Embeddings of integrable models in supergravity and their perturbative stability

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Abstract. We discuss the perturbative stability of an AdS_3 non-supersymmetric solution of the type-IIB supergravity, whose internal geometry is given by the direct product of a round three-sphere and two λ-deformed factors based on the coset CFTs $SU(2)/U(1)$ and $SL(2,\mathbb{R})/SO(1,1)$. This solution admits a two-dimensional parametric space spanned by the inverse radius of the AdS_3 and the deformation parameter λ. Reality of the background imposes restrictions on the values of these parameters. Further limitations on the values of the inverse radius and the parameter λ arise after requiring the stability of the solution. Our approach relies on the study of scalar perturbations around the AdS_3 vacuum of a three-dimensional effective theory. This reveals the existence of a region in the parametric space where the Breitenlohner-Freedman bound is not violated.

1. Introduction

During the past few years there has been a lot of interest in constructing and studying two-dimensional quantum field theories that exhibit remarkable properties. One such class refers to integrable deformations of WZW or gauged WZW models and comes with the name of "λ-deformations" [1, 2, 3, 4, 5, 6, 7, 8, 9]. A second class of integrable σ-models, which is known as "η-deformations" and is associated to the Principal Chiral Model (PCM), was introduced in [10, 11, 12] and in [13, 14, 15] for group and coset spaces respectively.

In the present work we are concerned about aspects relevant to the embeddings of λ-deformations in type-II supergravity. Elevating a σ-model into a full supergravity solution is a challenging task to perform. This combines the extension of the σ-model target space metric and antisymmetric field to ten dimensions and the inclusion of an appropriate dilaton and Ramond-Ramond (RR) fields that solve the equations of motion. Plenty of examples have been constructed in the literature until now [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. However, only those of [26] contain undeformed (unwarped) AdS spaces. Due to this feature the solutions of [26] might serve as candidates for employing the machinery of the AdS/CFT correspondence.

Besides the presence of an AdS space in the geometries [26], one should not ignore that these backgrounds are non-supersymmetric. This observation in conjunction with the conjecture by Ooguri and Vafa that *any non-supersymmetric AdS vacuum is unstable* [27], which is a stronger version of the weak gravity conjecture [28], suggests at least the study of the perturbative stability of the solutions found in [26].

A modern method for investigating the perturbative stability of supergravity solutions combines ideas of the exceptional field theory [29] and was developed in [30, 31] where the authors computed the Kaluza-Klein spectra of maximal gauged supergravity vacua. This method was also applied in [32] in order to prove the perturbative stability of the non-supersymmetric G_2 invariant $AdS_4 \times S^6$ background of the massive type-IIA supergravity. This result, together with the absence of a non-perturbative instability, named as "brane-jet instability" [33], in these backgrounds [34] poses a challenge to the Ooguri-Vafa conjecture.

In this work we study the perturbative stability of a type-IIB solution with geometry $AdS_3 \times S^3 \times CS^2_\lambda \times CH_{2,\lambda}$ where CS^2_λ and $CH_{2,\lambda}$ are the two-dimensional λ-deformed coset CFTs based on $SU(2)/U(1)$ and $SL(2,\mathbb{R})/SO(1,1)$ respectively. Our approach relies on a dimensional reduction of this solution to a three-dimensional effective theory of gravity with scalars and the analysis of the scalar fluctuations from the lower-dimensional point of view. A key characteristic of this solution is the existence of a two-dimensional parametric space spanned by the deformation parameter λ and the inverse radius of the AdS_3. The analysis shows that the parametric space is divided into a region where the Breitenlohner-Freedman (BF) bound [35] for the scalar fluctuations is violated and another one where no violation occurs.

The organization of the paper is as follows: in section 2 we review the field content of the supergravity solution of our interest. In section 3 we present the reduction of the supergravity solution to a three-dimensional theory of gravity with scalars and its perturbative stability analysis. Finally, in section 4 we conclude.

2. The supergravity solution

In this section we review the type-IIB solution on $AdS_3 \times S^3 \times CS^2_\lambda \times CH_{2,\lambda}$, where we use the notation CS^2_λ and $CH_{2,\lambda}$ for the λ-deformed cosets $SU(2)/U(1)$ and $SL(2,\mathbb{R})/SO(1,1)$, respectively. More specifically, for the λ-deformed model on $SU(2)/U(1)$ we get the following metric and dilaton

$$ds^2_{CS^2_\lambda} = e^{2\phi_y}\left(\lambda_+^2\, dy_1^2 + \lambda_-^2\, dy_2^2\right), \qquad \phi_y(y) = -\frac{1}{2}\ln\left(1 - y_1^2 - y_2^2\right), \tag{1}$$

where the coordinates (y_1, y_2) are restricted inside the unit disc $y_1^2 + y_2^2 < 1$. This space will be denoted as CS^2_λ. Similarly, the λ-deformed model on $SL(2,\mathbb{R})/SO(1,1)$ is

$$ds^2_{CH_{2,\lambda}} = e^{2\phi_z}\left(\lambda_+^2\, dz_1^2 + \lambda_-^2\, dz_2^2\right), \qquad \phi_z(z) = -\frac{1}{2}\ln\left(z_1^2 + z_2^2 - 1\right), \tag{2}$$

where now the coordinates (z_1, z_2) lie outside the unit disc, i.e. $z_1^2 + z_2^2 > 1$. For the latter space we will use the notation $CH_{2,\lambda}$.

The NS sector of this solution contains a metric that takes the form

$$ds^2 = \frac{2}{\ell}\Big(- r^2 dt^2 + r^2 dx^2 + \frac{dr^2}{r^2} + d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2\Big) + ds^2_{CS^2_\lambda} + ds^2_{CH_{2,\lambda}}\,, \tag{3}$$

where ℓ is a positive constant and the line elements for CS^2_λ and $CH_{2,\lambda}$ are given in (1) and (2), respectively. There is also a dilaton whose expression is

$$\Phi(y,z) = \phi_y(y) + \phi_z(z)\,, \tag{4}$$

where the functions $\phi_y(y)$ and $\phi_z(z)$ are given in (1) and (2). The NS two-form B_2 is trivial and so is its field strength H_3.

The above is supported by a RR sector whose content is

$$\begin{aligned} &F_1 = 0\,, \qquad F_3 = 0\,,\\ &F_5 = 2\,k\left(\frac{2}{\ell}\right)^{\frac{3}{2}} dz_1 \wedge dy_2 \wedge \left(\sqrt{\frac{\ell-\mu}{2}}\mathrm{Vol}(AdS_3) + \sqrt{\frac{\ell+\mu}{2}}\mathrm{Vol}(S^3)\right)\\ &\quad - 2\,k\left(\frac{2}{\ell}\right)^{\frac{3}{2}} dz_2 \wedge dy_1 \wedge \left(\sqrt{\frac{\ell+\mu}{2}}\mathrm{Vol}(AdS_3) + \sqrt{\frac{\ell-\mu}{2}}\mathrm{Vol}(S^3)\right), \end{aligned} \tag{5}$$

where we have defined the volume forms on AdS_3 and S^3 as

$$\mathrm{Vol}(AdS_3) = r\,dt\wedge dx\wedge dr\,,\qquad \mathrm{Vol}(S^3) = \sin^2\theta_1\,\sin\theta_2\,d\theta_1\wedge d\theta_2\wedge d\theta_3\,. \tag{6}$$

Moreover, we define the following set of parameters

$$\lambda_\pm = \sqrt{k\frac{1\pm\lambda}{1\mp\lambda}} = \frac{k}{\lambda_\mp}\,,\qquad \mu = \frac{4\lambda}{k\big(1-\lambda^2\big)}\,,\qquad \nu = \frac{4}{k}\frac{1+\lambda^2}{1-\lambda^2}\,. \tag{7}$$

In the above, k is the level of the associated CFTs, which is a positive number and in addition an integer in the compact case. The deformation parameter λ in principle takes values in the interval $[0,1)$. However, in order for the RR five-form to be real one has to require that $\ell \geqslant \mu$.

The background (3), (4) and (5) solve the equations of motion of the type-IIB supergravity summarized below

Dilaton and Einstein equations

$$\begin{aligned} &R\star\mathbb{1} + 4\,d\star d\Phi - 4\,d\Phi\wedge\star d\Phi - \frac{1}{2}H_3\wedge\star H_3 = 0\,,\\ &R_{MN} + 2\,\nabla_M\nabla_N\Phi - \frac{1}{4}\big(H_3^2\big)_{MN} = \frac{e^{2\Phi}}{2}\Bigg[\big(F_1^2\big)_{MN} + \frac{1}{2}\big(F_3^2\big)_{MN} + \frac{1}{48}\big(F_5^2\big)_{MN}\\ &\qquad\qquad - G_{MN}\Big(\frac{1}{2}F_1^2 + \frac{1}{12}F_3^2\Big)\Bigg]\,. \end{aligned} \tag{8}$$

Bianchi and flux equations

$$\begin{aligned} &dH_3 = 0\,,\qquad dF_1 = 0\,,\qquad dF_3 = H_3\wedge F_1\,,\qquad dF_5 = H_3\wedge F_3\,,\\ &d\big(e^{-2\Phi}\star H_3\big) - F_1\wedge\star F_3 - F_3\wedge F_5 = 0\,,\quad d\star F_3 + H_3\wedge F_5 = 0\,,\quad d\star F_1 + H_3\wedge\star F_3 = 0\ . \end{aligned} \tag{9}$$

3. Perturbative stability analysis

In this section we examine the stability of the type-IIB solution presented in the previous section. Our approach is based on the study of scalar fluctuations from a lower dimensional point of view. More specifically we will analyse scalar fluctuations in a three-dimensional theory of gravity with scalars. For this reason we will adopt a reduction ansatz which we introduce below.

3.1. The reduction ansatz

We will perform a reduction along the three-sphere and the two λ-deformed spaces of the ten-dimensional space (3). Thus, for the metric we consider the following ansatz (this method resembles that of [36, 37, 38])

$$\begin{aligned} d\hat{s}^2 = e^{2A}\Big[&ds^2_{\mathcal{M}_3} + e^{2\psi}\big(d\theta_1^2 + \sin^2\theta_1\,d\theta_2^2 + \sin^2\theta_1\sin^2\theta_2\,d\theta_2^2\big)\\ &+ e^{2\phi_y}\Big(\lambda_+^2\,e^{2\chi_1}\,dy_1^2 + \lambda_-^2\,e^{2\chi_2}\,dy_2^2\Big) + e^{2\phi_z}\Big(\lambda_+^2\,e^{2\chi_3}\,dz_1^2 + \lambda_-^2\,e^{2\chi_4}\,dz_2^2\Big)\Big]\,, \end{aligned}$$

where $\mathcal{M}_3$ is a three-dimensional space with metric

$$ds^2_{\mathcal{M}_3} = g_{\mu\nu}\,dx^\mu\,dx^\nu\ . \tag{10}$$

The scalars $A, \psi, \chi_1, \ldots, \chi_4$ depend exclusively on the coordinates x^μ of $\mathcal{M}_3$. The rest of the NS fields are taken to be

$$\widehat{H}_3 = 0\,, \qquad \widehat{\Phi}(x,y,z) = 4A(x) + \phi_y(y) + \phi_z(z)\,, \tag{11}$$

where $\phi_y(y)$ and $\phi_z(z)$ are given by (1) and (2), respectively. Finally, for the RR fields we allow

$$\begin{aligned} &\widehat{F}_1 = 0\,, \qquad \widehat{F}_3 = 0\,, \\ &\widehat{F}_5 = dz_1 \wedge dy_2 \wedge \Big(c_1 e^{\chi_2 - \chi_1 + \chi_3 - \chi_4 - 3\psi}\, \mathrm{Vol}(\mathcal{M}_3) + c_2 \mathrm{Vol}(S^3)\Big) \\ &\quad - dz_2 \wedge dy_1 \wedge \Big(c_2 e^{\chi_1 - \chi_2 - \chi_3 + \chi_4 - 3\psi} \mathrm{Vol}(\mathcal{M}_3) + c_1 \mathrm{Vol}(S^3)\Big)\,, \end{aligned} \tag{12}$$

with $\mathrm{Vol}(\mathcal{M}_3)$ being the volume form on $\mathcal{M}_3$ and $\mathrm{Vol}(S^3)$ is given by (6). The constants c_1 and c_2 are

$$c_1 = 2k\Big(\frac{2}{\ell}\Big)^{\frac{3}{2}} \sqrt{\frac{\ell - \mu}{2}}\,, \qquad c_2 = 2k\Big(\frac{2}{\ell}\Big)^{\frac{3}{2}} \sqrt{\frac{\ell + \mu}{2}}\,. \tag{13}$$

The solution of the previous section is recovered by taking the space $\mathcal{M}_3$ to be an AdS_3 with line element

$$ds^2_{AdS_3} = \bar{g}_{\mu\nu}\, dx^\mu\, dx^\nu = \frac{2}{\ell}\left(r^2 \eta_{\alpha\beta} dx^\alpha dx^\beta + \frac{dr^2}{r^2}\right)\,, \tag{14}$$

normalised as $\bar{R}_{\mu\nu} = -\ell\, \bar{g}_{\mu\nu}$ and by setting the scalars to

$$\bar{A} = \bar{\chi}_1 = \bar{\chi}_2 = \bar{\chi}_3 = \bar{\chi}_4 = 0\,, \quad \bar{\psi} = \frac{1}{2} \ln \frac{2}{\ell}\,. \tag{15}$$

3.2. The equations of motion

It is easy to check that the ansatz (10), (11) and (12) satisfies the Bianchi and flux equations (9). However, by inserting the ansatz into the dilaton and Einstein equations one obtains a set of differential equations for the scalars $A, \psi, \chi_1, \ldots, \chi_4$ and the metric $g_{\mu\nu}$ on $\mathcal{M}_3$. We will see this in more detail in the following lines. Notice that tensors constructed below and all contractions are performed with respect to the metric $g_{\mu\nu}$ on $\mathcal{M}_3$.

The dilaton equation: We sart with the dilaton equation (8) which reduces to

$$\begin{aligned} &R + 6\, e^{-2\psi} - 2\nabla_g^2\big(3\psi + \chi_1 + \chi_2 + \chi_3 + \chi_4\big) - \big(3\,\partial\psi + \partial\chi_1 + \partial\chi_2 + \partial\chi_3 + \partial\chi_4\big)^2 \\ &- 3\big(\partial\psi\big)^2 - \big(\partial\chi_1\big)^2 - \big(\partial\chi_2\big)^2 - \big(\partial\chi_3\big)^2 - \big(\partial\chi_4\big)^2 - 2\,\partial\big(3\psi + \chi_1 + \chi_2 + \chi_3 + \chi_4\big)\cdot\partial A \\ &- 2\,\nabla_g^2 A - 8\,\big(\partial A\big)^2 + 2\,\frac{e^{-2\chi_1}}{\lambda_+^2} + 2\,\frac{e^{-2\chi_2}}{\lambda_-^2} - 2\,\frac{e^{-2\chi_3}}{\lambda_+^2} - 2\,\frac{e^{-2\chi_4}}{\lambda_-^2} = 0\,. \end{aligned} \tag{16}$$

The directions along $\mathcal{M}_3$: Restricting ourselves to the components of the Einstein equations (8) along the $\mathcal{M}_3$ directions we get

$$\begin{aligned} &R_{\mu\nu} - \nabla_\mu\nabla_\nu\big(3\psi + \chi_1 + \chi_2 + \chi_3 + \chi_4\big) - 3\partial_\mu\psi\,\partial_\nu\psi - \partial_\mu\chi_1\,\partial_\nu\chi_1 - \partial_\mu\chi_2\,\partial_\nu\chi_2 \\ &- \partial_\mu\chi_3\,\partial_\nu\chi_3 - \partial_\mu\chi_4\,\partial_\nu\chi_4 - 8\,\partial_\mu A \partial_\nu A - g_{\mu\nu}\,\nabla_g^2 A - g_{\mu\nu}\,\partial\big(3\psi + \chi_1 + \chi_2 + \chi_3 + \chi_4\big)\cdot\partial A \\ &= -g_{\mu\nu}\frac{e^{-6\psi}}{4k^2}\Big(c_1^2 e^{-2\chi_1 - 2\chi_4} + c_2^2 e^{-2\chi_2 - 2\chi_3}\Big)\,. \end{aligned} \tag{17}$$

Taking the trace of the equation above with respect to the metric on $\mathcal{M}_3$ and using it in order to eliminate the Ricci scalar from (16) we arrive at

$$\begin{aligned}
&\nabla_g^2\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4-A\big)\\
&+\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4\big)\cdot\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4-A\big)\\
&-6e^{-2\psi}+3\frac{e^{-6\psi}}{4k^2}\Big(c_1^2e^{-2\chi_1-2\chi_4}+c_2^2e^{-2\chi_2-2\chi_3}\Big)-2\frac{e^{-2\chi_1}}{\lambda_+^2}-2\frac{e^{-2\chi_2}}{\lambda_-^2}+2\frac{e^{-2\chi_3}}{\lambda_+^2}+2\frac{e^{-2\chi_4}}{\lambda_-^2}=0\,.
\end{aligned}\tag{18}$$

It is convenient to use this equation instead of the equivalent one in (16).

The directions along S^3: From the Einstein equations along the sphere directions we get a single expression which is

$$\begin{aligned}
&\nabla_g^2\big(A+\psi\big)+\partial\big(A+\psi\big)\cdot\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4\big)-2e^{-2\psi}\\
&\qquad=-\frac{e^{-6\psi}}{4k^2}\Big(c_1^2e^{-2\chi_1-2\chi_4}+c_2^2e^{-2\chi_2-2\chi_3}\Big)\,.
\end{aligned}\tag{19}$$

The y-directions along the λ-deformed spaces: The y-components of (8) result to

$$\begin{aligned}
&\nabla_g^2\big(A+\chi_1\big)+\partial\big(A+\chi_1\big)\cdot\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4\big)-\frac{e^{-2\chi_1}}{\lambda_+^2}+\frac{e^{-2\chi_2}}{\lambda_-^2}\\
&\qquad=-\frac{e^{-6\psi}}{4k^2}\Big(c_1^2e^{-2\chi_1-2\chi_4}-c_2^2e^{-2\chi_2-2\chi_3}\Big)\,,
\end{aligned}\tag{20}$$

$$\begin{aligned}
&\nabla_g^2\big(A+\chi_2\big)+\partial\big(A+\chi_2\big)\cdot\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4\big)+\frac{e^{-2\chi_1}}{\lambda_+^2}-\frac{e^{-2\chi_2}}{\lambda_-^2}\\
&\qquad=\frac{e^{-6\psi}}{4k^2}\Big(c_1^2e^{-2\chi_1-2\chi_4}-c_2^2e^{-2\chi_2-2\chi_3}\Big)\,.
\end{aligned}\tag{21}$$

The z-directions along the λ-deformed spaces: Similarly, the $z-$components of (8) lead to

$$\begin{aligned}
&\nabla_g^2\big(A+\chi_3\big)+\partial\big(A+\chi_3\big)\cdot\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4\big)+\frac{e^{-2\chi_3}}{\lambda_+^2}-\frac{e^{-2\chi_4}}{\lambda_-^2}\\
&\qquad=\frac{e^{-6\psi}}{4k^2}\Big(c_1^2e^{-2\chi_1-2\chi_4}-c_2^2e^{-2\chi_2-2\chi_3}\Big)\,.
\end{aligned}\tag{22}$$

$$\begin{aligned}
&\nabla_g^2\big(A+\chi_4\big)+\partial\big(A+\chi_4\big)\cdot\partial\big(3\psi+\chi_1+\chi_2+\chi_3+\chi_4\big)-\frac{e^{-2\chi_3}}{\lambda_+^2}+\frac{e^{-2\chi_4}}{\lambda_-^2}\\
&\qquad=-\frac{e^{-6\psi}}{4k^2}\Big(c_1^2e^{-2\chi_1-2\chi_4}-c_2^2e^{-2\chi_2-2\chi_3}\Big)\,.
\end{aligned}\tag{23}$$

The mixed directions: Finally, the (μy)- and (μz)-directions give rise to the following first order equations, respectively

$$\partial_\mu\big(2A+\chi_1+\chi_2\big)=0\,,\qquad \partial_\mu\big(2A+\chi_3+\chi_4\big)=0\,,\tag{24}$$

which integrate to

$$2A+\chi_1+\chi_2=0\,,\qquad 2A+\chi_3+\chi_4=0\,.\tag{25}$$

The integration constants are fixed by the background values (15).

Using (25) it is easy to see that (20) and (21) are equivalent and similarly for (22) and (23). In the rest of this paper we will use (25) to eliminate χ_2 and χ_4 and the independent set of equations for the metric $g_{\mu\nu}$ and the scalars A,ψ,χ_1,χ_3 will be (17), (18), (19), (20) and (22).

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3.3. A change of frame and the stability analysis

The equations (17), (18), (19), (20) and (22) can be expressed conveniently in a different frame metric given by

$$g_{\mu\nu} = e^{8A-6\psi}\,\mathfrak{g}_{\mu\nu}\,. \tag{26}$$

In this frame, the equations for the rescaled metric $\mathfrak{g}_{\mu\nu}$ and the scalars A, ψ, χ_1, χ_3 can be derived from the action

$$S(\mathfrak{g}, X) = \frac{1}{2\kappa_3^2}\int d^3x\sqrt{|\mathfrak{g}|}\Big(\mathfrak{R} - \gamma_{ij}\partial X^i\cdot\partial X^j - V(X)\Big)\,, \tag{27}$$

where the scalars are encoded in a four-vector $X = (A, \psi, \chi_1, \chi_3)$. The matrix γ_{ij} is

$$\gamma_{ij} = \begin{pmatrix} 32 & -12 & 2 & 2 \\ -12 & 12 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix} \tag{28}$$

and the potential $V(X)$ reads

$$\begin{aligned} V(X) = &-6\,e^{8A-8\psi} - 2\,e^{8A-6\psi}\left(\frac{e^{-2\chi_1}}{\lambda_+^2} + \frac{e^{4A+2\chi_1}}{\lambda_-^2} - \frac{e^{-2\chi_3}}{\lambda_+^2} - \frac{e^{4A+2\chi_3}}{\lambda_-^2}\right) \\ &+ \frac{e^{12A-12\psi}}{2k^2}\Big(c_1^2 e^{2\chi_3-2\chi_1} + c_2^2 e^{2\chi_1-2\chi_3}\Big)\,. \end{aligned} \tag{29}$$

Notice that from (15) and (26)

$$\bar{\mathfrak{g}}_{\mu\nu} = \left(\frac{2}{\ell}\right)^3 \bar{g}_{\mu\nu} \tag{30}$$

which implies that $\bar{\mathfrak{g}}_{\mu\nu}$ amounts to an AdS_3 space with radius

$$L = \frac{4}{\ell^2}\,. \tag{31}$$

The background values (15) and (30), (14) correspond to a solution of the equations of motion.

In order to argue about the stability of the solution mentioned above one needs to study the linearized equations of motion. For this purpose we adopt the following perturbation scheme

$$\mathfrak{g}_{\mu\nu} = \bar{\mathfrak{g}}_{\mu\nu} + \delta\mathfrak{g}_{\mu\nu}\,, \qquad X^i = \bar{X}^i + \delta X^i\,, \qquad i = 1, \dots, 4\,, \tag{32}$$

with $\bar{X} = (\bar{A}, \bar{\psi}, \bar{\chi}_1, \bar{\chi}_3)$ determined by (14) and $\bar{\mathfrak{g}}_{\mu\nu}$ by (30), (14). The linearized equations for the scalar and metric [1] fluctuations then read

$$\nabla^2_{\bar{\mathfrak{g}}}\delta X^i - \left(M^2\right)^i{}_j\delta X^j = 0\,, \qquad \nabla^2\delta\mathfrak{g}_{\mu\nu} + \frac{2}{L^2}\delta\mathfrak{g}_{\mu\nu} = 0\,, \tag{33}$$

where

$$\left(M^2\right)^i{}_j = \frac{1}{2}\gamma^{ik}\partial_j\partial_k V(X)\Big|_{X=\bar{X}}\,. \tag{34}$$

In the gravity system with action (27), (28), (29) only the scalar fluctuations can be associated to perturbative instabilities around the AdS_3 vacuum. These can occur whenever any of the eigenvalues d_i ($i = 1, \dots, 4$) violates the inequality

$$d_i \geqslant -\frac{\ell^4}{16}\,, \qquad i = 1, \dots, 4\,. \tag{35}$$

The latter is the so-called *Breitenlohner-Freedman* (BF) bound [35] for scalars on AdS_3 with radius given by (31).

We now proceed with the stability analysis starting with the undeformed case, where $\lambda = 0$, which is easy to handle due to the simple form of the matrix M^2.

[1] Notice that for the metric fluctuations we used the transverse-traceless gauge $\nabla^\mu\delta\mathfrak{g}_{\mu\nu} = \bar{\mathfrak{g}}^{\mu\nu}\,\delta\mathfrak{g}_{\mu\nu} = 0$.

The undeformed case ($\lambda = 0$): The matrix M^2 is

$$M^2 = \frac{\ell^4}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -\frac{1}{\hat{\ell}} & 0 & \frac{1}{2}-\frac{1}{\hat{\ell}} & -\frac{1}{2} \\ \frac{1}{\hat{\ell}} & 0 & -\frac{1}{2} & \frac{1}{2}+\frac{1}{\hat{\ell}} \end{pmatrix}, \tag{36}$$

with eigenvalues

$$d_1 = 0\,, \quad d_2 = \frac{\ell^4}{4}\Big(1+\sqrt{1+\frac{4}{\hat{\ell}^2}}\Big)\,, \quad d_3 = \frac{\ell^4}{2}\,, \quad d_4 = \frac{\ell^4}{4}\Big(1-\sqrt{1+\frac{4}{\hat{\ell}^2}}\Big)\,, \tag{37}$$

where we define

$$\hat{\ell} := k\ell\ . \tag{38}$$

The BF bound (35) implies that

$$\hat{\ell} \geqslant \frac{8}{3}\,. \tag{39}$$

Therefore the inverse radius of the AdS_3 is bounded from below so that stability is not excluded, even though classically $\hat{\ell} > 0$.

The general case ($\lambda > 0$): The reality condition for the supergravity solution translates to

$$\hat{\ell} \geqslant \frac{4\lambda}{1-\lambda^2}\ . \tag{40}$$

However, stability might impose a stricter bound on $\hat{\ell}$ like it happened in the undeformed case. Indeed, we will show that this is also the case here.

To proceed it is convenient to define the matrix

$$B = \mathbb{1} + \frac{16}{\ell^4}M^2\ , \tag{41}$$

whose eigenvalues are written in terms of d_i (the eigenvalues of M^2) as

$$b_i = 1 + \frac{16}{\ell^4}d_i \geqslant 0\,, \qquad i = 1,\dots,4\,. \tag{42}$$

Stability requires that $b_i \geqslant 0$, $\forall i = 1,\dots,4$ according to (35). Due to the involved dependence of the matrix M^2 on λ and $\hat{\ell}$, it is more handy to work with the characteristic polynomial of the matrix B, which turns out to be

$$\begin{aligned} &p_4(s) = (s-1)p_3(s)\ , \\ &p_3(s) = s^3 - 19s^2 + \left(99 - 64\frac{1+18\lambda^2+\lambda^4}{\hat{\ell}^2(1-\lambda^2)^2}\right)s - \left(81 - 192\frac{3+22\lambda^2+3\lambda^4}{\hat{\ell}^2(1-\lambda^2)^2}\right)\ . \end{aligned} \tag{43}$$

From the factorisation of $p_4(s)$ it is obvious that one of the eigenvalues of B equals to one, say $b_1 = 1$. The constant term of the polynomial $p_3(s)$ coming with a minus sign equals the product [2] $b_2b_3b_4$. In the desired scenario of stability all of the eigenvalues (b_2, b_3, b_4) must be

[2] We use the fact that the polynomial $p_3(s)$ can also be written as

$$p_3(s) = (s-b_2)(s-b_3)(s-b_4) = s^3 - (b_2+b_3+b_4)s^2 + (b_2b_3+b_2b_4+b_3b_4)s - b_2b_3b_4\,.$$

non-negative and so must be their product. This tells us that a necessary but not sufficient condition for stability is that $\hat{\ell}$ satisfies the inequality

$$\hat{\ell} \geqslant \frac{8}{3\sqrt{3}} \frac{\sqrt{3 + 22\,\lambda^2 + 3\,\lambda^4}}{1 - \lambda^2}\,, \tag{44}$$

which is clearly a stricter bound than that in (40). This guarantees that $b_2 b_3 b_4 \geqslant 0$, but not the positivity of each eigenvalue separately. There is always a possibility that one eigenvalue is positive and two negative. However this can not be true in our case. In order to show this we assume, without loss of generality, that $b_2 \geqslant 0$ and $b_3, b_4 < 0$. From the coefficient of the quadratic term in $p_3(s)$ we have that $b_2 + b_3 + b_4 = 19$, which together with our assumption implies that $b_2 > 19$. In addition, it can be shown that the coefficient of the linear term in $p_3(s)$ is positive for all values of λ. Therefore, $b_2 b_3 + b_2 b_4 + b_3 b_4 > 0$. Trading b_4 from the aforementioned sum we have that $-(b_2 - 19)(b_2 + b_3) > b_3^2 > 0$. This is true for $b_2 + b_3 < 0$ yielding that $19 = b_2 + b_3 + b_4 < b_4$. The latter contradicts to our initial assumption where $b_4 < 0$. Hence we conclude that whenever $\hat{\ell}$ satisfies (44) all eigenvalues b_i, $i = 1, 2, 3, 4$ are non-negative.

The analysis above is also illustrated in figures 1 and 2. In figure 1 we plot the eigenvalues b_2 and b_3 of the matrix B as a function of λ and $\hat{\ell}$ parameters. The latter are confined between the horizontal axis and the curve in red, which is defined by the equality in (40). In this domain the eigenvalues b_2 and b_3 are positive, as it can be seen from the two contour plots, and thus they are not associated to unstable modes. The case of the eigenvalue b_4 is shown in figure 2. There exists a critical curve (dashed line), parametrized by the equality in (44), on which $b_4 = 0$. Therefore, the allowed region for the parameters λ and $\hat{\ell}$ is divided into two sub-regions, one that sits between the red and dashed lines with $b_4 < 0$ and one that sits on the right of the dashed line with $b_4 > 0$. Clearly, one should disregard the area of the parameter space with $b_4 < 0$, where the instability of the mode associated to the eigenvalue b_4 occurs.

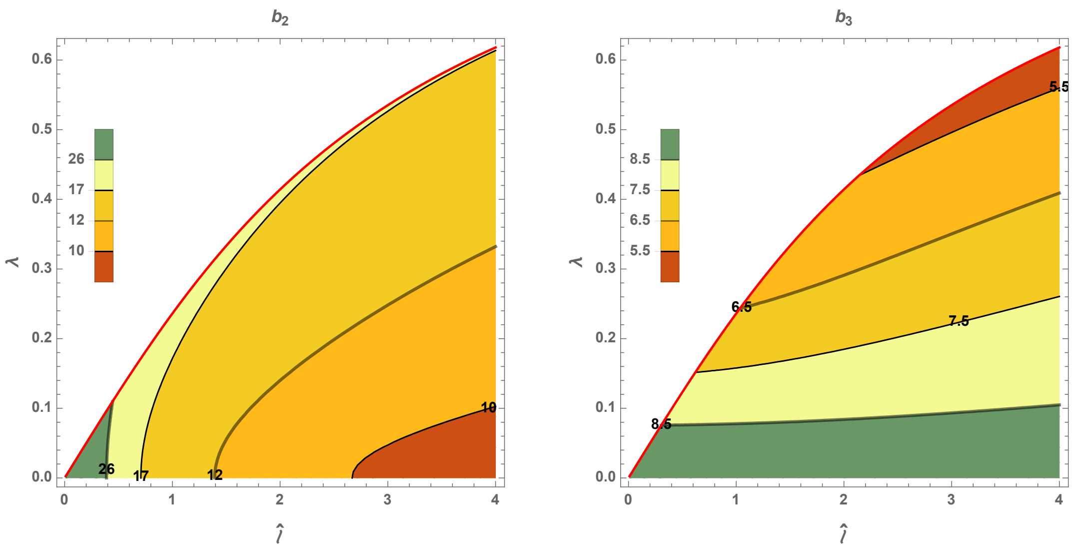

Figure 1. The eigenvalues b_2 (left) and b_3 (right) of the matrix B in eq. (41) are constant along the contours denoted by dark lines. In the coloured areas, the values of b_2 and b_3 are in between the values attached to the contours. The curve in red parametrized by the equality in eq. (40) defines the allowed region of the parameter space spanned by λ and $\hat{\ell}$.

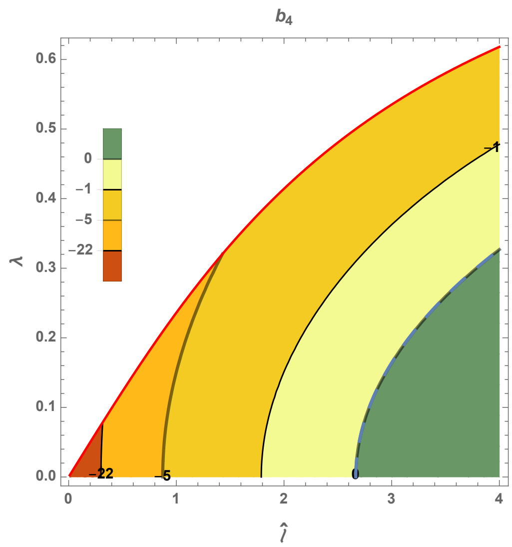

Figure 2. The eigenvalue b_4 as a function of λ and $\hat{\ell}$. Here, the classically allowed region in the parameter space is divided by a critical curve (dashed line) along which $b_4 = 0$. This curve is given by the equality in eq. (44). The area on the left of this contour is where instability occurs since b_4 is always negative. On the right of the dashed line (denoted by green colour) b_4 is always positive.

4. Conclusions

In this paper we studied the perturbative stability of a non-supersymmetric AdS_3 solution of the type-IIB supergravity, whose geometry contains a round three-sphere and the target spaces of the λ-deformed coset CFTs on $SU(2)/U(1)$ and $SL(2,\mathbb{R})/SO(1,1)$. This solution has two parameters, namely the deformation parameter λ and the radius of AdS_3. The stability analysis follows the reduction of the type-IIB solution to a three-dimensional theory of gravity with four scalars. A careful treatment of the scalar fluctuations in the lower dimensional theory shows the existence of a region in the parametric space where the BF bound is not violated. Although this is not a proof of the perturbative stability of the type-IIB solution, this result allows room for a more complete investigation in ten dimensions. Equally important is the possible existence of non-perturbative instabilities, which should also be examined.

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