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COMPARISON OF MUON-PROTON AND ELECTRON-PROTON DEEP INELASTIC SCATTERING*

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ABSTRACT

As a test of muon-electron universality we have compared muon-proton and electron-proton inelastic scattering cross sections for $|q^2|$ (square of the four-momentum transferred from the lepton) values up to 4.0 $(\mathrm{GeV}/c)^2$ and for lepton energy losses up to 9 GeV. There is no experimentally significant deviation from muon-electron universality. If the muon is assigned the form factor $(1.0+|q^2|/\Lambda_d^2)^{-1}$ relative to the electron, then with 97.7% confidence $\Lambda_d > 4.1$ GeV/c.

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In this Letter we compare our recent measurements[1] of 12 GeV/c muon-proton inelastic scattering with measurements[2] of electron-proton inelastic scattering in order to study one of the basic problems of elementary particle physics--the relationship between the muon and the electron usually called muon-electron universality. The muon and electron, neither of which are hadrons, have the same spin, electric charge and weak interaction coupling constant; they differ in their mass and in their lepton number. These relationships lead us to speculate about possible connections between the muon and electron. Are they manifestations of a single particle split into two mass levels by unknown forces? Or are the electron and muon the lowest mass members of a larger family of charged leptons? With no theoretical guidance as to how to answer these questions, the experimentalist seeks clues to the answer by measuring known properties of the muon with increasing precision or by studying hitherto unexplored properties of the muon and comparing the results with the corresponding measurements on the electron. The inelastic scattering of leptons on protons is such an unexplored interaction.

The study of muon-electron universality through inelastic scattering has three novel features. (1) In elastic scattering, $\nu = |q^2|/(2M)$. q^2 is the square of four-momentum transferred from the lepton, ν is the energy loss of the lepton in the laboratory frame and M is the proton mass. But in inelastic scattering where $\nu > |q^2|/(2M)$, ν and q^2 may be varied independently; thus allowing the exploration of a much larger kinematic region. (2) Measurements of inelastic lepton scattering in which only the scattered lepton is detected, place no restrictions upon the nature of the final hadronic state. It is conceivable that a violation of muon-electron universality involving hadrons would more easily be seen in inelastic scattering than in elastic scattering. (3) It is possible that one or both of the charged leptons, like the proton, have vertex form factors which are decreasing functions of $|q^2|$. In that case, the inelastic scattering cross sections would be reduced by the square of the lepton form factor. One of the more unexpected results of μ-p and e-p inelastic scattering was the large cross section, compared to elastic scattering, at high $|q^2|$. Hence inelastic scattering can provide a greater sensitivity to lepton form factors through the large range of q^2 which can be covered easily in a single experiment.

The inelastic scattering of leptons on protons occurs through the emission of a virtual photon by the lepton[3,4]; this photon interacts with the nucleon leading to the production of hadrons. For a point-like lepton the virtual photon emission is completely specified by quantum electrodynamics[3]. The interaction of the virtual photon with the nucleon, which must at present be experimentally determined, depends only on the kinematic variables q^2 and ν. Muon-electron universality may therefore be tested by comparing the properties of the virtual photon-proton interaction derived from muon-proton inelastic scattering with those properties derived from electron-proton inelastic scattering. If muon-electron universality is valid, those properties should be the same in both cases. In making such a comparison it is necessary to establish that known effects would not produce a difference. The weak interaction can easily be excluded because of the large difference in coupling strength between electromagnetic and weak interactions. Standard radiative effects have already been taken out in the analysis of both the muon and the electron data, and the contributions to the uncertainty in the results are included in the estimates of the errors. Finally, the contribution of two photon exchange to the inelastic interaction is at most of the order of a few percent[4].

The inelastic differential cross section[1,3] $d^2\sigma/dq^2 d\nu$ is the product of kinematic factors and two independent functions of q^2 and ν; these two functions must be experimentally determined. For these functions we use the virtual photon-proton total cross sections[1,3], $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$. For the comparison we use the combination

$$\sigma_{exp,\ell}(q^2, K, p_\ell) = \sigma_T(q^2, K) + \epsilon(q^2, K, p_\ell, m_\ell)\, \sigma_S(q^2, K)$$

$$= \sigma_T(q^2, K)\left[1 + \epsilon(q^2, K, p_\ell, m_\ell)\, R(q^2, K)\right]$$

where

$$R(q^2, K) = \sigma_S(q^2, K)/\sigma_T(q^2, K)$$

Here $K = \nu - |q^2|/(2M)$ and ϵ is a known function[1,3] of q^2, K, p_ℓ and m_ℓ. M is the proton mass, m_ℓ is the lepton mass, and p_ℓ is the laboratory momentum of the incident lepton, K is in GeV and q^2 is in $(\mathrm{GeV}/c)^2$. As $|q^2| \to 0$, $\sigma_S(q^2, K) \to 0$ and $\sigma_T(q^2, K) \to \sigma_{\gamma p}(K)$ (the total cross section for real photons of energy K on protons). Thus as $|q^2| \to 0$, $\sigma_{exp}(q^2, K, p_\ell) \to \sigma_{\gamma p}(K)$.

In comparing the muon cross section $\sigma_{exp,\mu}$ to the electron cross section $\sigma_{exp,e}$ we must note three factors. First, σ_{exp} depends on the momentum of the incident lepton and (very weakly) on the lepton mass. Second, the electron and muon data were obtained at different incident lepton energies. Third, the muon data was acquired over a continuous q^2, K kinematic region and then collected into q^2 and K bins; the electron data on the other hand was acquired in almost discrete points in that region. To allow for the first two factors we have modified the electron data through the equation

$$\sigma_{exp,e}(q^2,K,p_\mu) = \left[\frac{1+\epsilon_\mu(q^2,K,p_\mu,m_\mu)\,R(q^2,K)}{1+\epsilon_e(q^2,K,p_e,m_e)\,R(q^2,K)}\right]\sigma_{exp,e}(q^2,K,p_e)$$

This procedure is subject to error due to uncertainties in R. At $q^2 = 0$, R must equal zero, but measurements of R have only been made at a few values of q^2, K in the region of this experiment. These measurements are consistent[2] with R = .18 or with $R = |q^2|/16$ in the region of interest. Fortunately, for the data used in this comparison $\sigma_{exp,e}(q^2,K,p_\mu)$ is rather insensitive to R; even if $R = 1 \pm 1$, the uncertainty in $\sigma_{exp,e}$ is for the most part less than 1%. We have made the comparison assuming R = .18 and also with R = 0, 1 and $|q^2|/16$. The changes in the fits and the confidence levels, which we present later, are negligible. To take account of the third factor listed above, we interpolated and averaged the electron data to obtain $\sigma_{exp,e}(q^2,K,p_e)$ for K bins corresponding to those used for the muon data.

In Fig. 1, $\sigma_{exp,\mu}(q^2,K,p_\mu)$ and $\sigma_{exp,e}(q^2,K,p_\mu)$ for $p_\mu = 12$ GeV/c are shown as functions of q^2 for various K intervals. It is obvious that any possible muon-electron differences are small. To quantify those differences we define the ratio

$$\rho(q^2,K) = \sigma_{exp,\mu}(q^2,K,p_\mu)/\sigma_{exp,e}(q^2,K,p_\mu), \quad p_\mu = 12 \text{ GeV/c}.$$

To compute this ratio we have made a fit to the electron data, as represented by $\sigma_{exp,e}(q^2,K,p_\mu)$, and to the $\sigma_{\gamma p}(K)$ values. It was necessary to use $\sigma_{\gamma p}(K)$[5] because our muon data extends to lower $|q^2|$ values than the electron data used in this comparison[2]. This ratio is plotted in Fig. 1, the errors are the combined statistical errors only. We see that ρ is usually close to 1.0; but ρ is less than 1.0 more frequently than it is greater than 1.0.

To combine the data to search for less obvious differences, we need a model of how the two sets of measurements might differ. A common model assumes the leptons have a form factor $F_\ell(q^2) = (1.0 + |q^2|/\Lambda_\ell^2)^{-1}$. Then

$$\rho(q^2, K) = \frac{\sigma_{\exp,\mu}(q^2, K, p_\mu)}{\sigma_{\exp,e}(q^2, K, p_\mu)} = \frac{(1.0 + |q^2|/\Lambda_\mu^2)^{-2}}{(1.0 + |q^2|/\Lambda_e^2)^{-2}} \approx 1/(1.0 + |q^2|/\Lambda_d^2)^2 \qquad (1)$$

where

$$\Lambda_d^{-2} = \Lambda_\mu^{-2} - \Lambda_e^{-2}$$

Because this comparison uses data from two very different experiments, one might also allow for a normalization difference N^2 in the cross sections, generalizing Eq. 1 to

$$\rho(q^2, K) = N^2/(1.0 + |q^2|/\Lambda_d^2)^2 \qquad (2)$$

The overall normalization uncertainty in the muon data[1] is ±6%, excluding the statistical uncertainty in the number of events. The overall normalization uncertainty in the electron data[2] is about 4%. Thus the combined overall normalization uncertainty (excluding statistical errors) in the comparison is ±7% if the two uncertainties are combined in quadrature.

We have made a fit of $\rho(q^2, K)$ to Eq. 2, using all K bins at once. Since N^2 and Λ_d^{-2} are correlated parameters, we display the fit through the contour plot of Fig. 2 based on statistical errors only. The ±7% relative normalization uncertainty is not included. The effect of this normalization uncertainty is to allow the N^2 scale to be shifted up or down by an amount as large as 0.07. The best fit to Eq. 2 is $\Lambda_d^{-2} = .021 \pm .021 (\mathrm{GeV}/c)^{-2}$ and $N^2 = .946 \pm .042$ with $\chi^2 = 41.1$ for 42 degrees of freedom. These numbers, if one ignores the errors, mean that the overall muon-proton inelastic cross section is less than the electron-proton inelastic cross section; and that the muon cross section falls off very slightly faster with $|q^2|$ than the electron cross section. However, considering the normalization uncertainty and the extent of the one and two standard deviation elipses, it is quite possible that $N^2 = 1$ and $\Lambda_d^{-2} = 0$. If we constrain Λ_d^{-2} to be zero, then $N^2 = .917 \pm .024$ with a $\chi^2 = 42.1$ for 43 degrees of freedom. Finally it is conventional to quote a 2 standard deviation lower limit in Λ_d. Allowing N^2 to take <u>any</u> value, $\Lambda_d > 4.1$ (GeV/c) with 97.7% confidence. We are able to set this high lower limit on Λ_d because the muon data has such a "long lever arm" in $|q^2|$. Thus we have found no experimentally significant deviation from muon-electron unversality. On the other hand the agreement with muon-electron universality is not all that one might hope for. An exhaustive analysis of our data has not shown any additional sources of error beyond those which we have already taken into account.

Various other experiments have searched for muon-electron differences, but the only experiments which measure quantities similar to those measured in our experiment are the muon-proton elastic scattering experiments of Camilleri et al.[6] and Ellsworth et al.[7] Both of these experiments found that the μ-p elastic cross sections were smaller than the e-p elastic cross sections up to a maximum $|q^2|$ of about $1(\text{GeV/c})^2$. Camilleri et al. found $\Lambda_d > 2.4$ GeV/c, Ellsworth found $\Lambda_d > 2.0$, both with 95% confidence. In addition Camilleri gives a fit with $\Lambda_d^{-2} = 0$ of $N^2 = 0.92$. Our experiment cannot be compared directly with the precision measurements[8,9] of the gyromagnetic ratio of the muon (g_μ). However we note that if the muon is assigned an electromagnetic form factor $(1.0 + |q^2|/\Lambda_\mu^2)^{-1}$, then the g_μ experiment requires[9] with 95% confidence that $\Lambda_\mu > 7$ GeV/c.

We conclude with a speculative observation. We have analyzed our results using Eq. 2 which is just a simple function representing the belief that possible behavioral differences between the muon and the electron can be enhanced by going to larger values of $|q^2|$. Now if we consider our experiment and the two elastic experiments, we see that none of these experiments demand a muon-electron difference which increases steadily as $|q^2|$ increases. Therefore we should not rule out the possibility that any muon-electron differences which may exist will appear at relatively low $|q^2|$ values and will not increase steadily with $|q^2|$. Thus we might replace the form factor used in Eqs. 1 and 2 by

$$F_\mu(q^2) = (1-b) + b/(1 + |q^2|/\Lambda^2) \tag{3}$$

$$= 1 + (b\,|q^2|)/(1 + |q^2|/\Lambda^2), \qquad 0 \leq b \leq 1$$

If b were small, say 0.04, then in these scattering experiments all that we could see, with present statistics, is an apparent normalization difference when $|q^2|$ approaches Λ^2. But ρ would never fall below $(1-b)^2$. Such a form factor might result from a model in wnich most of the muon mass was, like the electron, concentrated into a point particle; but where some of the mass was distributed in a halo. Of course the parameters of such a model must not contradict the results of the g_μ experiment.

An alternative way to obtain Eq. 3 is to postulate that the muon has a special interaction which connects the muon to the hadrons[10]; an interaction not possessed by the electron. The interference of this special interaction with the electromagnetic interaction can then lead to the second form of Eq. 3 and an apparent muon form factor. Since the postulated special interaction is between the muon and hadrons, the g_μ experiment, with its present precision, may not substantially limit the parameters which can be used in this model. These speculations suggest that experimenter might search for muon-electron differences in elastic and inelastic scattering by making high precision measurements at moderate q^2 values, rather than going to high q^2

values, as was done in the present experiment. In such a high precision, moderate q^2 experiment, the limits on the systematic errors would have to be substantially reduced below the limits which now hold for present muon and electron scattering experiments.

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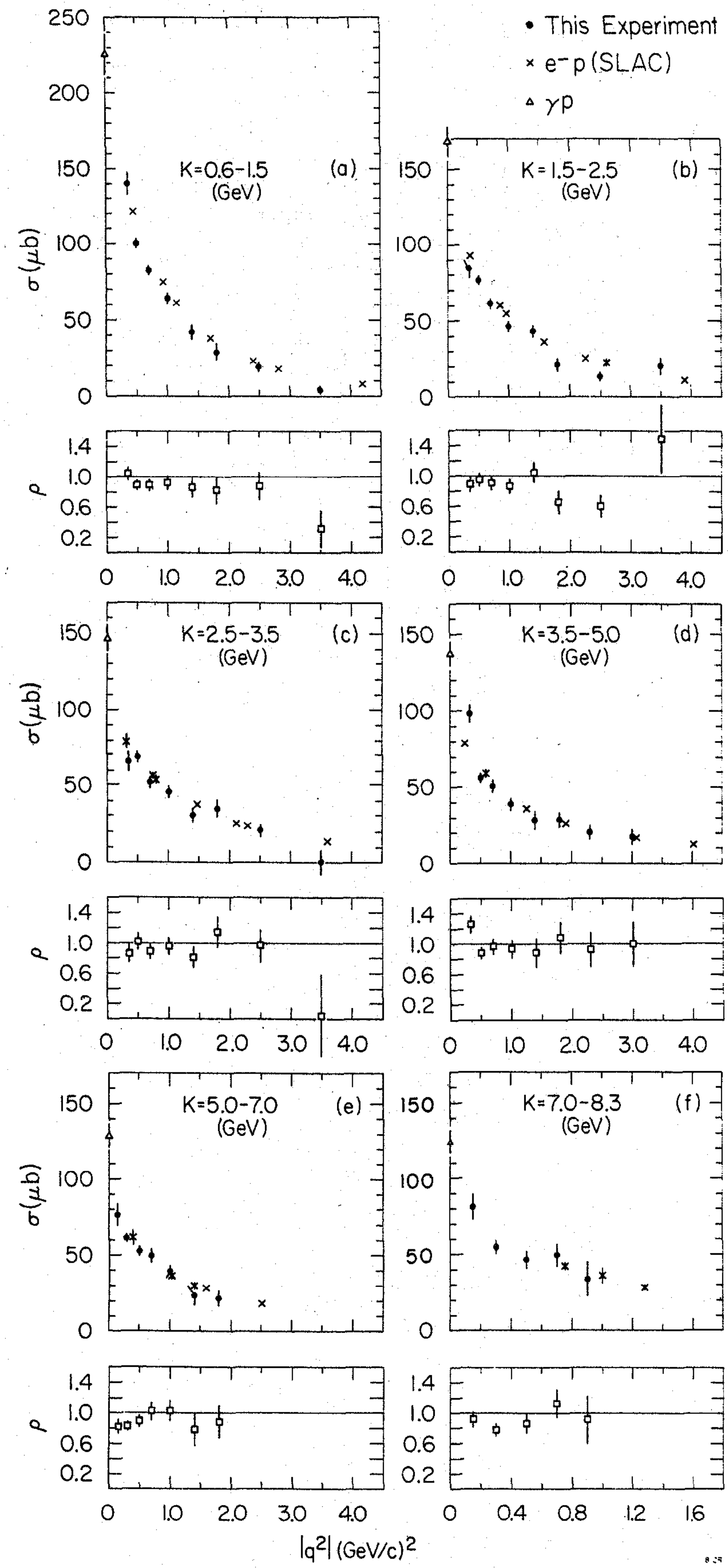
FIGURE CAPTIONS

1. For each K interval the upper plot gives the experimental values of $\sigma_{exp,\mu}(q^2, K, p_\mu)$ denoted by a solid circle, $\sigma_{exp,e}(q^2, K, p_\mu)$ denoted by an x and $\sigma_{\gamma p}(K)$ denoted by a triangle; p_μ = 12 GeV/c. These quantities are defined in the text. $\sigma_{exp,e}(q^2, K, p_\mu)$ is taken from Ref. 2 as described in the text. For each K interval the lower plot gives the values of $\rho(q^2, K) = \sigma_{exp,\mu}(q^2, K, p_\mu)/\sigma_{exp,e}(q^2, K, p_\mu)$. The error bars represent only statistical errors. In most cases the errors in $\sigma_{exp,e}$ are too small to be displayed.

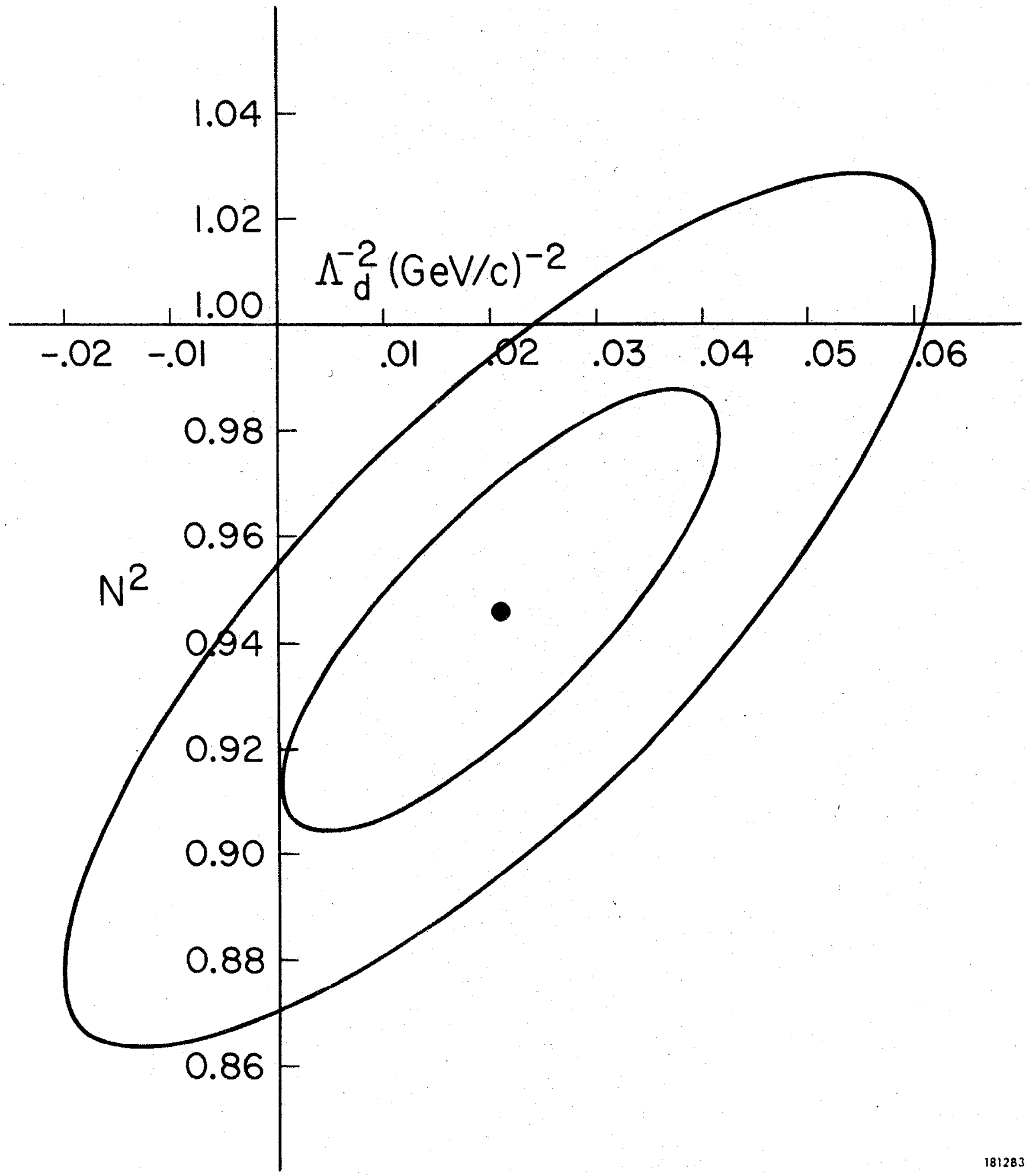
2. Contour plots for the parameters N^2 and Λ_d^{-2} obtained by fitting the experimental values of the ratio $\rho(q^2, K)$ to the equation $\rho(q^2, K) = N^2/(1.0+|q^2|/\Lambda_d^2)^2$. The inner ellipse represents one standard deviation and the outer ellipse represents two standard deviations in the fit.



• This Experiment
× e⁻p (SLAC)
▵ γp
K=0.6-1.5 (GeV) (a)
K=1.5-2.5 (GeV) (b)
K=2.5-3.5 (GeV) (c)
K=3.5-5.0 (GeV) (d)
K=5.0-7.0 (GeV) (e)
K=7.0-8.3 (GeV) (f)
σ(μb)
ρ
|q²| (GeV/c)²

1812R1

Fig. 1



1.04
1.02
1.00
Λ_d^{-2} (GeV/c)$^{-2}$
-.02
-.01
.01
.02
.03
.04
.05
.06
0.98
0.96
0.94
0.92
0.90
0.88
0.86
N^2
181283

Fig. 2