Photonic Maxwell's Demon

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We report an experimental realization of Maxwell's demon in a photonic setup. We show that a measurement at the few-photons level followed by a feed-forward operation allows the extraction of work from intense thermal light into an electric circuit. The interpretation of the experiment stimulates the derivation of an equality relating work extraction to information acquired by measurement. We derive a bound using this relation and show that it is in agreement with the experimental results. Our work puts forward photonic systems as a platform for experiments related to information in thermodynamics.

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Maxwell's demon made its appearance in 1867 as part of a thought experiment discussing the limitations of the second law of thermodynamics [1]. James Clerk Maxwell imagined the demon as a microscopic intelligent being, able to control a small door in the wall separating two boxes, both enclosing a gas in thermal equilibrium. The demon would use the door to filter particles based on their energy achieving an unbalanced gas distribution, an operation that appeared to be in violation of the second law of thermodynamics, decreasing the entropy of the gas without any investment of work. The discussions emerging due to the apparent paradox played a fundamental role in revealing the relation between information and thermodynamics: the amount of work extractable from the imbalance of energy created between the boxes by the sorting operation is limited by the information acquired through the demon's measurement of individual particle energies. By Landauer's principle, the erasure of this information from the demon's memory cannot be implemented without using at least as much work as can be extracted [2–5]. Maxwell's demon has seen many reinterpretations [6–8], denoting, in general, a system that either decreases entropy or extracts mechanical work by applying measurement and control to a medium in thermal equilibrium. Various physical realizations have recently been demonstrated experimentally [9–12].

Spurred by the advancement of experimental techniques, which allow for the control of physical systems down to the single particle level, there has been significant progress in the theoretical analysis of thermodynamics in microscopic systems, including the description of small thermal engines consisting of only a few energy levels [13,14], fluctuation theorems [5,15–20], the role of quantum coherence [21–26], and resource theories of thermodynamic transformations [27]. As was the case in the 19th century, when the steam engine demanded the development of thermodynamics of macroscopic systems, the modern analogues involving few or single particles have led to the emergence of new ideas in microscopic thermodynamics.

Here, we put forward photonics as a promising experimental platform for investigating the role of information in statistical mechanics by realizing a photonic Maxwell's demon. In analogy with the original thought experiment with gas particles on two sides of a wall, we prepare thermal states in two spatial light modes. We show that a measurement on these modes at the few-photons level and a simple conditional operation (feed forward) can lead to a difference in average energy between the two light modes. In order to extract work using this imbalance, we then let the light fall on two photodiodes connected to a capacitor. Thereby, the prepared imbalance leads to a charging of the capacitor, a genuine practical energy storage device. This demonstrates that microscopic measurements can be used to extract macroscopic work.

As the measurement devices are randomized in this process, the experiment cannot violate the second law of thermodynamics. Our further theoretical analysis of the setup stimulates the derivation of a work-information equality inspired by methods in Ref. [5]. This equality is applicable to work extraction scenarios that, like the one described here, do not take place at thermal equilibrium. It is thus quite general and, to our knowledge, new. We employ the equality to derive a bound on the work distribution in our setup which allows us to test it experimentally, demonstrating its physical relevance.

Setup.—The experiment depicted in Fig. 1 corresponds to a Maxwell's demon operating on a thermalized system via measurement, conditional operations, and work extraction.

The thermalized system consists of two pulsed light modes, each prepared in a thermal state described by the

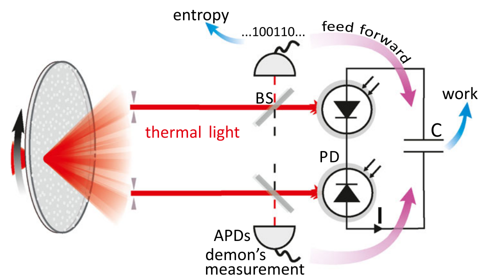

FIG. 1. Experimental setup. In analogy with Maxwell's original thought experiment, the setup uses energy from a thermal system, measurements and feed forward, in order to extract work. Thermal light is produced by collecting laser pulses scattered from a spinning glass diffuser wheel. The demon's measurement is implemented by high transmittance beam splitters (BS) and highly sensitive avalanche photodiodes (APDs). The two final linear photodiodes are the work extraction mechanism, acting as an electromotive source that charges a capacitor (C). A nonzero average voltage across C can be obtained by feed forward of the demons measurement, swapping its polarity according to the APD measurement outcomes.

density matrix $(1-e^{-\beta h\nu})\sum_{n=0}^{\infty} e^{-\beta h\nu n}|n\rangle\langle n|$ in the photon number basis where $h\nu$ is the single-photon energy and $\beta = 1/(k_B T)$. Light pulses in the two modes have undefined phase and energy distributed according to the Boltzmann distribution. We prepare thermal states by collecting light from a variable laser speckle pattern produced by a spinning glass diffuser (Arecchi's wheel). This type of source is known to produce light with thermal fluctuations [28–31] at much higher intensities that those achievable by selecting a single mode in the emission of a thermal lamp.

For the demon's measurement, each thermal light mode propagates through a high transmittance beam splitter from which the reflected light is coupled to a highly sensitive photodetector. In our setup we use avalanche detectors which can signal the presence of at least one photon, giving a simple binary measurement output. We treat the energy consumption of these detectors as pertaining to the demon, not subtracting it from the extracted work.

The state inferred after observing a detection event has a different expected number of photons than the initial light mode, increased or decreased depending on whether the incident light has super-Poissonian or sub-Poissonian statistics. Consider the extreme case in which an incident light pulse contains either a vacuum or a large number of photons, at equal probabilities. Detection of even a single photon from this incident pulse would imply that the transmitted beam has high energy. The expected number of photons in the transmitted beam conditioned on registering a photon detection would rise by a factor of 2, minus the number of photons that were detected. In a similar manner, our measurement can resolve the energy of a thermal light mode, as the states inferred after filtering according to measurement outcomes have a modified expected number of photons. This has been observed [32,33] in the limit of small photon-detection probabilities, where the measurement approximates photon subtraction [34,35], in which case the filtering of a single-mode thermal state based on successful photon detections generates a state with double the initial expected energy.

In our setup the change in the expected energy signaled by the measurement can be tuned by choosing the amount of light sent towards the avalanche photodetectors (see the Supplemental Material [36]), which sets the photon-detection rate. If the detection outcomes are ignored, the effect of the measurement amounts to a negligibly small loss introduced by the high transmittance beam splitters.

The role of the measurements is to change the light state by conditional updating. Feed forward of the output of the demon's measurement can be implemented by swapping the two thermal light modes based on avalanche photodiode (APD) detections, so that on average there is more energy on one of the two sides. By this operation, an asymmetric energy distribution can be created from two equally populated thermal modes.

To extract work we let the two light modes fall on two photodiodes, connected with opposing polarities such that, on average, they produce zero voltage when the light intensities are balanced. This photodiode circuit includes a capacitor which charges according to the fluctuating energy difference between the two light modes. When an unbalanced energy distribution of the two modes is produced by feed forward according to the APD measurement outcomes, the capacitor will have a nonzero average voltage which can be used to charge a battery, as detailed below in the section deriving a bound on the extractable work. We propose this setup for its conceptual simplicity, without aiming to realize the optimal work extraction strategy.

We make the following simplifications to provide a proof-of-principle implementation. The measurement can be implemented with imperfect photodetectors and, as shown in the Supplemental Material [36], its effect does not depend on the detection efficiency. We implement a beam splitter with reflectance 5×10^{-3}, small enough for the average effect on the transmitted light to be negligible compared to the thermal fluctuations and, in order to regulate the detection rates of the demon's measurement, we use variable absorbers. As the two initial light modes are well balanced, the symmetry of the setup is such that swapping the two light modes is indistinguishable from switching the polarity of the capacitor. Thus, we measure the capacitor voltage, replacing feed forward by a logical operation on the measured value: switching its sign as a function of the APD output.

Nonequilibrium work-information equality.—In order to understand the role of information in the work extraction scenario described above, we introduce a theoretical model that pertains to our experiment. The arguments that we use will apply quite generally to demons using measurement and controlled operations in open quantum systems.

We aim to obtain a relation between work and information via a nonequilibrium work relation. This type of relation, like the Jarzynski equality [15], links nonequilibrium processes to equilibrium quantities, like a system's free energy. One such equality, derived in Ref. [5] incorporates the effect of measurement and conditional operations, providing a way to obtain work-information bounds in Maxwell's demon–type scenarios. The effectiveness of a demon's measurement is included in this equality by means of the mutual information quantifying the correlations created between measurement outcomes and the measured system. When the initial energy state of the measured system is denoted n and the measurement outcome m, the pointwise mutual information is $I = \log[p(m|n)] - \log[p(m)]$. Here, by $p(m)$ we denote the probability of outcome m and by $p(m|n)$ the conditional probability of outcome m given energy state n. The theorem by Sagawa and Ueda reads $\langle e^{\beta(W-\Delta F)-I}\rangle = 1$, where W is the work extracted and ΔF the equilibrium free energy difference between the initial and final states of the working system. Jensen's inequality applied here gives a bound on the extracted work, $\beta\langle W\rangle \leq \beta\Delta F + \langle I\rangle$, showing that mutual information allows for work extraction without free energy consumption. The entropy of the measurement register, which can be readily estimated from a set of measurement outcomes, provides an upper bound to the mutual information I, which links this result to Landauer's principle.

The theorem described holds for work extraction scenarios where the system is in contact with a single thermal bath and detailed balance is assumed. This does not model well the work extraction setup presented here, due to the fact that the photodiodes and the capacitor are in contact with an environment at room temperature, which is significantly smaller than the temperature that characterizes the energy distribution of the light pulses. To find a relation between information gained and work extracted that applies to our experiment, we require a theoretical model describing a general work extraction operation. We turn to the experimental setup, noting that work extraction can only be performed by applying suitable feed forward and we aim to incorporate in the theoretical model the situation in which no feed forward is performed. This leads us to the first theoretical result:

$$\langle e^{\beta W - I}\rangle_f = \langle e^{\beta W}\rangle_0. \tag{1}$$

Here, the left-hand term is an average (denoted f) corresponding to the situation with feed forward, controlled by the output of the measurement, which is characterized by the mutual information I. The right-hand side (denoted 0) is an average corresponding to the same system, but where the measurement and feed-forward steps are missing. When the measurement is, on average, nondisturbing (for example, the effect of the high transmittance beam splitter in our setup is negligible), this means that measurement outcomes are simply ignored. Equation (1) is derived assuming only that the conditional operation is an energy conserving unitary transformation acting on the light modes and that a nondisturbance condition applies to the measurement, which is indeed the case in our setup. The theoretical model is detailed further in the final section of this Letter.

Information bound on work extraction.—Let U_k denote the voltage created across the capacitor C in the kth pulse of the experiment, each experiment looking at N pulses. Work extraction occurs when the energy stored in the capacitor during each run is accumulated in a work reservoir. When the capacitor is charged, connecting it to a battery with voltage U_0 transfers the charge $C(U_k - U_0)$ to the battery, increasing its energy by $CU_0(U_k - U_0)$; the capacitor can then be discharged, making the voltage 0 again. The effect on U_0 after a finite number of pulses can be made arbitrarily small by choosing a battery with high capacity, so, for simplicity, we take U_0 to be constant. Over N pulses the work extracted is $W = CU_0\sum_{k=1}^{N}(U_k - U_0)$ and the mutual information gain is $I^{(N)} \equiv \sum_{k=1}^{N} I_k$. We define the sum of voltages across the capacitor over the N pulses $U^{(N)} \equiv \sum_{k=1}^{N} U_k$. Applying equality (1), we get $\langle \exp[\beta CU_0(U^{(N)} - NU_0) - I^{(N)}]\rangle_f = \langle \beta CU_0(U^{(N)} - NU_0)\rangle_0$ and defining $x \equiv \beta CU_0$, $\langle \exp(xU^{(N)} - I^{(N)})\rangle_{\rm fb} = \langle \exp(xU^{(N)})\rangle_0$. For a large N, $U^{(N)}$ becomes normally distributed with the average $\langle U^{(N)}\rangle \equiv N\langle U\rangle$ and a standard deviation $\sigma(U^{(N)}) \equiv \sqrt{N}\sigma(U)$, where $\langle U\rangle$ and $\sigma(U)$ are the mean and standard deviation of the single-pulse voltages. For a normal distribution, we have $\langle \exp(xU^{(N)})\rangle_0 = \exp[x\langle U^{(N)}\rangle + x^2\sigma(U^{(N)})^2/2]$. Using Jensen's inequality and the convexity of the exponential, we get $x\langle U^{(N)}\rangle_f - \langle I^{(N)}\rangle \leq x\langle U^{(N)}\rangle_0 + x^2\sigma(U^{(N)})^2/2$ and a reordering, $\langle U^{(N)}\rangle_f - \langle U^{(N)}\rangle_0 \leq \langle I^{(N)}\rangle/x + x/2\sigma(U^{(N)})^2$. Since x can take any value due to the U_0 factor, we can optimize the right-hand term with respect to x to get the tightest bound, which is $(\langle U^{(N)}\rangle_f - \langle U^{(N)}\rangle_0)/\sigma(U^N) \leq \sqrt{2\langle I^{(N)}\rangle}$. The dependence on N cancels out and we get

$$\frac{|\langle U\rangle_f - \langle U\rangle_0|}{\sigma(U)_0} < \sqrt{2\langle I\rangle}, \tag{2}$$

which can be rewritten in terms of the extracted work, $(|\langle W\rangle_f - \langle W\rangle_0|/\sigma(W)_0) < \sqrt{2\langle I\rangle}$, with $\langle I\rangle$ being the single-measurement mutual information. We have thus obtained a bound on the work distribution in terms of the information gained by measurement. The left-hand side of this relation can be interpreted as a measure of distinguishability between work distributions with and without measurement and feed forward. This measure relates the power produced by the setup to the fluctuations of this power and can be linked to the concept of strength of work, introduced in

Ref. [14]. If the battery is used to drive a load system, the left-hand side of Eq. (2) quantifies how small the source's power fluctuations will be. Our relation approaches the regime of Landauer's bound [2] when the work fluctuations are of the same order as the characteristic thermal energy defining the initial state of the equilibrium system on which measurement and feed forward are applied. In the following section, we show how experimentally measured quantities relate to this bound. Since the amount of work that can be extracted is proportional to the voltage across the capacitor, we focus on measuring the distribution of this voltage in relation to the APD measurements.

Experimental results.—In our pseudothermal light source [28–31], we used 4 μJ pulses from an amplified Ti:sapphire laser focused on a fine diffuser to produce laser speckle, and multimode optical fibers with a core size of 25 mm to collect the light, 15 cm away from the diffuser. The source yielded on the order of 10^8 photons per pulse. The intensity of the two light beams was balanced such that the bias of the photodiode voltage was less than 0.3% of its standard deviation. Single-mode thermal states are certified by estimating the intensity autocorrelation at zero delay, $g^{(2)}(0)$ [37]. In the ideal case, this should yield $g^{(2)}(0) = 2$. In a repeatable manner, we obtained $1.9 \lesssim g^{(2)}(0) < 2$ by measuring the pulse energy distribution and we observed no cross-correlation of the pulse energies in the two modes.

We recorded oscilloscope traces of the voltage across the capacitor ($C = 2$ pF) as well as outcomes of the APD detection (the demon's measurement). In Fig. 2(a) we illustrate how the voltage distribution depends on the APD signals. The imbalance of this distribution can be used to create a nonzero mean voltage by changing the capacitor polarity conditioned on the APD measurement outcomes. Figure 2(b) shows that the average voltage is close to zero when the APD signals are ignored but significantly displaced when this conditional operation is applied.

The demon's measurement can be tuned by varying the amount of light sent towards the APDs, which changes photon-detection rates from zero to one detection per pulse. As we show in the Supplemental Material [36], the imbalance of energy that can be created between the two optical modes depends only on these rates. One might expect this imbalance to be highest when the detection probabilities are around 1/2, corresponding to the highest entropy (information content) of the measurement register. However (see the Supplemental Material [36]), the probabilities that maximize the average imbalance are asymmetric: 1/3 and 2/3 for the two arms, respectively. In order to maximize the observable effect in our setup, we set one of the arms to yield approximately one detection every three pulses and scan the detection rate corresponding to the other arm. This allows us to relate the effect of the measurement and conditional operation to the information gained by measurement (which also depends on the detection rates detailed in the Supplemental Material [36]) using the bound

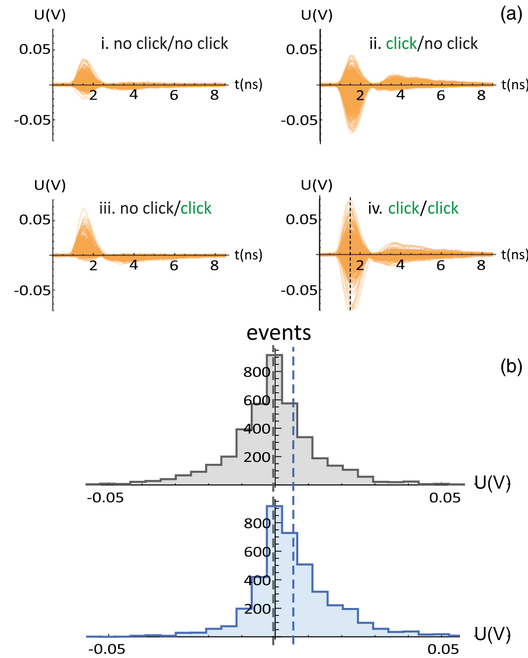

FIG. 2. Measured voltage across the capacitor. (a) Oscilloscope traces showing the voltage created by the photodiodes, filtered by the demon's measurement outcomes. 4000 traces are sorted according to binary APD signals (click for photon detection). The black dashed line depicted only in quadrant (iv) indicates the time at which the maximum voltage is sampled. (b) Histogram of the maximum voltage. Above, the APD outputs are ignored; below, the sign of the voltage is changed if the measurement outcome is click–no click [quadrant (ii) in (a)]. The dashed vertical lines show the average of each of the two distributions. The number of photon detections per pulse—$p_1 = 0.702 \pm 0.008$ and $p_2 = 0.311 \pm 0.008$—as well as the control strategy, is chosen aiming to optimize the displacement of the mean voltage, as detailed in the Supplemental Material [36].

given by Eq. (2). The results are reported in Fig. 3. The two optimum strategies, corresponding to a change of polarity every time one of the asymmetric APD detection outcomes is registered, are both depicted in this figure.

The main imperfection of the experiment is in the statistics of the thermal light modes. The states prepared are slightly multimode due to the finite coherence length of the speckle pattern from which the two modes are collected with optical fibers. In our setup, we produced a signal with intensity autocorrelation $g^{(2)}(0) \approx 1.9$, as opposed to 2 for single-mode thermal states. This results in a reduced correlation between the APD measurement outcomes and the voltage across the capacitor, which can be accounted for by the reduced mutual information characterizing the APD measurement. We work out the dependence of both the work distribution and the measurement mutual information on the number of modes in the thermal states in the Supplemental Material [36]. Figure 3 shows the very high agreement between the experimental data and the

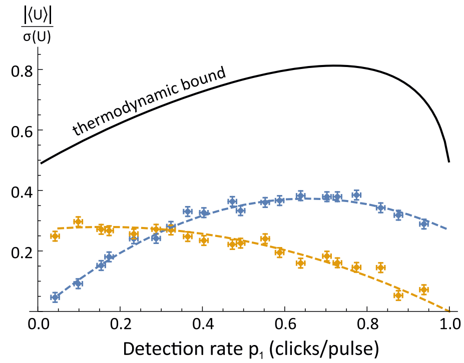

FIG. 3. Extracted work and information. $|\langle U\rangle|/\sigma(U) \sim \Delta\langle W\rangle/\sigma(W)$ [see the left-hand side of Eq. (2)], as a function of p_1, the number of photon detections per pulse registered by the demon's measurement on the first mode. The measurement on the second mode yields 0.311 ± 0.008 detections per pulse. Blue and orange points correspond to two types of feedback: flipping the voltage when the measurement yields click–no click (blue) or no click–click (orange). Error bars are estimated by binning the experimental data. The black line gives the bound established in terms of mutual information [see the right-hand side of Eq. (2)]. Dashed lines give predictions based on the average number difference between the two thermal states after feed forward, modeling states with imperfect autocorrelation, $g^{(2)}(0) = 1.9$. The calculations of the photon number difference and the mutual information are detailed in the Supplemental Material [36].

prediction based on a model that is ideal, apart from taking into account the limited $g^{(2)}(0)$.

Comparing the left-hand and right-hand terms of Eq. (2), we find that for different measurement and feed-forward settings, the extracted work weighted by the work fluctuations is below the bound given in terms of mutual information, yet of the same order. The gap between the bound and the data is not due to the experimental imperfections, and setting $g^{(2)}(0) = 2$ in the model does not change this gap significantly. It is instead due to the bound not being analytically tight in this regime (for example, because Jensen's inequality is not tight in general). The bound is tight when no information is acquired and when there is a sharp work distribution. Nevertheless, being derived from a general model, the bound is independent of the exact details of the work extraction mechanism and it gives the right order of magnitude in our example, demonstrating its relevance for establishing the performance of feed-forward experimental strategies.

Discussion.—We have presented an experiment similar to Maxwell's demon, in which work extraction is ultimately bound by the amount of information acquired through measurement on a system in equilibrium. We derived a bound relating work extraction to mutual information characterizing the measurement setup by employing an information-theoretic analysis, building on previous literature and considering less idealized levels of complexity, as dictated by the experimental arrangement. Unlike previous studies, the focus of our analysis is not the absolute energy efficiency [2,3,5,6], but the ratio between the average extracted work and the fluctuations of the work distribution in the unperturbed setup. This measure is relevant when the amount of fluctuation produced by each cycle of work extraction [14] is an important figure. The derivation presented here could be adapted to impose limits on the way in which the distribution of parameters other than work can be affected by measurement and feed-forward strategies.

The ability of addressing light in both the single-photon and intense-field regimes has been key in our experiment, and this constitutes one of the main advantages of a photonic approach. Our demonstration has been carried out in the classical regime, exploiting the properties of second-order coherence. The setup can be adapted to include nonclassical states and measurements, investigating the role of quantum coherence in thermodynamic processes. Furthermore, single particle measurement techniques similar to that presented here are pertinent to optomechanical [38] or spin-ensemble [39] systems, and they can be used to investigate the effect of weak probing in the dynamics of open systems.

Derivation of the work-information equality.—We divide the theoretical model into three systems. (i) The first system, $\mathcal{S}$, starts in thermal equilibrium, with an inverse temperature β, separable from the rest; this represents the two thermal light modes in our setup. The initial and final energy eigenstates this occupies are labeled by $|s\rangle$ and $|S\rangle$, respectively, and their Hamiltonians are indicated by $H_{\mathcal{S}}$. Measurement and feed forward operate only on this system, with feed forward described by a unitary transformation with no energy cost. The conditional mode swapping in our setup is well described by such an operation (in principle, the feed forward can be implemented by a variable-reflectivity beam splitter). The measurement outcome is labeled by m, with the corresponding set of measurement operators $M_j^{(m)}$, where j allows for the possibility of several measurement operators associated with a given measurement outcome (e.g., if the outcome is a click, there could be several possible physical processes associated with that). The measurement operators are normalized according to $\sum_{m,j} M_j^{(m)\dagger} M_j^{(m)} = \mathbb{1}$. The feed-forward operation is denoted by U_m. (ii) The second system is the work reservoir or battery $\mathcal{B}$, with the initial and final energy levels occupied being $|b\rangle$ and $|B\rangle$, respectively, with energies E_b and E_B; its Hamiltonian is $H_{\mathcal{B}}$. We define the extracted work as the energy increase of this system: $W = E_B - E_b$. (iii) The third system comprises everything else, the rest $\mathcal{R}$, corresponding in our setup to the photodiodes, the capacitor, and the environment. This occupies the initial and final energy levels r and R, respectively, and has the Hamiltonian $H_{\mathcal{R}}$. We shall also find it convenient to label the final joint energy of $\mathcal{S}$ and $\mathcal{B}$ as $E_{SR} \coloneqq E_S + E_R$. We denote the set of random variables of interest as $\xi \equiv \{s, b, r, m, SR\}$.

We assume that, owing to decoherence, the total initial state is diagonal in the product basis of the free Hamiltonians so that we can treat the initial energies of the three systems as classical variables. We also assume that the work reservoir undergoes decoherence in its energy eigenbasis at the end, so its final energy is well defined. There is also an interaction Hamiltonian. It is *a priori* not necessarily the case that the sum of the free Hamiltonian energies of the systems is conserved. Either of two conditions suffices for this energy conservation to hold: $[H, H_{\rm int}] = 0$, or the initial and final states have zero (or otherwise equal) interaction energy. In our experiment, both of these are respected, as we can model the photodetection with the Jaynes-Cummings (rotating-wave approximation) Hamiltonian plus decoherence in the energy eigenbasis, and in both the initial and final states there is no interaction energy between the light and the detectors and between the capacitor and the battery (these can be simply disconnected). We shall accordingly assume that $E_s + E_b + E_r = E_S + E_B + E_R$. This implies that $W = E_s + E_r - E_{SR}$. The evolution of the entire closed system is given by the unitary operator V.

Using these definitions, the left-hand side of Eq. (1) is $\langle e^{\beta W - I}\rangle_f = \sum_\xi p(s,b,r,m,SR) e^{\beta W - I} = \sum_\xi p(m,SR|s,b,r) \times p(s,b,r) e^{\beta W - I}$, using Bayes's rule in the second step. The probability distribution of the initial system energies is $p(s,b,r) = (1/Z) e^{-\beta E_s} p(b,r)$. Introducing the expressions for W and I, we get $\langle e^{\beta W - I}\rangle_f = \sum_\xi p(m,SR|s,b,r)(1/Z) e^{-\beta E_s} p(b,r) e^{\beta(E_s + E_r - E_{SR})}\{[p(m)]/[p(m|s)]\} = \sum_\xi p(m,SR|s,b,r)\{[p(m)]/[p(m|s)]\}(1/Z) \times p(b,r) e^{\beta(E_r - E_{SR})}$. Isolating the sum over s, we have $\sum_s p(m,SR|s,b,r)/p(m|s) = \mathrm{Tr}\{|SR\rangle\langle SR| V[U_m(\sum_s\{(\sum_j M_j^{(m)}|s\rangle\langle s|M_j^{(m)\dagger})/[p(m|s)]\}) U_m^\dagger \otimes |br\rangle \times \langle br|]V^\dagger\}$. The sum over s is a sum over all normalized states produced by the demon's measurement given a sharp energy input. If the measurement is nondisturbing, it leaves the state $|s\rangle\langle s|$ unchanged and the sum reduces to the identity. We show in the Supplemental Material [36] that, for our measurement—although it is not nondisturbing—we have $\sum_s\{(\sum_j M_j^{(m)}|s\rangle\langle s|M_j^{(m)\dagger})/[p(m|s)]\} = \mathbb{1}$. Using this and $U_m U_m^\dagger = \mathbb{1}$, we get $\sum_s p(m,SR|s,b,r)/p(m|s) = \mathrm{Tr}[|SR\rangle\langle SR| V(\mathbb{1} \otimes |br\rangle\langle br|)V^\dagger] = \sum_s p_0(SR|s,b,r)$, where p_0 denotes the probability distribution corresponding to the evolution of the systems when the measurement and conditional operations are not implemented. Plugging this into the expression for the left-hand side of Eq. (1), we get $\langle e^{\beta W - I}\rangle_f = \sum_\xi p_0(SR|s,b,r) p(s,b,r) e^{\beta W} = \langle e^{\beta W}\rangle_0$, yielding Eq. (1). This equation is thus simply an information-theoretic equality, like that presented in Ref. [40], applied to an energy transfer scenario, which exposes the information-theoretic underpinning of thermodynamics.

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