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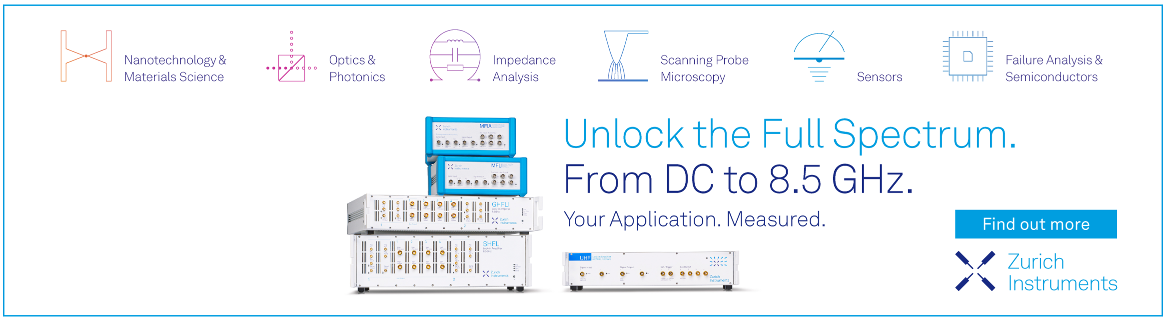

SUCCESS IN ONE-TURN MAPS FOR DYNAMIC APERTURE STUDIES—A BRIEF REVIEW[1]

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Abstract

Progress in the use of one-turn maps for long-term tracking of particles—useful in dynamic aperture studies on large circular accelerators—is reviewed briefly. It is recommended that long-term tracking of particles for large circular accelerators such as the Superconducting Super Collider (SSC) or its high-energy booster be performed with one-turn maps. The advantages are twofold: not only is tracking speed dramatically enhanced (by about two orders of magnitude for the SSC), but one-turn-map tracking also offers an easier reliability check of the tracking results and easier order-by-order analysis.

Since my last comment on one-turn maps for long-term tracking,[1] there has been more progress in this field. Indeed, it is time to recommend and encourage our colleagues to consider using one-turn maps for particle orbit advancement, especially for long-term tracking of large circular accelerators. In this letter the author wishes to give a very brief review of the progress of the one-turn map concept for tracking.

Given the advantage that the Lie transformations can be concatenated (to form a one-turn map) via the Campbell-Baker-Hausdorf theorem,[2] I believe that the concept of using one-turn maps for particle tracking in circular accelerators should have been planted in Dragt's mind since he and his associates introduced the Lie-algebraic treatment of beam dynamics in the mid-1970s.[3] However, one-turn maps for particle tracking, particularly for long-term tracking, were not seriously tried—at least not for practical cases—until the late 1980s. It was then that Alex Chao headed the accelerator physics division of the Universities Research Association (URA)/Superconducting Super Collider (SSC) Central Design Group (CDG) at Berkeley. Thanks to Chao's encouragement and collaboration, several individuals were able to get together at Lawrence Berkeley Laboratory for a period of time (a review paper was written that summarize work done during that period).[4] Berz introduced the computational use of differential algebra, which allows computational extraction of one-turn maps more easily and in a more general way than before.[5] Forest, having been associated with Dragt, was able to see the advantage of using these differential algebras for computational Lie-algebraic formulations and to develop the Lie-algebraic normal form in collaboration with Berz and Irwin.[6] After performing the long-term dynamic aperture

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studies for the SSC with the time-consuming (both computer time and human time) single-particle trackings, Irwin and I knew that in order to achieve a more complete study of the dynamic apertures of large circular accelerators such as the SSC, advancements in tracking techniques must be made. I went on to take advantage of the supercomputer vectorization and parallelization (multi-tasking) capabilities for multi-particle tracking and was able to create "Livingston dynamic aperture plots."[7] (The name was changed to "survival plots" at Richard Talman's suggestion.) Meanwhile, Irwin factorization was developed,[8] enabling one to convert a series of Lie transformations of different orders into a reasonable number of kicks and rotations for symplectic tracking. Forest, even though it was against his philosophy of not using one-turn maps for tracking, was kind enough to collaborate with me in programming the Irwin factorization, hoping to make the study of the SSC dynamic aperture more efficient. About the same time, some of the Zlib[9] vectorized subroutines were developed, making possible the programming of Zmaps[10] for extracting high-order, one-turn maps of the SSC.

Tracking with Irwin factorization for the SSC dynamic aperture study was tried during the transition period in which the CDG was merged with the SSC Laboratory; it was not quite successful. Part of the reason, it was suspected, was that the Irwin factorization bases were chosen randomly instead of by following Irwin's description that would have helped minimize the high-order spurious terms. However, for various reasons this effort was not continued.

In the spring of 1990, the issue of whether a one-turn map can be used for long-term tracking and thus for dynamic aperture study was raised at the Capri nonlinear-beam dynamics workshop.[11] This topic attracted many colleagues to a somewhat controversial discussion, but no conclusion was reached because of a lack of sufficient practice. After the workshop, we tested an 11th-order Taylor map of the SSC by advancing the phase-space orbits of particles turn-by-turn via direct evaluation of the truncated 11th-order Taylor map (not exactly symplectic due to truncation). We found that the survival plot was roughly the same as that of the previous results with element-by-element tracking.[12] This was the first time, to my knowledge, that the one-turn map showed some promise for dynamic aperture study, although there were still some concerns about the non-exact symplecticity. The same one-turn map was also tested with 10th-order Taylor-map tracking, resulting in somewhat different survival plots. However, the 10th-order Taylor map—after it had been Lie-transformed (by Dragt-Finn factorization)[3] and re-expanded into an 11th- or 12th-order Taylor map to gain a higher degree of symplecticity—showed correct dynamic aperture up to 10^6 turns.[13] What we have learned from these results is that a moderate-order (lower than 11th-order), one-turn differential algebraic Taylor map is usually accurate enough for dynamic aperture studies, but its degree of symplecticity is usually not enough for long-term tracking.[14] The wrong survival plots obtained with the direct Taylor-map tracking of 10th order are due not to inaccuracy of the maps but to artificial diffusion of the particle orbits because of the lack of sufficient symplecticity. How to symplectify the Taylor map without imposing large spurious errors in the map becomes the key to success when using one-turn maps for long-term tracking.

An accurate symplectification method for a truncated one-turn Taylor map is a series of order-by-order Lie transformations called Dragt-Finn factorization.[3] Unfortunately, nobody yet knows how to directly evaluate a Lie transformation in general. Therefore, re-formation of a series of order-by-order general Lie transformations into a special kind of Lie transformation that can be evaluated has attracted a great deal of effort. The methods will, in general, induce high-order spurious terms, thereby reducing the accuracy of the map. Irwin factorization, mentioned above, is one of these symplectification methods. Monomial factorization is another worth mentioning.[15] Since neither method guarantees no symmetry violation, the symplectified map will, in general, preserve a strong image imposed by how the method is used. Therefore the map, although symplectic, can be inaccurate for dynamic aperture study unless it is extended to an order higher than what one would consider adequate. Recently, we have been investigating a method for the evaluation of Lie transformations that will take care of the symmetry property. It is hoped that this method is to be more accurate than both the Irwin factorization and the monomial factorization at the same order.[16] However, before we can know how useful this method is for long-term tracking and thus for dynamic aperture study, it must be tested for practical cases. Such testing will be performed once a numerical program has been completed.

Before I go on to review another accurate symplectification method, I wish to acknowledge the efforts contributed by colleagues in Europe.[17] They have recently tested an interesting method of evaluating one-turn Taylor maps with dynamic rescaling, using linear corrections to suppress the artificial diffusion due to non-exact symplecticity of the truncated Taylor map.[18] Although not mathematically rigorous, this method, in my opinion, can be used for the study of certain cases, especially given the fact that accelerators are not exactly static.

Another symplectification method for "order-by-order symplectic" but truncated Taylor maps (thus not exactly symplectic) is the use of generating functions for converting the explicit truncated Taylor maps into implicit ones. Since symplecticity means canonical transformation, it is very likely that many colleagues may have considered the possibility of using generating functions for symplectification. For example, we know Dragt and his associates have considered such a method.[19]

Evaluation of these implicit maps can lead to numerical instabilities. Large spurious errors can also be generated if one imposes a certain form (not type) of generating functions. It is, perhaps, due to these drawbacks that such an implicit method has been overlooked in the past.

Recently, Yan, Channell, and Syphers have considered, tested, and successfully used an implicit Taylor-map tracking scheme.[20] First, the one-turn map is separated into two maps, a linear Courant-Snyder matrix followed by a nonlinear truncated Taylor map to enhance numerical stability. Then, differential algebras are used to convert the truncated nonlinear Taylor map into an implicit type of truncated Taylor map without imposing a certain form of generating function. (Of course, one of the four types of generating functions exists implicitly.) Because this method does not impose a pre-determined form (not type) of generating function, it provides the same degree of accuracy as the explicit Taylor map at the same order, and it has shown much success in practical use. It has been used to save enormous amounts of computer time for long-term tracking of the SSC and its high-energy booster.[21] There are two advantages to using such a one-turn-map tracking scheme over the traditional element-by-element tracking scheme: it not only provides faster tracking speed (about two orders of magnitude faster for the SSC), thus allowing lifetime tracking of large circular accelerators, but it also provides easier order-by-order analysis, thus allowing easier reliability checks of the global results such as the survival plots. Note that due to slow tracking speed, one of the drawbacks of element-by-element tracking is that one usually assigns nonlinear multipole errors up to certain orders without checking whether they are adequate. (Although one-turn-map tracking cannot be more accurate than element-by-element tracking, due to fast tracking speed, tracking with various orders can be easily performed to make sure that the order of the map and the order of multipole errors are indeed adequate.)

In summary, since one is interested only in phase-space regions where one-turn Taylor maps converge, it is fine to use one-turn maps not only for order-by-order analysis but also for turn-by-turn tracking. It is especially economical to use one-turn maps for long-term tracking of large circular accelerators. In certain cases, dynamical (time-dependent) one-turn maps are also workable.[22]

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