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A nonperturbative approach to Hawking radiation and black hole quantum hair

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Abstract

We present a nonperturbative derivation of the subleading order in Hawking radiation based on diffeomorphism symmetry breaking during black hole evaporation. The diffeomorphism group of horizon admits a nontrivial phase factor which encodes information about infalling matter during formation. This nonintegrable phase represents the black hole quantum hair as it arises from the diffeomorphisms that change the physical state of the black hole. During evaporation, the decrease in total area breaks the diffeomorphism symmetry and leads to a dynamical shift in that phase factor. This shift affects the usual Hawking spectrum via dispersion relation and results in the subleading term in Hawking radiation. The higher order terms are locally insensitive to the Unruh radiation due to the lack of diffeomorphism groups on the local Rindler horizon at the low energy scale. This explains the generic difference between Hawking radiation and Unruh radiation. In addition, this phase shift indicates the decrease of the total number of degrees of freedom in horizon phase space during evaporation as past Page time. This enables us to escape from the firewall paradox and provide an account for the resolution to the information paradox.

Keywords: black hole, diffeomorphism symmetry, Hawking radiation, information paradox, quantum hair, nonperturbative effect

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1. Introduction

Hawking radiation is one of the most striking effects of the combination of quantum mechanics and general relativity [1]. There are several derivations of Hawking radiation based on different mechanisms including path integral approach [2], quantum tunnelling [3], splitting of entangled modes as the horizon forms [4, 5]; tidal forces pulling apart virtual pairs [6, 7]; the effects of non-stationarity of the background metric field [8]; and quantum anomaly [9, 10]. The universality of Hawking radiation has also been discussed in [11]. Based on these studies, Hawking radiation is a very plausible effect despite the lack of experimental evidence.

Nonetheless, the information paradox [12] (for a recent review see [13].), challenges the above derivations as none of them takes information recovery into account. Also, the firewall paradox [14] suggests a nonperturbative effect when a black hole is in a maximally entangled state. This raises the attention to the diffeomorphism of horizon [15–26], the derivation of Page curve [27–32], and the asymptotic structure [33].

Previous studies focused on either the information problem or the universality of Hawking radiation, but no previous studies covered both aspects. As the subleading order may play a role in the information problem [34], the universality of Hawking radiation is tied to the information problem. Therefore, a satisfactory derivation of Hawking radiation must contain the subleading order and offer an account for the information problem. In this paper we will fill this gap.

The purpose of this paper is to show that diffeomorphism symmetry breaking, as a nonperturbative effect during the dynamical process, could be naturally introduced to derive the subleading order of Hawking radiation. We consider Schwarzschild black hole to investigate the diffeomorphism group of the horizon in dynamical geometry. We do not appeal to any particular theory of quantum gravity and asymptotic symmetry. The idea is that there exists a nontrivial relation between quasi-normal modes on the horizon during evaporation. This relation is expressed in terms of a nonintegrable phase factor arising from the diffeomorphism group of the horizon as a global gauge transformation on the horizon. This phase factor represents the black hole quantum hair as it encodes quantum information about the infalling matter during formation. Contrast to the result from the previous studies in diffeomorphism of horizon [16, 24, 25, 35, 36] which shows that diffeomorphism of horizon is associated with soft charges in non-dynamical process, we found that in dynamical process due to diffeomorphism symmetry breaking, this phase factor nontrivially contributes to the sub-leading term of Hawking radiation and leads to the information recovery. This novel derivation of Hawking radiation enables us to escape from the information paradox and firewall paradox.

This paper is organised as follows. In section 2 we establish the relation between diffeomorphism group of horizon and black hole hair. Then in section 3, the sub-leading order of Hawking radiation from diffeomorphism symmetry breaking. In section 4 we argue that this nonperturbative effect evades the information paradox and firewall paradox. We conclude in section 5. Throughout this paper, we set $\hbar = c = G = k = 1$ and follow the convention of $(-,+,+,+)$, unless otherwise stated.

2. Diffeomorphism group of horizon and black hole quantum hair

To start with, we review that the local quantum field theory misses to take quantum hair into account in. Hawking's original derivation. For two quasi-normal modes b_i and b_j on the horizon, we have the commutation relation $[b_i, b_j] = 0$. This means that there are no correlations between the two modes. This assumption is made in Hawking's original argument, see page 2467 in [12]. However, in the dynamical process, this is simply not the case because the emission of the ith mode changes the thermal probability of the emission of the jth mode by reducing the total mass of the black hole. Hence, the temporal ordering of emissions from different modes matters, and in general we have

$$|k_1 k_2\rangle = b_1^\dagger b_2^\dagger |0\rangle \neq b_2^\dagger b_1^\dagger |0\rangle = |k_2 k_1\rangle \tag{1}$$

where $|k_i k_j\rangle$ is the state in momentum space and $b_i^\dagger$, $b_j^\dagger$ are the corresponding creation operators. This means that there exists a nontrivial relation between the two modes. In general, this relation could be understood as a geometrical formulation in phase space (see for instance [37]), but it was not taken into account by Hawking's original argument of the information paradox.

To see how black hole quantum hair arises from this geometric relation, note first, that there is a diffeomorphism group of horizon, i.e. the diffeomorphism of 2-sphere (assuming spherically symmetric black holes.). In non-dynamical process, i.e. area-preserving diffeomorphism of 2-sphere, this is equivalent to the BMS group (either on the 2-sphere or at null infinity) [16, 35, 36]. The associated generators are soft charges. Those soft charges lead to the energy degeneracy as they carry zero energy and the leading order in Hawking radiation is perfectly thermal. The thermal vacuum is the Kruskal vacuum in which Hawking radiation could be derived from the conformal symmetry of quasi-normal modes near the horizon [17, 38, 39]. In dynamical process, i.e. with area non-preserving diffeomorphism, this gauge group on the 2-sphere is spontaneously broken due to the diffeomorphism symmetry breaking. We cannot identify this horizon symmetry as the asymptotic symmetry since there is no global conformal Killing vector on the shrinking horizon, i.e. no global conformal Killing vectors in Painleve metric. The vacuum is the Painleve vacuum that is not strictly thermal as its dynamics reflects gravitational back-reaction. The small division to the thermal spectrum arises from the conformal symmetry breaking of quasi-normal modes near the horizon, for relevant studies in conformal symmetry breaking and back-reaction, see [40, 41]. Only the leading order term of near-horizon modes that lead to thermal radiation is conformal invariant. The conformal symmetry breaking of the higher order terms leads to the small division from the usual thermal spectrum of Hawking radiation and those soft charges carry information out of the black hole. Therefore, we should adopt the dynamical framework to work with Painleve metric rather than Kruskal metric.

To work within the dynamical framework, we consider the Schwarzschild black hole and use fibre bundle theory to describe the diffeomorphism group of 2-sphere. This was first used to study the Dirac monopole in U(1) gauge theory [42]. Analogous to this, we adopt minimal coupling and use the integral formalism in gauge theory [43] to describe the gravitational flux across the horizon. In this formalism, the total flux across the sphere depends only on the gauge type (principal fibre bundle), i.e. how the gauge fields transform, and they are not tied to any specific gauge potential (connection on principal fibre bundle). However, due to general covariance, gravitational field also plays the role of coordinate. The gravitational gauge potential determines how gauge fields transform on the sphere as it serves as a coordinate system. As a result, the global gauge (principal coordinate bundle) and gauge potential (connection on principal bundle) are physically equivalent in the sense that given the principal coordinate bundle there exists a unique gauge type (principal fibre bundle) associated with a connection if and only if the existence of the gravitational gauge type with a connection uniquely determines the principal coordinate bundle. Hence, we do not separate the two roles of the gravitational gauge field. Therefore, we need not specify gauge fixing conditions since only the nonintegrable phase factor arising from the global gauge transformation is of interest.

In addition, the global formulation of gauge fields [43] automatically exhibits quantisation of the flux across the 2-sphere. The gauge transformation on the 2-sphere is well-defined if and only if the total flux across the 2-sphere is quantised as $g = N$ where g is the flux across the 2-sphere and N is the integer [42]. This quantisation condition results from the gauge transformation on the 2-sphere and thus serves as a natural origin of black hole quantum states. As the area of horizon is associated with the surface gravity, the change in area during evaporation is in line with the discrete spectrum. This quantisation condition perfectly explains the cutoff in area. What we actually quantise is not the coordinate, but the total flux across the surface, i.e. curvature. The cutoff originates from global gauge and we conjecture that this the relationship between the black hole microstates and macroscopic temperature roots in the diffeomorphism symmetry of horizon. A similar cutoff can also be found in the soft hair implants [16] and the investigation of Bekenstein–Hawking entropy from hidden conformal symmetry on the soft horizon of Kerr black hole [44]. From this aspect, the cutoff in the area law arises from the gauge symmetry on the 2-sphere. Therefore, the diffeomorphism group of horizon naturally represents the black hole quantum hair.

To see how this allows quantum information to be encoded in S^2, consider the following thought argument: a photon falls into a large Schwarzschild black hole, and if we neglect the small change in black hole mass, then Bekenstein–Hawking entropy is the same during the process, and diffeomorphism symmetry of the horizon is preserved. However, when the photon passes through the horizon, the entire horizon would feel the 'force' effect $F_{\mu\nu}$ corresponding to the gauge potential A_μ. The quantum degrees of freedom corresponding to two helicities of the photon are converted into black hole quantum hair and which, at a later time, affects Hawking radiation. In purely gravitational cases, black holes have intrinsic quantum degrees of freedom arising from gravitations. The corresponding effect is dynamically exhibited through the global gauge. Similar to soft graviton effect [45], one could imagine that a graviton with zero energy falls into a black hole, and the black hole would acquire its two intrinsic degrees of freedom. This suggests that a black hole could have quantum degrees of freedom beyond the domain of the three classical parameters: mass, charge, and angular momentum. The diffeomorphism group of horizon represents black hole quantum degrees of freedom. However, we do not appeal to soft horizon, anti-de Sitter/conformal field theory (AdS/CFT) duality, or other asymptotic effects as we assume diffeomorphism symmetry breaking and consequently there are no global conformal Killing vectors on dynamical horizon. This framework is general and open to any theory of quantum gravity. For instance, in loop gravity, there is no asymptotic region [46], but it is still possible to construct black hole quantum hair from the diffeomorphism of horizon. To remain neutral to quantum gravity theories, we do not assume a specific framework of quantum gravity.

In dynamical process, since the area of the horizon is not preserved, the diffeomorphism invariance of the 2-sphere is spontaneously broken. The nonintegrable phase factor is no longer ignorable. To see this, consider a Schwarzschild black hole with mass M and a massless scalar field $\psi(x^\mu)$. We can choose the evaporation to be a quasi-stationary process such that the metric field could be approximated by a sequence of time-independent spherically symmetric solutions. As a result, we have $\mathrm{M} = M_i$ where i denotes the ith mass in a sequence of masses during evaporation, and the diffeomorphism group of horizon is the gauge group of S^2. Therefore we can simply focus on the wavefunctional defined on S^2 i.e. $\psi = \psi(x^\mu), \forall x^\mu \in S^2$. To avoid singularities, we restrict our gauge transformation on $\theta = \frac{\pi}{2}$. We define the diffeomorphism as usual: $x^\mu \to x^\mu + \epsilon^\mu$, $\forall x^\mu \in (t = 0, r = 2M_i, \theta = \frac{\pi}{2}, \phi \in [0, 2\pi])$. The global gauge transformation is

$$\begin{aligned} \partial_\mu &\to \partial_\mu - iB_{\mu ab}\sigma^{ab} \\ \psi &\to \psi e^{i\chi} \\ B_{\mu ab}\sigma^{ab} &\to B_{\mu ab}\sigma^{ab} - i\partial_\mu\chi \\ f_{\mu\nu} &= i\left[D_\mu, D_\nu\right] \end{aligned} \tag{2}$$

where σ^{ab} stands for the usual Dirac matrices and $B_{\mu ab}$ is the spin connection in the vierbein formalism [47, 48], while $f_{\mu\nu}$ is its corresponding field strength, i.e. curvature. Note that in S^2 we have periodic boundary condition $\phi = 2\pi$ with a closed transported curve of S^1. So,

$$\psi \to e^{i\int_{S^1} B_{\mu ab}\sigma^{ab}\mathrm{d}x^\mu}\psi = e^{i\iint_{S^2} f_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu}\psi. \tag{3}$$

Hence, the scalar field acquires a nonintegrable phase factor related to the total gravitational flux across the surface. In the non-dynamical process, this phase factor is insensitive to observables since we could absorb it by refining the overall phase in phase space. Therefore, we could identify Bekenstein–Hawking entropy as Noether charge [49, 50]. Similar results could be found in [24, 25] where the soft charges on black hole horizon are found to acquire a nonintegrable factor arising from singular supertranslations. This nonintegrable factor commutes with all charges in the Dirac bracket algebra and trivially contributes to observables in non-dynamical process. In the dynamical process, however, since the radiation causes a decrease in the total area of S^2, the diffeomorphism symmetry (the gauge symmetry on S^2) is spontaneously broken. In such a case, there is a shift in χ due to the flux across S^2 (i.e. the change in curvature).

$$\delta\psi = e^{i\delta\chi}\psi. \tag{4}$$

Consequently, χ records the nonperturbative effect in the dynamical process, and the change in it modifies the usual wave equation $\Box\psi = 0 \to \Box\psi + F(\chi)\psi = 0$ where $F(\chi)$ is the function representing the nontrivial dispersion relation. As a result, χ becomes sensitive to observables since the total degrees of freedom in the phase space of the horizon is not conserved and thus we are unable to eliminate $\delta\chi$. This suggests that the quantum degrees of freedom, corresponding to the gauge group of the horizon, escape, via radiation, from the black hole during evaporation. As a result, the nonperturbative effect on the entire horizon is dynamically exhibited through the global gauge during evaporation.

3. The subleading order of Hawking radiation

We now turn to the demonstration of the derivation of the subleading order in Hawking radiation from diffeomorphism symmetry breaking during evaporation. First, the modified wave equation reads,

$$\Box\psi + F(\chi)\psi = 0 \tag{5}$$

where $F(\chi)$ is the function representing nontrivial dispersion relation from the phase factor shift. We could understand the horizon shrinking effect as the nonlinear dispersion relation from Lorentz symmetry breaking [11]. The shift represents the conversion of high-frequency modes to low-frequency modes as Hawking radiation during evaporation. The nonlinear dispersion term in the modified wave equation generally depends on the field species because gauge type determines the total flux across S^2 [42], in agreement with the argument that black hole entanglement entropy depends on particle species and could not be identified as

Bekenstein–Hawking entropy [51]. One might naively expect to solve the modified wave equation by directly expanding ψ. However, this is simply not the case since the dynamical change in χ results in the frequency shift in the time translation Killing vector ∂_t defined in terms of the initial mass due to the decrease of the horizon area.

$$\psi(\chi) = f_i(\chi)\, a_i + \tilde{f}_i(\chi)\, a_i^\dagger \tag{6}$$

where $\{f_i\}$ are the complete set of the orthogonal family of solutions of the wave equation. Hence, the frequencies depend on χ. To define the vacuum state to derive Hawking radiation, one needs a consistent definition of time. But in the dynamical process, the 'function of time' becomes dependent on the total mass and varies with the decrease of area $t = t(M_i)$. This causes the nonlinear shift in Kruskal time [52] and Painlevé time $t = t_s + 2\sqrt{2Mr} + 2M \ln \frac{\sqrt{r}-\sqrt{2M}}{\sqrt{r}+\sqrt{2M}}$ where t_s is the Schwarzschild time [53], and the corresponding time translation Killing vectors ∂_t breaking. Consequently, we cannot consistently define the positive frequency with respect to the above time coordinates. In other words, diffeomorphism symmetry (time translation symmetry) breaking leads to inconsistency in the definition of positive frequency and the vacuum state.

To resolve this problem, we need to choose a particular frame in which the time translation symmetry is preserved. We shall call this type of frame 'comoving' frame. To define comoving frames, we first note that in Painleve metric the local geometry seen by fiducial observers is

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + 2\sqrt{\frac{2M}{r}} + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \tag{7}$$

where $\sqrt{\frac{2M}{r}}$ is the velocity of local fiducial observers with respect to the Killing vector ∂_t in that stationary metric. With the equivalence principle, this could be interpreted as local Lorentz symmetry breaking. When expanding quasi-normal modes, the frequencies of quasi-normal modes are dynamically changing as the gauge potential depends on the total black hole mass, i.e. curvature, which causes the inconsistency in the definition of time. To see this, consider the modified wave equation,

$$\left(\partial_t + \partial_i v(r,M)\right)\left(\partial_t + v(r,M)\,\partial_i\right)\psi = \left(\partial_r^2 + F(\delta\chi)\right)\psi \tag{8}$$

where the corresponding dispersion relation is $(\omega + vk)^2 = k^2 - F$. When we expand ψ into stationary modes, the separation between positive frequency modes and negative frequency modes depends on the velocity factor resulting from the non-linear dispersion relation. Since the velocity factor is not time-independent, the separation between positive frequency modes and negative frequency modes is not consistent through black hole evaporation. Thus, we cannot consistently define the Painleve vacuum during evaporation. To resolve this, we simply require the flux seen by local fiducial observers to be constant such that the frequency change rates of quasi-normal modes on the shrinking horizon, relative to the Killing vector ∂_t is zero. This means that local fiducial observers are comoving at the constant velocity $v_{O'} = \sqrt{\frac{d(2M)}{\mathrm{d}r_{O'}}}$ (i.e. we require $\frac{d(2M)}{\mathrm{d}r_{O'}} = 1$.). The dynamical shift in frequency during evaporation is therefore compensated by the dynamics of the comoving frames. Thus, the function F is constant now. Because of this, the frequency shifts in ψ in k space are constant.

$$\psi = f_i a_i + \tilde{f}_i a_i^{\dagger} \rightarrow \psi' = f_i' a_i + \tilde{f}_i' a_i^{\dagger} \tag{9}$$

where $\{f_i\}$ are the complete sets of the orthogonal family of solutions of the wave equation and the unprime and prime denote the observers at rest and comoving observers respectively. We could redefine ψ as $\psi' = e^{i\sqrt{F}}$ by a phase shift factor in k space to obtain the usual wave equation.

$$\Box\psi' = 0 \tag{10}$$

The nonlinear term in the dispersion relation from diffeomorphism symmetry breaking is cancelled out by the dynamics of the choice of the frame as this shift arises from the backreaction of metric as a consequence of energy conservation. This enables us to derive Hawking radiation by directly expanding ψ'. However, we need to consider the constant frequency shift caused by the dynamics of comoving frames. Since the frequency shift is constant, the difference between the frequencies defined in comoving frames and rest frames is the Doppler factor of $\sqrt{\frac{1+v}{1-v}}$ in each polynomial when expanding ψ in the calculation of thermal probability. Hence,

$$\langle n_i' \rangle = \langle b_i' b_i' \dagger \rangle = \sqrt{\frac{1+v}{1-v}}^2 \langle b_i b_i^{\dagger} \rangle = \frac{1+v}{1-v} \langle n_i \rangle \tag{11}$$

where $\langle n_i \rangle$ is the expectation value for the hole to emit particles and $\langle b_i b_i^{\dagger} \rangle$ the corresponding operators with respect to distant observers at rest in usual black hole thermodynamics [1], while the prime denotes the comoving observer.

To calculate $\frac{1+v}{1-v}$, we rewrite v as $v = \frac{\mathrm{d}r_{O'}}{\mathrm{d}t_P} = \frac{\mathrm{d}r_{O'}}{d(2M)} \frac{d(2M)}{\mathrm{d}t_P} = 2\frac{\mathrm{d}M}{\mathrm{d}t_P}$ where we used the comoving condition of $\frac{\mathrm{d}r_{O'}}{d(2M)} = 1$ and $\frac{\mathrm{d}M}{\mathrm{d}t_P}$ is the radiation rate with respect to Painleve time. Note that the velocity of local freefalling frames reaches to 1, thus the Doppler factor is divergent. However, because outgoing particles of Hawking radiation are expected to undergo extreme redshifts to reach the asymptotic regions, this divergence is only apparent. Since the velocity is now expressed in terms of radiation rate and we are free to express the radiation rate in any coordinate system, we drop the subscript in the radiation rate $\frac{\mathrm{d}M(t_P)}{\mathrm{d}t_P} = \frac{\mathrm{d}M(t)}{\mathrm{d}t}$. We work out that factor within frames at distance in which the singularity vanishes. Viewed from the distant observers, the radiation rate is slow relative to the time translation Killling vector, we keep only the leading order of O(v) when expanding the shift factor and ignore the higher order in v, therefore we have $\frac{1+v}{1-v} = 1 + \frac{2v}{1-v} \sim 1 + 2v$. So, $\frac{1+v}{1-v} \sim 1 + 4\frac{\mathrm{d}M}{\mathrm{d}t}$. Therefore, the frequency shift is expressed in terms of the radiation rate. So, we have the emission relation $\frac{\mathrm{d}M}{\mathrm{d}t} = \alpha\frac{\omega}{M}$, where α is a numerical value depending on the species of the emitted particle [54] and ω is the energy of the emitted particle. Hence,

$$\langle n_i' \rangle = \left(1 + 4\alpha\frac{\omega}{M}\right) \langle n_i \rangle. \tag{12}$$

The expectation value of $\langle n_i \rangle$ is the Hawking spectrum viewed in the frame of the distant observer.

$$\langle n_i \rangle = \Gamma\left(e^{8\pi M\omega} - 1\right)^{-1} \tag{13}$$

where Γ is the greybody factor. Hence,

$$\begin{aligned}
\langle n_i' \rangle &= \left(1+4\alpha\frac{\omega}{M}\right)\Gamma\left(e^{8\pi M\omega}-1\right)^{-1} \\
&= \Gamma\left(1+\frac{32\pi\alpha\omega^2}{8\pi M\omega}\right)\left(e^{8\pi M\omega}-1\right)^{-1} \\
&= \Gamma\left(\frac{8\pi M\omega+32\pi\alpha\omega^2}{8\pi M\omega}\right)\left(\frac{8\pi M\omega-4\pi\omega^2}{8\pi M\omega-4\pi\omega^2}\right)\left(e^{8\pi M\omega}-1\right)^{-1} \\
&= \Gamma\left(\frac{8\pi M\omega+32\pi\alpha\omega^2}{8\pi M\omega-4\pi\omega^2}\right)\left(1-\frac{4\pi\omega^2}{8\pi M}\right)\left(e^{8\pi M\omega}-1\right)^{-1} \\
&\sim \Gamma\left(\frac{8\pi M\omega+32\pi\alpha\omega^2-4\pi\omega^2}{8\pi M\omega-4\pi\omega^2}\right)\left(e^{8\pi M\omega}-1\right)^{-1} \\
&\sim \Gamma\left(\frac{8\pi M\omega+32\pi\alpha\omega^2-4\pi\omega^2}{8\pi M\omega}\right)\left(e^{8\pi M\omega-4\pi\omega^2}-1\right)^{-1} \\
&= \Gamma\left(1+\frac{32\pi\alpha\omega^2-4\pi\omega^2}{8\pi M\omega}\right)\left(e^{8\pi M\omega-4\pi\omega^2}-1\right)^{-1}.
\end{aligned} \tag{14}$$

We keep only up to $O(\omega^2)$. For the fifth line, we leave out $O(\omega^4)$, while for the sixth line, we use $e^x \sim 1+x$ and again discard higher order terms. Setting $\alpha = 0.125$ approximately corresponding to the peak in thermal radiation spectrum for the cross-section of $27\pi M^2$ [54], we get

$$\langle n_i' \rangle = \Gamma\left(e^{8\pi M\omega-4\pi\omega^2}-1\right)^{-1}. \tag{15}$$

Remarkably, we obtain the subleading order of Hawking radiation in agreement with the approach of quantum tunnelling [3]. This shows that the dynamical effect leading to the subleading order of Hawking radiation is nonperturbative.

4. Quantum hair and information paradox

To see how the above derivation avoids implausible assumptions that lead to the information paradox, we first sharpen the definition of the information paradox. We focus on the firewall paradox [14] to avoid the issues around the final stage of evaporation, such as remnant [55]. Namely, we assume that the evaporation process follows the Page curve [56] and we encounter the firewall paradox after the Page time. In this aspect, one usually attempts to obtain the Page curve for the entire evaporation, such as [18, 27, 29, 30]. However, it has been pointed out that information recovery might not follow the Page curve as the assumptions in Hawking's original argument are inappropriate in quantum gravity [57, 58]. Page curve assumes the uniqueness of vacuum and thus information recovery follows the Page curve. But the vacuum in quantum gravity is not unique and we cannot consistently define black hole entanglement entropy during evaporation. Therefore, Page curve is not a general resolution to the information paradox as it assumes the uniqueness of vacuum. The main issue in this paradox is the violation of quantum monogamy (past Page time). The essential point is that we could formulate the same paradox without mentioning the black hole entanglement entropy as well as Page curve. To see this, recall that the original firewall argument [14] against complementarity [59] argues that a lately emitted (past Page time) Hawking quantum could be correlated with the black hole and an earlier emitted quantum at the same time. A full resolution must evade this aspect of the firewall argument. But the derivation of the Page curve is not necessary to resolve the quantum monogamy as it might not prevent the multi-correlations after the Page time. The violation of quantum monogamy arises from the correlation between the later Hawking quantum and the black hole. The nonperturbative effect, no matter what that is, must explain the information recovery and evade the late time correlations between Hawking quanta and the black hole.

Hence, our purpose is not to rely on the Page curve to resolve the paradox, but to explain the information recovery and avoid the violation of quantum monogamy. Therefore, here we will explain why the late time (past Page time) correlations between black hole and Hawking quanta are implausible. First, recall why local quantum field theory leads to the violation of quantum monogamy. The late time correlations between Hawking quanta and black hole arise from the locality of quasi-normal modes near the horizon. For two quasi-normal modes b_i and b_j near the horizon, we have the commutation relation $[b_i, b_j] = 0$. However, after Page time, this is simply not the case because the black hole is maximally entangled with the early emitted quanta,

$$|\Psi\rangle = |\psi_{\mathrm{BH}}, \psi_{\mathrm{radiation}}\rangle. \tag{16}$$

This means that the evaporation past Page time is nonlocal and it is unreasonable to expect the local field theory to hold. In other words, we should replace the local field theory with the non-perturbative effect to obtain the information recovery. To see how our approach can explain the information recovery, recall that the black hole microstates are quantised according to the total flux across the 2-sphere. This means that during evaporation the total degrees of freedom in the horizon phase space decreases during evaporation. Because of this, the late time correlations between the black hole and lately emitted quanta are impossible since they are incompatible with the decrease of number of the total degrees of freedom in horizon phase space. Also, since the black hole is maximally correlated with the earlier radiation, it is reasonable to expect the information transfer between the black hole and outgoing quanta. As a result, the derivation of Hawking radiation from the diffeomorphism symmetry breaking naturally takes the information recovery into account and avoids the violation of quantum monogamy.

From the S-matrix perspective, we need to take topological transition into account to obtain a unitary evolution. In the language of AdS boundary algebra, one could describe gravity in terms of asymptotic observables. Since $R^3 - \{0\}$ and S^2 are of the same homotopic type, the asymptotic observable in quantum gravity admits this phase factor. However, one cannot simply describe the black hole quantum hair from the boundary observables and directly obtain a unitary evolution. The reason is that the non-integrable phase is defined on the horizon, not the boundary. The same boundary could have different bulk contributions. For instance, the topology for a AdS Schwarzschild black hole is $S^2 \times D^2$ with the boundary topology of $S^1 \times S^2$ where D stands for disk. But the topology of $S^1 \times D^3$ has the same boundary. The global conserved charge corresponding to that non-integrable phase could not be described in terms of boundary observables as one cannot find a global time translation for the topological transition. Therefore, we need to consider all possible topologies during evaporation to prevent information loss [60]. For instance, for a large Schwarzschild black hole in AdS, the dominant contribution to path integral is initially $S^2 \times D^2$ including the quantum hair of non-integrable phase on S^2; while at the late stage of evaporation, the dominant topology is $S^1 \times D^3$ from the radiation. The black hole quantum degrees of freedom corresponding to the diffeomorphism group of the horizon are converted into quantum states possessed by Hawking quanta. To describe this process, we need to sum over all possible topological metrics. If one considers only asymptotic observables, the small corrections from soft charges is shown insufficient to resolve information paradox [61]. This is why we do not appeal to asymptotic symmetry and start with the 2-sphere. This verifies the conjecture that black hole quantum hair could not be spatially localised and we might appeal to diffeomorphism invariance to prevent information loss [18].

5. Concluding remarks

The subleading order of Hawking radiation arising from energy conservation seems to break this equivalence principle because the equivalence principle implies that the Unruh vacuum [62] is indistinguishable from Hartle-Hawking vacuum at a low-frequency limit. The difference between Hawking radiation and Unruh radiation is that there is no diffeomorphism group of 2-sphere on a local Rindler horizon from which Unruh radiation is derived. Therefore, the Unruh vacuum in flat spacetime is perfectly thermal since there is no quantum information restored on the local Rindler horizon, and local observers cannot feel the phase shift effect as they cannot, under the demand of the equivalence principle, distinguish whether the horizon is the event horizon or the local Rindler horizon. Note that the quantum tunnelling approach [3] is indeed nonlocal as the notion of black hole mass is a global notion, though the surface gravity could be locally formulated. The energy conservation results in the mass change in dynamical process is not a local process. The noncommutativity of near-horizon modes is locally not detectable, which ensures that the equivalence principle is upheld and confirms that the subleading term is nonlocal.

On the other hand, we emphasise that our derivation from the diffeomorphism symmetry breaking is in agreement with the argument of the universality of Hawking radiation that starts with Lorentz symmetry breaking at Planck scale [11]. The higher order terms could be viewed as the UV effect in Hawking radiation, in agreement with the investigation of time-dependence of Hawking radiation [63]. At high energy scale, it is possible to detect the Lorentz violating effect in Unruh radiation and the resulting spectrum would not be strictly thermal. In this sense, once we ignore information transfer, there is no difference between Hawking radiation and Unruh radiation even at high energy scale. Also, this small deviation from the thermal spectrum would not suffice to the potential violation of generalised second law from Lorentz symmetry breaking [64], as the higher order terms would not contribute to Bekenstein–Hawking entropy. This is because Bekenstein–Hawking entropy, as a Noether charge [49], ignores the dynamical contributions from quantum hair.

In summary, we have demonstrated the nonperturbative derivation of the subleading order in Hawking radiation and shown that quantum information is encoded in the diffeomorphism group of horizon. A black hole would release its quantum degrees of freedom as a consequence of diffeomorphism symmetry breaking during evaporation. A unitary evolution is expected and the relevant paradoxes are evaded. We believe that this result is of fundamental significance for our understanding of black holes.

Data availability statement

The data cannot be made publicly available upon publication because no suitable repository exists for hosting data in this field of study. The data that support the findings of this study are available upon reasonable request from the authors.

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