

EXACT SOLUTIONS FOR A VECTOR MESON IN QUANTIZED FIELD OF A MONOCHROMATIC ELECTROMAGNETIC PLANE WAVE

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The exact solutions for a vector meson in a quantized plane-wave external field are obtained. The expressions for 4-momentum of the particle and wave system are derived for each of three particle spin states.

The exact solutions of the Klein-Gordon and Dirac equations for particles in a quantized field of a monochromatic electromagnetic plane wave with linear polarization have been found for the first time by Berson [1]. Later these solutions have been generalized to the case of an arbitrarily polarized electromagnetic wave [2]. Some different aspects of the problem have been considered in Refs [3–5].

One encounters great mathematical difficulties trying to obtain the exact wavefunctions describing particles with higher spins in an external electromagnetic field. The covariant method [6, 7] for deriving the wavefunctions of particles with arbitrary spin in a classical electromagnetic plane-wave field enables us to avoid, to a certain extent, the difficulties in solving similar problems for a quantized electromagnetic field. The present authors used this method to obtain the wavefunctions for a vector meson (spin 1) in a quantized field of a linearly polarized monochromatic electromagnetic plane wave.

We start by considering (see [8]) the equation

$$\left[\beta_l (\partial_l - ie_0 A_l) + \frac{i}{2} (\kappa_1 P + \kappa_2 \bar{P}) \beta_{ln} F_{ln} + m \right] \psi = 0. \quad (1)$$

Here β_l are the Duffin-Kemmer matrices, $\beta_{ln} = \beta_l \beta_n - \beta_n \beta_l$ and κ_1, κ_2 , respectively, are the particle parameters expressed in terms of the static anomalous magnetic moment (AMM) and electric quadrupole moment (EQM); P and $\bar{P}$ are the projective operators denoting, respectively, the vector and tensor parts of the 10-component wavefunction ψ ,

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$\partial_l = \partial/\partial x_l$, $x = (x_l) = (x, it)$, $l = 1, 2, 3, 4$. The vector potential A_l satisfies the Lorentz gauge $\partial_l A_l = 0$ and is expressed through the creation c^+ and annihilation c^- operators of photons with the 4-momentum $k = (\mathbf{k}, ik_0)$ and polarization e' :

$$A = \frac{e'}{(2k_0 V)^{1/2}} (c^- e^{i\varphi} + c^+ e^{-i\varphi}), \quad \varphi = kx = k_l x_l, \quad k^2 = 0. \quad (2)$$

The $c^\pm$ operators in (2) are taken in the coordinate representation [9]

$$c^\pm = (1/\sqrt{2}) (\xi \pm \partial_\xi).$$

According to [1] the wavefunction is defined as follows

$$\psi(x, \xi) = U\chi, \quad U = \exp \{i[qx + \frac{1}{2} \varphi(\partial_\xi^2 - \xi^2)]\}, \quad (3)$$

where q is a constant 4-vector. We use (1) to derive an equation for the unknown function $\chi(\xi)$

$$\hat{k}\chi'' + 2i(\kappa_1 P + \kappa_2 \bar{P}) [\hat{k}\hat{a}]\chi' - 2i(i\hat{c} + m)\chi = 0 \quad (4)$$

with the help of the operator identity

$$U^+ (\xi \pm \partial_\xi) U = e^{\mp i\varphi} (\xi \pm \partial_\xi), \quad U^+ = \exp \{-i[qx + \frac{1}{2} \varphi(\partial_\xi^2 - \xi^2)]\}.$$

Here

$$\begin{aligned} [\hat{k}\hat{a}] &= \hat{k}\hat{a} - \hat{a}\hat{k}, \quad \hat{a} = a_l \beta_l, \quad a = e'/(k_0 V)^{1/2}, \\ c &= b - \frac{1}{2} \xi^2 k, \quad b = q - e_0 \xi a, \quad \chi' = d\chi/d\xi. \end{aligned}$$

The function χ as well as the result of action of an arbitrary operator $\hat{r}$ can be written in the form [7]

$$\chi = \begin{pmatrix} u \\ \alpha \end{pmatrix}, \quad \hat{r}\chi = \begin{pmatrix} \alpha r \\ [u \cdot r] \end{pmatrix}, \quad (5)$$

where u is a 4-vector, $\alpha = (\alpha_{ln}) = -\tilde{\alpha}$ is an antisymmetric tensor of the second rank with six independent components, $[u \cdot r] = ([u \cdot r]_{ln}) = (u_l r_n - r_l u_n) = u \cdot r - r \cdot u$ is the alternating dyad, $\alpha r = (\alpha_{ln} r_n)$. Equation (4) is thus reduced to a system of two equations

$$\alpha'' k + 2\alpha c + 2im\kappa_1 [k \cdot a]u' - 2imu = 0, \quad (6)$$

$$\alpha = m^{-1} \{ (i/2) [k \cdot u]'' + i[c \cdot u] + \kappa_2 ([\alpha' a \cdot k] + [a \cdot \alpha' k]) \}. \quad (7)$$

From (7) we can derive the tensor α . Indeed, multiplying (7) by a and differentiating it with respect to ξ , we obtain

$$\alpha' a = i(2m)^{-1} (au)''' k + \varrho_2 [(\alpha k a) a - a^2 \alpha k]'', \quad \varrho_2 = \kappa_2/m. \quad (8)$$

Similarly, we derive

$$\alpha'' k a = -i\mu m^{-1} (du)'', \quad \mu = kq, \quad d = a - (ab)\mu^{-1} k, \quad (9)$$

$$\alpha'' k = im^{-1} [\frac{1}{2} (ku)'' k + [c \cdot u] k - \mu \varrho_2 (du) k]'.$$

From (8) and (9) we have

$$[\alpha' a \cdot k] = im^{-1} \{ [[c \cdot u] a \cdot k] + \varrho_2(\mu(du) [k \cdot a] + a^2 [k \cdot [c \cdot u] k])' \}'.$$

In the same manner we obtain

$$[\alpha' k \cdot a] = im^{-1} \{ \frac{1}{2}(ku)'' [k \cdot a] + [[c \cdot u] k \cdot a] + \mu \varrho_2(du)' [a \cdot k] \}'.$$

As a result, we have

$$\begin{aligned} \alpha = im^{-1} \{ & \frac{1}{2}[k \cdot u]'' + [c \cdot u] + \varrho_2(\frac{1}{2}(ku)'' [a \cdot k] + [[c \cdot u] a \cdot k] \\ & + [a \cdot [c \cdot u] k])' + \varrho_2^2(2\mu(du) [k \cdot a] + a^2 [k \cdot [c \cdot u] k])' \}. \end{aligned} \quad (10)$$

The tensor α is expressed through the unknown vector u and the constant 4-vectors k , a , q . The substitution of (10) into Eq. (6) results in a fourth-order equation for the unknown vector u

$$\begin{aligned} & \frac{1}{2}(ku)^{(4)}k + [c \cdot u]''k + [k \cdot u]''c + 2([c \cdot u]c - m^2u + m\kappa_1[k \cdot a]u') \\ & + \mu \varrho_2[(ku)'''d - (du)'''k] + 2\varrho_2^2\{-2\mu^2(du)''d + \mu[u \cdot d]'c \\ & + a^2(\mu[u \cdot c]''k + ([c \cdot u]''kc)k) + ((au)[c \cdot k])'c + ((ku)[a \cdot c])'c\} = 0. \end{aligned} \quad (11)$$

Subsequent multiplication of this equation by the vector k gives the relations

$$ku'' + 2[gku - cu - \mu \varrho_2(du)'] = 0, \quad g = \mu^{-1}(c^2 + m^2),$$

which along with (11) provide the second-order equation for the unknown vector $u(\xi)$

$$\begin{aligned} & [(c - gk)u]'k + [c \cdot u]''k + [k \cdot u]''c + 2[(cu)c - \mu gu + m\kappa_1[k \cdot a]u'] \\ & + 2\varrho_2\{\mu[(c - gk)u]'d + \mu[u \cdot d]'c + ((au)[c \cdot k])'c + ((ku)[a \cdot c])'c\} \\ & + 2\varrho_2^2[-\mu^2(du)''d + a^2([c \cdot u]''kc)k + \mu a^2[u \cdot c]''k] = 0. \end{aligned} \quad (12)$$

According to [7] we introduce the following basic vectors

$$k, d, e = c - \mu^{-1}c^2k, \quad v = (v_l) = (i\mu^{-1}\varepsilon_{lmnr}k_m a_n q_r). \quad (13)$$

The vector u is expanded now as follows

$$u = \eta_1 k + \eta_2 d + \eta_3 e + \eta_4 v. \quad (14)$$

Here the coefficients $\eta_1, \eta_2, \eta_3, \eta_4$ are the unknown scalar functions of ξ . These functions satisfy the system of four equations resulting from (12)

$$\eta_1 = (1/2)\eta_3'' + g\eta_3 - a^2\varrho_2\eta_2', \quad (15)$$

$$\eta_2'' + (e_0 a^2 \varrho_2 + g)\eta_2 - \tau_- \eta_3' = 0, \quad (16)$$

$$\eta_3'' + (e_0 a^2 \varrho_1 + g)\eta_3 + a^2 m^{-1} \tau_- \eta_2' = 0, \quad (17)$$

$$(1 - \mu a^2 \varrho_2^2)\eta_4' + (-e_0 a^2 \varrho_2 + g)\eta_4 = 0, \quad (18)$$

where $\varrho_1 = \kappa_1/m$, $\tau_- = e_0 - \kappa_+$, $\kappa_+ = m(\kappa_1 + \kappa_2)$.

Equation (18) gives

$$\eta_4 = C_1 \beta^{-1/2} H_n(\sigma) \exp(-\sigma^2/2). \quad (19)$$

Here

$$\begin{aligned} \beta &= 1 - \mu a^2 \varrho_2^2, & \sigma &= \beta^{-1/4} \zeta, & \zeta &= \tau(\xi + \kappa), \\ \tau^4 &= 1 - e_0^2 a^2 / \mu, & \kappa &= e_0 q a / \mu \tau^4, & n &= 0, 1, 2, \dots \end{aligned}$$

$H_n(\sigma)$ are the Hermite polynomials and C_1 is the normalization constant. From the finiteness of the solution of (18) we have

$$q^2 = -m^2 + 2\mu\tau^2[(n + \frac{1}{2})\beta^{1/2} - \tau^2\kappa^2/2 + e_0 a^2 \varrho_2/2\tau^2]. \quad (20)$$

Due to (5), (10), (14), (19) we obtain a unique solution of the problem

$$\begin{aligned} \chi_1 &= \begin{pmatrix} im\eta_4 v \\ \alpha_1 \end{pmatrix}, \\ \alpha_1 &= [v \cdot (c - e_0 a^2 \varrho_2 k)] \eta_4 + \mu \varrho_2 [d \cdot v] \eta'_4 + (\frac{1}{2} - \mu \varrho_2^2 a^2) [v \cdot k] \eta''_4. \end{aligned} \quad (21)$$

Now (16) and (17) ought to be solved in order to construct two other solutions. With (16) and (17) solved we readily obtain from (15) the function η_1 . Below we deal with some particular cases because the solution of the system for a general case is very cumbersome in practice.

1. Neglecting the particle interaction due to the particle extra moments, i. e. considering $\kappa_1 = \kappa_2 = 0$, we find from (16) and (17)

$$\begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}^\pm = C_\pm \begin{pmatrix} \pm im^{1/2}/|a| \\ 1 \end{pmatrix} H_n(\zeta) \exp[-\frac{1}{2}(\zeta^2 \pm ie_0|a|m^{-1/2}\zeta)], \quad (22)$$

$$q^2 = -m^2 + 2\mu\tau^2(n + 1/2 - \tau^2\kappa^2/2 + e_0^2 a^2/8m\tau^2). \quad (23)$$

2. Let us consider the neutral vector particle ($e_0 = 0$). In this case the particle interaction with the electromagnetic field is due solely to the extra moments κ_1 and κ_2 . From (16) and (17), with $e_0 = 0$, we have

$$\begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}^\pm = C_\pm \begin{pmatrix} \pm im^{1/2}/|a| \\ 1 \end{pmatrix} H_n(\xi) \exp[\frac{1}{2}(\pm i|a|m^{-1/2}\kappa_+ - \xi)\xi], \quad (24)$$

$$q^2 = -m^2 + 2\mu(n + \frac{1}{2} + a^2\kappa_+^2/8m). \quad (25)$$

The above results describe exactly the solution of the problem for the neutral vector meson.

3. It should be noted that the electric charge e_0 and particle extra moments enter, as additive parameters, the exact wavefunction of a vector meson interacting with a classical electromagnetic plane wave [10]. Equations (16) and (17) for the case of interaction with a quantized electromagnetic field contain nonadditive terms (with respect to these

parameters) which can be neglected provided the electromagnetic wave frequency is sufficiently large. From (16) and (17) under conditions $e_0 a^2 \varrho_i \ll 1$ ($i = 1, 2$) we have

$$\begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}^{\pm} = C_{\pm} \begin{pmatrix} \pm i m^{1/2} / |a| \\ 1 \end{pmatrix} H_n(\xi) \exp \left[-\frac{1}{2} (\zeta^2 \pm i |a| m^{-1/2} \tau_- \xi) \right], \quad (26)$$

$$q^2 = -m^2 + 2\mu\tau^2(n + \frac{1}{2} - \tau^2\kappa^2/2 + a^2\tau_-^2/8m\tau^2). \quad (27)$$

It is worth noting that the smallness of the parameter $\lambda = e_0 a^2 / m$ at high electromagnetic field frequencies enables us to express the unknown functions η_2 and η_3 in the form of an infinite series.

The 4-vector q represents the total momentum of the particle and electromagnetic field system. From (20), in particular, we find

$$q = p + \tau^2 [(n + 1/2)\beta^{1/2} - \tau^2\kappa^2/2 + e_0 a^2 \varrho_2 / 2\tau^2] k, \quad (28)$$

$$p^2 = -m^2,$$

where p is the free particle energy-momentum 4-vector. In particular cases one can derive from (23), (25), (27) the expressions for q for each of the three spin states.

The wavefunctions of the vector meson in the classical electromagnetic wave field contain the tachyonic modes (see [11–13]). The problem of existence of tachyonic modes in the case of a quantized field has not yet been solved.

REFERENCES

- [1] I. Ya. Berson, *Zh. Eksp. Teor. Fiz.* **56**, 1627 (1969).
- [2] V. G. Bagrov, P. V. Bozrikov, D. M. Gitman, *Teor. Mat. Fiz.* **14**, 202 (1973).
- [3] M. V. Fedorov, A. E. Kazakov, *Phys. Lett.* **261**, 191 (1973).
- [4] V. G. Bagrov, D. M. Gitman, P. M. Lavrov, *Izv. VUZ Fiz.* No 6, 89 (1974).
- [5] V. G. Bagrov, P. V. Bozrikov, D. M. Gitman, *Izv. VUZ Fiz.* No 6, 129 (1974).
- [6] F. I. Fedorov, *Dokl. Akad. Nauk SSSR* **174**, 334 (1967).
- [7] F. I. Fedorov, *Dokl. Akad. Nauk BSSR* **18**, 17 (1974).
- [8] A. A. Bogush, L. G. Moroz, *Introduction to Classical Field Theory*, Minsk 1968 (in Russian).
- [9] E. Henley, W. Thirring, *Elementary Quantum Field Theory*, New York 1962.
- [10] A. F. Radyuk, F. I. Fedorov, *Dokl. Akad. Nauk BSSR* **18**, 118 (1974).
- [11] G. Velo, D. Zwanziger, *Phys. Rev.* **A186**, 1337 (1969).
- [12] G. Velo, D. Zwanziger, *Phys. Rev.* **B188**, 2218 (1969).
- [13] W. Becker, *J. Phys. A* **9**, 149 (1976).