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On parametrization of neutrino mass matrix

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Discussed the experimental situation and parametrization of neutrino mass matrix in connection with the neutrino properties.

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1. Introduction: experimental status

The discovery of neutrino oscillations was reported by several experiments [1–4]. This event renews a great interest to neutrino properties and mixing [5]. Neutrino oscillation explains so called solar neutrino puzzle. The oscillations of neutrino species may be caused by non-diagonality of mass matrix in interaction basis of weak processes and by interactions with matter [6]. Short range experiments point on mass matrix as on the origin of observed oscillations.

First of all we briefly look at experimental data from SNO experiment [7]. This experiment is sensitive to neutrino flux from ^8B reaction with energy about 6-7 GeV. The detector can register three reactions:

$$\begin{aligned}
\nu_e + d \to e^- + p + p, \qquad &=> \qquad \phi(\nu_e),\\
\nu + d \to \nu + p + n, \qquad &=> \qquad \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau),\\
\nu + e^- \to \nu + e^- \qquad &=> \qquad \phi(\nu_e) + \frac{\phi(\nu_\mu) + \phi(\nu_\tau)}{6.5},
\end{aligned}$$

where in second column indicated the result of corresponding measurement, $\phi(\nu_\alpha)$ denotes the flux of neutrino of α-type. The comparison of SNO results with SSM (Standard Solar model) looks as

$$\text{SNO:} \qquad \frac{\phi(\nu_e)}{\phi(\nu_e)+\phi(\nu_\mu)+\phi(\nu_\tau)} = 0.340 \pm 0.023(\text{stat}) \begin{matrix}+0.029\\-0.031\end{matrix} (\text{syst});$$

$$\text{SNO:} \ \ \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = [4.94 \pm 0.21(\text{stat}) \begin{matrix}+0.38\\-0.34\end{matrix} (\text{syst})] \times 10^6 cm^{-2}s^{-1};$$

$$\text{SSM:} \ \ \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = [(5.49 \begin{matrix}+0.95\\-0.81\end{matrix}) - (4.34 \begin{matrix}+0.71\\-0.61\end{matrix})] \times 10^6 cm^{-2}s^{-1};$$

and shows that

- only one third of total neutrino flux is a one of electron neutrino (first row), which indirectly support the existence of neutrino oscillations,

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- the prediction of SSM (third row) agrees well with SNO experimental data (second row),
- so, we obtain a possibility to determine more precisely the parameters of SSM from SNO results.

The resent results of KamLAND [8] demonstrates fine graphical proof of oscillations.

Most of experimental data may be summarized by next fundamental parameters, which depend upon parametrization of mixing matrix and interpretation of data (experimentalists are using very often simple two-particles mixing picture) [9]:

$$\begin{aligned} \sin^2(2\theta_{12}) &= 0.86 \begin{array}{c} +0.03 \\ -0.04 \end{array}, \\ \Delta m^2_{21} &= (8.0 \pm 0.03) \times 10^{-5} eV^2; \\ \sin^2(2\theta_{23}) &> 0.92, \\ \Delta m^2_{32} &= (1.9 to 3.0) \times 10^{-3} eV^2; \\ \sin^2(2\theta_{13}) &< 0.19. \end{aligned} \tag{1}$$

We will define the above parameters in the next parts.

2. Masses and mixings

Let's start from several definitions:

- (a) Neutrino has only weak interaction of the form (we use Pauli metric with imaginary time)

$$\begin{aligned} \mathcal{L}^{\rm CC}_{g\ell\nu} &= \frac{ig}{\sqrt{2}} W^+_\mu \bar{\nu}_\ell \gamma_\mu \frac{1+\gamma_5}{2} \ell + \frac{ig}{\sqrt{2}} W_\mu \bar{\ell} \gamma_\mu \frac{1+\gamma_5}{2} \nu_\ell, \\ \mathcal{L}^{\rm NC}_{g\nu} &= \frac{ig}{2cos\theta_W} Z_\mu \bar{\nu}_\ell \gamma_\mu \frac{1+\gamma_5}{2} \nu_\ell. \end{aligned} \tag{2}$$

- (b)Dirac bispinor fermion field is a sum of two Weyl spinors:

$$\psi = \frac{1+\gamma_5}{2}\psi + \frac{1-\gamma_5}{2}\psi = \psi_L + \psi_R = \begin{pmatrix} \xi \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \eta \end{pmatrix}, \tag{3}$$

  where last expression take place in spinor representation. So, only half of dirac neutrino bispinor components participate in SM interactions.

- (c) Dirac bispinor fermion field is a sum of two Majorana fields:

$$\begin{cases} \psi = \frac{1}{\sqrt{2}}(\chi_\xi + \gamma_5 \chi_\eta), \\ \psi^C = \frac{1}{\sqrt{2}}(\chi_\xi - \gamma_5 \chi_\eta); \end{cases} \qquad \begin{cases} \psi = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2), \\ \psi^C = \frac{1}{\sqrt{2}}(\chi_1 - i\chi_2); \end{cases} \qquad \chi^C = \chi. \tag{4}$$

  Here we use definition $\psi^C = C\psi^*$, where matrix C has the next properties $C\gamma^*_\mu C^{-1} = \gamma_\mu(\eta_\mu)$, $C\gamma^T_\mu C^{-1} = -\gamma_\mu(\eta_\mu)$ and $\eta_\mu = diag(\eta_{\mu\nu}) = (1,1,1,-1)$ is a sign of using 4-d imaginary component.

- So, weak interaction does not distinguish Majorana and Dirac fermions. Neutrino may be Dirac or Majorana fermion. This give us more possibilities for the mass term.

Even for a single neutral Dirac field we can write down two mass terms :

$$L_D^{mM} = im\bar{\psi}\psi + i\frac{M}{2}[\bar{\psi^C}\psi + \bar{\psi}\psi^C] = i\frac{M_\xi}{2}\bar{\chi_\xi}\chi_\xi + i\frac{M_\eta}{2}\bar{\chi_\eta}\chi_\eta, \tag{5}$$

where the first term is named "Dirac mass" and the second one – "Majorana mass". It is evident that standard electromagnetic phase transformation have place only for Dirac term. Majorana mass term can exists only for electrically neutral fermion. This Lagrangian in terms of Majorana fields (4) becomes diagonal, masses of Majorana fermions are the following:

$$m = \frac{M_\xi - M_\eta}{2}, \qquad M = \frac{M_\xi + M_\eta}{2}. \tag{6}$$

These relations give rise to so called "see-saw mechanism": two very heavy Majorana fermions manifest themselves at low energy as Dirac fermion with mass equal to splitting of the initial masses.

The reason of such decomposition of Dirac field is its bispinor character, i.e. Dirac bispinor is a reducible representation of spinor group (locally isomorphic to Lorenz group), it is a direct sum of fundamental representation and conjugated one. Majorana field may be treated as fundamental representation of extended spinor group, which has one additional operator - complex conjugation. Of course, this additional operator has no direct manifestation in real Lorenz group and is connected with charge conjugation in field theory.

Taking into account that Dirac and Majorana fermions can not be distinguished by weak interaction it is more "economical" to treat neutrino as Majorana field. This also corresponds to Occam's principle: do not introduces new essences.

In case of three fermion family the general mass matrix has the next form:

$$M = \begin{pmatrix} m_{\nu e} & ae^{i\vartheta_a} & ce^{i\vartheta_c} \\ ae^{-i\vartheta_a} & m_{\nu\mu} & be^{i\vartheta_b} \\ ce^{-i\vartheta_c} & be^{-i\vartheta_b} & m_{\nu\tau} \end{pmatrix} = M^+, \tag{7}$$

and defined by 9 parameters. The hermiticity requirement on mass matrix is a sequence of common demand of Lagrangian hermiticity.

3. Quark mixing matrix — Dirac fermion case

For the charged fermions we may use only Dirac fields. In case of Dirac fermions (quarks) we can choose the relative phases of the fields to compensate two of three phase parameters. One of phase parameters can not be canceled by global phase redefinition and is a reason of $CP-$ violation in quark sector.

Then we can transform these fields by unitary transformation

$$\psi_\alpha = V_{\alpha i}\psi'_i, \qquad \alpha = (d, s, b); \quad i = (1, 2, 3), \tag{8}$$

to the diagonal form, i.e. from general mass Lagrangian

$$-2i\mathcal{L}_\nu^m = \sum_{\alpha,\alpha'} \bar{\psi}_\alpha M_{\alpha,\alpha'}\psi_{\alpha'}, \tag{9}$$

we obtain the next simple form

$$-2i\mathcal{L'}_\nu^m = \sum_i \bar{\psi}_i m_i \psi_i, \qquad M_\nu^{diag} = \text{diag}(m_1, m_2, m_3). \tag{10}$$

For a free fields this transformation means nothing, but for interacting fields it produces observable mixing parameters. This transformation was proposed by M.Kobayashi and T.Maskawa [10] as generalization of Cabibbo quark mixing in the next form:

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3 - \cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}. \quad (11)$$

It is well known under the name of CKM quark-mixing matrix.

Now this matrix is often used in parametrization independent form

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad (12)$$

or in next standard form of CKM quark-mixing matrix [9]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (13)$$

where $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$ and δ is the $CP-$violating phase. It is evident that any parametrization of this kind is not unique and may be changed without any influence on physics. After more precise measurement of a matrix element you have to ensure the consistency of parametrization, e.g. by method of unitary triangles.

Taking into account the numerical values of CKM-matrix it was proposed [11] more transparent parametrization using decomposition in series on parameter λ

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \quad (14)$$

where parameters are $\lambda = 0.2257 \begin{matrix} +0.0009 \\ -0.1110 \end{matrix}$, $A = 0.814 \begin{matrix} +0.021 \\ -0.022 \end{matrix}$, $\bar{\rho} = 0.135 \begin{matrix} +0.031 \\ -0.016 \end{matrix}$, $\bar{\eta} = 0.349 \begin{matrix} +0.015 \\ -0.017 \end{matrix}$.

4. Neutrino mixing matrix — Majorana fermions?

Because we still do not know is neutrino a Dirac or Majorana fermion we have to take into account both cases. If neutrino fields are Dirac origin, we may use the same parametrization as in quark case. If neutrino are Majorana fermion, we have to extend parametrization. First of all Majorana particles have no phase invariance, the particle coincides with antiparticle. In Majorana representation of $\gamma-$matrices the bispinor describing the fermion is real (not complex as in Dirac case) function or operator. Thereby such particle has no electromagnetic interactions.

For mixing matrix (7) we have no possibility by choosing relative phases to cancel $CP-$violating phases and have to take into account all three phases.

Solar and atmospheric neutrino experiments have shown that neutrino oscillations are due to a mismatch between the flavor and mass eigenstates of neutrinos. The relationship between these eigenstates is given by

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle, \qquad |\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle,$$

where $|\nu_\alpha\rangle$ is a neutrino with definite flavor ($\alpha = e$ (electron), μ (muon) or τ (tau)); $|\nu_i\rangle$ is a neutrino with definite mass ($i = 1, 2, 3$).

$U_{\alpha i}$ represents the Pontecorvo-Maki-Nakagawa-Sakata matrix (shortly it is called the "PMNS matrix", "neutrino mixing matrix"). It is the analogue of the CKM matrix for quarks.

When the standard three neutrino theory is considered, the matrix is 3×3. If only two neutrinos are considered, a 2×2 matrix is used. If one or more sterile neutrinos are added it is 4×4 or larger. In the case of 3×3 form, it is given by:

$$\begin{aligned} U = & \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ = & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ = & \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (15)$$

where $s_{12} = \sin\theta_{12}$, $c_{12} = \cos\theta_{12}$, etc. The phase factors α_1 and α_2 are non-zero only if neutrinos are Majorana particles. It's important to notice that phase parameters have no any influence on oscillation phenomena. So, phases are completely undefined now.

5. Vector parametrization of mixing matrix

Neutrino is created in weak interaction process in the couple with lepton of one of three known generations and belongs to one of three weak isotopical doublets $\nu_{L\alpha}$, where index $\alpha = e, \mu, \tau$ has one of three meanings. However, mass matrix of interacting neutrino has not be diagonal one $\mathcal{L}_m^0 = \frac{i}{2}\sum_{\alpha,\alpha'} \bar{\chi}_\alpha M_{\alpha,\alpha'} \chi_{\alpha'}$. The presence of non diagonal terms means the absence of lepton number conservation.

The further propagation of created neutrino to detector is a movement of free particle, and is described as movement of definite mass particle. So, for the right description of neutrino propagation we have to transform neutrino field in diagonal mass basis by the unitary transformation (unitarity is needed to not disturb the kinetic part of Lagrangian).

The transformation $\nu_\alpha = U_{\alpha i}\chi_i$ has to be unitary in general. However, in case of Majorana fermions, which may be treated as real bispinors (this is easy to see in real Majorana representation of Dirac matrices, therefore, this property has to be valid in any other representation), it's more natural to limit ourself by orthogonality condition on matrix U: $U^T U = 1$. This assumption decreases the number of parameters to 3 angles and is a step from unitary to orthogonal group. Of course, we neglect all CP-violation phases by this assumption, but Majorana neutrino by its nature does not respect many conservation laws.

We propose [12] to use for neutrino and quarks mixing matrix parametrization the vector-parameter of rotational group $SO(3)$ by one 3-dimensional vector-parameter $\vec{\rho}$, proposed by Gibbs, reopened and developed by F.I. Fedorov [13]. In this case to every 3-dimensional vector $\vec{\rho}$ corresponds orthogonal matrix:

$$O(\vec{\rho}) = \frac{1 - \vec{\rho}^2 + 2\vec{\rho}\cdot\vec{\rho} + 2\vec{\rho}^{\times}}{1 + \vec{\rho}^2} =$$
$$= \frac{1}{1+\vec{\rho}^2}\begin{pmatrix} 1+\rho_1^2-\rho_2^2-\rho_3^2 & 2(\rho_1\rho_2-\rho_3) & 2(\rho_1\rho_3+\rho_2) \\ 2(\rho_2\rho_1+\rho_3) & 1+\rho_2^2-\rho_1^2-\rho_3^2 & 2(\rho_2\rho_3+\rho_1) \\ 2(\rho_3\rho_1+\rho_2) & 2(\rho_3\rho_2-\rho_1) & 1+\rho_3^2-\rho_1^2-\rho_2^2 \end{pmatrix}, \qquad (16)$$

and to every orthogonal matrix O corresponds the 3-dimensional vector-parameter $\vec{\rho}$, obtained by the next prescription:

$$\vec{\rho}^{\times} = (\varepsilon_{aib}\rho_i) = \begin{pmatrix} 0 & -\rho_3 & \rho_2 \\ \rho_3 & 0 & -\rho_1 \\ -\rho_2 & \rho_1 & 0 \end{pmatrix} = \frac{O - O^T}{1 + Tr(O)}, \qquad (17)$$

$$\rho_1 = (\vec{\rho}^{\times})_{32}, \quad \rho_2 = (\vec{\rho}^{\times})_{13}, \quad \rho_3 = (\vec{\rho}^{\times})_{21}, \qquad (18)$$

based on the following relations

$$1 + Tr(O) = \frac{4}{1+\vec{\rho}^2}, \qquad O - O^T = \frac{4\vec{\rho}^{\times}}{1+\vec{\rho}^2}. \qquad (19)$$

This parametrization has additional advantage, vector-parameter has physical meaning: direction of vector $\vec{\rho}$ coincides with the direction of rotation, defined by transformation O, and the value $|\vec{\rho}|$ is defined by the magnitude of rotation angle α: $|\vec{\rho}| = \tan\frac{\alpha}{2}$. This parametrization also is coordinate-independent.

For quark we have very small value of phase δ, so the new parametrization will be a good approximation. As is known, average experimental values of quark mixing matrix [9] are:

$$V = (V_{ff'}) = \begin{pmatrix} 0.9745 & 0.224 & 0.0037 \\ 0.224 & 0.9737 & 0.0415 \\ 0.0094 & 0.040 & 0.9991 \end{pmatrix}, \qquad (20)$$

where $CP-$violation phase is so small that it even not written down, in this world-average matrix elements. It's easy to obtain the numerical value of vector-parameter for quark mixing matrix : $\vec{\rho} = (-0.00038, -0.00144, 0)$.

For the neutrino mass matrix (besides of analogous to quark matrix) people use the next parametrization, which mainly takes into account experimental data:

$$U = \begin{pmatrix} c & s & s_{13} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \qquad (21)$$

where $s = \sin\theta_{sun} \approx \sqrt{0.3}$, $c = \cos\theta_{sun}$, $s_{13} = \sin\theta_{13} \approx 0$. ($CP-$violation phase will be introduced if one find that neutrino is Dirac fermion, and particle is not coincide with antiparticle.) It's easy to obtain corresponding vector-parameter:

$$\vec{\rho} = \left(-\frac{1}{1+\sqrt{2}}, -\frac{s - s_{13}}{(1+\sqrt{2})(1+c)}, -\frac{s}{(1+c)}\right) = \qquad (22)$$
$$= (-0.414, -0.124 + 0.226 s_{13}, -0.298) \approx$$
$$= (-0.414, -0.124, -0.298).$$

So, vector parametrization of rotational group may be used for Majorana neutrino mixing matrix and for Dirac quark mixing matrix. Of course, to take into account additional $CP-$violation we have to expand this transformation by including $CP-$violation phase.

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