

Theoretical progress in QCD

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Abstract. I review recent theoretical developments in perturbative QCD, with particular attention fixed order calculations of collider processes, and to shower Monte Carlo event simulation.

1. Introduction

QCD is by now the established theory of strong interactions. It provides a framework for the computation of hard processes, that has been applied to several different contexts. The computation of hard processes is carried out in perturbation theory, using the QCD lagrangian. The results of these calculations are formulated in terms of fundamental fields, i.e. quarks and gluons. Some further assumptions are needed to tell us how to apply a result expressed in terms of fundamental fields (i.e. quarks and gluons) to scattering phenomena measured in terms of hadrons. This inability to deal rigorously with hadron production is essentially rooted in the asymptotic freedom and infrared slavery aspects of the strong interactions. The coupling becomes small at short distances, and large at large distances. We can compute the small coupling part of a process, but not the large coupling (i.e. long distance, large time) part. It is usually assumed that the long distance part of the process has only a minor effect on observables of calorimetric type, that is to say, the coarse energy and momentum distribution of the final state should be calculable in terms of fundamental fields. Mainly because of these assumptions, our confidence in perturbative QCD prediction must be based upon validation with data. On the other hand, several years of experience with e^+e^- colliders, as well as lepton-hadron and hadron hadron colliders, where tests on several different areas of perturbative QCD have been carried out, have convinced us that QCD and the PQCD framework are reliable. Several examples of comparisons between QCD calculations (often carried out at NLO (Next-to-Leading Order) accuracy) and data have been presented in the parallel sessions. The agreement is generally quite good.

With the forthcoming of the LHC, the emphasis of QCD theoretical work is shifting from QCD tests to QCD applications for SM (Standard Model) and BSM (Beyond the Standard Model) physics studies. At hadron colliders, QCD plays a dominant role in all kind of interactions. Furthermore, the accurate computation of physics signals and backgrounds at the LHC is typically very challenging, mainly because of the complex signals, and because higher energy means more open thresholds. As a typical example, we may recall that top production is a common background at the LHC, providing leptons, missing energy, and many jets. It is thus important to be able to compute cross section for complex processes. Since it is known that NLO corrections can be important (ranging from 10% to 100% in typical hadron collider processes),

they should be computed, at least for the most important processes. Furthermore, these results should be made available, if possible, in the framework of Shower Monte Carlo event generators.

In this report I will review the current status of LO (Leading order), NLO and NNLO (Next-to-next-to Leading Order) calculations. Furthermore, I will review the techniques adopted to embed LO and NLO results in shower Monte Carlo programs.

There are several important QCD topic that I will not cover in this review, like small x QCD, diffraction, and various topics on jet definition and algorithms that are particularly relevant to future LHC jet studies [1][2][3][4].

2. Matrix element calculations

Several methods and programs for the calculation of complex processes at the tree level have been available for quite a long time. Some of these programs are fully automatized, i.e. the user enters a description of the process, and the program delivers the matrix element as a computer program, or even the cross section. As a rule of thumb, there are programs that aim at total automation (like MadGraph [5]), and programs that aim at speed (like ALPGEN [6]). The speed issue for these programs is very important, since it is directly related to the maximum level of complexity that the program can deal with. The Berends-Giele recursion method [7] for the computation of $W + n$ jets has been available for a long time, and is capable to compute processes involving up to 10 final state gluons. The more general ALPGEN [6] program (based upon the ALPHA [8] algorithm) and the HELAC [9] program, have comparable speed. On the other hand, programs like MadGraph (based upon the HELAS program [10]) use the more straightforward helicity amplitude method, and do not perform as well. It is foreseeable that at some point even the fast algorithm may be implemented in a fully automated way. At present, in fast programs like ALPGEN, the new processes are added by the authors.

Recently, new approaches to the calculation of tree level amplitude have been developed: the so called CSW [11] [12] [13] recursion relations and the BCFW recursions [14] recently extended for the case of massive partons in [15] [16]. These new methods can provide very compact formulae. It is unclear, at present, whether they can also provide algorithmic simplicity in computer code implementations. In [17] and [18] it was shown that traditional algorithms (i.e. Berends-Giele recursion relations) perform much better than the new ones. Table 1, taken from

Final state	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
$2g$	0.24	0.28	0.28	0.33	0.31	0.26
$3g$	0.45	0.48	0.42	0.51	0.57	0.55
$4g$	1.20	1.04	0.84	1.32	1.63	1.75
$5g$	3.78	2.69	2.59	7.26	5.95	5.96
$6g$	14.20	7.19	11.9	59.10	27.80	30.60
$7g$	58.50	23.70	73.6	646.00	146.00	195.00
$8g$	276.00	82.10	597	8690.00	919.00	1890.00
$9g$	1450.00	270.00	5900	127000.00	6310.00	29700.00
$10g$	7960.00	864.00	64000		48900.00	

Table 1. Computation time (in seconds) of the Berends-Giele (BG), BCF and CSW algorithms for both colour ordered and colour disordered amplitudes.

[18], a comparison of the different algorithms is displayed. It is quite possible, however, that the full potential of the new approaches has yet to be achieved.

In summary, the computation of very complex matrix element can be achieved today with several methods. A trade-off between speed and flexibility is still present in current programs. It is however conceivable that fully automated implementations of fast algorithms will become available in a reasonable time.

3. NLO calculations

The inclusion of NLO (next-to-leading order) corrections to QCD processes implies the calculation of some extra unresolved parton emission, or virtual exchange. If the extra parton emission is resolved (as in a process with the requirement of an extra jet) we still speak about a leading order calculation. In fig 1 we show a representative graph for Born level heavy flavour

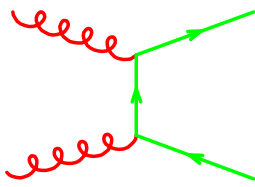
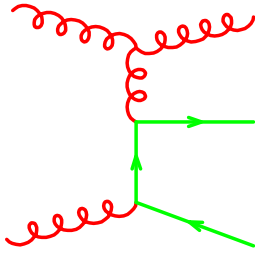
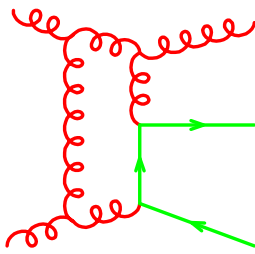
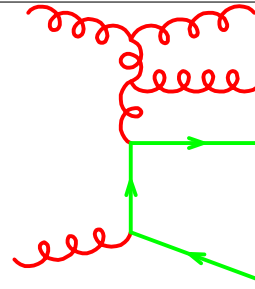
Born $Q\bar{Q}$	Born $Q\bar{Q} + \text{Jet}$	NLO virtual $Q\bar{Q} + \text{Jet}$	NLO real $Q\bar{Q} + \text{Jet}$
			
	$k_T > p_T^{\text{cut}}$	$k_T > p_T^{\text{cut}}$	$k_T > p_T^{\text{cut}}, l_T \text{ unrestr.}$

Figure 1. Example of NLO correction to a higher order process.

production, Born level $Q\bar{Q} + \text{jet}$ production, and some NLO corrections to $Q\bar{Q} + \text{jet}$ production. Notice that the Born level process $Q\bar{Q} + \text{jet}$ must be defined with a transverse momentum cut on the jet. In fact, $Q\bar{Q}$ production with an unresolved jet is a NLO correction to the $Q\bar{Q}$ production process. Also, the $Q\bar{Q} + \text{jet}$ process is of higher order with respect to the Born $Q\bar{Q}$ process, but with a given transverse momentum cut can be consider as a Born level process. Its NLO correction include the possible radiation of an extra light parton, with no restriction upon its transverse momentum. Unrestricted emission of light partons yield divergences in the cross section, that are cancelled against corresponding divergences in the virtual correction graphs.

Given that we already know the leading contribution to a process, the first question to answer is whether (and why) we need the NLO corrections at all. It is obvious that we should compute them for important processes. The size of NLO correction is typically a few 10% of the Born contribution. For higgs production in gluon fusion, NLO corrections turn out to be of the same size as the Born contribution. Since automation of NLO calculation has not yet been achieved, the processes to be computed should be selected carefully. In table 2, the list of processes selected in ref. [19] has been reported. A few new, interesting results have become available in recent time. Radiative corrections to the production of a heavy quark pair plus a jet have been completed [20]. This is an extremely complex calculation, and is very far from the Les Houches wish list, requiring $t\bar{t} + 2\text{jets}$. A first result on three vector boson production has been obtained, for the case of three Z bosons in ref. [21]. The same authors have also computed the NLO corrections to $t\bar{t}Z$ production in the gluon fusion channel [22], a process that can be used to study the Z couplings to the top quark. The process of production of $WW + \text{jet}$ has been computed at NLO in ref. [23].

NLO corrections have been recently computed for several processes relevant to the study of the vector boson fusion Higgs production, and to vector-boson scattering. For these processes one typically requires the presence of two (opposite) forward jets. Typical cuts require $|\eta_1 - \eta_2| > 4.2$, $\eta_1 \cdot \eta_2 < 0$, $m_{12} > 600 \text{ GeV}$, where $\eta_{1/2}$ are the jets pseudorapidity, and m_{12} is the invariant mass

process, $V \in \gamma, W^\pm, Z$	background to	As of now
$pp \rightarrow VV + 1 \text{ Jet}$	$t\bar{t}H$, new physics	
$pp \rightarrow t\bar{t} + b\bar{b}$	$t\bar{t}H$	
$pp \rightarrow t\bar{t} + 2 \text{ Jets}$	$t\bar{t}H$	$t\bar{t} + 1 \text{ Jet}$
$pp \rightarrow VV + b\bar{b}$	VBF $\rightarrow VV$, $t\bar{t}H$, new physics	
$pp \rightarrow VV + 2 \text{ Jets}$	VBF $\rightarrow VV$	($\alpha_{\text{weak}}^6 \alpha_s$, no S channel)
$pp \rightarrow V + 3 \text{ Jets}$	new physics signatures	
$pp \rightarrow VVV$	SUSY trilepton	ZZZ

Table 2. LHC priority wish list of the 2005 Les Houches workshop.

of the two jet system. Furthermore, a central jet veto effectively suppresses the background, since in the vector fusion process the two opposite hadronic systems that produce the virtual vector boson are colour singlet systems, and thus very little QCD activity is expected in the central region. Gluon fusion Higgs production in association with two jets provides a background to the vector fusion process, roughly a factor of two below the signal after the VBF cuts have been applied. NLO corrections to this process are needed for safe subtraction, and have been computed in [24]. Furthermore, vector boson scattering, besides being a process of interest by itself, is a background to the VBF signal; the calculation of NLO corrections to all vector boson scattering process has been completed recently [25] [26] [27] [28]. Furthermore, full electroweak plus strong corrections to the VBF process have been computed [29].

It can be noticed that many processes in the Les Houches wish list involve four particles in the final state. This implies that, in the virtual corrections to such processes, there is an hexagon graph, i.e. a loop with six vertexes attached, two of them corresponding to the incoming partons, and four of them to the final state. Hexagon graphs are thus an important benchmark for NLO calculation. In particular, a considerable effort has gone in the computation of the 6-gluon hexagon graph. Let me remark that all one-loop Feynman graphs can be expressed in terms of known basic integrals by the Passarino-Veltman [30] tensor reduction method. The same can be said about the six-gluon hexagon graph. The only problem is complexity: the number of terms generated in the Passarino-Veltman approach is too large to deal with, even with modern computer algebra systems. Theoreticians have thus tried different approaches in order to develop a winning strategy for the computation of loop integrals. The first computation of the 6-gluon hexagon has been performed in [31]. In this work, the tensor reduction was performed numerically, rather than algebraically, so that the method was called semi-numerical. A different strategy aims at the full numerical computation of the virtual integral. This approach is motivated by the fact that the accuracy of the virtual contribution has only to match the accuracy of the real emission contribution, that has to be performed numerically, since it depends upon the experimental cuts that are imposed upon the final state. In ref. [32] the six photon amplitude with a massless fermion has been computed fully numerically. Furthermore, in ref. [33], a numerical method based upon sector decomposition [34] [35] and contour distortion has been set up, and used to reproduce known analytic results on $gg \rightarrow h$ at two loops [36]. The same method was used independently in the ZZZ and $t\bar{t}Z$ calculations of refs. [21] and [22].

The so called on-shell methods [37][38][39] (for a review see [40]) seem to be very promising approaches for the computation of one loop amplitude. These method aim at the reconstruction of the full one loop amplitude sawing together on-shell tree amplitudes, using unitarity techniques. The analytical calculation of all helicity contributions to the six gluon amplitude has been completed recently mostly within this framework [41][42][43][44][45][46]. Furthermore, in ref. [47], a very promising procedure for the reduction of full one loop amplitudes at the integrand level has been presented, that can be used in conjunction with unitarity techniques.

First attempts to automate the computation of one loop amplitudes, based upon these methods, have appeared in the literature [48][49]. In particular, in [49], exploiting also the results of ref. [47], an algorithm for the computation of the cut-constructable part of one loop amplitudes was developed and tested.

In summary, very interesting developments have taken place in the past few years for the computation of one loop amplitudes, both in the numerical and in the analytical approaches. At this stage, full automation of the calculation of NLO cross sections for processes with up to four particles in the final state does not seem impossible, although certainly not near in time.

4. New NNLO result: $e^+e^- \rightarrow 3 \text{ jets}$

A very important development has taken place this year, and has been presented in this conference by N. Glover: the computation of $e^+e^- \rightarrow 3 \text{ jets}$ at NNLO (next-to-next-to leading order) has been completed [50][51][52]. This result is the conclusion of a collective effort that spanned several years of work, with many researchers involved [53] [54] [54] [55] [56] [57] [58][59][60] [61].

We remind the reader that most hadronic events in e^+e^- annihilation into hadrons are two jet events, with a fraction of order α_S of three jet events. Three-jet sensitive shape variables were used at LEP to carry out α_S measurements, and to test the basics of perturbative QCD. These tests were carried out by comparing experimental measurements with the NLO calculation of $e^+e^- \rightarrow 3 \text{ jets}$ of ref. [62]. In fig. 2, a Feynman graph for the three jet production process at the

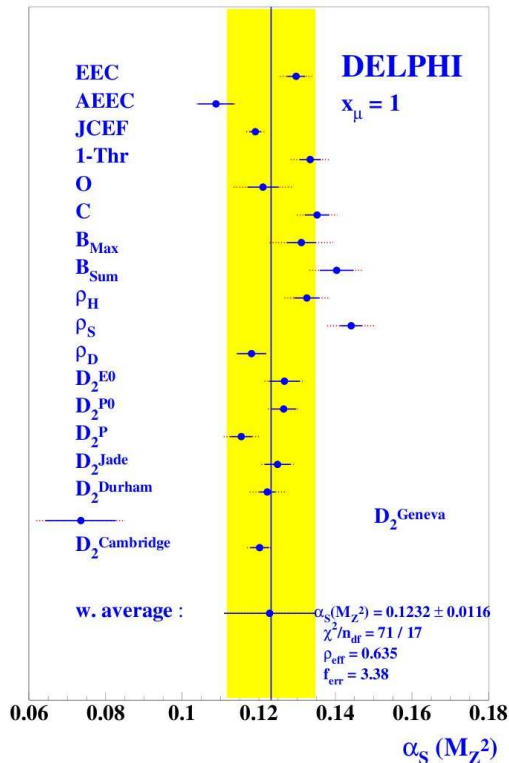


Figure 3. DELPHI results [63] for α_S measurements extracted using several shape variables, at the NLO level.

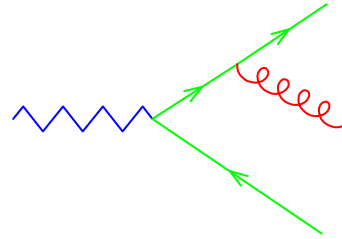


Figure 2. The LO elementary process for the calculation of $e^+e^- \rightarrow 3 \text{ jets}$.

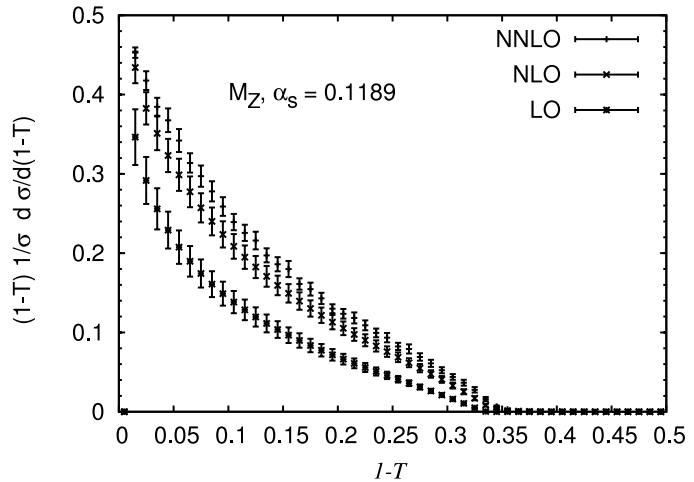


Figure 4. Scale dependence of the thrust distribution at LO, NLO and NNLO.

LO (leading order) level is shown. These kind of graphs give the leading contribution (of order α_S) to the shape variables that are used to measure the strong coupling. The computation of the NLO process requires the inclusion of a virtual gluon attached in all possible ways to the coloured lines of the LO graph, and the inclusion of the emission of an extra gluon. NNLO corrections require the inclusion of two virtual gluon corrections to the LO graph, one virtual gluon corrections to the graphs with the emission of an extra gluon, and the graphs with the emission of two extra gluons. In fig. 3, α_S determinations carried out at the NLO level by the DELPHI collaboration [63] are shown. The determinations performed using different shape variables do not really look consistent among each other, a consequence of the fact that theoretical errors were not included. If theoretical errors are included, for example by varying the renormalization scale in the calculation, the various determinations would become consistent. In other words, the spread among the various determinations is an indication of a large theoretical uncertainty, and on the existence of much room for improvement in the determinations by improving the theory alone.

At present it is not clear whether the theory uncertainty is due to unknown higher order terms or to power suppressed hadronization effects. If new NNLO calculation will lead to a spectacular improvement in the consistency of the various determination, we will conclude that perturbative calculation alone give a very good description of hadronic phenomena, and power suppressed effects are small. We expect that we will soon learn if this is the case.

In fig. 4, the uncertainty in the thrust distribution, obtained by varying the renormalization scale between $M_Z/2$ and $2M_Z$ is shown for the leading order, the next-to-leading order and the new next-to-next-to-leading order calculation. A full NNLO analysis of LEP data (not yet performed) is now possible.

5. Shower Monte Carlo and fixed order calculations

Shower Monte Carlo programs provide a description of strong processes that is complementary to the one given by fixed order calculation. In a Shower Monte Carlo, an arbitrary number of Feynman graphs is evaluated, but with a limited accuracy, i.e. including only the dominant component of the cross section. This corresponds to the regions where there are small angle emission of partons, and where soft gluon are emitted. While in fixed-order calculations one has to sum up processes with different final states in order to achieve the cancellation of infrared effects, in shower Monte Carlo these effects are resummed to all orders in perturbation theory. It follows then that in the Shower Monte Carlo it is possible to give an exclusive (rather than inclusive) description of the final states, provided that, besides the shower algorithm, some low energy model for hadron fragmentation is also included. Shower Monte Carlo programs play a key role in the analysis of collider experiments, and they will be even more fundamental at the LHC physics. These programs undergo continuous development, and grow in complexity with time. In order to improve the extensibility of the code, a considerable effort has gone into rewriting the old FORTRAN programs HERWIG [64] and PYTHIA [65] in c++ [66] [67] [65] [68].

5.1. Matrix element matching

Shower Monte Carlo generally fail for large angle emission, since they do not contain the exact matrix elements for large angle radiation. Similarly, they do not include exact NLO corrections, and so their accuracy does not normally go beyond LO. On the other hand, matrix element (tree level) calculations for complex processes are available today, and so are many NLO calculations for important processes. The question arises whether one can merge matrix element and shower approaches in such a way that the best features of both methods are retained. A first answer to this problem was given in the so called CKKW method, formulated in [69] for e^+e^- annihilation, and extended to hadronic collisions in [70]. In the CKKW prescription one introduces a separation scale, with the dimension of an energy. For radiation at transverse momenta below

this scale, the Shower Monte Carlo is used. For transverse momenta above this scale the exact tree level matrix elements are used. The CKKW approach, furthermore, gives a prescription for the assignment of the scales for the evaluation of the running coupling constants in the exact matrix elements. These scales have to match the scales used by the shower Monte-Carlo in the limiting configurations where the Shower algorithm is accurate. Furthermore, appropriate factors (called Sudakov form factors) must be supplied for the same reason. These corrections are motivated by the fact that tree level calculations, computed at fixed coupling constant, are valid only as long as all invariant are of the same order of magnitude, and the coupling is evaluated at a scale of the order of these invariants. If some invariant become small, also virtual corrections become important. The Shower algorithms are designed to include these dominant virtual corrections, precisely by appropriately choosing the scale of the coupling constant at the splitting vertexes, and by introducing Sudakov form factors. A matrix element computation to be matched with a shower Monte Carlo must also be corrected in the same way. The CKKW method also requires that the shower Monte Carlo, if not ordered in transverse momentum, must be capable to veto radiation with a transverse momentum below the separation cut.

It has become apparent that matrix element accuracy are needed at the LHC in order to give reliable estimated of important backgrounds (for an example, see [71]). Furthermore, matched matrix element results seem also to give a better description of TEVATRON data (for an example see ref. [72]). Because of these reasons, a considerable effort has gone in improving Shower and matrix element generators, in order to make it easier to interface them together. Most noticeably, a standard for interfacing parton showers and matrix element calculations have been set up [73]. New versions of the PYTHIA program [74] offer a new showering scheme that is more suited to matrix element matching.

The ALPGEN team have used a simplified matching method (the so called MLM matching) that does not require the calculation of Sudakov form factors [75]. In fig. 5, the W transverse

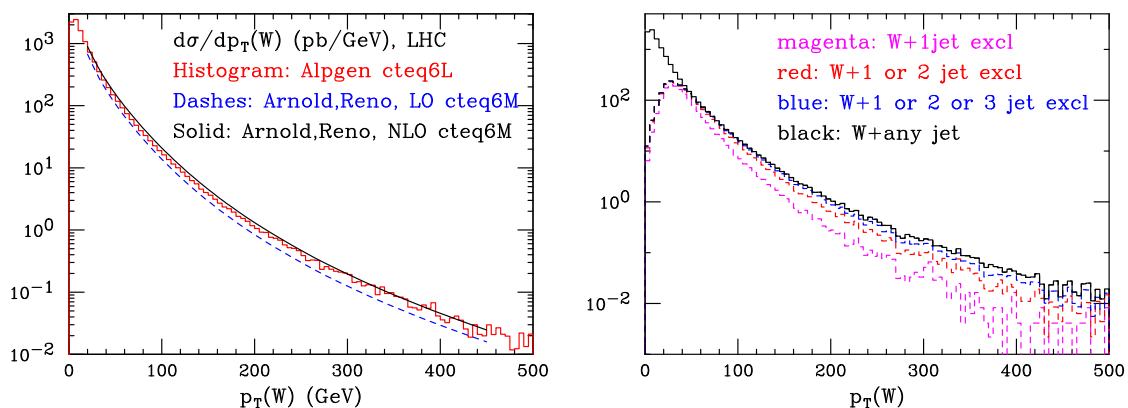


Figure 5. The W transverse momentum spectrum using ALPGEN, compared to the fixed order calculation. On the right, the composition of the inclusive distribution in terms of $W + n$ jets components.

momentum computed at LO, NLO and ALPGEN is shown. The ALPGEN result is also shown for $W + n$ jets, with $1 \leq n \leq 1, 2$ or 3 , in order to show the composition of the full inclusive cross section in terms of multijet samples. We emphasize that such results can only be obtained within a matched matrix-elements and parton shower calculation, since fixed order calculation alone only yield inclusive cross sections. In fact, for example, the $W + 1$ jet cross section in ALPGEN (on the right figure) is dumped at large p_T with respect to the LO calculation (dashes, left figure), and the dumping is due to the Sudakov form factors introduced in the matching procedures. We

also remark that the full ALPGEN result does not correctly predict the total spectrum, since it does not include NLO corrections, and is thus rescaled by a K factor.

In ref [76] a comparative study was carried out of the available matrix element merging schemes and programs. In particular, the following schemes and programs were considered: the CKKW scheme, as implemented in SHERPA [77] [78]; the ARIADNE program [79], with its variant of the CKKW method; the MLM scheme, as implemented in ALPGEN [6] [80]; the MLM procedure as implemented in MadEvent [81] [5]; the MLM procedure implemented in HELAC [9] [82]. The process $W + n$ jets was examined at both the Tevatron and the LHC. The different approaches are studied and compared, and the systematic effects within each approach are also considered. It is found that, although the methods considered differ widely from an analytical point of view, they show reasonable agreement among each other. Typically, the differences among the approaches is comparable to the size of the systematics, so that it is conceivable that they may be all successfully tuned to Tevatron or LHC data.

6. NLO+Shower

In view of the positive experience with NLO QCD calculations and with Shower Monte Carlo programs at e^+e^- and hadron colliders, the natural question arises whether we can merge in some way the two approaches. At the time of QCD tests at LEP this was not considered a very important issue. Convincing tests of QCD required a simple framework, with the least possible contamination of model dependent assumptions. Pure NLO calculations provide such framework. On the other hand, today we are more interested in QCD applications for the description of collider phenomena. Unlike the case of matrix element calculations, NLO results are not so readily available for complex processes, as we have already seen. Furthermore, NLO accuracy is not always needed, or, at least, there are processes for which NLO corrections are more important. A typical example is heavy flavour production. In order to describe realistically heavy flavour production using standard Shower Monte Carlo tools, one has to combine three different production mechanisms: gluon fusion, gluon splitting and flavour excitation, where the last two processes are sensitive to some cutoff parameters that have to be tuned. All these mechanisms arise at NLO, so that, with an NLO Monte Carlo, one would be able to give a much simpler description of the process. In general, NLO accuracy is desirable for very common LHC processes, that are either very important for physics (like vector bosons or top production) or for calibration (like simple processes involving jets).

When trying to merge NLO calculations with Shower Monte Carlo, one is facing a delicate overcounting problem. The Monte Carlo sums QCD correction at all orders in the coupling constant. Thus, it includes its own NLO correction, that is only accurate in certain kinematic regions. An NLO calculation includes the exact NLO matrix element, and thus includes also the contribution computed by the Shower Monte Carlo. A first approach to the problem, that helped to better focus the requirement of an NLO+showers implementation, was given in [83]. The first acceptable solution to the overcounting problem is the so called MC@NLO approach [84] [85] [86], that is now available for several collider processes [87]. In essence, in the MC@NLO approach one subtract the parton shower approximate NLO correction from the exact one, and uses this difference as a correction process to the shower LO process. This difference is regular, if the shower Monte Carlo reproduces exactly the collinear and soft singularities of the emission amplitude. It may be negative, so that in this approach one cannot avoid negative weighted events. Several other proposals have appeared in the literature. In refs. [88] [89] schemes suitable for e^+e^- annihilation are discussed. In [90] [91] an approach based upon effective theories is discussed. In [92] an attempt to formulate a shower algorithm with full colour and spin interference effects is given. The authors of [92] [93] [94] propose to formulate the shower in such a way that the NLO terms in the Monte Carlo correspond to the subtraction terms in popular subtraction methods for NLO calculations, so that the implementation of MC@NLO

type of corrections would be easier in these frameworks.

In ref. [95] a radically different proposal for NLO+shower was made. According to this method, (called POWHEG, for Positive Weights Hardest Emission Generator) the hardest emission in the shower should be performed as the first emission, with full control over the NLO accuracy, and according to an algorithm that avoids the appearance of negative weights. The shower Monte Carlo program to which the NLO calculation is matched is only required to continue the showering with transverse momenta smaller than the transverse momentum of the first emission. Mismatch for shower Monte Carlo programs that do not adopt transverse momentum ordering, like HERWIG and the older showering scheme of PYTHIA, should be dealt with appropriately, although some studies have shown that it is unlikely that this mismatch will lead to important effects. The proposal of [95] has been implemented in [96] for ZZ production in hadronic collisions, in [97] for heavy flavour production, and in [98] for e^+e^- annihilation. Furthermore, a comprehensive description of the method was given in ref. [99]. Besides being the only method that yields positive weights, the POWHEG method has the advantage that it fully separates the NLO part of the calculation from the showering algorithm. Thus, the POWHEG implementation of a process can be carried out without any reference to a particular showering program, and the resulting program can be interfaced to any Shower Monte Carlo. In fact, the implementations of refs. [96] and [97] have been interfaced to both HERWIG and PYTHIA.

At present, only the MC@NLO and POWHEG approaches have produced useful implementations of hadron collider processes. Comparisons of ZZ distributions obtained with the two methods have shown remarkable agreement [96]. Distributions for the $t\bar{t}$ pair generators also display a remarkable consistency (see fig. 6), and only minor discrepancies. On the other

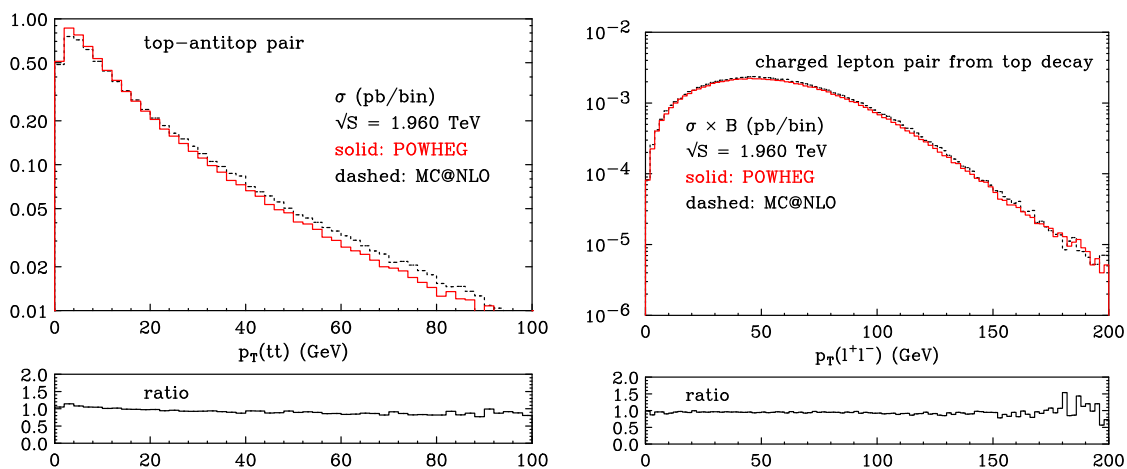


Figure 6. Comparison of MC@NLO and POWHEG for $t\bar{t}$ production (and semileptonic decay) at the Tevatron.

hand, for bottom pair production there are non-negligible differences, as shown in fig. 7.

One expects that the MC@NLO and POWHEG approaches should agree up to corrections of order α_S^4 . In particular, when looking at distributions that are dominated by effects of order α_S^3 , like the distribution of an extra jet, we expect relative differences of the order of 10%. In ref. [100] a detailed comparison of ALPGEN and MC@NLO was carried out. In principle, the two codes should have comparable accuracy for heavy flavour production in association with a jet. A difference was found in the shape of the differential distribution in the rapidity of the emitted jet, MC@NLO showing a dip that is not present in ALPGEN. The dip is an effect of the order of 10% in magnitude, and thus can be attributed to higher order effects. POWHEG confirms the ALPGEN result, showing no dip, as can be seen in fig. 8. This issue is still under

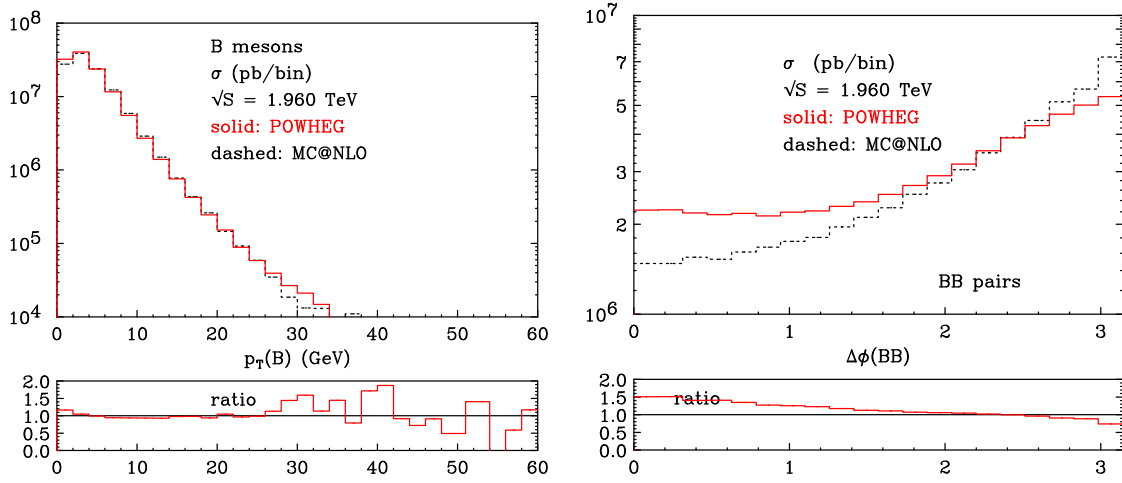


Figure 7. Comparison of MC@NLO and POWHEG for $b\bar{b}$ production at the Tevatron.

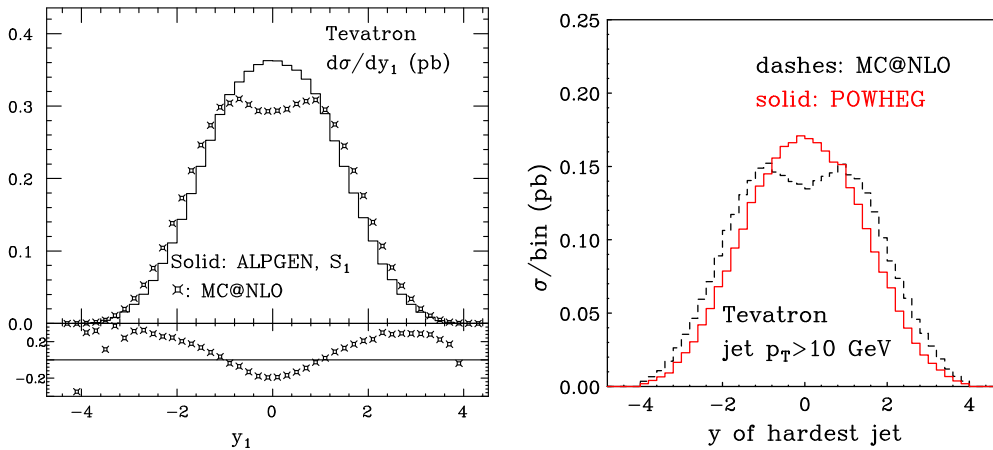


Figure 8. Rapidity distribution of the hardest jet: MC@NLO versus ALPGEN and POWHEG.

study.

7. Conclusions

Much theoretical activity in perturbative QCD has taken place in the past few years. New developments in techniques for tree-level and one-loop calculation are being studied, in preparation for LHC physics. Furthermore, new developments in Shower Monte Carlo algorithms have taken place, with a remarkable increase of interest in the theoretical community. These new developments are changing the way experimental collaborations prepare their simulated data samples, and will deeply influence the analysis of experimental data.

The applicability of two-loop calculations to LHC physics is still quite limited to a handful of simple processes. However, this year, the completion of the calculation of the e^+e^- jet cross sections at two-loop opens up new perspectives in NNLO developments. Thanks to this calculation, it has in fact become possible to test perturbative QCD at the two loop level in a complex framework, involving several different distributions, where it should be possible to clearly see whether an increase in the level of consistency of the different measurements is achieved.

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