Low Energy Particle Physics and Cosmology of Nonlinear Supersymmetric General Relativity

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Abstract

Basic ideas of nonlinear supersymmetric general relativity(NLSUSY GR) are explained and some cosmological and the low emergy particle physics consequences are discussed.

1 NLSUSY GR

By extending the geometric arguments of Einstein general relativity (EGR) on Riemann space-time to new space-time inspired by NLSUSY, where tangent space-time is specified not only by x_a for $SO(1,3)$ but also by the Grassmanian ψ_α for isomorphic $SL(2C)$ of NLSUSY, the fundamental action (called nonlinear supersymmetric general relativity) has been constructed[1]:

$$L(w) = \frac{c^4}{16\pi G}|w|(\Omega(w) - \Lambda), \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^i\gamma^a\partial_\mu\psi^i - \partial_\mu\bar{\psi}^i\gamma^a\psi^i), (N = 1, 2, \cdots, N), \tag{2}$$

where $w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi)$, $e^a{}_\mu$, $t^a{}_\mu(\psi)$ and $\Omega(w)$ are the invertible unified vierbein of new spacetime, the ordinary vierbein of EGR, the stress-energy-momentum of N NG fermion $\psi(x)$ (called *superons* as hypothetical spin 1/2 objects constituting all observed particles) and the the unified scalar curvature of new(SGM) spacetime, respectively. $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$ and $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) w^{\nu a}(x)$ are unified metric tensors of SGM spacetime. New space-time is the generalization of the compact isomorphic groups SU(2) and SO(3) for the gauge symmetry of 't Hooft-Polyakov monopole into the noncompact isomorphic groups SO(1,3) and SL(2C) for space-time symmetry. NLSUSY GR action possesses promissing large symmetries isomorphic to SO(10) SP[2]. Note that the so called no-go theorem is overcome (circumvented) in a sense that the non-tivial N-extended SUSY gravity theory with $N > 8$ has been constructed in a NLSUSY invariant way.

2 Cosmology and Low Energy Physics

NLSUSY GR $L(w)$ on new *empty* space-time written in the form of the *vacuum* EH type is unstable due to NLSUSY structure of tangent space-time and decays (called *Big Decay* [3]) spontaneously into ordinary Einstei-Hilbert(EH) action with the cosmplogical constant Λ, NLSUSY action for N superons and their gravitational interactions on ordinary Riemann space-time written formally as follows, which ignites Big Bang of the present universe.

$$L(e, \psi) = \frac{c^4}{16\pi G}|e|\{R(e) - \Lambda + \tilde{T}(e, \psi)\}, \tag{3}$$

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which is called superon-graviton model(SGM) from composite viewpoint.

Considering SGM action reduces to N-extended NLSUSY action in asymptotic Riemann-flat ($e^a{}_\mu \to \delta^a_\mu$) space-time, we can study the low energy contents of SGM by constructing the N-extended LSUSY theory equivalent to N-extended NLSUSY model. We have shown explicitly by the heuristic arguments for simplicity in two space-time dimensions ($d=2$) [4] that $N=2$ LSUSY interacting QED is equivalent to $N=2$ NLSUSY model. (Note that the minimal realistic SUSY QED in SGM composite scenario is described by $N=2$ SUSY.)

$N=2$ NLSUSY action for two superons ψ^i $(i=1,2)$ in $d=2$ is written as follows,

$$L_{N=2\mathrm{NLSUSY}} = -\frac{1}{2\kappa^2}\,|w| = -\frac{1}{2\kappa^2}\left\{1 - i\kappa^2\bar{\psi}^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4(\bar{\psi}^i \partial\!\!\!/ \psi^i \bar{\psi}^j \partial\!\!\!/ \psi^j - \bar{\psi}^i\gamma^a\partial_b\psi^i\bar{\psi}^j\gamma^b\partial_a\psi^j)\right\} \tag{4}$$

where $\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$ in the SGM scenario and $|w| = \det(w^a{}_b) = \det(\delta^a_b + t^a{}_b)$, $t^a{}_b = -i\kappa^2\bar{\psi}^i\gamma^a\partial_b\psi^i$. The most general massless $N=2$ LSUSY QED action in $d=2$ is written as follows,

$$\begin{aligned}
&L_{N=2\mathrm{SUSYQED}} \\
&= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a\phi)^2 + \frac{1}{2}D^2 - \frac{1}{\kappa}\xi D \\
&+\frac{i}{2}\bar{\chi}\partial\!\!\!/\chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/\nu + \frac{1}{2}(F^i)^2 + f(A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + A^2 D - \phi^2 D - \epsilon^{ab}A\phi F_{ab}) \\
&+e\left\{iv_a\bar{\chi}\gamma^a\nu - \epsilon^{ij}v^a B^i\partial_a B^j + \bar{\lambda}^i\chi B^i + \epsilon^{ij}\bar{\lambda}^i\nu B^j - \frac{1}{2}D(B^i)^2 + \frac{1}{2}(\bar{\chi}\chi + \bar{\nu}\nu)A - \bar{\chi}\gamma_5\nu\phi\right\} \\
&+\frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2.
\end{aligned} \tag{5}$$

We have shown explicitly[4]: $L_{N=2\mathrm{NLSUSY}}$ + [suface terms] = $L_{N=2\mathrm{SUSYQED}}$. Therefore the low energy particle physics contents in asymptotic flat space-time of $N=2$ SGM can be read from $N=2$ SUSY QED action equivalent to $N=2$ NLSUSY action.

Now we study the vacuum structure of $N=2$ SUSY QED action (5). The vacuum is determined by the minimum of the potential $V(A,\phi,B^i,D)$,

$$V(A,\phi,B^i,D) = -\frac{1}{2}D^2 + \left\{\frac{\xi}{\kappa} - f(A^2-\phi^2) + \frac{1}{2}e(B^i)^2\right\} D. \tag{6}$$

Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2 - \phi^2 - \frac{e}{2f}(B^i)^2 - \frac{\xi}{f\kappa}\right\}^2 \geq 0. \tag{7}$$

The vacuum field configurations possess SO(1,3) or SO(3,1) isometries in (A,ϕ,B^i)-space depending upon the signatures of the parameters. One of the vacua in SO(3,1) isometry for

$$ef<0,\quad \frac{\xi}{f\kappa}>0,\quad A^2-\phi^2+(\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}}B^i,\ \ k^2 = \frac{\xi}{f\kappa}\right) \tag{8}$$

can be studied by substituting the following expressions

$$\begin{aligned}
&A = -(k+\rho)\cos\theta\cos\varphi\cosh\omega, \quad \phi = (k+\rho)\sinh\omega, \\
&\tilde{B}^1 = (k+\rho)\sin\theta\cosh\omega, \quad \tilde{B}^2 = (k+\rho)\cos\theta\sin\varphi\cosh\omega,
\end{aligned}$$

into the action and expanding the results around the vacuum. We obtain

$$\begin{aligned}
L_{N=2\mathrm{SUSYQED}} &= \frac{1}{2}\{(\partial_a\rho)^2 - 4f^2k^2\rho^2\} + \frac{1}{2}\{(\partial_a\theta)^2 + (\partial_a\varphi)^2 - e^2k^2(\theta^2+\varphi^2)\} - \frac{1}{4}(F_{ab})^2 \\
&+\frac{1}{2}(\partial_a\omega)^2 - \frac{1}{4}(F_{ab})^2 + \frac{1}{2}(i\bar{\lambda}^i\partial\!\!\!/\lambda^i - 2fk\bar{\lambda}^i\lambda^i) + \frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/\chi + \bar{\nu}\partial\!\!\!/\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdot
\end{aligned} \tag{9}$$

and the following mass spectra with mass hierarchy by the factor $\frac{e}{f}$ indicating the spontaneous SUSY breaking as anticipated;

$$m_\rho^2 = m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \quad m_\theta^2 = m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \quad m_{v_a} = m_\omega = 0. \qquad (10)$$

The local $U(1)$ gauge symmetry is not broken. The massless scalar ω is a NG boson for the degeneracy of the vacuum in $(A, \tilde{B}_2)$-space, which is gauged away provided the gauge symmetry between the vector and the scalar multiplet is introduced. As for the cosmological significances of $N = 2$ SUSY QED in SGM scenario, the above vacuum produces the same interesting predictions as already pointed out in $N = 2$ pure SUSY QED in SGM scenario [5], which may simply explain the observed mysterious (numerical) relations

$$((dark)\ energy\ density\ of\ the\ universe)_{obs} \sim 10^{-12} \sim (m_\nu)_{obs}{}^4 \sim \frac{\Lambda}{G} \sim g_{sv}{}^2,$$

and give a new insight into the origin of mass provided $f\xi \sim O(1)$ and λ^i is identified with neutrino. (Λ, G and g_{sv} are the cosmological constant of NLSUSY GR (SGM) on *empty* new space-time, the Newton gravitational constant and the superon-vacuum coupling constant via the supercurrent, respectively.) Here we just mention that there is another physical (apparently pathological) vacuum in $SO(3,1)$ isometry, which contains off-diagonal fermion mass terms[6]. The similar investigations in $d = 4$ are urgent and the extension to large N, especially to $N = 5$, is important for *superon quintet hypothesis* in SGM scenario with $N = \underline{10} = \underline{5} + \underline{5^*}$[7]. Also NLSUSY GR in extra space-time dimensions is an interesting problem, which can describe all observed particles as elementary *a la* Kaluza-Klein.

Our analysis shows that the vacua of N-extended NLSUSY GR action in SGM scenario possess rich structures promissing for the unified description of nature.

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