Investigation of alpha decay in ^{146}Sm, ^{150}Gd, ^{154}Dy and ^{156}Er nuclei using two-potential approach and modified Wood Saxon nuclear potential

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Introduction

Gurvitz [1] proposed a two-potential approach (TPA) to address the decay of quasistationary states. Currently, the TPA has been extended to study α decay, cluster decay, and proton radioactivity using various nuclear potentials, including cosh-type, Skyrme-Hartree-Fock, folding potentials, and the Wood-Saxon potential. N. Wang et al. [2] proposed a modified Woods-Saxon potential (MWS) model based on the Skyrme energy density functional, together with the extended Thomas-Fermi approach. In the modified Wood-Saxon nuclear potential, fusion barriers are determined within the framework of the Skyrme energy density functional, combined with the semi-classical extended Thomas–Fermi method. The potential barriers calculated using this modified nuclear potential are similar to those calculated using phenomenological models that employ proximity potentials. The Unified Fission Model with the Modified Woods-Saxon nuclear potential (UFMMWS) has successfully explained and predicted the half-lives of various decay processes, including cluster decay, alpha decay, and proton decay. In the present study, we investigated the alpha decay of the ^{146}Sm, ^{150}Gd, ^{154}Dy, and ^{156}Er nuclei using the two-potential approach and the modified Wood-Saxon nuclear potential [2].

The model

The total interaction potential is the sum of the coulomb potential, the angular potential, and the nuclear potential

$$V_{(r)} = V_C(r) + V_l(r) + V_N(r) \qquad (1)$$

The coulomb potential is determined by treating the nucleus as a uniformly charged sphere and is calculated as

$$V_C(\mathrm{r}) = \frac{2Z_d e^2}{2R_0}\left[3 - \frac{r^2}{{R_0}^2}\right] \qquad \text{for } r \le R_0 \quad (2)$$

$$V_C(\mathrm{r}) = \frac{2Z_d e^2}{2r} \qquad \text{for } r > R_0 \quad (3)$$

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Where the atomic number Z_d corresponds to the daughter nucleus, and A is the mass number of the parent. The Woods–Saxon type [2] potential is utilized to calculate the nuclear potential in the present model. The Woods-Saxon nuclear potential is calculated as

$$V_N(r) = \frac{V_0}{1+exp[(r-R_0)/a]} \qquad (4)$$

where the depth of potential (V_0) and diffuseness parameter (a) are computed as

$$V_0 = -44.16[1 - 0.40(\mathrm{I}_D + \mathrm{I}_\alpha)\frac{A_D^{1/3}A_\alpha^{1/3}}{A_D^{1/3}+A_\alpha^{1/3}} \qquad (5)$$

$$\mathrm{a} = 0.5 + 0.33 \cdot \mathrm{I}_D \qquad (6)$$

$$I_i = \frac{N_i - Z_i}{A_i} \quad (i = \alpha, D) \qquad (7)$$

In eq. (5) are the isospin asymmetries of projectile and target nuclei, respectively. R_0 is calculated as

$$R_0 = R_D + R_\alpha - 1.06 \qquad (8)$$

The nuclear charge radii R_i (i=D and α) using the formula R_i = $1.2A^{1/3}$ where i=D and α denote the daughter and alpha cluster nuclei, respectively. The centrifugal potential is determined using the Langer-modified centrifugal barrier and it is calculated as

$$V_l(r) = \frac{\hbar^2(l+\frac{1}{2})^2}{2\,\mu\, r^2} \qquad (9)$$

Where l and μ represent the value of the alpha fragment's angular momentum and the reduced mass, respectively. The decay constant depends on the preformation probability, the penetration probability, and the normalized factor. It can be expressed as

$$\lambda = \frac{\mathrm{P}_\alpha \mathrm{FP}}{\mathrm{h}} \qquad (10)$$

the hindrance factor h can be estimated by the simple formula with five parameters proposed by Zhang et al.[3] The hindrance factor is calculated as,

$$\log_{10}\mathrm{h} = \mathrm{a}+\mathrm{b}(Z-Z_1)(Z_2 - Z)+\mathrm{c}(N- N_1)(N_2 -N)+\mathrm{d}A + \mathrm{e}(Z -Z_1)(N-N_1) \qquad (11)$$

where Z, N, and A are the proton, neutron, and mass numbers of the parent nucleus. Z_1 and Z_2 (N_1 and N_2) are the proton(neutron) magic numbers around Z (N).

$$F = \frac{1}{\int_{r_1}^{r_2} \frac{1}{2k(r)} dr} \quad (12)$$

The penetration probability P is calculated using the WKB approximation.

$$P = \exp\{-2\int_{r_2}^{r_3} k(r)\mathrm{dr} \quad (13)$$

The conditions V (r) = Q are used to find the classical turning points r_1, r_2, and r_3. The wave number of the alpha decay can be calculated as.

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}[Q - V(r)]} \quad (14)$$

Where $\mu = M_\alpha M_D/(M_\alpha + M_D)$ is the reduced mass with M_α and M_D being the mass of the alpha particle and the residual daughter nucleus, respectively, and Q is the decay energy. The calculation of the half-life of alpha radioactivity is determined using the following equation.

$$T_{1/2} = \frac{ln2}{\lambda} \quad (17)$$

Result, Discussions and Conclusion

The total interaction potential is the combined contributions from the Coulomb, centrifugal, and nuclear potentials. In Fig. 1 we studied the nature of interaction potential for the alpha decay from the ^{156}Er nucleus as a function of the internuclear separation distance (r). We identified three coinciding points corresponding to classical turning points where the total potential equals the experimentally observed decay energy of 3.483 MeV. The parent and daughter nuclei are assumed to be perfect spheres without deformation in spherical cases.

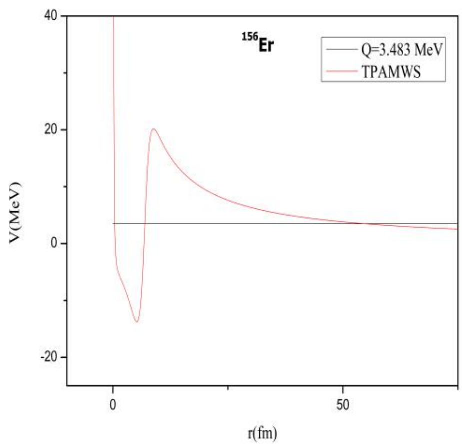

Fig. 1 The variation of the interaction potential of ^{156}Er nucleus computed using TPAMWS with the internuclear separation distance r**.**

The atomic number and neutron number for the selected parent nuclei in this study are in the ranges 50 ≤ Z ≤ 82 and 82 ≤N ≤126. The adjustable parameters are selected as a=-0.265, b=-0.0009, c=-0.0016, d=0.0035, and e=0.0008 [3]. The calculated hindrance factors are given in the third column of Table 1. The α-particle preformation probabilities were assumed to be constant. In the present work, the preformation factor P_0 is taken as 0.43.

In Table 1, the predicted decay half-lives of ^{146}Sm, ^{150}Gd, ^{154}Dy and ^{156}Er using the present model are close to the experimental values. The potential barriers calculated using the modified Wood-Saxon nuclear potential are similar to those computed using phenomenological models that utilize proximity potentials. These potential barriers depend on the isospin effect of the daughter nucleus and alpha particle, as well as the mass number of the daughter nucleus and the alpha particle. Studying alpha decay using the TPA allows for a simple formula to calculate the decay constant. This combination of TPA and MWS improves the accuracy of half-life predictions. Therefore, we hope that this model will also be useful for studying alpha decay in the future.

Table 1: Comparison of the experimentally observed and computed logarithmic half-lives of alpha decay.

Nuclei	Q (MeV)	h	$\log_{10}T_{1/2}$(s)	
			(Exp.)[4]	(Cal.)
^{146}Sm	2.528	0.822	2.20×10^{15}	1.70×10^{15}
^{150}Gd	2.808	0.664	5.60×10^{13}	1.72×10^{13}
^{154}Dy	2.945	0.57	9.50×10^{13}	7.03×10^{13}
^{156}Er	3.483	0.597	6.70×10^{9}	1.60×10^{9}

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