

Quartet unconstrained formulation for massless higher spin fields

A.V. Galajinsky

*Laboratory of Mathematical Physics, Tomsk Polytechnic University,
634050 Tomsk, Russia
galajin@mph.phtd.tpu.edu.ru*

Abstract

Starting from the triplet equations of Francia and Sagnotti, we introduce an auxiliary field and find gauge invariant constraints that single out the spin- s mode and allow one to construct simple unconstrained Lagrangian formulations for higher spin fields.

Recently, the issue of constructing an unconstrained Lagrangian formulation for massless higher spin fields in flat space and on (anti) de Sitter background was intensively studied. By the term "unconstrained" one means a Lagrangian formulation such that off-shell higher spin gauge fields and gauge parameters are not subject to any trace conditions. All the necessary constraints intrinsic to the conventional approach of Fang and Fronsdal [1, 2] arise as equations of motion, possibly after partial gauge fixing, and follow directly from a Lagrangian.

By now there are two approaches leading to consistent unconstrained Lagrangian formulations for massless higher spin fields. The first approach, known as the BRST approach (see [3]–[5] and references therein), automatically yields Lagrangian formulations in terms of completely unconstrained fields. However, the number of auxiliary fields, i.e. the fields which should be either gauged away or eliminated with the use of their (algebraic) equations of motion in order to reproduce the constrained formulations of Fang and Fronsdal [1, 2], grows with the value of spin. The second approach, called the geometric approach, relies upon a modification of the original Fang–Fronsdal equations [6]–[9]. Here the trace constraints on the gauge parameter intrinsic to the conventional formulations of [1, 2] are forgone at the price of allowing nonlocal terms in the equations of motion. A local version of the geometric approach was proposed in [8] which incorporates the trace condition on the higher spin gauge field as the constraint. However, the minimal local Lagrangians of [8], [9] involve higher derivative terms.

The purpose of this work is to construct a modified formulation, which is local, free from higher derivative terms and uses equal number of auxiliary fields for an unconstrained description of any value of spin.

We shall use the notation, which suppresses the vector indices and automatically takes care of symmetrizations. Conventionally, this is done by introducing an auxiliary variable y^μ such that

$$\phi_{\mu_1 \dots \mu_s}(x) \Leftrightarrow \phi^{(s)}(x, y) = \phi_{\mu_1 \dots \mu_s}(x) y^{\mu_1} \dots y^{\mu_s}. \quad (1)$$

Denoting $p_\mu = \frac{\partial}{\partial x^\mu}$, $\pi_\mu = \frac{\partial}{\partial y^\mu}$, one has π^2 for the trace, $(p\pi)$ for the divergence and (yp) for the derivative of a field followed by symmetrization of indices. In this notation the

Fronsdal equations [1] which describe a massless spin- s boson read

$$\left(p^2 - (yp)(p\pi) + (yp)(yp)\frac{\pi^2}{2}\right)\phi^{(s)}(x, y) = 0, \quad \pi^2\pi^2\phi^{(s)} = 0. \quad (2)$$

These are invariant under the gauge transformation

$$\delta\phi^{(s)} = (yp)\epsilon^{(s-1)}, \quad \pi^2\epsilon^{(s-1)} = 0. \quad (3)$$

In order to put the equation of motion and the gauge transformation in an unconstrained form let us introduce an auxiliary field $\alpha^{(s-3)}$ whose gauge transformation law is related to the Fronsdal constraint on the gauge parameter (in this respect see also [6]–[8])

$$\delta\alpha^{(s-3)} = \frac{\pi^2}{2}\epsilon^{(s-1)}. \quad (4)$$

Then we consider two more auxiliary fields $C^{(s-1)}$, $D^{(s-2)}$ and impose the equations of motion, which reproduce the Fronsdal formulation, when the compensator $\alpha^{(s-3)}$ is gauged away

$$\begin{aligned} p^2\phi^{(s)} - (yp)C^{(s-1)} &= 0, & C^{(s-1)} - (p\pi)\phi^{(s)} + (yp)D^{(s-2)} &= 0, \\ D^{(s-2)} - \frac{\pi^2}{2}\phi^{(s)} + (yp)\alpha^{(s-3)} &= 0, & \frac{\pi^2}{2}D^{(s-2)} - (p\pi)\alpha^{(s-3)} &= 0. \end{aligned} \quad (5)$$

One can easily verify that the system holds invariant under the gauge transformation accompanying (4)

$$\delta\phi^{(s)} = (yp)\epsilon^{(s-1)}, \quad \delta C^{(s-1)} = p^2\epsilon^{(s-1)}, \quad \delta D^{(s-2)} = (p\pi)\epsilon^{(s-1)}, \quad (6)$$

with an unconstrained $\epsilon^{(s-1)}$. For our subsequent discussion it proves convenient to display the differential consequence of eqs. (5)

$$p^2D^{(s-2)} - (p\pi)C^{(s-1)} = 0. \quad (7)$$

Let us take a closer look at the system (5), (7). An important observation is that omitting the second line in (5), i.e. the trace conditions, one gets precisely the bosonic triplet of [7, 10]. As is well known, the triplet describes a chain of spins $s, s-2, \dots, 0$ or 1 depending on whether s is even or odd. Thus, the restrictions that comprise the second line in (5) can be viewed as gauge invariant constraints that single out the spin- s mode from the chain of fields described by the triplet. Notice that the first of the constraints appeared previously in [7, 10]. The observation suggests that in order to build an unconstrained Lagrangian formulation for a massless spin- s boson it suffices to start with the Lagrangian density corresponding to the triplet (see e.g. [10]) and enforce the second line in (5) by introducing two Lagrange multipliers $\lambda^{(s-2)}$, $\lambda^{(s-4)}$. In component form the action reads

$$\begin{aligned} S = & -(-1)^s \int d^N x \left\{ \frac{1}{2}\phi_{\mu_1 \dots \mu_s} \square \phi^{\mu_1 \dots \mu_s} - s\phi_{\mu_1 \dots \mu_s} \partial^{\mu_1} C^{\mu_2 \dots \mu_s} - \frac{1}{2}s C_{\mu_1 \dots \mu_{s-1}} C^{\mu_1 \dots \mu_{s-1}} - \right. \\ & - s(s-1) C_{\mu_1 \dots \mu_{s-1}} \partial^{\mu_1} D^{\mu_2 \dots \mu_{s-1}} - \frac{1}{2}s(s-1) D_{\mu_1 \dots \mu_{s-2}} \square D^{\mu_1 \dots \mu_{s-2}} + \\ & + \lambda^{\mu_1 \dots \mu_{s-2}} \left(\frac{1}{2}\phi_{\nu \mu_1 \dots \mu_{s-2}}^\nu - D_{\mu_1 \dots \mu_{s-2}} - \frac{1}{2}(s-2)\partial_{\mu_1} \alpha_{\mu_2 \dots \mu_{s-2}} \right) + \\ & \left. + \lambda^{\mu_1 \dots \mu_{s-4}} \left(D_{\nu \mu_1 \dots \mu_{s-4}}^\nu - \partial^\nu \alpha_{\nu \mu_1 \dots \mu_{s-4}} \right) \right\}. \end{aligned} \quad (8)$$

It is straightforward to verify that the Lagrange multipliers vanish on-shell. It should be appreciated that, in contrast to other approaches, this formulation is local, free from higher derivative terms and uses equal number of auxiliary fields for any value of spin. A coupling of this Lagrangian to anti de Sitter background was constructed in [11].

An unconstrained description of fermionic higher spin fields is constructed in a similar way. One starts with a compensator field $\alpha^{(n-2)}$ whose transformation law is related to the constraint on gauge parameter within the Fang-Fronsdal approach [2]

$$\delta\alpha^{(n-2)} = (\gamma\pi)\epsilon^{(n-1)}. \quad (9)$$

Then one introduces two auxiliary spin-tensors $C^{(n-1)}$, $D^{(n-2)}$ and imposes the equations of motion, which are designed so as to reproduce the Fang-Fronsdal equations [2] when the compensator $\alpha^{(n-2)}$ is gauged away

$$\begin{aligned} (\gamma p)\Psi^{(n)} - (yp)C^{(n-1)} &= 0, \quad C^{(n-1)} - (\gamma\pi)\Psi^{(n)} + (yp)\alpha^{(n-2)} = 0, \\ D^{(n-2)} - \frac{1}{2}(\gamma\pi)C^{(n-1)} - \frac{1}{2}(\gamma p)\alpha^{(n-2)} &= 0, \quad (\gamma\pi)D^{(n-2)} - (p\pi)\alpha^{(n-2)} = 0. \end{aligned} \quad (10)$$

Accompanying the transformation law (9) by

$$\delta\Psi^{(n)} = (yp)\epsilon^{(n-1)}, \quad \delta C^{(n-1)} = (\gamma p)\epsilon^{(n-1)}, \quad \delta D^{(n-2)} = (p\pi)\epsilon^{(n-1)}, \quad (11)$$

with an unconstrained gauge parameter $\epsilon^{(n-1)}$, one can demonstrate that the system (10) is invariant.

In order to construct a Lagrangian formulation reproducing equations (10), we first consider their differential consequences

$$(\gamma p)C^{(n-1)} - (p\pi)\Psi^{(n)} + (yp)D^{(n-2)} = 0, \quad (\gamma p)D^{(n-2)} - (p\pi)C^{(n-1)} = 0. \quad (12)$$

An important observation is that, being combined with the first equation in (10), these give precisely the fermionic triplet considered in [7]. As the triplet describes a chain of half-integer spins [7], the last three equations in (10) can be viewed as gauge invariant constraints extracting the spin-s mode.

The discussion above also makes clear how to construct an unconstrained Lagrangian formulation for a massless spin-s fermion. One has to take the Lagrangian density describing the triplet (see e.g. [7]) and enforce the last three equations in (10) by introducing the Lagrange multipliers $\lambda^{(n-1)}$, $\lambda^{(n-2)}$, $\lambda^{(n-3)}$

$$\begin{aligned} S = \int d^N x \Big\{ & i\bar{\Psi}_{\mu_1 \dots \mu_n} (\not{\partial} \Psi^{\mu_1 \dots \mu_n} - n\partial^{\mu_1} C^{\mu_2 \dots \mu_n} + n\gamma^{\mu_1} \lambda^{\mu_2 \dots \mu_n}) + in\bar{C}_{\mu_1 \dots \mu_{n-1}} (\not{\partial} C^{\mu_1 \dots \mu_{n-1}} - \\ & - \partial^\nu \Psi_{\nu}^{\mu_1 \dots \mu_{n-1}} + (n-1)\partial^{\mu_1} D^{\mu_2 \dots \mu_{n-1}} - \lambda^{\mu_1 \dots \mu_{n-1}} + \frac{1}{2}(n-1)\gamma^{\mu_1} \lambda^{\mu_2 \dots \mu_{n-1}}) + \\ & + in(n-1)\bar{D}_{\mu_1 \dots \mu_{n-2}} (-\not{\partial} D^{\mu_1 \dots \mu_{n-2}} + \partial^\nu C_{\nu}^{\mu_1 \dots \mu_{n-2}} - \lambda^{\mu_1 \dots \mu_{n-2}} - (n-2)\gamma^{\mu_1} \lambda^{\mu_2 \dots \mu_{n-2}}) + \\ & + in(n-1)\bar{\alpha}_{\mu_1 \dots \mu_{n-2}} (-\frac{1}{2}\not{\partial} \lambda^{\mu_1 \dots \mu_{n-2}} + \partial^\nu \lambda_{\nu}^{\mu_1 \dots \mu_{n-2}} - (n-2)\partial^{\mu_1} \lambda^{\mu_2 \dots \mu_{n-2}}) + \\ & + in\bar{\lambda}_{\mu_1 \dots \mu_{n-1}} (C^{\mu_1 \dots \mu_{n-1}} - \gamma^\nu \Psi_{\nu}^{\mu_1 \dots \mu_{n-1}} + (n-1)\partial^{\mu_1} \alpha^{\mu_2 \dots \mu_{n-1}}) + \\ & + in(n-1)\bar{\lambda}_{\mu_1 \dots \mu_{n-2}} (D^{\mu_1 \dots \mu_{n-2}} - \frac{1}{2}\gamma^\nu C_{\nu}^{\mu_1 \dots \mu_{n-2}} - \frac{1}{2}\not{\partial} \alpha^{\mu_1 \dots \mu_{n-2}}) + \\ & + in(n-1)(n-2)\bar{\lambda}_{\mu_1 \dots \mu_{n-3}} (\gamma^\nu D_{\nu}^{\mu_1 \dots \mu_{n-3}} - \partial^\nu \alpha_{\nu}^{\mu_1 \dots \mu_{n-3}}) \Big\}. \end{aligned} \quad (13)$$

The Lagrange multipliers prove to vanish on-shell. The coupling of this model to anti de Sitter background was constructed in [11].

To summarize, in this work we have constructed simple unconstrained Lagrangian formulations for free massless higher spin fields in flat space. The formulations are local, free from higher derivative terms and use equal number of auxiliary fields for an unconstrained description of any value of spin. In this setting an irreducible representation of the Poincaré group is described in terms of a quartet of fields. The models occupy an intermediate position between the geometric formulations of [6]–[9] and the BRST models of [3]–[5] and enjoy all the standard features of a conventional classical field theory. Our considerations also highlight the important role of the bosonic and fermionic triplets [7, 8] in higher spin gauge theory.

Acknowledgements

Collaboration with I.L. Buchbinder and V.A. Krykhtin on the results reported in this work is gratefully acknowledged.

References

- [1] C. Fronsdal, Phys. Rev. D **18** (1978) 3624.
- [2] J. Fang, C. Fronsdal, Phys. Rev. D **18** (1978) 3630.
- [3] A. Pashnev, M. Tsulaia, Mod. Phys. Lett. A **13** (1998) 1853.
- [4] I.L. Buchbinder, A. Pashnev, M. Tsulaia, Phys. Lett. B **523** (2001) 338.
- [5] I.L. Buchbinder, V.A. Krykhtin, A.I. Pashnev, Nucl. Phys. B **711** (2005) 367.
- [6] D. Francia, A. Sagnotti, Phys. Lett. B **543** (2002) 303;
- [7] D. Francia, A. Sagnotti, Class. Quant. Grav. **20** (2003) S473.
- [8] D. Francia, A. Sagnotti, Phys. Lett. B **624** (2005) 93.
- [9] D. Francia, J. Mourad, A. Sagnotti, Nucl. Phys. B **773** (2007) 203.
- [10] A. Sagnotti, M. Tsulaia, Nucl. Phys. B **682** (2004) 83.
- [11] I.L. Buchbinder, A.V. Galajinsky, V.A. Krykhtin, Nucl. Phys. B **779** (2007) 155.