they will measure observables that are combinations of these creation and annihilation operators, and all the averages involving these operators in Minkowki's vacuum are equivalently given by a thermal ensemble.

The Unruh effect, as described by this method, relies on the entanglement present in the vacuum when understood as a chain of harmonic oscillators. The thermal behavior in this case is associated with the ignorance of accelerated observers of the observables that lay in their causal complement, and therefore to the formation of a horizon. It also puts special emphasis on the two different notions of particle possessed by inertial and accelerated observers. This is an important conceptual point of this method, for the Unruh effect is usually claimed to reveal that the notion of particle is not fundamental, as it is observer dependent.

The Bogoliubov method is problematic in the sense that faces apparently technical difficulties that hide deep truths. Note that all sums over modes that have been written in this section are merely schematic. When written properly, one realizes that one cannot write Minkowski's vacuum as a unitary operator times the Rindler vacuum, which makes the Bogoliubov transformation ill defined. Indeed, the two quantizations, namely the one associated with the Minkowski modes and the one associated with the Rindler modes, are not unitarily equivalent. This implies that the partial trace over the region II of the vacuum does not lead to a trace-class operator, and that there is not a density matrix describing the partial state seen by an accelerated observer. This puts into question the claim that accelerated observers experience the vacuum as a thermal reservoir, for thermal reservoirs in quantum mechanics are given by Gibbs states.

6.1.1 The KMS condition

To claim that Minkowski's vacuum is seen as a thermal bath by accelerated observers can be made mathematically rigorous through the concept of Kubo-Martin-Schwinger (KMS) states. Indeed, thermal states in quantum statistical mechanics [91, 92] are described by Gibbs' distribution. This distribution is well defined for systems in which the Hamiltonian has a point spectrum, for instance when considering systems with finite degrees of freedom. However, for quantum fields, the Gibbs distribution is not always well defined [93]. In those cases, it is still possible to define thermal states by considering large, but finite systems and then taking the thermodynamic limit.

KMS states have been thoroughly studied in analysis and C^*-algebras, as well as modular theory [94], and including here a detailed account of the mathematical subtleties involved in their definition would take us too much afar. However, we will show in the following how the KMS condition, which defines KMS states, captures many of the characteristics demanded from thermal states. In this spirit, the results and derivations presented here are meant to illustrate a simplified version of the the ones regarding KMS states that have been rigorously derived in the literature, and they should *not* be taken as a reference.

The definition that can be found in AQFT texts, for instance, makes explicit reference to the algebra of observables of the theory, and a one parameter group of automorphisms $\alpha_\tau(\cdot)$ representing the time evolution of the system. It involves the average of products of any two observables, say $\hat{A}$ and $\hat{B}$. A state is said to be a KMS state, with inverse temperature β, if

$$F_{AB}(\tau) := \langle \alpha_\tau(\hat{A})\hat{B}\rangle_\beta \tag{6.40}$$

is a holomorphic function of τ in the lower strip, $\{-\beta \le \mathrm{Im}\tau \le 0\}$ that fulfills

$$F_{AB}(\tau - \mathrm{i}\beta) = \langle \hat{B}\alpha_\tau(\hat{A})\rangle_\beta = F^*_{A^\dagger B^\dagger}(\tau). \tag{6.41}$$

After some manipulations it can be checked that Gibbs states are KMS states, but the KMS condition can hold in more general scenarios where Gibbs states may not be available.

KMS states are stationary, that is $\langle \alpha_\tau(\hat{A})\rangle_\beta = \langle \hat{A}\rangle_\beta$, for all τ and all $\hat{A}$. It is easy to see that this is the case by choosing $\hat{A}$ self-adjoint, so that $\langle \alpha_\tau(\hat{A})\rangle_\beta$ is a real, bounded function of τ. Then, using Schwarz's reflection principle, and the KMS condition particularized to $F_{A\mathbb{I}}$, we find that we can analytically extend $\langle \alpha_\tau(\hat{A})\rangle_\beta$ to an imaginary periodic function, which hence is a bounded function on the whole complex plane. Finally, by Liouville's theorem, this implies that the function is constant [90].

Moreover, condition (6.41) has implications for the Fourier transform

$$\tilde{F}_{AB}(\omega) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\tau e^{-\mathrm{i}\omega\tau} F_{AB}(\tau). \tag{6.42}$$

The fact that F_{AB} is analytic in the strip allows us to lower the integration contour down the imaginary axis by an amount β:

$$\tilde{F}_{AB}(\omega) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\tau e^{-\mathrm{i}\omega\tau} F_{AB}(\tau) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\tau e^{-\mathrm{i}\omega(\tau - \mathrm{i}\beta)} F_{AB}(\tau - \mathrm{i}\beta)$$
$$= \frac{e^{-\beta\omega}}{\sqrt{2\pi}} \int \mathrm{d}\tau e^{-\mathrm{i}\omega\tau} F^*_{A^\dagger B^\dagger}(\tau) = \frac{e^{-\beta\omega}}{\sqrt{2\pi}} \left(\int \mathrm{d}\tau e^{\mathrm{i}\omega\tau} F_{A^\dagger B^\dagger}(\tau) \right)^* = e^{-\beta\omega} \tilde{F}^*_{A^\dagger B^\dagger}(-\omega). \tag{6.43}$$

The relation given by (6.43) is sometimes called the detailed balance condition.

Note that for any stationary state, in particular KMS states, and any operator $\hat{A}$, it holds that

$$0 \leq \langle \left(\int \mathrm{d}\tau f(\tau) \alpha_\tau(\hat{A}) \right)^\dagger \left(\int \mathrm{d}\tau f(\tau) \alpha_\tau(\hat{A}) \right) \rangle_\beta = \iint \mathrm{d}\tau \mathrm{d}\tau' f^*(\tau) f(\tau') \langle \alpha_\tau(\hat{A}^\dagger) \alpha_{\tau'}(\hat{A}) \rangle_\beta$$
$$= \iint \mathrm{d}\tau \mathrm{d}\tau' f^*(\tau) f(\tau') \langle \alpha_{\tau - \tau'}(\hat{A}^\dagger) \hat{A} \rangle_\beta = \iint \mathrm{d}\tau \mathrm{d}\tau' f^*(\tau) f(\tau') F_{A^\dagger A}(\tau - \tau') \tag{6.44}$$
$$= \int \mathrm{d}\tau \bar{f} \star f(\tau) F_{A^\dagger A}(\tau). \tag{6.45}$$

But this means, given the definition of the Fourier transform, that

$$\int \mathrm{d}\omega |\tilde{f}|^2(\omega) \tilde{F}_{A^\dagger A}(\omega) \geq 0. \tag{6.46}$$

Given that $|\tilde{f}|^2$ is positive and arbitrary, this further implies that $\tilde{F}_{A^\dagger A}(\omega)$ is a real, positive function. Therefore, equation (6.43) can be particularized to

$$\tilde{F}_{A^\dagger A}(\omega) = e^{-\beta\omega} \tilde{F}_{A A^\dagger}(-\omega). \tag{6.47}$$

Equation (6.47) can be used to further prove that states KMS states fulfill the following inequality [95], which states that

$$\mathrm{i}\beta \langle \delta(\hat{A}) \hat{A}^\dagger \rangle_\beta \geq \langle \hat{A} \hat{A}^\dagger \rangle_\beta \ln \left(\frac{\langle \hat{A} \hat{A}^\dagger \rangle_\beta}{\langle \hat{A}^\dagger \hat{A} \rangle_\beta} \right), \tag{6.48}$$

where we have defined the operator

$$\delta(\hat{A}) := \frac{\mathrm{d}}{\mathrm{d}\tau} \alpha_\tau(\hat{A}) \bigg|_{\tau=0}. \tag{6.49}$$

This is a consequence of Jensen's inequality [96] which, in an adapted version, states that given a positive, integrable function $g(\omega)$, and a convex function $f(\omega)$, it holds that

$$f\left(\frac{\int \omega g(\omega)\mathrm{d}\omega}{\int g(\omega)\mathrm{d}\omega}\right) \leq \frac{\int f(\omega)g(\omega)\mathrm{d}\omega}{\int g(\omega)\mathrm{d}\omega}. \tag{6.50}$$

Substituting g with $\tilde{F}_{AA^\dagger}(-\omega)$, and f with $e^{-\beta\omega}$, we get

$$e^{-\beta\frac{\int \mathrm{d}\omega\omega\tilde{F}_{AA^\dagger}(-\omega)}{\int \mathrm{d}\omega\tilde{F}_{AA^\dagger}(-\omega)}} \leq \frac{\int \mathrm{d}\omega e^{-\beta\omega}\tilde{F}_{AA^\dagger}(-\omega)}{\int \mathrm{d}\omega\tilde{F}_{AA^\dagger}(-\omega)} = \frac{\int \mathrm{d}\omega\tilde{F}_{A^\dagger A}(\omega)}{\int \mathrm{d}\omega\tilde{F}_{AA^\dagger}(-\omega)} \tag{6.51}$$

where we have used the detailed balance condition (6.47) in the last equality. But

$$\int \mathrm{d}\omega\tilde{F}_{A^\dagger A}(\omega) = F_{A^\dagger A}(0) = \langle\hat{A}^\dagger\hat{A}\rangle_\beta\,, \tag{6.52}$$

$$\int \mathrm{d}\omega\tilde{F}_{AA^\dagger}(-\omega) = F_{AA^\dagger}(0) = \langle\hat{A}\hat{A}^\dagger\rangle_\beta\,, \tag{6.53}$$

and

$$-\int \mathrm{d}\omega\omega\tilde{F}_{AA^\dagger}(-\omega) = -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\tau}F_{AA^\dagger}(0) = -\mathrm{i}\,\langle\delta(\hat{A})\hat{A}^\dagger\rangle_\beta\,. \tag{6.54}$$

Taking into account that the logarithm is a monotonically increasing function, it is immediate to arrive at the entropy-energy inequality from (6.51) by taking logarithms on both sides and changing a sign.

The entropy-energy inequality may seem rather abstract, but has the important consequence that KMS states are passive. A state is passive if $\mathrm{i}\,\langle\delta(\hat{U})\hat{U}^\dagger\rangle \geq 0$ for all unitary operators $\hat{U}$. Taking into account that $\hat{U}^\dagger\hat{U} = \hat{U}\hat{U}^\dagger = \mathbb{I}$, one can check that KMS states are passive by direct substitution in (6.48), with $\hat{A} = \hat{U}$. The interpretation of passivity can be made more transparent when considering that $\hat{U}$ is implementing an operation on the state of the system, in such a way that the averages transform as

$$\langle\cdot\rangle \to \langle\cdot\rangle_U = \langle\hat{U}\cdot\hat{U}^\dagger\rangle\,. \tag{6.55}$$

Taking into account that

$$\delta(\hat{U}) = \mathrm{i}[\hat{H},\hat{U}], \tag{6.56}$$

where $\hat{H}$ is the Hamiltonian associated with α_τ,

$$\mathrm{i}\,\langle\delta(\hat{U})\hat{U}^\dagger\rangle = -\,\langle[\hat{H},\hat{U}]\hat{U}^\dagger\rangle = \langle\hat{U}\hat{H}\hat{U}^\dagger\rangle - \langle\hat{H}\rangle = \langle\hat{H}\rangle_U - \langle\hat{H}\rangle \geq 0. \tag{6.57}$$

One concludes from here that perturbations acting over the system always increase the energy on average. This is related to thermality in the sense that thermal states are defined, at least in finite dimensions, as minimizers of the free energy. For a detailed account of these facts related to passivity, see [97].

Finally, one characteristic of thermal baths is that observables fulfill fluctuation-dissipation relations [98]. Consider a self-adjoint operator $\hat{A}$, from equation (6.47) and some manipulations one can relate the symmetric part and the antisymmetric part of $\tilde{F}_{AA}$ as

$$\tilde{F}_{AA}(\omega) + \tilde{F}_{AA}(-\omega) = \mathrm{ctgh}(\beta\omega/2)(\tilde{F}_{AA}(\omega) - \tilde{F}_{AA}(-\omega)). \tag{6.58}$$

A fluctuation dissipation relation is a concept from linear response theory, which is concerned with the response of, say, materials at a certain temperature to weak external fields. The name alludes to the fact that, through the Fourier transform of the expressions in (6.58), one can relate the function

$$\langle\{\alpha_\tau(\hat{A}),\hat{A}\}\rangle_\beta\,, \tag{6.59}$$

i.e. the fluctuations of the observable $\hat{A}$ over time, to the function

$$-\mathrm{i}\,\langle[\alpha_\tau(\hat{A}),\hat{A}]\rangle_\beta\,, \tag{6.60}$$

that is, the variation of the observable $\hat{A}$ when introducing a small interaction term of the form $\hat{H}(\tau) = \lambda\hat{A}(\tau)\chi(\tau)$, which corresponds to dissipation.

Further, the fluctuation dissipation relation can be used to determine the dependence with temperature of some relevant functions, like the two-point function of a quantum field. Indeed, if $[\alpha_\tau(\hat{A}),\hat{A}] \propto \mathbb{I}$, as happens e.g. with the field's amplitude (see chapter 1),

$$\tilde{C}(\omega) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}\tau e^{-\mathrm{i}\omega\tau}\,\langle[\alpha_\tau(\hat{A}),\hat{A}]\rangle\,, \tag{6.61}$$

then for a KMS state,

$$\tilde{C}(\omega) = \tilde{F}_{AA}(\omega) - \tilde{F}_{AA}(-\omega) = (1 - e^{\beta\omega})\tilde{F}_{AA}(\omega), \tag{6.62}$$

which implies

$$\tilde{F}_{AA}(\omega) = -\frac{\tilde{C}(\omega)}{e^{\beta\omega} - 1}. \tag{6.63}$$

This will play a role in chapter 7, where we will investigate some aspect of the Unruh effect that have to do with the dependence of field's fluctuations with the acceleration.

6.1.2 Some comments on the algebraic derivation of the Unruh effect

We now return our attention the Unruh effect. When discussing accelerated observers from the point of view of von Neumann algebras, one can derive the predictions of the Unruh effect in a fully rigorous manner. This approach shares many of the conclusions of the analysis performed with the Bogoliubov method. Nonetheless, it introduces new concepts that help to avoid the problematic notion of particle, that as we pointed out earlier, admits no comparison between inertial and accelerated observers.

It can be shown that the restriction of Minkowki's vacuum to the algebra of observables localized in the Rindler wedge II is a KMS state with respect to the generator of boosts, with temperature 2π. When rescaled with the acceleration, this result reaches the conclusion that Minkowski's vacuum is a thermal bath for accelerated observers, where thermal has to be understood as a KMS state. This is true as far as we associate accelerated observers with the evolution generated by the boosts and the subalgebra of operators localized in the Rindler wedge.

The relation of this result with entanglement is still present, yet in a more obscure form than the one obtained with Bogoliubov transformations. The idea is that Minkowski's vacuum is a cyclic and separating vector for the local algebras, a result known as Reeh-Schlieder theorem [94]. Cyclic vectors are those such that the action of the subalgebra acting over the vector is dense in the Hilbert space, whereas separating means that there is no element in the subalgebra that annihilates the vector. For instance, when considering a finite dimensional bipartite system, a pure state is cyclic and separating with respect to the subalgebra of operators acting over one of the parties if the state has full-rank entanglement [94], and therefore, cyclic and separating vectors constitute the analog of full-rank entangled states in field theory.

Tomita-Takesaki's modular theory [88] is precisely the theory that studies this type of state in general von Neumann algebras. One of the central theorems in modular theory is Tomita-Takesaki's theorem, which states that, under some conditions, a state that is cyclic and separating with respect to a subalgebra is KMS with respect to the modular group associated with that state and that subalgebra. The modular group is a one-parameter group of isomorphisms of the subalgebra (e.g. in the case of a bipartite system these would be given by local unitaries), which can be calculated with a mathematical object called modular operator. It turns out that the modular group associated with Minkowki's vacuum (and with respect to the Rindler wedge) is precisely the group of boosts, a result stemming from the aforementioned Bisognano-Wichmann's theorem. In the same way that KMS states extend the notion of Gibbs state to situations in which it is not possible to define density matrices, this procedure generalizes the scheme followed with the Bogolioubov method to relate the correlations in Minkoswki's vacuum to the emergence of a thermal reduced state. However, this method does not rely in any notion of particle. Moreover, this method can be extended to curved spacetimes and allows one to collect a series of effects under the umbrella of the Unruh effect. This method points out that the presence of the thermal bath is related to a bifurcate Killing horizon [26] in spacetime, in a theory that has the CRT symmetry in the appropriate coordinates [99].

6.2 The Unruh effect and the response of particle detectors

In this section we describe the Unruh effect from the perspective of Unruh-DeWitt detectors. We shall analyze the problem within the point-like UDW model exclusively. Particle detectors provide us with a way to characterize the Unruh effect through the physical process of thermalization of accelerated particle detectors. If the accelerating detector reaches a thermal state after its characteristic thermalization time scale at a temperature proportional to its acceleration, it is claimed that the detector has experienced the Unruh effect.

Within the formalism developed in section 3, we want to study a detector interacting with Minkowki's vacuum through an accelerated trajectory in $3+1$ dimensions, which takes the form

$$\mathsf{x}(\tau) = (t(\tau), \boldsymbol{x}(\tau)) = \left(\frac{1}{a}\sinh(a\tau), \frac{1}{a}\cosh(a\tau), \boldsymbol{x}_\perp\right) \tag{6.64}$$

where a is the magnitude of the acceleration, which we will consider positive without loss of generality, and $\boldsymbol{x}_\perp$ is a constant 2-dimensional vector representing the coordinates perpendicular to the acceleration. Further, we shall consider the paradigmatic case of the massless field.

following chapter 3, the statistics of the detector are determined by the pull-back of the one and two-point functions of the field in Minkowki's vacuum, i.e.

$$\langle 0|\hat{\phi}(\mathsf{x}(\tau))|0\rangle = 0, \tag{6.65}$$

and, from (2.17),

$$W(\tau,\tau') = \frac{1}{4\pi}\,\langle 0|\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))|0\rangle = \frac{1}{(\mathsf{x}(\tau)-\mathsf{x}(\tau'))^2}. \tag{6.66}$$

Further, the pull-back on the accelerated trajectory gives

$$\begin{aligned}
&(\mathsf{x}(\tau)-\mathsf{x}(\tau'))^2 = -\frac{1}{a^2}(\sinh{(a\tau)}-\sinh{(a\tau')})^2 + \frac{1}{a^2}(\cosh{(a\tau)}-\cosh{(a\tau')})^2\\
&= -\frac{4}{a^2}(\sinh(a(\tau-\tau')/2)\cosh(a(\tau+\tau')/2))^2 + \frac{4}{a^2}(\sinh(a(\tau-\tau')/2)\sinh(a(\tau+\tau')/2))^2\\
&= \frac{4\sinh^2(a(\tau-\tau')/2)}{a^2}(-\cosh^2(a(\tau+\tau')/2))^2 + \sinh^2(a(\tau+\tau')/2))\\
&= -\frac{4\sinh^2(a(\tau-\tau')/2)}{a^2},
\end{aligned} \tag{6.67}$$

where we have used basic properties of the hyperbolic functions.

Therefore, the pull-back of the Wightman function will be given by

$$W(\tau,\tau') = -\frac{a^2}{16\pi\sinh^2(a(\tau-\tau')/2)} = W(\tau-\tau'). \tag{6.68}$$

Note that, perhaps unsurprisingly in the light of previous sections, the pullback of the Wightman function is stationary. Moreover, since $\sinh(z)$, with $z\in\mathbb{C}$, is an analytic function with no zeroes in the strip $-\mathrm{i}\pi < \mathrm{Im}(z) < 0$, the function $W(z)$ can be extended analytically to the strip $-\mathrm{i}\frac{2\pi}{a} < \mathrm{Im}(z) < 0$. Finally, since $\sinh(x-\mathrm{i}\pi) = \sinh(-x)$ for $x\in\mathbb{R}$, we get that

$$W\left(\tau - \mathrm{i}\frac{2\pi}{a}\right) = W(-\tau) = W^*(\tau), \tag{6.69}$$

so $W(\tau)$ fulfills the KMS condition with temperature $\beta = \frac{2\pi}{a}$.

This fact does not imply that the whole state is KMS, because the condition has only been verified for a single observable, namely $\hat{\phi}(\mathsf{x}(0))$. However, and most importantly, the detailed balance condition (6.47) still holds

$$\tilde{W}(\omega) = e^{-\frac{2\pi\omega}{a}}\tilde{W}(-\omega) \tag{6.70}$$

Remember from chapter 3 that the response of a detector can always be expanded as

$$\mathcal{E}[\hat{\rho}_{\mathrm{D}}] = \hat{\rho}_{\mathrm{D}} + \lambda\Xi[\hat{\rho}_{\mathrm{D}}] + \lambda^2\Theta[\hat{\rho}_{\mathrm{D}}] + \mathcal{O}(\lambda^3). \tag{6.71}$$

Since the field's mean value vanishes, $\Xi = 0$. Regarding the second-order contribution, the map Θ_{pt} takes the form

$$\begin{aligned}&\Theta_{\mathrm{pt}}[\hat{\rho}_{\mathrm{D}}]\\ &= -\mathrm{i}[\int \mathrm{d}\tau \hat{V}_{\mathrm{pt}}(\tau), \hat{\rho}_{\mathrm{D}}] + \sum \gamma_{ab}(\hat{A}_a\hat{\rho}_{\mathrm{D}}\hat{A}_b^\dagger - \frac{1}{2}\{\hat{A}_b^\dagger\hat{A}_a, \hat{\rho}_{\mathrm{D}}\}).\end{aligned} \tag{6.72}$$

where the operators $\hat{A}$ are given by

$$\hat{A}_{1,2} = |e\rangle\langle g|\,, |g\rangle\langle e|\,, \tag{6.73}$$

and the coefficients γ_{ab}

$$\gamma_{ab} = \begin{pmatrix} \mathcal{F}(\Omega,\chi) & \mathcal{R}(\Omega,\chi) \\ \mathcal{R}^*(\Omega,\chi) & \mathcal{F}(-\Omega,\chi)\end{pmatrix} \tag{6.74}$$

which, since W is stationary, has entries

$$\mathcal{F}_{\mathrm{pt}}(\Omega,\chi) = \int \mathrm{d}\omega |\tilde{\chi}|^2(\omega)\tilde{W}(\omega+\Omega) \tag{6.75}$$

$$\mathcal{R}_{\mathrm{pt}}(\Omega,\chi) = \int \mathrm{d}\omega \tilde{\chi}(\omega+\Omega)\tilde{\chi}(\omega-\Omega)\tilde{W}(\omega), \tag{6.76}$$

Consider now the adiabatic limit, as described in chapter 3. In this limit the matrix γ becomes asymptotically diagonal

$$\gamma_{ab} \sim \sigma \begin{pmatrix} \tilde{W}(\Omega) & 0 \\ 0 & \tilde{W}(-\Omega))\end{pmatrix} = \sigma\tilde{W}(\Omega)\begin{pmatrix} 1 & 0 \\ 0 & e^{-\frac{2\pi\Omega}{a}}\end{pmatrix} \tag{6.77}$$

where σ is the adiabatic interaction time, and where we have used the detailed balance condition. The map thus behaves asymptotically as

$$\Theta_{\text{pt}}[\hat{\rho}_{\text{D}}] \sim -\mathrm{i}[\int \mathrm{d}\tau \hat{V}_{\text{pt}}(\tau), \hat{\rho}_{\text{D}}] + \sigma \tilde{W}(\Omega) \Big((\langle g|\hat{\rho}_{\text{D}}|g\rangle - e^{-\frac{2\pi\Omega}{a}} \langle e|\hat{\rho}_{\text{D}}|e\rangle)(|e\rangle\langle e| - |g\rangle\langle g|) - \frac{1+e^{-\frac{2\pi\Omega}{a}}}{2} (\langle e|\hat{\rho}_{\text{D}}|g\rangle \, |e\rangle\langle g| + \langle g|\hat{\rho}_{\text{D}}|e\rangle \, |g\rangle\langle e|) \Big) . \tag{6.78}$$

We are not interested in the form of $\hat{V}_{\text{pt}}(\tau)$ as $\sigma \to \infty$, only on the fact that it remains diagonal in the basis of the free Hamiltonian. This allows us to find the states for which of the map Θ vanishes. Indeed, if the state of the detector is diagonal, it follows that

$$-\mathrm{i}[\int \mathrm{d}\tau \hat{V}_{\text{pt}}(\tau), \hat{\rho}_{\text{D}}] = 0, \tag{6.79}$$

which implies

$$\Theta_{\text{pt}}[\hat{\rho}_{\text{D}}] \sim \sigma \tilde{W}(\Omega) \big(\langle g|\hat{\rho}_{\text{D}}|g\rangle - e^{-\frac{2\pi\Omega}{a}} \langle e|\hat{\rho}_{\text{D}}|e\rangle \big)(|e\rangle\langle e| - |g\rangle\langle g|),$$

i.e. only the diagonal elements change under the interaction. Moreover, if

$$\langle g|\hat{\rho}_{\text{D}}|g\rangle = e^{-\frac{2\pi\Omega}{a}} \langle e|\hat{\rho}_{\text{D}}|e\rangle \, , \tag{6.80}$$

then $\Theta_{\text{pt}}[\hat{\rho}_{\text{D}}] = 0$. This singles out the state of the detector as the thermal state at inverse temperature $\frac{2\pi}{a}$.

Therefore, in the long adiabatic limit, the detector's state is invariant up second order in perturbation theory if and only if the state is a thermal state at the Unruh temperature $T_U = \frac{a}{2\pi}$ independently of its characteristic energy gap Ω.

Although it is not possible to show that the detector reaches a thermal state using perturbation theory, this result strongly suggests that the detector is undergoing a thermal interaction with the quantum field at a temperature T_U. There are methods that allow one to better characterize the thermalization process, such as master equations [62] or the return to the equilibrium described in more mathematically sophisticated grounds, which is based on the stability of KMS states [63].

The perturbative calculation, while not being well-suited for analyzing the thermalization of a single particle detector, can nonetheless be used to reach the conclusion for ensembles of detectors. After the interaction, the probability of the detector to transition from its ground state to its excited state is given by

$$P_{g\to e} \sim \sigma\lambda^2 \tilde{W}(\Omega) \tag{6.81}$$

and the opposite process, the decay from the excited state to the ground state

$$P_{e\to g} \sim \sigma\lambda^2 \tilde{W}(-\Omega). \tag{6.82}$$

These processes have a relatively low probability that justifies the use of perturbation theory, but they are balanced

$$\frac{P_{g\to e}}{P_{e\to g}} \sim \frac{\tilde{W}(\Omega)}{\tilde{W}(-\Omega)} = e^{-\frac{2\pi\Omega}{a}}. \tag{6.83}$$

This implies that as the interaction time increases the populations of excited and ground detectors in a given ensemble will balance too, which we can associate to macroscopic thermal equilibrium.

Chapter 7

The Anti-Unruh effect

This chapter is devoted to reviewing the results in [3], a work concerned with non trivial behavior of the response of accelerated detectors that departs from standard thermal behavior.

What was first referred as Anti-Unruh corresponds to the phenomenology first reported in [100] under the name of *Anti-Unruh* phenomena. There, the surprising result was that, in the context of accelerated, point-like detectors, *"the transition probability can actually decrease with acceleration"*. Besides finding these results, the authors of [100] further discussed that if one were to define an effective temperature using the Excitation-to-Deexcitation Ratio (EDR) $T_{\text{EDR}} = -\Omega[\log(P_{g\to e}/P_{e\to g})]^{-1}$, T_{EDR} was effectively independent of the detector's energy gap when the Anti-Unruh effect is present, a behaviour usually associated with stationarity. From the results of [100], two lines of thought arise naturally:

- Is the finite time interaction thermal? How can one characterize thermality for finite time physics? Can one interpret this phenomenon as a process in which the detector cools down as the acceleration increases?
- Is the adiabatic limit thermal in general? Given that the state should be asymptotically thermal, it would be cumbersome if the behavior with acceleration were not thermal in this limit.

In [3] we analyzed the Anti-Unruh effect, studied its relationship with the KMS condition [91, 92, 64], and further discussed implications for the thermality of the Unruh effect in the adiabatic limit.

The Anti-Unruh phenomena appeared in [100] when studying situations that depart from the usual analysis of the Unruh effect as presented in chapter 6. Namely, they studied accelerated detectors coupled to a massless scalar field in two scenarios: *a)* Under a hard-IR momentum cutoff in free 1+1D flat spacetime, and *b)* for a detector in a periodic cavity from which the zero mode is removed. In those cases the Wightman function is not stationary, thus, strictly speaking, the Wightman function is not KMS.

In order to distill the origin of this phenomenology, we wanted to study situations in which the field's correlations are stationary with respect to accelerated observers, and further, fulfil the KMS condition. This implies that we needed to study the vacuum of a Lorentz invariant theory, but to maintain the similarities with the preceding results we studied the case of 1+1 dimensional fields with an extra infrared scale, namely massive Klein-Gordon fields. We will see that, for small interaction times, the response function decreases as the acceleration increases. Moreover, we showed the existence of regimes where the effective EDR temperature decreases as the KMS temperature increases. The non-monotonicity of T_{EDR} as a function of the KMS temperature disappears for long interaction times or large temperatures. Surprisingly, the observation that the transition probability can decrease as the detector's acceleration increases with the detector's acceleration survives even in the limit of infinitely long times under KMS. Therefore this modality of Anti-Unruh effect cannot be associated with transient behaviour in any way.

Interestingly, we will show that under the KMS condition and when the trajectory of the detector does not depend on the KMS temperature (e.g. when the temperature is not related to the dynamics of the observer, such as in ordinary thermal baths), the Anti-Unruh effect cannot appear at all. We will therefore show that the perception of the Anti-Unruh effect is linked to accelerated observers and it is not present for inertial observers coupled to generic thermal baths. In this chapter, we constrain our analysis to the point-like UDW model.

7.1 The Anti-Unruh effect and the KMS condition

To shed some light on this Anti-Unruh effect, let us introduce the following two definitions. One is concerned with the excitation probability of the detector, encoded in the function $\mathcal{F}(\Omega, \chi)$, the other other one is concerned with the excitation-to-decay ratio:

- **Weak Anti-Unruh:** We define the weak Anti-Unruh regime as the set of values of the detector gap Ω, interaction times σ, and field's (KMS) temperature $T_{\text{KMS}} = 1/\beta$, for which the detector's response function decreases as the KMS temperature increases, i.e.

$$\partial_\beta \mathcal{F}_\beta(\Omega, \chi_\sigma) > 0. \tag{7.1}$$

  In plain words: the detector detects fewer field excitations when the temperature increases.

- **Strong Anti-Unruh:** We define the strong Anti-Unruh regime as the set of values of the detector gap Ω, interaction times σ, and KMS temperatures $T_{\text{KMS}} = 1/\beta$ for which the effective EDR temperature

$$T_{\text{EDR}} = -\Omega[\log(P_{g\to e}/P_{e\to g})]^{-1} \tag{7.2}$$

  decreases as the KMS temperature increases, i.e.

$$\partial_\beta \beta_{\text{EDR}} < 0. \tag{7.3}$$

It is possible to have weak Anti-Unruh phenomena and yet not observe strong Anti-Unruh phenomena. It is unclear, however, if losing the monotonicity of the effective temperature will always lead to a decrease of the excitation probability with the KMS temperature.

Given that the excitation probability for finite time interactions is

$$\mathcal{F}_{\text{pt}}(\Omega, \chi) = \int \mathrm{d}\omega |\tilde{\chi}|^2(\omega) \tilde{W}(\omega + \Omega), \tag{7.4}$$

it is clear a necessary condition for the weak Anti-Unruh condition (7.1) to hold is that the Fourier transform of the Wightman function $\tilde{W}$ has to grow as β increases somewhere in its domain.

As we discussed in chapter 6, if the KMS condition is satisfied, $\tilde{W}$ is the product (6.63) of the Planckian distribution and the Fourier transform of the commutator. Therefore, the necessary condition for having weak Anti-Unruh phenomena when the KMS condition is satisfied can be simply written as

$$\partial_\beta \big(\tilde{C}(\omega, \beta) P(\omega, \beta)\big) < 0, \tag{7.5}$$

where

$$P(\omega, \beta) = \frac{1}{e^{\beta\omega} - 1} \tag{7.6}$$

is the Planckian factor.

Let us analyze the relationship between the KMS condition and the presence of Anti-Unruh phenomena in two different scenarios: *a)* when the commutator is independent of the KMS parameter (e.g., inertial detectors in a thermal background), and *b)* when the commutator depends explicitly on the KMS parameter (e.g., accelerated detectors coupled to the vacuum of a massive field).

7.1.1 $\tilde{C}$ does not depend on the KMS parameter

If $\tilde{C}$ does not depend on the KMS parameter, we see from (7.5) that the necessary condition for weak Anti-Unruh effect is

$$\tilde{C}(\omega)\partial_\beta \mathcal{P}(\omega,\beta) < 0. \tag{7.7}$$

But it can checked that

$$\mathrm{sgn}\big(\partial_\beta \mathcal{P}(\omega,\beta)\big) = -\mathrm{sgn}(\omega). \tag{7.8}$$

This implies that the necessary condition for weak Anti-Unruh (7.5) can be simplified in this case as

$$\omega\tilde{C}(\omega) > 0. \tag{7.9}$$

Because the commutator is pure imaginary, it is clear that $\tilde{C}(-\omega) = -\tilde{C}(\omega)$, which means that $\omega\tilde{C}(\omega)$ is even. On the other hand, since $\tilde{W}$ is positive [64], from (6.63) we see that $\mathrm{sgn}\,\tilde{C}(\omega) = -\mathrm{sgn}\,\omega$. Therefore $\omega\tilde{C}(\omega) < 0$ for all $\omega \in \mathbb{R}$ and the condition (7.9) will never be satisfied.

This leads to the following general result: For KMS states with respect to a time-like vector ∂_τ generating trajectories for which the commutator is independent of the KMS temperature there is no weak Anti-Unruh effect. In other words, the probability of detector excitation is monotonically increasing with the KMS temperature. This is the case of the following examples:

- An inertial detector coupled to a thermal state of a scalar field of mass $m \geq 0$ in arbitrary spatial dimensions, even in the presence of an IR cutoff Λ. Explicitly, in this case the commutator is

$$\tilde{C}(\omega) = -\frac{\pi^{\frac{2-d}{2}}\mathrm{sgn}(\omega)}{2^{d-1}\Gamma(d/2)}(\omega^2 - m^2)^{\frac{d-2}{2}}\Theta(|\omega| - m)\Theta(|\omega| - \Lambda), \tag{7.10}$$

  where we recall m is the field mass, d is the number of spatial dimensions and Λ is an IR cutoff.

- Uniformly accelerated detectors coupled to the vacuum state of a massless scalar field in $d = 1$ or $d = 3$ spatial dimensions. In these cases it can be shown [21] that for $m = 0$, $\Lambda = 0$, the commutator is the same as in the inertial case thus leading to the same conclusion.

Since in these cases the commutator is independent of the KMS temperature, the fact that there is no weak Anti-Unruh effect implies that there is no strong Anti-Unruh effect either. In other words, the EDR temperature increases monotonically with the KMS temperature.

7.1.2 $\tilde{C}$ depends on the KMS parameter

Although the commutator is independent of the field state, it may still depend on the KMS temperature through the trajectory $\mathsf{x}(\tau)$. In this case it is not straightforward to derive a general result as in the previous case. Let us consider some critical examples. If the field state is the Minkowski vacuum of a scalar field, trajectories with constant acceleration $a \geq 0$, yield Wightman functions that satisfy the KMS condition with KMS temperature $a/(2\pi)$.

For the massive case, however, the commutator depends explicitly on the acceleration [21]. Indeed, the Wightman function has a nontrivial dependence on $\beta = 2\pi/a$:

$$\tilde{W}_d(\omega, \beta) = \frac{\beta e^{-\frac{\beta\omega}{2}}}{2\pi^2} \int \frac{\mathrm{d}^{d-1}\boldsymbol{k}}{(2\pi)^{d-1}} \left| K_{\mathrm{i}\frac{\beta\omega}{2\pi}} \left(\frac{\beta}{2\pi}\sqrt{m^2 + \boldsymbol{k}^2} \right) \right|^2, \tag{7.11}$$

which, for $d \geq 2$ becomes

$$\begin{aligned} \tilde{W}_d(\omega, \beta) =& \frac{\beta e^{-\frac{\beta\omega}{2}}}{2^{d-1}\pi^{\frac{d+3}{2}}\Gamma(\frac{d-1}{2})} \\ & \times \int \mathrm{d}|\boldsymbol{k}|\, |\boldsymbol{k}|^{d-2} \left| K_{\mathrm{i}\frac{\beta\omega}{2\pi}} \left(\frac{\beta}{2\pi}\sqrt{m^2 + \boldsymbol{k}^2} \right) \right|^2, \end{aligned} \tag{7.12}$$

while for $d = 1$ the expression (7.11) reduces to

$$\tilde{W}_1(\omega, \beta) = \frac{\beta e^{-\frac{\beta\omega}{2}}}{2\pi^2} \left| K_{\mathrm{i}\frac{\beta\omega}{2\pi}} \left(\frac{\beta m}{2\pi} \right) \right|^2. \tag{7.13}$$

In these cases the necessary condition (7.5) for weak Anti-Unruh can be fulfilled. In fact it is easy to check explicitly that this condition can actually be satisfied both in the $1 + 1$D and $3 + 1$D cases. Let us first focus on the 1+1D case.

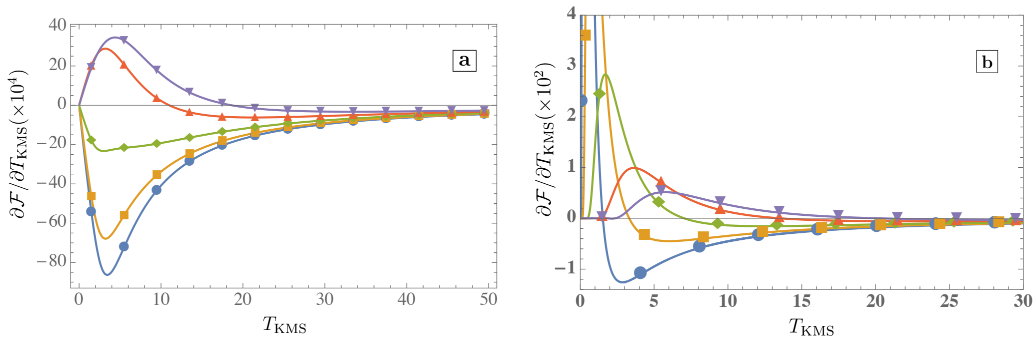


Figure 7.1: *Weak Anti-Unruh effect*: Derivative of the response function with respect to the KMS temperature $T_{\text{KMS}} = 1/\beta$ for 1+1D, $m = 1$. The different lines correspond to values of $\Omega = 15$ (inverted purple triangles), $\Omega = 10$ (red triangles), $\Omega = 5$ (green rhombi), $\Omega = 2$ (orange squares), $\Omega = 0.5$ (blue circles). The two plots represent the short and the long time regimes. Namely **Right.** $\sigma = 1$ with a Gaussian switching $\chi(\tau/\sigma) = \pi^{-1/4}e^{-\tau^2/(2\sigma^2)}$, and **Left.** $\sigma \to \infty$, independently of the switching. We see that for a broad range of the parameters this derivative is negative (i.e., the response function decreases as the KMS temperature increases), even for adiabatic (eternal) switching.

In the massive 1+1D case we can see (along the same lines as in the massless 1+1D case with an IR cutoff studied in [100]) that the accelerated detector experiences the weak Anti-Unruh effect: That is, the detector's response function can decrease as the KMS temperature $T_{\text{KMS}} = 1/\beta$ increases, as illustrated in Fig. 7.1a for a Gaussian switching $\chi(\tau/\sigma) = \pi^{-1/4}e^{-\tau^2/(2\sigma^2)}$, and in Fig. 7.1b for any square integrable switching $\chi(\tau/\sigma)$ in the infinitely adiabatic limit $\sigma \to \infty$.

Remarkably, this voids one of the major possible criticisms that could have been raised against the relevance of the Anti-Unruh phenomena reported in [100]. Namely, it could have been argued that in [100], the introduction of a hard IR cutoff, which, rigorously speaking, yields non-stationary Wightman functions, was the responsible for the appearance of transients that give rise to the Anti-Unruh effect. However, we see that we do not require a breakdown of the KMS condition to see the Anti-Unruh effect. Specifically, an accelerated detector coupled to a massive field vacuum will experience the weak Anti-Unruh effect in spite of the fact that the KMS condition is satisfied in this case. In other words, we can have a detector that, when switched on for finite times, can decrease its transition rate as the KMS temperature increases.

More so, this weak Anti-Unruh behaviour also shows up even in the limit of detectors adiabatically switched on for an infinite amount of time. Indeed, in this

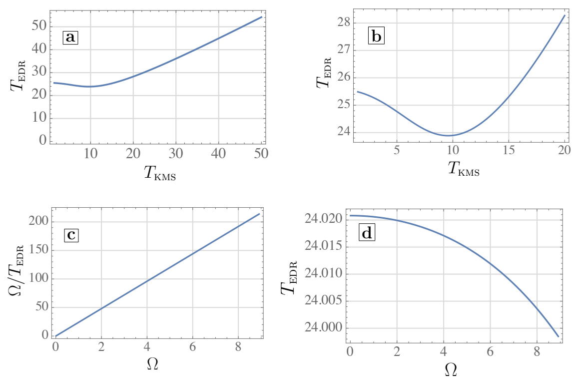


Figure 7.2: *Strong Anti-Unruh effect*: Figure *a)* shows the EDR temperature $T_{\text{KMS}} = 1/\beta$ as a function of the KMS temperature for 1+1D, $m = 1$, $\sigma = 0.04$, and $\Omega = 1$. For large KMS temperatures, $T_{\text{EDR}} \simeq T_{\text{KMS}}$, while for small ones the EDR temperature actually decreases, as seen in the zoomed Fig. *b)*. Figure *c)* displays (for $m = 1$, $\sigma = 0.04$, and $T_{\text{KMS}} = 8$) the apparent linearity of Ω/T_{EDR} with Ω and hence the almost independence of the EDR temperature with Ω. Figure *d)* shows (for the same parameters as Fig. *c)*) that this dependence is actually present although it is extremely weak.

limit, we know that the expression of the response function is particularly simple. We show in Fig. 7.1b that the weak Anti-Unruh effect is present in the strict limit $\sigma \to \infty$, independently of the particular form of the switching function χ (even including non-adiabatic switchings for which the transition rate is well defined). Therefore the weak Anti-Unruh effect cannot be associated with transient behaviour.

The strong Anti-Unruh behaviour, on the other hand, is confined to short interaction times and small accelerations (i.e. KMS temperatures), as shown in Fig. 7.2. In the figure we see that in the regime of small KMS temperatures, the EDR temperature decreases as the KMS temperature increases. We also see that for larger KMS temperatures, the EDR temperature approaches the KMS temperature. Finally, this figure also shows that the EDR temperature depends very weakly on the gap frequency Ω, despite the detector not being in equilibrium with the field. This behaviour is entirely the same as that found in [100]. There, a hard-IR cutoff (either removing the zero mode in a periodic cavity, or imposing a cutoff Λ in the continuum case) causes the Wightman function to not satisfy the KMS condition, but β_{EDR} as defined in (7.2) behaves as a function of acceleration exactly in the same way described above.

In particular, we have proven that this is a genuine effect of the acceleration of the detector, even when KMS is satisfied, and that it would not be seen by an inertial detector interacting for a finite timescale with a thermal bath regardless of the number of spacetime dimensions and the presence of cutoffs.

7.2 Parameter space dependence of the Anti-Unruh phenomena

In this section we analyze in more detail in what region of the parameter space we can find Anti-Unruh phenomena.

One legitimate question that one may ask is whether this effect may be related with the fact that even though the response of a static detector in a thermal bath and the response of an accelerated detector coupled to the field vacuum are statistically identical, the two responses come from fundamentally different physical effects.

In the inertial thermal case, the main contribution to the detector's excitation rate for sufficiently long times comes from rotating-wave contributions (those involving processes where the detector gets excited by emitting a field quantum [101]). However, in the Unruh effect, the contribution of the rotating-wave and counter-rotating wave terms are comparable. This is the fundamental difference in the two processes and this is ultimately the reason why the two scenarios are different despite the fact that in both cases the detectors display a thermal response.

To answer this question, let us first consider the response function of an accelerated detector coupled to a massive field prepared in the vacuum state in the long time limit ($\sigma \to \infty$). The response function is given by the Wightman function evaluated at $\omega = \Omega$. Specifically, for the 1+1D case the response function is given by (7.13) evaluated at $\omega = \Omega$.

Let us consider two different asymptotic limits of this equation, the large mass limit and the small mass limit. Let us begin with the the large mass limit. Using the leading order of the asymptotic expansion of the Bessel function for large values of its argument

$$K_{\mathrm{i}\frac{\beta\Omega}{2\pi}}\left(\frac{\beta m}{2\pi}\right) \sim \frac{\pi}{\sqrt{\beta m}} e^{-\frac{\beta\Omega}{2\pi}}, \tag{7.14}$$

which is valid under the condition

$$\left(\frac{\beta\Omega}{2\pi}\right)^2 + \frac{1}{4} \ll \frac{\beta m}{2\pi}, \tag{7.15}$$

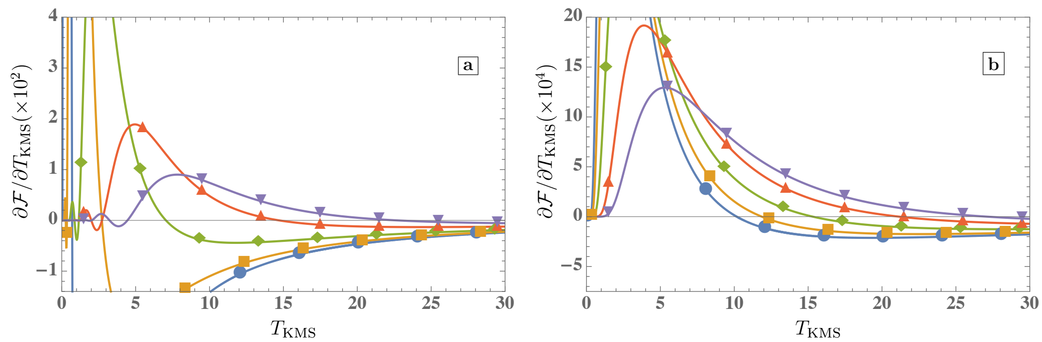


Figure 7.3: *Anti-Unruh effect dependence on the mass*: For any switching function shape, in the limit of infinite interaction time $\sigma \to \infty$, we show the derivative of the response function with respect to the KMS temperature $T_{\text{KMS}} = 1/\beta$ for 1+1D and for $m = 0.1$ (Left) and $m = 10$ (Right). The different lines correspond again to values of $\Omega = 15$ (inverted purple triangles), $\Omega = 10$ (red triangles), $\Omega = 5$ (green rhombi), $\Omega = 2$ (orange squares), $\Omega = 0.5$ (blue circles). The right figure shows how for $m \ll T_{\text{KMS}}$ the oscillations of the derivative generate Anti-Unruh effect in the low temperature zone, whereas the left shows that when the KMS temperature drops below tha mass scale (in this case $m = 10$), the AntiUnruh effect disappears. We see in both figures that the Anti-Unruh effect can exist for values of Ω below and above the mass scale m.

we get the following response function in the limit $\sigma \to \infty$ [21]:

$$\mathcal{F}(\Omega, \beta) \approx \frac{\sigma e^{-\beta(\Omega/2+m/\pi)}}{4\pi m}. \tag{7.16}$$

The response function (7.16) is a monotonically increasing function of the temperature, and thus does not exhibit any kind of Anti-Unruh phenomena. This allows us to reach to the conclusion that in the asymptotic limit of field mass much larger than the detector gap for constant KMS temperature, there cannot be any Anti-Unruh phenomena.

On the other hand, as shown in [21], the asymptotic behaviour of the response function in the limit of small mass is given by

$$\mathcal{F}(\Omega, \beta) \sim \frac{\sigma}{\Omega(e^{\beta\Omega} - 1)}\left[1 + \cos\left(\frac{\beta\Omega}{\pi}\log\frac{\beta m}{4\pi} + \phi\left[\frac{\beta\Omega}{2\pi}\right]\right)\right], \tag{7.17}$$

with $\phi(z) = 2\,\mathrm{Arg}\,\Gamma(\mathrm{i}z)$, in the regime where

$$\left(\frac{\beta\Omega}{2\pi}\right)^2 + 1 \gg \left(\frac{\beta m}{2\pi}\right)^4 \tag{7.18}$$

is satisfied.

In the light of (7.17), we see that the response function in the limit of small βm, where $\beta\Omega$ is kept constant, is not a monotonically increasing function of β. In fact, (7.17) becomes highly oscillatory as βm goes to zero and, as such, its derivative with respect to the KMS temperature will take negative values. The Anti-Unruh phenomena will appear therefore for sufficiently small βm regardless of the constant value of β and Ω.

The conclusion that we extract is that although there may be some relationship between the Anti-Unruh phenomena and the ratio between Ω and m, the existence of the Anti-Unruh effect is independent of the scale of Ω, since, for sufficiently small mass, we can find Anti-Unruh phenomena regardless of the value detector gap. Instead the relevant figure of merit ruling the appearance of the phenomena is the ratio between the field mass and the acceleration.

We illustrate this in Fig. 7.3, where we show that the Anti-Unruh phenomena for detectors interacting for long times ($\sigma \to \infty$) can exist for a diverse range of parameters. In particular, it can exist when m is more than an order of magnitude larger than Ω (Fig. 7.3a) and also when m is more than an order of magnitude below Ω (Fig. 7.3b). In both cases it can be seen that the Anti-Unruh effect ceases to appear when $\beta m \gtrsim 1$,

7.3 Discussion

To shed light into the Anti-Unruh phenomena [100], we analyzed the role of the interaction time in the thermalization of an Unruh-DeWitt particle detector. In particular, it is well-known that, for infinitely long interaction times and if the Wightman function satisfies the KMS condition, the Excitation-to-Deexcitation Ratio (EDR) is determined by the detailed balance condition.

The Anti-Unruh effect can be characterized in terms of the behaviour of the response function and the effective EDR temperature with the KMS temperature T_{KMS}. On the one hand, we have called *weak* Anti-Unruh effect those situations in which the excitation probability decreases as T_{KMS} increases (a detector clicks less often as the KMS temperature of the field increases). On the other hand, we have called *strong* Anti-Unruh effect situations where the effective EDR temperature almost independent of the gap frequency decreases as T_{KMS} increases.

We have seen that the weak and strong Anti-Unruh effects do not appear at all under some general conditions. Namely, that the Wightman function satisfies the KMS condition with respect to translations along the proper time of a detector whose trajectory does not depend on the KMS temperature. In particular, this implies that the Anti-Unruh effects (both weak and strong) are absent for inertial detectors coupled to massless or massive scalar fields in a KMS state (for example, a thermal state) with or without a momentum cutoff and for any spatial dimensions. It is also absent for accelerated detectors in the Minkowski vacuum of a massless scalar field for one and three spatial dimensions.

The situation is entirely different for an accelerated detector coupled to the Minkowski vacuum in two different but related cases, namely, there can be Anti-Unruh phenomena for a massive scalar field or when an IR cutoff is in operation. We showed that in these cases there appear clear signatures of both weak and strong Anti-Unruh behaviour in 1+1 spacetime dimensions.

For the massive case, for small interaction times, and well within the regime of validity of perturbation theory, we see that *i)* the response function decreases as the acceleration (the KMS temperature) increases, and *ii)* the effective EDR temperature decreases with the KMS temperature, depends also on the interaction time, but is almost independent of the gap frequency. Furthermore, for long interaction times or large KMS temperatures, the strong Anti-Unruh effect disappears but, remarkably, the weak version of it is still at work, i.e. the derivative of the response function with respect to the KMS temperature is negative. This is true even in the strict limit of infinitely long adiabatic switching for any square integrable switching function.

The massless case with an IR momentum cutoff was studied in [100] with the same results. It must be stressed that although in [100] the Wightman function was not stationary and hence was not KMS, the Anti-Unruh effect cannot be associated with this fact because it is also present in the massive case, which certainly is KMS. This effect cannot be dismissed as a transient either since, as we have discussed the (weak) Anti-Unruh effect (i.e., a detector 'seeing less particles' as the temperature of the medium increases) is present even for infinitely long times.

Finally, let us note that the fact that the Anti-Unruh effect can be seen by relativistic accelerated observers but not by inertial observers coupled to a thermal bath is a distinctive signature of perceived particle creation by accelerated observers, that can be singled out from the behaviour of detectors coupled to thermal backgrounds.

As a final comment, we note that Anti-Unruh effect has been further studied in the context of gravitational physics, such as BTZ blackholes [102].

Chapter 8

The Unruh effect without thermality

This chapter reviews the results published in [4], which contains a detailed analysis about the preservation of the Unruh effect in scenarios that deviate from the the usual situation described in chapter 6. Therefore, we will examine the question of robustness in the Unruh effect, by which the detector approach will play an essential role.

8.1 Motivation

Quantum field theory is considered to be an effective theory that is valid above the quantum gravity scale, typically introduced in terms of a length scale ℓ [103]. It is therefore expected that QFT will become less precise as this scale is approached, perhaps eventually failing completely as a correct description of nature. In most cases, a hierarchy of scales prevents deep UV physics from playing a crucial role for physics at scales below ℓ. Interestingly, this is not always the case and it is also expected that ultraviolet deformations of the structure of quantum field theory can actually have an effect at lower scales [104, 105, 106, 107, 108]. The study of these phenomena becomes relevant as they can set the ground for experimental predictions of deep UV physics, such as, e.g. quantum gravitational effects.

When considering detector models, examples of this behavior that have been discussed in the literature include the response of particle detectors along inertial trajectories in the framework of polymer quantization [109, 110, 111] and in non-local field theories [112], or the transmission of information through non-local fields [113]. Determining the deformations that lead to this "intrussion", and finding the predictions that are affected, is of clear importance for quantum gravity phenomenology.

With respect to the Unruh effect, the analysis of UV deformations can be made even more concise. There have been multiple discussions on how deformations of the UV structure of QFT can affect the Unruh effect, [114, 115, 116, 117, 118, 119, 120, 121, 122]. These pieces of research, however, focus in concrete deformations, which besides cannot be compared since the authors of the different works use different mathematical formalisms. In contrast, we studied this problem with a unified approach. Our starting point is the definition of the Unruh effect as the thermalization of accelerated particle detectors. Setting universality issues for the detector model aside, we further claim that the Unruh effect is present provided the thermalization of UDW, point-like accelerated particle detectors, and we understand thermalization as the thermal balance of excitations and deexcitations.

8.1.1 Deformations of the two-point correlator

As we saw in chapter 6, the Unruh effect only requires the two-point correlator, or Wightman function of the field, which determines the excitation and deexcitation probabilities of particle detectors. On general grounds, the introduction of an additional length scale will lead to deformations of the functional form of the Wightman function. These deformations encode the leading modifications arising from the particular ultraviolet completion chosen. Let us make the following technical assumptions:

1. There is an effective continuum description of flat spacetime in which deformed Wightman functions can be written as functions of the spacetime coordinates x.

2. The deformed Wightman functions reduce to their standard Poincaré invariant form $W_0(\mathsf{x}, \mathsf{x}')$ in the formal limit $\ell \to 0$.

3. The functional form of deformed Wightman functions may break explicitly the invariance under Lorentz transformations, while keeping spacetime translations and spatial rotations as symmetries.

4. The deformed Wightman functions are polynomially bounded in $|\Delta t|$ and $|\Delta x|$ when these absolute values tend to infinity.

Note that condition 1 above permits us to include in our analysis discrete or quantum-mechanical features of the spacetime structure. On the other hand, we can exploit conditions 2 and 3 in order to write the deformed Wightman function $W_\ell(\mathsf{x},\mathsf{x}')$ as

$$W_\ell(\mathsf{x},\mathsf{x}') = W_0(\mathsf{x},\mathsf{x}') + D_\ell(t-t', \|\boldsymbol{x}-\boldsymbol{x}'\|). \tag{8.1}$$

Condition 3 imposes, first, translation invariance, which implies that 2-point Wightman functions are functions of the differences $t''-t'$ and $\boldsymbol{x}''-\boldsymbol{x}'$. Invariance under rotations further implies that the Wightman function can only depend on the spacial coordinates through the norm $\|\boldsymbol{x}-\boldsymbol{x}'\|$. Moreover,

$$\lim_{\ell\to 0} D_\ell(t-t', \|\boldsymbol{x}-\boldsymbol{x}'\|) = 0. \tag{8.2}$$

The function D_ℓ has been introduced on phenomenological grounds, and its specific form will depend on the origin of the deformations.

Note that we have also implicitly assumed that the one point function of the field is zero, since the Unruh effect, the phenomenon our interest, has to do with the fluctuations of the field rather that with its mean value.

8.2 Infinitely adiabatic limit for uniformly accelerated detectors

The excitation probability of a UDW detector following an accelerated trajectory has certain characteristics that, together with our assumptions about the deformations of the Wightman function, allow us to perform a detailed analysis of the adiabatically switched, uniformly accelerated detector.

Since the excitation probability is linear in the Wightman function of the field, we can distill the contributions to the excitation probability coming form the deformations D_ℓ:

$$\mathcal{F}_\ell(\Omega,\chi) = \mathcal{F}_0(\Omega,\chi) + \iint \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau)\chi(\tau')D_\ell\left(t(\tau)-t(\tau'), \|\boldsymbol{x}(\tau)-\boldsymbol{x}(\tau')\|\right)e^{-\mathrm{i}\Omega(\tau-\tau')}. \tag{8.3}$$

Henceforth we will slightly abuse the notation by defining

$$D_\ell(\tau,\tau') = D_\ell\left(t(\tau)-t(\tau'), \|\boldsymbol{x}(\tau)-\boldsymbol{x}(\tau')\|\right). \tag{8.4}$$

Expression (8.2) actually should be understood as

$$\lim_{\ell\to 0}\mathcal{F}_\ell(\Omega,\chi) = \mathcal{F}_0(\Omega,\chi) \tag{8.5}$$

for all test functions χ, whenever the pointwise limit is not well defined.

In order to discuss the existence of the Unruh effect, it is necessary to analyse the adiabatically switched statistics of the detector. Note that we are not analyzing stationary correlations in general, so the adiabatically switched statistics cannot be written as simply as in chapter 3.

Some simplifications can be made, however, when particularizing the switching function used to perform the adiabatic limit. First, note that the excitation probability (3.70) can be written as a double integral of the difference of proper times, $z=\tau-\tau'$, and the sum of proper times $w=\tau+\tau'$, by means of a simple change of variables

$$\begin{aligned}\mathcal{F}_\ell(\Omega,\chi) &= \iint \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau)\chi(\tau')W_\ell(\tau,\tau')e^{-\mathrm{i}\Omega(\tau-\tau')}\\ &= \frac{1}{2}\iint \mathrm{d}w\mathrm{d}z\chi\left((w+z)/2\right)\chi\left((w-z)/2\right)W_\ell(z,w)e^{-\mathrm{i}\Omega z},\end{aligned} \tag{8.6}$$

where again we have abused the notation by writing $W_\ell(z,w) = W_\ell\left((w+z)/2,(w-z)/2\right)$.

Now, we would like to separate the part of the switching function that regulates the duration of the interaction. In order to do so, we choose the switching function to be a Gaussian,

$$\chi(\tau)_\sigma = \frac{1}{\pi^{1/4}}e^{-\tau^2/2\sigma^2}, \tag{8.7}$$

since then

$$\chi\left((w+z)/2\right)\chi\left((w-z)/2\right) = \frac{e^{-w^2/\sigma^2}}{\sqrt{\pi}}\,e^{-z^2/\sigma^2}. \tag{8.8}$$

With this choice, we get

$$\mathcal{F}_\ell(\Omega,\chi_\sigma) = \frac{1}{2}\int_{-\infty}^{\infty}\mathrm{d}w\,\frac{e^{-w^2/\sigma^2}}{\sqrt{\pi}}\int_{-\infty}^{\infty}\mathrm{d}z\,e^{-z^2/\sigma^2}W_\ell(w,z)e^{-i\Omega z}. \tag{8.9}$$

Although this choice of switching function is in conflict with the assumption that the interaction region between detector and field is spacetime compact, the calculations are more apparent due to the algebraic characteristics of Gaussian functions, and we assume that the physical content of the predictions does not differ substantially when performing the calculations with compact switching functions. We will assume that the Gaussians lie in the distributional domain of the Wightman function (and its deformations), since failing to do so is related to the infrared characteristics of the theory that do not concern us here.

What makes equation (8.9) so appealing is that, as σ increases beyond any other scale of the problem, it represents the response function $\mathcal{F}$ as a time-average of, basically, the Fourier transform of the Wightman function for each w. This Fourier transform can be fully characterized by the behavior of the poles of the two-point function on the complex plane, and indeed that is what allows one to discuss deformations of different nature within a single formalism. But that is not the end of the story, the particular set of assumptions that we have taken allows one to conclude further that the adiabatic response of accelerated observers depends not on the poles located on the whole complex plane, but on a finite-width strip of the upper complex plane. We develop this argument in what follows.

8.2.1 Accelerated trajectories

Consider indeed the trajectory followed by an accelerated observer

$$t(\tau) = \frac{1}{a}\sinh(a\tau), \qquad x(\tau) = \frac{1}{a}\cosh(a\tau). \tag{8.10}$$

One has then, using the relevant identities involving hyperbolic functions, that

$$\Delta t(\tau'', \tau') = \frac{\sinh(a\tau'') - \sinh(a\tau')}{a} = \frac{2}{a}\sinh(az/2)\cosh(aw/2) \tag{8.11}$$

and, for $\tau'' \geq \tau'$,

$$\sqrt{\Delta \boldsymbol{x}(\tau'', \tau')^2} = \frac{\cosh(a\tau'') - \cosh(a\tau')}{a} = \frac{2}{a}\sinh(az/2)\sinh(aw/2). \tag{8.12}$$

It follows that,

$$-\Delta t^2 + \Delta \boldsymbol{x}^2 = -\frac{4}{a^2}\sinh^2(az/2). \tag{8.13}$$

We are going to exploit the fact that hyperbolic functions are periodic in the complex plane. The transformation that is useful for our purposes is

$$z \longrightarrow z + \frac{4\pi\mathrm{i}}{a}, \tag{8.14}$$

because under this transformation both hyperbolic functions $\sinh(az/2)$ and $\cosh(az/2)$ are invariant. Taking into account equations (8.11) and (8.12), this implies that any deformations D_ℓ that are a function of the difference of the coordinates only are invariant under the transformation (8.14).

Now, consider the following integral

$$\int_{-\infty}^{\infty} \mathrm{d}z\, e^{-z^2/\sigma^2} W_\ell(w, z) e^{-i\Omega z}, \tag{8.15}$$

and define

$$f_\ell(z, w) = e^{-z^2/\sigma^2} W_\ell(w, z) e^{-\mathrm{i}\Omega z}. \tag{8.16}$$

Under the transformation (8.14), the integrand f_ℓ transforms as

$$f_\ell(z, w) \longrightarrow e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} f_\ell(z, w) e^{-8\pi\mathrm{i}z/a\sigma^2}. \tag{8.17}$$

This identity can be exploited in order to evaluate the real integral in z in equation (8.15). Let us consider the following complex integral on the contour γ defined as the rectangle with horizontal sides on the real line and $z = 4\pi\mathrm{i}/a$, and vertical sides at $\pm\zeta \in \mathbb{R}$:

$$\oint_\gamma f_\ell(z, w) = 2\pi\mathrm{i} \sum_{k\in I} \mathrm{Res}[f_\ell(z, w), z_k]. \tag{8.18}$$

In this expression, we have used the residue theorem, so that $\{z_k\}_{k\in I}$ is the finite set of poles enclosed by γ. The integral on the left-hand side can be decomposed in four integrals and, using equation (8.17), one has

$$\begin{aligned}\oint_\gamma f_\ell(z, w) = &\int_{-\zeta}^{\zeta} \mathrm{d}z\, f_\ell(z, w) - e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \int_{-\zeta}^{\zeta} \mathrm{d}z\, f_\ell(z, w) e^{-8\pi\mathrm{i}z/a\sigma^2} \\ &+ \mathrm{i}\int_0^{4\pi/a} \mathrm{d}x\, f_\ell(\zeta + \mathrm{i}x) - \mathrm{i}\int_0^{4\pi/a} \mathrm{d}x\, f_\ell(-\zeta + \mathrm{i}x).\end{aligned} \tag{8.19}$$

Our next step will be showing that the two integrals on the second line vanish in the $\zeta \to \infty$ limit. It is enough to consider explicitly one of them, for instance

$$\int_0^{4\pi/a} \mathrm{d}x\, f_\ell(\zeta + \mathrm{i}x) = e^{-\zeta^2/\sigma^2} e^{-\mathrm{i}\Omega\zeta} \int_0^{4\pi/a} \mathrm{d}x\, e^{-2\mathrm{i}\zeta x/\sigma^2} e^{x^2/\sigma^2} e^{\Omega x}\, W_\ell(w, \zeta + \mathrm{i}x). \tag{8.20}$$

Now let us restrict our attention to Wightman functions that, in the large $|\Delta t|$ or $|\Delta x|$ limits, display a polynomial dependence on these variables i.e. that satisfy the condition 4. Recalling equations (8.11) and (8.12) this would imply that the behavior of $W_\ell(w,z)$ satisfies

$$W_\ell(w,z) \leq c(w)\sinh^m(az/2), \tag{8.21}$$

for some value of $m \in \mathbb{Z}$ and a function $c(w)$. The worst-case scenarios are those for which $m \geq 0$; using $|\sinh(\alpha + \mathrm{i}\beta)| \leq \cosh(\alpha)$ for $\alpha, \beta \in \mathbb{R}$, one has

$$\left| \mathrm{i}\int_0^{4\pi/a} \mathrm{d}x\, f_\ell(\zeta + \mathrm{i}x) \right| \leq \frac{4\pi c(w)}{a} e^{-\zeta^2/\sigma^2} e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \cosh^m(a\zeta/2). \tag{8.22}$$

Hence, this integral vanishes for $\zeta \to \infty$. The sum of the residues of f_ℓ in the strip equals the two first integrals in (8.19),

$$2\pi\mathrm{i}\sum_{k\in I} \mathrm{Res}[f_\ell(z,w), z_k] = \int \mathrm{d}z\, f_\ell(z,w) - e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \int \mathrm{d}z\, f_\ell(z,w) e^{-8\pi\mathrm{i}z/a\sigma^2} \tag{8.23}$$

Integrals of these terms, weighted appropriately with a Gaussian function, are precisely related to the excitation probability. Indeed,

$$\int \mathrm{d}w\, \frac{e^{-w^2/\sigma^2}}{\sqrt{\pi}} \int \mathrm{d}z\, f_\ell(z,w) = 2\mathcal{F}_\ell(\Omega, \chi_\sigma) \tag{8.24}$$

and similarly

$$\begin{aligned} &- e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \frac{1}{2\sigma} \int \mathrm{d}w\, \frac{e^{-w^2/\sigma^2}}{\sqrt{\pi}} \int \mathrm{d}z\, e^{-z^2/\sigma^2} W_\ell(w,z) e^{-i\Omega z} e^{-4\pi\mathrm{i}z/a\sigma^2} \\ &= -2e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \mathcal{F}_\ell(\Omega + 8\pi/a\sigma^2, \chi_\sigma). \end{aligned} \tag{8.25}$$

This implies the following relation

$$\begin{aligned} &\mathrm{i}\pi \int \mathrm{d}w\, \frac{e^{-w^2/\sigma^2}}{\sqrt{\pi}} \sum_{k\in I} \mathrm{Res}[f_\ell(z,w), z_k] \\ &= \mathcal{F}_\ell(\Omega, \chi_\sigma) - e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \mathcal{F}_\ell(\Omega + 8\pi/a\sigma^2, \chi_\sigma). \end{aligned} \tag{8.26}$$

Finally, in the long-time adiabatic limit, the second expression (8.26) is asymptotically equivalent[1] to

$$\mathcal{F}_\ell(\Omega, \chi_\sigma) - e^{16\pi^2/a^2\sigma^2} e^{4\pi\Omega/a} \mathcal{F}_\ell(\Omega + 8\pi/a\sigma^2, \chi_\sigma) \sim (1 - e^{4\pi\Omega/a}) \mathcal{F}_\ell(\Omega, \chi_\sigma), \tag{8.27}$$

[1] Two functions depending on a parameter λ are asymptotically equivalent in the limit $\lambda \to \lambda_0$ if the limit of their quotient is 1.

which allows one to identify the leading asymptotic behavior of the transition probabilities, in terms of the residues of f_ℓ, as

$$\mathcal{F}_\ell(\Omega, \chi_\sigma) \sim \frac{\mathrm{i}\pi}{1 - e^{4\pi\Omega/a}} \int \mathrm{d}w\, \frac{e^{-w^2/\sigma^2}}{\sqrt{\pi}} \sum_{k\in I} \mathrm{Res}[f_\ell(z,w), z_k] \tag{8.28}$$

for large σ. In expression (8.28) the residues are to be located in the strip $S := \{0 < \mathrm{Im}(z) < \frac{4\pi}{a}\}$.

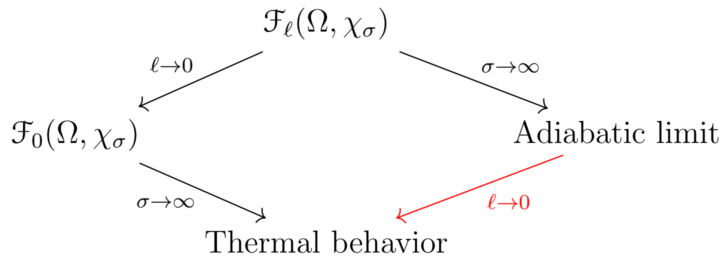
This is the main result of this section. We have found that for general deformations satisfying the conditions 1-4, the adiabatic, long-time statistics of the detectors can be written in terms of integrals of the poles of the deformed Wightman function.

We just mention a last technical point. We have implicitly assumed that the integrals can be manipulated as if the Wightman function were a common function, rather than a distribution, but this is not completely general. In relativistic quantum field theory, and for ordinary, non-pathological states, the two-point function is the boundary value of an ordinary function, a fact that is related to the spectrum condition [123]. This is not generally the case, and in what follows we will assume that the deformed Wightman function is also the boundary value of a regular function.

8.3 Preservation of the Unruh effect

In standard Lorentz-invariant quantum field theory, the thermal behavior of the response function holds exactly under the KMS condition. However, demanding that the KMS condition is strictly satisfied is, arguably, unnecessarily restrictive from a physical perspective, an observation that has attracted much attention. In the presence of a deformation with typical length scale ℓ, it is reasonable to expect that small deviations from exact thermal behavior, involving this new scale, would appear. This broader set of scenarios cannot be characterized by the KMS condition, which will be generally violated. Taking this into account, in the following we determine the minimal requirements that single out the scenarios in which this violation is mild enough so that the Unruh effect is preserved. The relation between the KMS condition and the preservation of the Unruh effect is detailed in Sec. 8.4.4.

On general grounds, we consider that the Unruh effect is preserved if the statistics of the detector along uniformly accelerated trajectories has the right $\ell \to 0$ limit, in the adiabatic regime, namely if $\lim_{\ell\to 0} \mathcal{F}_\ell(\Omega, \chi_\sigma) = \mathcal{F}_0(\Omega, \chi_\sigma)$ for the leading asymptotic terms as $\sigma \to \infty$. This can be alternatively defined in terms of a commutative diagram involving the double integration in w and z and the $\ell \to 0$ limit.



Note that, in the adiabatic limit, the only possible dimensionless combinations of the physical quantities involved are ℓa and $\ell\Omega$. Hence, if this condition is satisfied, the $\ell = 0$ expressions for the response functions are recovered up to small corrections when $\ell a \ll 1$ and $\ell\Omega \ll 1$. In other words, appreciable deviations from the Unruh effect would only exist for accelerations or frequencies that are of the same scale of the parameter of the deformation ℓ. This realizes the decoupling of scales that we alluded to in the introductory discussion.

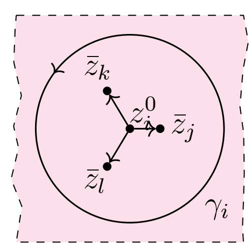


Figure 8.1: The contour γ_i encloses all the poles $\{\bar{z}_j, \bar{z}_k, \bar{z}_l\}$ that are originated from the isolated pole z_i^0 of the undeformed Wightman function.

Let us define the set of poles $\{\bar{z}_l\}_{l\in K}$ that are obtained as a continuous deformation of the original set of poles $\{z_m^0\}_{m\in K}$ originally in $S \subset \mathbb{C}$, with the possible addition or splitting of poles.

A. *Local uniform convergence:* The integral along each of the contours γ_i containing all the deformed poles that stem from each of the poles z_0 of the undeformed Wightman function, but not from other poles of the latter, recovers the undeformed contribution in the $\ell \to 0$ limit.

B. All the poles $\{\bar{z}_l\}_{l\in K}$ must remain in the horizontal strip.

C. The sum of the residues of the poles of the Wightman function in z, times $e^{-\frac{w^2}{\sigma^2}}$, must be integrable with respect to x in the adiabatic limit, $\sigma \to \infty$.

These three necessary conditions (A,B,C) are, in fact, sufficient when holding simultaneously in order to preserve the Unruh effect.

Let us sketch the proof of this statement. Condition B implies that the right-hand side of equation (8.28) is finite in the $\sigma \to \infty$ limit. On the other hand, condition C implies that all the deformed poles that stem from undeformed poles z_i^0 inside the horizontal strip $S \subset \mathbb{C}$ remain in S. Therefore, the corresponding residues are all taken into account in the right-hand side of equation (8.28). Finally, condition A ensures that the sum of these residues has the right $\ell \to 0$ limit.

8.4 Examples

8.4.1 No deformation

As a consistency check of equation (8.28) let us show that we obtain the usual results for the undeformed Wightman function

$$W_0(z) = -\frac{1}{4\pi^2}\frac{a^2}{4\sinh^2\left[a\left(z-\mathrm{i}\epsilon\right)/2\right]}, \tag{8.29}$$

with $\epsilon > 0$. There is an infinite set of second-order poles in the imaginary axis, namely

$$z_k^0 = \mathrm{i}\epsilon + \mathrm{i}\frac{2\pi k}{a}, \qquad k \in \mathbb{Z}. \tag{8.30}$$

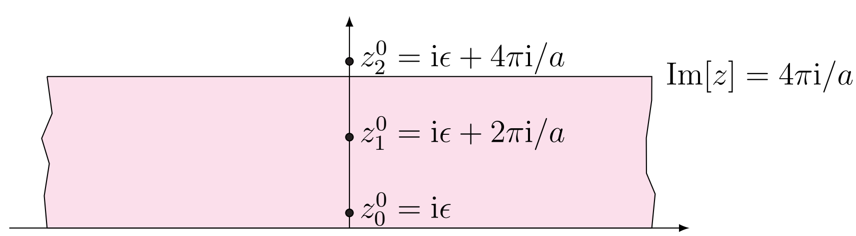


Figure 8.2: Complex poles of the undeformed Wightman function.

From all these poles, we just need the two first ones, namely $k = 0$ and $k = 1$ (the remaining ones are outside the horizontal strip $S \subset \mathbb{C}$; see Fig. 8.2). The corresponding residues are

$$\begin{aligned} \mathrm{Res}[f_{0,\sigma}(z), z_0^0] &= \frac{\mathrm{i}e^{\epsilon\Omega+\epsilon^2/\sigma^2}(\Omega + 2\epsilon/\sigma^2)}{4\pi^2}, \\ \mathrm{Res}[f_{0,\sigma}(z), z_1^0] &= e^{2\pi\Omega/a}\frac{\mathrm{i}e^{\epsilon\Omega+\epsilon^2/\sigma^2+4\pi\epsilon/a\sigma^2+4\pi^2/a^2\sigma^2}(\Omega + 2\epsilon/\sigma^2 + 4\pi/a\sigma^2)}{4\pi^2}. \end{aligned} \tag{8.31}$$

Application of equation (8.28) leads then to

$$\frac{1}{\sigma}\mathcal{F}_0(\Omega, \chi_\sigma) = -\frac{\Omega}{4\pi}\frac{1 + e^{2\pi\Omega/a}}{1 - e^{4\pi\Omega/a}} = \frac{\Omega}{4\pi}\frac{1}{e^{2\pi\Omega/a} - 1}, \tag{8.32}$$

which is the usual expression for $\frac{1}{\sigma}\mathcal{F}_0(\Omega, \chi_\sigma)$. This expression satisfies

$$\frac{1}{\sigma}\mathcal{F}_0(-\Omega, \chi_\sigma) = e^{2\pi\Omega/a}\frac{1}{\sigma}\mathcal{F}_0(\Omega, \chi_\sigma) \tag{8.33}$$

for large σ. In physical terms, this implies that the quotient between the probabilities of excitation and de-excitation of the Unruh-DeWitt detector satisfies the detailed balance condition.

8.4.2 Lorentz-invariant deformations

For deformations that are invariant under Lorentz transformations, equation (8.28) simplifies further. In these cases, the pull-back of the Wightman function to uniformly accelerated trajectories is a function of the variable z only. The necessary condition B of integrability with respect to w is therefore trivially satisfied. The integral in equation (8.28) can be directly evaluated, leading to

$$\frac{1}{\sigma}\mathcal{F}_\ell(\Omega, \chi_\sigma) = \frac{\pi\mathrm{i}}{1 - e^{4\pi\Omega/a}}\sum_{k\in I}\mathrm{Res}[f_\ell(z), z_k], \tag{8.34}$$

where

$$f_\ell(z) = W_\ell(z)e^{-\mathrm{i}\Omega z}. \tag{8.35}$$

Let us consider several examples that illustrate different aspects:

(a) *Splitting the poles along the real axis:* The deformation $D_\ell(\Delta t, \Delta \boldsymbol{x}) = -\ell^2/((\Delta \mathsf{x})^2 + \ell^2)$ leads to the Wightman function

$$W_\ell(z) = -\frac{1}{4\pi^2}\frac{a^2}{4\sinh^2\left[a\left(z - \mathrm{i}\epsilon\right)/2\right] - (\ell a)^2}. \tag{8.36}$$

This is one of the minimal natural options for the introduction of the length scale ℓ [124, 117]. This deformation splits the poles and displaces each of them along the real axis (see Fig. 8.3), so that each second-order pole is split into two first-order poles. This deformation satisfies the sufficient conditions for the preservation of the Unruh effect. An explicit calculation of the response function, using equation (8.28), shows that

$$\frac{1}{\sigma}\mathcal{F}_\ell(\Omega, \chi_\sigma) = \frac{\sin\left[2\Omega\,\mathrm{argsinh}(\ell a/2)/a\right]}{4\pi\ell\sqrt{1 + (\ell a/2)^2}}\frac{1}{e^{2\pi\Omega/a} - 1}. \tag{8.37}$$

It is then straightforward to show that equation (8.32) is recovered at leading order for $\ell a \ll 1$ and $\ell\Omega \ll 1$, in agreement with our previous general discussion, so that the Unruh effect is preserved.

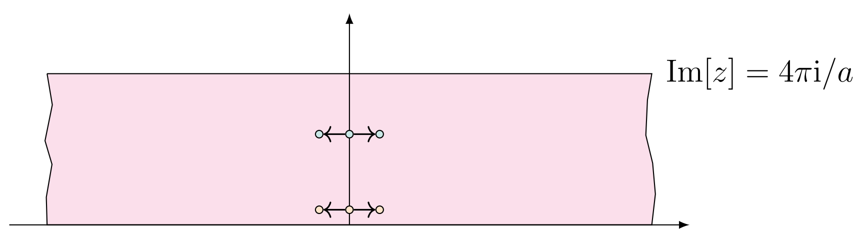


Figure 8.3: Displacement of the poles for the deformation in equation (8.36).

(b) *Adding new poles:* The more general deformation $D_\ell(\Delta t, \Delta \boldsymbol{x}) = -\beta\ell^2/(\Delta \mathsf{x}^2 + \ell^2)$ creates new poles when $\beta \in \mathbb{R} \setminus \{1\}$. These deformations can arise from, e.g. considering a conformally coupled detector in AdS [125]. The sufficient conditions are still satisfied, and the response function is given by

$$\frac{1}{\sigma}\mathcal{F}_\ell(\Omega, \chi_\sigma) = \frac{\Omega}{4\pi}\frac{1-\beta}{e^{2\pi\Omega/a} - 1} + \beta\frac{\sin[2\Omega\mathrm{argsinh}(\ell a/2)/a]}{4\pi\ell\sqrt{1 + (\ell a/2)^2}}\frac{1}{e^{2\pi\Omega/a} - 1}. \tag{8.38}$$

This expression leads to equation (8.32) in the $\ell \to 0$ limit, as expected. As in the previous case, the Unruh effect is preserved.

(c) *Splitting the poles along the imaginary axis:* A seemingly innocent change of sign $D_\ell(\Delta t, \Delta\boldsymbol{x}) = -\ell^2/(\Delta \mathsf{x}^2 - \ell^2)$ leads however to completely different physics (see [114, 117] for a complementary discussion). This deformation splits the poles along the imaginary axis, and therefore fails to satisfy the necessary condition C. An explicit calculation shows that, for $\ell a < 2$,

$$\frac{1}{\sigma}\mathcal{F}_\ell(\Omega, \chi_\sigma) = \frac{\sinh\left[2\Omega \arcsin(\ell a/2)/a\right]}{4\pi\ell\sqrt{1-(\ell a/2)^2}} \frac{1}{e^{2\pi\Omega/a}-1} + \frac{e^{-2\Omega \arcsin(\ell a/2)/a}}{8\pi\ell\sqrt{1-(\ell a/2)^2}}. \quad (8.39)$$

It is straightforward to check that this response function does not reduce to equation (8.32) for $\ell a \ll 1$ and $\ell\Omega \ll 1$, which can be anticipated graphically (Fig. 8.4). As a consequence, the Unruh effect is not preserved in this case.

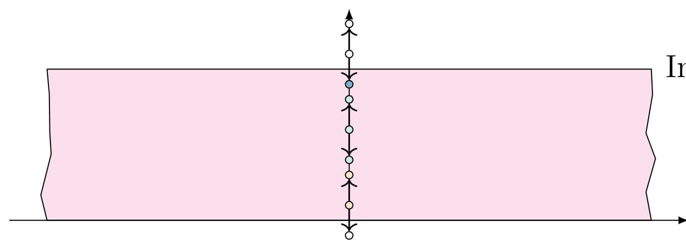


Figure 8.4: Displacement of the poles for the deformation that is obtained by changing the sign in front of ℓ^2 in equation (8.36).

(d) *Non-polynomial decay:* It is interesting to mention a deformation that does not satisfy the condition 4 given above, for instance $D_\ell(\Delta t, \Delta\boldsymbol{x}) = -e^{-\Delta \mathsf{x}^2/\ell^2}/\Delta \mathsf{x}^2$ (see [116, 126] for a motivation of this deformation). The decay of this deformation in $|\Delta \mathsf{x}|$ is not always polynomial (take, for instance, $\mathrm{Im}[z] = \mathrm{i}\pi/a$). As condition 4 above is not satisfied, equation (8.28) does not apply, and this example cannot be included in our analysis.

8.4.3 Lorentz-violating deformations

In the same spirit of the previous section, let us consider several distinct examples:

(e) *Changing the order of the poles:* The first example is given by $D_\ell(\Delta t, \Delta\boldsymbol{x}) =$

$\ell^2/\Delta\mathsf{x}^2\Delta t^2$, which leads to

$$W_\ell(w,z) = -\frac{1}{4\pi^2}\frac{a^2}{4\sinh^2[a(z-\mathrm{i}\epsilon)/2]}\left\{1+\frac{(\ell a)^2}{4\cosh^2(aw/2)\sinh^2[a(z-\mathrm{i}\epsilon)/2]}\right\}. \tag{8.40}$$

In this case the poles in z are at the same location, but these are now of fourth order. As a consequence, it can be shown by direct calculation that the sufficient conditions are met (note that the only relevant conditions are A and B, as C is trivially satisfied). Using equation (8.28), it follows that the response function in the adiabatic limit reduces to equation (8.32) identically (even for $\ell \neq 0$). The Unruh effect is therefore preserved.

(f) *Adding new poles:* This has been discussed in the Lorentz-invariant case, but deformations such as $D_\ell(\Delta t, \Delta\boldsymbol{x}) = \ell/\Delta\mathsf{x}^2(\Delta t - \ell)$ also add new poles, the position of which is not Lorentz invariant. It can be shown that the necessary condition A is not satisfied by this deformation (in particular, around the pole $\bar{z}_1 = \mathrm{i}\epsilon + 2\pi\mathrm{i}/a$), so that the Unruh effect is not preserved.

(g) *Adding poles in w:* This kind of behavior is associated with inverse powers of $\Delta\boldsymbol{x}$ occurring in the deformed Wightman function, such as $D_\ell(\Delta t, \Delta\boldsymbol{x}) = \ell^2/\Delta\mathsf{x}^2\Delta\boldsymbol{x}^2$. For this particular case, one has

$$W_\ell(w,z) = -\frac{1}{4\pi^2}\frac{a^2}{4\sinh^2[a(z-\mathrm{i}\epsilon)/2]}\left\{1+\frac{(\ell a)^2}{4\sinh^2[a(w\pm\mathrm{i}\epsilon)/2]\sinh^2[a(z-\mathrm{i}\epsilon)/2]}\right\}. \tag{8.41}$$

For both signs of the regulator in the integral over the variable w, the additional piece vanishes due to the fact that the residues on the poles on w vanish identically, so that

$$\frac{1}{\sigma}\int_{-\infty}^{\infty}\frac{\mathrm{d}w\, e^{-w^2/\sigma^2}}{\sinh^2[a(w\pm\mathrm{i}\epsilon)/2]} \leq \frac{1}{\sigma}\int_{-\infty}^{\infty}\frac{\mathrm{d}w}{\sinh^2[a(w\pm\mathrm{i}\epsilon)/2]} = 0. \tag{8.42}$$

This deformation satisfies the three sufficient conditions for the preservation of the Unruh effect.

(h) *Non-integrability in w:* We have discussed examples that violate conditions A and C. Let us now consider an example that does not satisfy the condition B above, for instance, $D_\ell(\Delta t, \Delta\boldsymbol{x}) = \mathrm{i}\ell\Delta t/\Delta\mathsf{x}^2$. The imaginary unit i in front of Δt implies that this deformation is invariant under time reversal, which is an anti-unitary transformation. In this case,

$$W_\ell(w,z) = -\frac{1}{4\pi^2}\frac{a^2}{4\sinh\left[a\left(z-\mathrm{i}\epsilon\right)/2\right]^2}\left\{1-\frac{\mathrm{i}\ell a}{2}\frac{\cosh(aw/2)}{\sinh\left[a\left(z-\mathrm{i}\epsilon\right)/2\right]}\right\}. \quad (8.43)$$

It is straightforward to show that the sum of the residues of the poles in z is not integrable with respect to w. Hence, the Unruh effect is not preserved in this case.

Note that a similar expression has appeared in recent explorations of polymer quantization [121]. However, it is crucial to keep in mind that the equation above is valid for arbitrary values of w and z, which in general is a requirement in order to be able to determine unambiguously whether or not the Unruh effect is preserved. The expressions obtained in [121] are valid only for restricted values of w, and therefore the question of preservation of the Unruh effect in that particular scenario is still open.

(i) Violating the imaginary periodicity: In the next section just below, we discuss the relation between the preservation of the Unruh effect and the KMS condition, which involves the imaginary periodicity of the Wightman function. The example above violates both condition B for the preservation of the Unruh effect and this imaginary periodicity (which will be defined in precise terms later). In order to break the degeneracy of the example above, and clarify the role of these two conditions, it is convenient to provide an example that only violates the imaginary periodicity. This is given for instance by $D_\ell(\Delta t, \Delta \boldsymbol{x}) = \ell/\mathrm{i}\Delta t$, which satisfies all the conditions for the preservation of the Unruh effect.

8.4.4 Regarding the KMS condition

These are all the ingredients that are needed in order to compare the KMS condition with the definition for the preservation of the Unruh effect given in this paper. This is summarized in Table 8.1 below.

Example	KMS				Preservation
	Imaginary periodicity	Stationarity	Holomorphicity	Polynomial	
a	✓	✓	✓	✓	✓
b	✓	✓	✓	✓	✓
c	✓	✓	✗	✓	✗
d	✓	✓	✓	✗	?
e	✓	✗	✓	✓	✓
f	✗	✗	✓	✓	✗
g	✓	✗	✓	✓	✓
h	✗	✗	✓	✗	✗
i	✗	✗	✓	✓	✓

Table 8.1: Comparison of the KMS condition and the sufficient conditions for the preservation of the Unruh effect in the adiabatic limit. Note that "Imaginary periodicity" refers to the property of the Wightman function, not its pull-back

.

Whether or not the Unruh effect can be sensitive to the short-distance structure of quantum field theory is a question that has been asked recently in several scenarios, typically associated with quantum gravity. However, a strict determination of the conditions that guarantee that the Unruh effect is preserved (or is, otherwise, washed away) in deformations of quantum field theory was lacking. In [4], we gave a detailed justification and description of the conditions that guarantee that the adiabatic response function of uniformly accelerated detectors reduces to the standard Lorentz-invariant result in the limit in which the deformation scale ℓ vanishes. We have shown that these can be formulated in terms of the analytic structure of the deformed Wightman functions and their asymptotic behavior. We have kept the discussion fairly general, in order to encompass the examples considered previously in the literature and to ensure the broad applicability of our results.

Moreover, we analyzed the interplay between the preservation of the Unruh effect and the more traditional KMS condition. We have illustrated that the latter is more restrictive, as it is a sufficient, but not necessary, condition to ensure that the response function displays a thermal behavior. In order to illustrate that the latter approach is not adequate, we have provided explicit examples in which the KMS condition is violated in different ways, with the response function in the adiabatic limit still displaying a thermal behavior. This provides further insight on the meaning of the KMS condition and its relation to the Unruh effect.

Chapter 9

Conclusions

Detectors are a useful conceptual framework that allows us to interpret some mathematical objects in QFT, and discuss some facts derived from the formalism in a physically motivated fashion. From the purely interpretational side, the absence of minimal projectors in the local algebras in QFT may be seen as problematic, as discussed by Ruetsche and Earman in [11]. This can be seen as a consequence of the universal divergence of the entanglement entropy, or equivalently, from the fact that the local algebras are type III Von Neumann algebras. The fact that states cannot be disentangled with a finite energy cost prevents the existence of states holding properties locally, in the sense that there will be no state vectors that can be written as a combination of operators belonging to local regions.

This is in stark contrast to non relativistic quantum mechanics where entanglement is a resource usually thought of as the exception to the norm. Therefore, the intuitive notion of detector, which is an object that is localized and with well-defined properties, e.g. with a ground state that the detector transitions away as a result of the interaction with the field, seems to be doomed to fail in a purely relativistic QFT scenario, and can be only reached through an approximation. The lack of minimal projectors in the local algebras and the intepretational challenge this supposes have been discussed in [15], and we are not in a position to say that non-relativistic detectors models offer the only alternative to tackle this issue, although they definitely offer one.

When resorting to local, non-relativistic probes that couple locally to the quantum field, seeking to have well-defined properties of an (arguably) local object, one faces the dichotomy of singularity v.s. non-locality of the detector. Namely, one has to choose between interactions that are singular, because the detector interacts with the field on a world line and interactions that are nonlocal because the detector undergoes nonrelativistic dynamics over an extended region of spacetime. We addressed the first point in chapter 4, in which we dealt with the UV divergences of the UDW model in the case of the pointlike model with delta-like interactions. The second point was fully addressed in chapter 5.

One can still take the approach of modelling the detector with other quantum fields, as done e.g. by Fewster and Verch in [29]. This approach, however, seems to become redundant regarding the interpretation of the local operators, as far as they restrict to local measurements of the probe. Of course, their approach has many other qualities that make it desirable. For instance, they give a dynamical interpretation to a whole class of operations that can be performed on a quantum field without violating causality.

When resorting to field-theoretic probes, one does not need to restrict the analysis to local operations on the probe, so one can actually enjoy the interpretational advantages of the projectors acting over the probe's global algebra, e.g the eigenstates of the number operator. One may very well be interested in interpreting local quantities of the system field in terms of global properties of the probe field given by one-rank projectors; this is legitimate. However, although in some cases the actual experimental setting may resemble this situation, e.g. in particle physics where one claims to have found a photon with a certain momentum $\boldsymbol{k}$, it does not account for all types of experimental situations involving, say, atomic transitions, which cannot be described within QFT.

These considerations lead to a rich and interesting debate, because they are related to separation of scales and universality considerations. Indeed, according to our ordinary experience, all detectors should be point-like when probing long enough wavelenghts. More concretely, the pointlike model should be able to reproduce model-independent results depending on the energy scale of interest, for instance the energy gap. If one is looking at energies such that the wavelengths involved are comparable to the interaction region, one will have to specify the spacial structure of the detector. It is true that the set-up will have to resort to a QFT description eventually, but it is an open question how to transition from a fully relativistic description of nature to, say, the type of protocols appearing in quantum information processing. The field description can thus become very cumbersome. As the energy scale increases, it may not even make sense to distinguish a system-field and a probe-field, and the very notion of measurement (in the Von Neumann measurement sense, in which one requires system and probe to statr uncorrelated) may not be available.

With respect to the Unruh effect, the first methods exposed at the beginning of

6, namely Bogoliubov transformations and modular theory, depart from each other in technical details and also some conceptual gaps. However, they share the notion that an accelerated observer is associated with a set of operations constrained to the Rindler wedge, and that this observer's evolution is represented in the Hilbert space through Lorentz boosts. Also, they stress that the Unruh effect has its origin in the correlations exhibited by Minkowki's vacuum between the Rindler wedge and its causal complement.

The particle detector approach departs from these ideas, as it puts all interpretational weight on the detector's physics. Indeed, particle detector models were first introduced to shed some light over the problematic concept of particle as revealed by the Unruh effect. Indeed, particles are not only observer-dependent, but different observers cannot compare their notions of particle due to unitary inequivalence. Beyond the claim, which may seem circular to some, it makes sense to turn the burden of the conclusion on a detector, thereby switching from claiming that "Minkowski's vacuum is experienced as a thermal bath by accelerated observers" to "an accelerated detector interacting with a quantum field in its vacuum state responds in the same way as in a thermal reservoir".

Regarding the more mathematical analysis of the Unruh effect, which also avoids the notion of particle, it becomes unclear what the elements of the local algebra represent. Indeed, the statistics of a particle detector have a clear interpretation in terms of transition probabilities. Beyond interpretational problems, perhaps the main issue of the approach that uses modular theory is its lack of robustness. One would like to define the Unruh effect in a way that does not depend drastically on the KMS condition, because this one cannot be tested in any realistic experiment. The only way to test the KMS condition, actually, is through the detailed balance condition, which involves the spectral analysis of the all the correlation functions of the field.

The type of observables that one may be able to test are, first, finite in space and time, and second they cannot be arbitrarily complex. This means that the detailed balance condition, which involves the correlation functions at all times, and for all observables, becomes untestable on it own. Moreover, one could imagine situations in which one does not expect the KMS condition to happen, but such that the scales involved are so that the relevant physical quantities are indistinguishable from the ones associated with an accelerated observer. We addressed this point in chapter 7, where we wondered whether a detector sensing the Anti-Unruh effect was actually probing some sort of time-dependent Unruh effect.

Generally speaking, detectors provide a very solid ground to discuss questions of robustness in the context of the Unruh effect. For instance, one can have an experiment setting in an optical cavity, in which an accelerated detector travels within a small distance compared to the size of the cavity. Since the cavity is not Lorentz invariant, the vacuum fluctuations associated with the field in the cavity cannot be stationary with respect to the the evolution associated with accelerated observers, and hereby the state cannot be KMS. Still, one would expect the detector to react to the cavity's vacuum in the same way it would to free space's vacuum.

Further, one would expect the Unruh effect to be preserved in situations in which the state is not globally KMS, but it is asymptotically for long times. An example of this would be a coherent state that is localized in some region of space. If one has an accelerating detector, one can conclude that as the detector's speed increases the state of the field will red-shift and blue-shift, and all non Lorentz invariant contributions will eventually be out of the band of frequencies that the device can probe. Therefore, the detector will eventually interact with this non-KMS state as if it was Minkowski's vacuum. Another example would be a detector that accelerates away from a reflecting mirror, in this case, even though the vacuum is not KMS with respect to Lorentz boosts, one would expect the detector to be insensitive to the presence of the mirror as it moves away. We investigated this problem in chapter 8, where we studied the effect of UV deformations in the Unruh effect.

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