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Research Article

An Analysis of a Minimal Vectorlike Extension of the Standard Model

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We analyze an extension of the Standard Model with an additional $SU(2)$ hypercolor gauge group keeping the Higgs boson as a fundamental field. Vectorlike interactions of new hyperquarks with the intermediate vector bosons are explicitly constructed. We also consider pseudo-Nambu–Goldstone bosons caused by the symmetry breaking $SU(4) \rightarrow Sp(4)$. A specific global symmetry of the model with zero hypercharge of the hyperquark doublets ensures the stability of a neutral pseudoscalar field. Some possible manifestations of the lightest states at colliders are also examined.

1. Introduction

The experimental detection of the Higgs boson [1, 2] with mass $M_H \approx 125\,\text{GeV}$ leaves unanswered many questions of the Standard Model (SM) (see [3], for example). A part of the SM puzzles can be solved by supersymmetry (SUSY) [4, 5]. Unfortunately, there are no clear indications that SUSY manifests itself in the experiments near a "naturalness" scale ~1 TeV. Obviously, SUSY is not rejected at all, but sparticles and their interactions are now expected to be observed at a much higher scale, ~5–10 TeV, because the parameter space of SUSY models is increasingly constrained by the LHC data [6–8].

Besides SUSY, a lot of ways are proposed to enlarge SM: an addition of extra $U(1)$ groups, multi-Higgs and technicolor (TC) models, and many others (see reviews [3, 9] and references therein). However, we currently have not found any comprehensive variant of the theory of "everything" (excepting, possibly, string theory which has no phenomenological applications for now), so all problems of SM cannot be solved simultaneously. An origin of Dark Matter (DM) is also one of the known SM problems. At the moment we are skeptical of any manifestations of (sufficiently light) neutralino as the DM particle [10]. Note that there are a lot of other DM candidates which are suggested and discussed [11–19]. For example, DM can originate from the Higgs sector too (e.g., the inert Higgs model) [20, 21].

From a "technical" viewpoint, technicolor scenario [22–25] means a "duplication" of an analog of the QCD sector at a higher energy scale with confinement of the extra technifermions and technigluons. Originally, TC models were suggested to introduce dynamical electroweak (EW) symmetry breaking (EWSB) without fundamental Higgs scalars. Corresponding scalar boson arises in this case as a bound state of techniquarks—these models are Higgsless (note also the so-called "see-saw" mechanism giving a light scalar boson in TC) [26–31]. In this way both structure and interactions of the T-strong confined sector are considered as extra options to solve some SM problems (see [32–36]). It seems that the discovery of the Higgs boson closes some Higgsless technicolor scenarios and many investigations concentrate now on extra fermion sectors in confinement (the so-called hypercolor models) as a source of composite states and Dark Matter candidates.

Contributions of additional fields to the SM precision parameters are crucial for the models—variety of them is constrained [26] by the experimentally required values of Peskin–Takeuchi (PT) parameters [35, 37–41]. So, to select a realistic and reasonable extension, it is necessary to calculate EW polarization operators with an account of the model contributions. Then, the comparison of calculated values of S, T, and U parameters with the experimental data gives some constraint on the structure of the model. As a rule, in the models with chirally nonsymmetric fermions, there appear unacceptable contributions to the PT parameters. It is the main reason why vectorlike models have been under consideration recently [35, 36, 42–44].

Thus, multiplet and chiral structure of the new fermion sector is a principal characteristic of SM extension. In the framework of technicolor models, as a rule, such multiplets have a standard-like $SU(2)_L$ structure, namely, left-hand doublets and right-hand singlets [45, 46]. In the hypercolor models, chirally symmetric (with respect to the weak group) set of new fermions is used [47]. However, this chirally symmetric fermion sector crucially differs from the standard one, so interpretation of the gauge fields as standard vector bosons is hypothetical.

In this work, we suggest a construction of vectorlike weak interaction which starts from standard-like chirally nonsymmetric set of new fermions doublets. This program has been carried out for zero hypercharge in the simplest model with two hyperquark (H-quark) generations and two hypercolors (HC), $N_{\mathrm{HC}} = 2$ [44, 48]. We consider this scenario for the case of nonzero hypercharge and show that two left doublets of H-quarks can be transformed into one doublet of Dirac H-quarks with vectorlike weak interaction. This possibility can be realized if the hypercharges of generations have the same value and opposite signs. Importantly, this condition is in accordance with the absence of anomalies in the model. To form the Dirac states which correspond to constituent quarks, we have used a scalar field having nonzero vacuum expectation value (v.e.v.). This field is introduced as a scalar singlet pseudo-Nambu–Goldstone (pNG) boson in the framework of the simplest linear sigma model. We consider in detail the structure of the pNG multiplet which is defined by the global symmetry breaking $SU(4) \to Sp(4)$. It is also shown that the Lagrangian of this minimal extension has specific global symmetries making neutral H-baryon and H-pion states stable.

The paper is organized as follows. In Section 2, we construct vectorlike interactions for the case of $SU(2)$ H-color and EW groups with even generations. The total Lagrangian together with the pNG bosons is considered in Section 3. The principal part of the physical Lagrangian of the model is presented in Section 4, where we demonstrate the presence of a specific discrete symmetry that leads to the stability of a pseudoscalar state. In Section 5, we analyze the main phenomenological consequences of the model.

2. Vectorlike Interaction of the Gauge Bosons with H-Quarks

An essential point is the choice of chiral structure of the H-quark multiplets. It is known that chirally nonsymmetric interaction of the extra fermions with the SM bosons may contradict to restrictions on Peskin–Takeuchi parameters. Thus, it is reasonable to consider vectorlike (chiral-symmetric) interaction of (initially standard-like) H-quarks with Z- and W-bosons. We construct such interactions explicitly for the case of even generations of two-color ($N_{\mathrm{HC}} = 2$) H-quarks.

In the simplest scenario with two generations ($A = 1, 2$) of left-handed H-quarks, the bidoublet of these quarks is presented as a matrix $Q^{a\underline{a}}_{L(A)}$, where $a = 1, 2$ and $\underline{a} = 1, 2$ are indices of $SU(2)_L$ and $SU(2)_{\mathrm{HC}}$ fundamental representations, respectively. (In the following all indices related to the hypercolor group are underlined.)

This bidoublet transforms under $U(1)_Y \otimes SU(2)_L \otimes SU(2)_{\mathrm{HC}}$ as

$$\begin{aligned}\left(Q^{a\underline{a}}_{L(A)}\right)' = Q^{a\underline{a}}_{L(A)} + i g_B Y_A \theta Q^{a\underline{a}}_{L(A)} + \frac{i}{2} g_W \theta_k \tau_k^{ab} Q^{b\underline{a}}_{L(A)} \\ + \frac{i}{2} g_{\mathrm{HC}} \varphi_{\underline{k}} \tau_{\underline{k}}^{\underline{a}\,\underline{b}} Q^{a\underline{b}}_{L(A)}.\end{aligned} \quad (1)$$

Here $Q^{1\underline{a}}_{L(A)} = U^{\underline{a}}_{L(A)}$, $Q^{2\underline{a}}_{L(A)} = D^{\underline{a}}_{L(A)}$, and the H-quarks charges $q_{U,D}$ are defined by the arbitrary hypercharges Y_A. The right-handed singlets (with respect to electroweak $SU(2)_L$ group) have the following group transformations:

$$\left(S^{\underline{a}}_{R(A)}\right)' = S^{\underline{a}}_{R(A)} + i g_B Y_{R(A)} \theta S^{\underline{a}}_{R(A)} + \frac{i}{2} g_{\mathrm{HC}} \varphi_{\underline{k}} \tau_{\underline{k}}^{\underline{a}\,\underline{b}} S^{\underline{b}}_{R(A)}, \quad (2)$$

where $A = 1, 2$ and $Y_{R(A)}$ are hypercharges of singlets. Now, the charge conjugation operation, $\widehat{\mathbf{C}}$, is applied to the fields of the second generation keeping the first generation of H-quarks unchanged:

$$Q^{Ca\underline{a}}_{L(2)} = \widehat{\mathbf{C}} Q^{a\underline{a}}_{L(2)}. \quad (3)$$

The transformation properties of the charge conjugated fields have the form

$$\begin{aligned}\left(Q^{Ca\underline{a}}_{L(2)}\right)' = Q^{Ca\underline{a}}_{L(2)} - i g_B Y_2 Q^{Ca\underline{a}}_{L(2)} - \frac{i}{2} g_W \theta_k \left(\tau_k^{ab}\right)^* Q^{Cb\underline{a}}_{L(2)} \\ - \frac{i}{2} g_{\mathrm{HC}} \varphi_{\underline{k}} \left(\tau_{\underline{k}}^{\underline{a}\,\underline{b}}\right)^* Q^{Ca\underline{b}}_{L(2)}.\end{aligned} \quad (4)$$

Then, we redefine the H-quark fields (the fermion chirality is changed by the charge conjugation):

$$Q^{a\underline{a}}_{R(2)} = \epsilon^{ab} \epsilon^{\underline{a}\,\underline{b}} Q^{Cb\underline{b}}_{L(2)}, \quad \epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (5)$$

Further, we multiply both sides of (4) by $\epsilon^{ab}\epsilon^{\underline{a}\,\underline{b}}$ and use the following properties of $SU(2)$ group matrices:

$$\begin{aligned}\epsilon^{ac}\epsilon^{bc} = \delta^{ab}, \\ \epsilon^{ab}\left(\tau_k^{bc}\right)^* \epsilon^{cf} = \tau_k^{af}.\end{aligned} \quad (6)$$

Using redefinition (5), from (4), we get

$$\begin{aligned}\left(Q^{a\underline{a}}_{R(2)}\right)' = Q^{a\underline{a}}_{R(2)} - i g_B Y_2 Q^{a\underline{a}}_{R(2)} + \frac{i}{2} g_W \theta_k \tau_k^{ab} Q^{b\underline{a}}_{R(2)} \\ + \frac{i}{2} g_{\mathrm{HC}} \varphi_{\underline{k}} \tau_{\underline{k}}^{\underline{a}\,\underline{b}} Q^{a\underline{b}}_{R(2)}.\end{aligned} \quad (7)$$

This transformation law coincides with the one given by formula (1) for the first generation ($A = 1$) when $Y_2 = -Y_1$.

Thus, we have constructed the right-handed field partner of the first generation, using the second generation of the left-handed fields in two steps: charge conjugation and redefinition. Therefore, composing these fields we have a Dirac state:

$$Q^{a\underline{a}} = Q^{a\underline{a}}_{L(1)} + Q^{a\underline{a}}_{R(2)} = Q^{a\underline{a}}_{L(1)} + \epsilon^{ab}\epsilon^{\underline{a}\underline{b}}Q^{Cb\underline{b}}_{L(2)}. \tag{8}$$

Because both parts (left- and right-handed) of the field have the same transformation properties, namely, (1), then the Dirac H-quarks interact with the EW vector bosons as chiral-symmetric fields.

Analogously, the right-handed field $S^{\underline{a}}_{R(2)}$ is redefined as follows:

$$S^{\underline{a}}_{L} = \epsilon^{\underline{a}\underline{b}}\widehat{\mathbf{C}}S^{\underline{b}}_{R(2)}. \tag{9}$$

This redefined field transforms as the right-handed singlet $S_{R(1)}$ if $Y_{R(2)} = -Y_{R(1)}$ in full analogy with the previous case. This representation of the H-fields allows us to get a usual Dirac mass term after the summation of left and right parts. Both current and constituent H-quark masses can be introduced because the mass term does not violate the model symmetry. The simplest way to do this is to use a singlet real scalar, s, which has a nonzero v.e.v., $s = \tilde{\sigma} + u$, where $u = \langle s\rangle$. Just interaction of the H-quarks with this scalar field provides Dirac type mass term for H-quarks. Note that, to get a Dirac state with the vectorlike interaction from two Weyl spinors, we should require the initial fields for the first and second families to have opposite hypercharges, $Y_1 = -Y_2$. The same requirement follows from the condition of the absence of anomalies in the model. It should be noted that the suggested construction of vectorlike interaction is valid due to unique properties of $SU(2)_{\mathrm{HC}}$ group and for the case of an even number of generations.

The gauge part of the model Lagrangian directly follows from (1) and (2):

$$\begin{aligned} L(Q,S) = &-\frac{1}{4}T^{\underline{k}}_{\mu\nu}T^{\mu\nu}_{\underline{k}} \\ &+ i\overline{Q}\gamma^{\mu}\left(\partial_{\mu} - ig_B Y_1 B_{\mu} - \frac{i}{2}g_W W^{k}_{\mu}\tau_k - \frac{i}{2}g_{\mathrm{HC}}T^{\underline{k}}_{\mu}\tau_{\underline{k}}\right) \\ &\cdot Q - m_Q\overline{Q}Q \\ &+ i\overline{S}\gamma^{\mu}\left(\partial_{\mu} - ig_B Y_{R(1)} B_{\mu} - \frac{i}{2}g_{\mathrm{HC}}T^{\underline{k}}_{\mu}\tau_{\underline{k}}\right)S - m_S\overline{S}S, \end{aligned} \tag{10}$$

where $T^{\underline{k}}_{\mu}$ is a H-gluon field. The mass terms are formally included in (10) because they do not break $SU(2)_{\mathrm{HC}}$-symmetry of the model. The status of the $SU(2)_L$-singlet H-quark significantly differs from that of the standard quarks. The standard quark singlet is a right-handed part of the Dirac fermion state, while S-quark consists of the two initial chiral singlets. It should be noted that the singlet S can be useful since a composite H-meson $\overline{Q}S$ is a representation of the groups $U(1)_Y \otimes SU(2)_L$. The standard Higgs doublet is the same representation, that is, the Higgs field can be considered as a composite state of the singlet and doublet H-quarks. However, due to the fields Q and S being independent, from now on, the $SU(2)_L$ singlet states can not be included in the consideration.

3. Fundamental Higgs Boson, Two-Color Fermions, and Pseudo-Nambu–Goldstone Bosons in the Linear Sigma Model

Here, we construct a linear sigma model involving the constituent H-quarks and lowest pseudo(scalar) H-hadrons—σ H-meson, pNG states, and their opposite-parity partners [45, 46, 49–51]. As it was shown in [51–53] (see also more recent papers [54, 55]), Lagrangian (10) in the limit $m_Q \to 0$, $g_W \to 0$ has a global $SU(4)$ symmetry corresponding to rotations in the space of the four initial chiral fermion fields. The Lagrangian with nonzero m_Q can be rewritten in the form which explicitly reveals the violation of symmetry $SU(4) \to Sp(4)$ by the mass term [54, 55]. For $m_Q = 0$ the Lagrangian retains the full $SU(4)$ symmetry but, in an analogy with QCD, one might expect the dynamical symmetry breaking by vacuum expectation value $\langle \overline{U}U + \overline{D}D\rangle \neq 0$. This v.e.v. has the mass term structure and leads to the dynamical breaking of the symmetry $SU(4) \to Sp(4)$. As a result, the broken generators of $SU(4)$ would be accompanied by a set of pNG states. The spectrum of the pNG states depends on the way of symmetry breaking.

The global symmetry of two-color QCD with $N_{\tilde{F}}$ Dirac quarks in the limit of zero masses is $SU(2N_{\tilde{F}})$, with the chiral group being its subgroup, $SU(N_{\tilde{F}})_L \otimes SU(N_{\tilde{F}})_R \subset SU(2N_{\tilde{F}})$ (this statement is valid for any symplectic gauge theory [56]; the group $SU(2)$ is isomorphic to the group $Sp(2)$) [52, 53]. This global symmetry is often called the Pauli–Gürsey symmetry. The quark condensate breaks the Pauli–Gürsey symmetry to its subgroup $Sp(2N_{\tilde{F}})$ [51, 57]. In the following we will consider the simplest case of two flavors $N_{\tilde{F}} = 2$.

We have only two possibilities to assign EW quantum numbers to the two fundamental fermion constituents (for the general case a classification of physically relevant ultraviolet completions of composite Higgs models based on the coset $SU(4)/Sp(4)$ is given in [56, 58], which considers different gauge groups with arbitrary numbers of flavors and colors, $N_{\tilde{F}}$ and N_{HC}). These possibilities are determined by the cancellation of gauge anomalies.

(i) *V-A ultraviolet completion*. We can introduce a left-handed weak doublet $Q_L = \binom{U_L}{D_L}$ and two right-handed weak singlets U_R and D_R with opposite hypercharges $Y(U_R) = -Y(D_R)$. It is the case that is considered in most papers dealing with a new two-flavor confined sector [55, 59–63].

(ii) *Vectorlike ultraviolet completion*. Both left- and right-handed fermions are grouped as fundamental representations of the weak $SU(2)_L$ group, $Q_L = \binom{U_L}{D_L}$ and $Q_R = \binom{U_R}{D_R}$ [44, 48]. The hypercharges of the doublets should be the same, $Y(Q_L) = Y(Q_R)$. In this case the

Dirac mass term, $\overline{Q}_L Q_R + \overline{Q}_R Q_L$, is permitted by the EW symmetry.

In this paper, we study the case of the vectorlike ultraviolet completion with zero hypercharges of the doublets.

At the fundamental level, the Lagrangian of two-flavor and two-color QCD (10) can be written in terms of a left-handed quartet field:

$$L = -\frac{1}{4} T^{\underline{k}}_{\mu\nu} T^{\mu\nu}_{\underline{k}} + i\overline{P}^{\underline{a}}_L \not{D}_{\underline{a}\,\underline{b}} P^{\underline{b}}_L - \frac{1}{2} m_Q \left(\overline{P}^{\underline{a}}_L M_0 P^{\underline{a}}_R + \overline{P}^{\underline{a}}_R M_0^\dagger P^{\underline{a}}_L \right), \tag{11}$$

$$D^\mu_{\underline{a}\,\underline{b}} = \partial^\mu \delta_{\underline{a}\,\underline{b}} - \frac{i}{2} g_{\rm HC} T^\mu_{\underline{k}} \tau^{\underline{k}}_{\underline{a}\,\underline{b}} - \sqrt{2} i g_W W^\mu_k \Sigma_k \delta_{\underline{a}\,\underline{b}}, \tag{12}$$

where

$$P^{\underline{a}}_L = \begin{pmatrix} Q^{\underline{a}}_{L(1)} \\ Q^{\underline{a}}_{L(2)} \end{pmatrix}, \qquad P^{\underline{a}}_R = \epsilon^{\underline{a}\,\underline{b}} \left(P^{\underline{b}}_L \right)^C \tag{13}$$

are left- and right-handed quartet fields ($Q_{L(1)}$ and $Q_{L(2)}$ are left-handed doublets introduced in the previous section). The EW term in the covariant derivative (12) involves the matrices

$$\Sigma_k = \frac{1}{2\sqrt{2}} \begin{pmatrix} \tau_k & 0 \\ 0 & \tau_k \end{pmatrix}, \quad k = 1, 2, 3, \tag{14}$$

that are three of ten $Sp(4)$ generators Σ_α satisfying the following conditions:

$$\begin{gathered} \operatorname{Tr} \Sigma_\alpha = 0, \\ \Sigma^\dagger_\alpha = \Sigma_\alpha, \\ \operatorname{Tr} \Sigma_\alpha \Sigma_\beta = \frac{1}{2} \delta_{\alpha\beta}, \\ \Sigma^T_\alpha M_0 + M_0 \Sigma_\alpha = 0, \\ \alpha, \beta = 1, 2, \ldots, 10. \end{gathered} \tag{15}$$

The mass term in Lagrangian (11) introduces the antisymmetric 4×4 matrix

$$M_0 = -M_0^T = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}. \tag{16}$$

We have used the matrix M_0 also to define the algebra of the $Sp(4)$ generators. Although M_0 has a noncanonical form, it can be brought into the form $\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ or $\left(\begin{smallmatrix} \epsilon & 0 \\ 0 & \epsilon \end{smallmatrix}\right)$ by a unitary transformation.

The equivalence of the Lagrangians (10) and (11) was proved in the previous section. It should be noted that the similar rearrangement of the Lagrangian in terms of the left-handed fields would be possible in any sort of techni- or hyperchromodynamics with T/H-quarks in self-contragredient representation of T/H-confinement group. The fundamental representation of $SU(2)_{\rm HC}$, which is symplectic and pseudoreal representation, is just the simplest case. An aspect of this property is that the global symmetry group of the massless theory is larger than the chiral symmetry.

In the limit of vanishing m_Q and g_W the global symmetry group of Lagrangian (11) is the Pauli–Gürsey group $SU(4)$ [52, 53], the chiral symmetry being a subgroup of the Pauli–Gürsey group:

$$\begin{gathered} P^{\underline{a}}_L \longrightarrow U P^{\underline{a}}_L, \\ P^{\underline{a}}_R \longrightarrow U^* P^{\underline{a}}_R, \\ U \in SU(4). \end{gathered} \tag{17}$$

The mass term of the current H-quarks breaks the group $SU(4)$ explicitly. Indeed, if we consider infinitesimal transformations $U = 1 + i\theta_\alpha \Sigma_\alpha$, $\theta_\alpha \ll 1$, it is readily seen that the mass term in Lagrangian (11) is left invariant by the generators satisfying conditions (15); that is, the mass term is invariant under the subgroup $Sp(4)$ of the Pauli–Gürsey group (see [54, 55]). H-quark condensate $\langle \overline{Q} Q \rangle$ has the same spinor structure as the mass term. Thus, the dynamical breaking by the condensate $\langle \overline{Q} Q \rangle$ should be also $SU(4) \to Sp(4)$ [51, 57]. If the current H-quark masses are significantly smaller than the scale of the spontaneous breaking of the Pauli–Gürsey group, we have the situation similar to the one in well-established QCD of light quarks. Putting it in terms natural to the quark-meson sigma models, there are five pNG bosons associated with the five "broken" generators of the group $SU(4)$; these bosons acquire small masses due to the small explicit breaking of the global symmetry of the model, while the constituent masses of the H-quarks are generated mostly by the dynamical symmetry breaking.

Before leaving our consideration of the Lagrangian of the fundamental current H-quarks, we should note that apart from the Pauli–Gürsey group $SU(4)$ Lagrangian (11) possesses an additional global $U(1)$ symmetry as well as a new discrete symmetry. The former symmetry leads to conservation of an analog of the baryon number, while the latter one is a generalization of the G-parity of QCD. The important consequences of these symmetries are discussed at the end of this section and in the next one.

Now, we proceed to construct an effective Lagrangian of a linear quark-hadron sigma model $SU(4) \cong SO(6) \to SO(5) \cong Sp(4)$. This model describes the interactions of the constituent H-quarks and lightest (pseudo)scalar H-hadrons. The Lagrangian of the H-quark sector of the model reads

$$L = i\overline{P}_L \not{D} P_L - \sqrt{2}\kappa \left(\overline{P}_L M P_R + \overline{P}_R M^\dagger P_L \right), \tag{18}$$

$$D_\mu P_L = \partial_\mu P_L - \sqrt{2} i g_W W^k_\mu \Sigma_k P_L. \tag{19}$$

Here κ is a H-quark–H-hadron coupling constant. The matrix M of spin-0 H-hadrons is antisymmetric. Its transformation law under the global symmetry $SU(4)$ is

$$M \longrightarrow U M U^T, \quad U \in SU(4). \tag{20}$$

Being a complex antisymmetric matrix with 12 independent components, the field M can be conveniently expanded in terms of five "broken" generators $\beta_{\dot\alpha}$ of the Pauli–Gürsey group:

$$M = \left[\frac{1}{2\sqrt{2}}\left(A_0 + iB_0\right) + \left(A_{\dot\alpha} + iB_{\dot\alpha}\right)\beta_{\dot\alpha}\right]M_0. \tag{21}$$

The generators $\beta_{\dot\alpha}$ are subjected to the conditions

$$\begin{aligned}
\operatorname{Tr}\beta_{\dot\alpha} &= 0,\\
\beta_{\dot\alpha}^{\dagger} &= \beta_{\dot\alpha},\\
\operatorname{Tr}\beta_{\dot\alpha}\beta_{\dot\gamma} &= \frac{1}{2}\delta_{\dot\alpha\dot\gamma},\\
\operatorname{Tr}\Sigma_{\alpha}\beta_{\dot\alpha} &= 0,\\
\beta_{\dot\alpha}^{T}M_0 - M_0\beta_{\dot\alpha} &= 0,\\
&\dot\alpha, \dot\gamma = 1, 2, \ldots, 5,\ \alpha = 1, 2, \ldots, 10,
\end{aligned} \tag{22}$$

and can be written explicitly as

$$\begin{aligned}
\beta_k &= \frac{1}{2\sqrt{2}}\begin{pmatrix}\tau_k & 0\\ 0 & -\tau_k\end{pmatrix}, \quad k = 1, 2, 3,\\
\beta_4 &= \frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\\
\beta_5 &= \frac{i}{2\sqrt{2}}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}.
\end{aligned} \tag{23}$$

Now the Lagrangian of constituent H-quarks (18) can be put into the following form:

$$\begin{aligned}
L = {} & i\overline{Q}\not{D}Q - \kappa u\overline{Q}Q - \kappa\Big[\sigma'\overline{Q}Q + i\tilde\eta\overline{Q}\gamma_5 Q + \tilde a_k\overline{Q}\tau_k Q\\
& + i\tilde\pi_k\overline{Q}\gamma_5\tau_k Q + \frac{1}{\sqrt{2}}\Big(A^0\overline{Q}_{a\underline{a}}\epsilon_{ab}\epsilon_{\underline{a}\,\underline{b}}Q_{b\underline{b}}{}^{C}\\
& + iB^0\overline{Q}_{a\underline{a}}\epsilon_{ab}\epsilon_{\underline{a}\,\underline{b}}\gamma_5 Q_{b\underline{b}}{}^{C} + \text{h.c.}\Big)\Big],
\end{aligned} \tag{24}$$

$$D_\mu Q = \partial_\mu Q - \frac{i}{2}g_W W_\mu^k\tau_k Q, \tag{25}$$

where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and

$$\begin{aligned}
\sigma' &= A_0 - u,\\
\tilde\eta &= B_0,\\
\tilde a_k &= A_k,\\
\tilde\pi_k &= B_k,\\
&\qquad k = 1, 2, 3,\\
A^0 &= \frac{1}{\sqrt{2}}\left(A_4 + iA_5\right),\\
B^0 &= \frac{1}{\sqrt{2}}\left(B_4 + iB_5\right).
\end{aligned} \tag{26}$$

From now on we use tildes to distinguish hypermesons from usual ones. The v.e.v. $u = \langle A_0\rangle \sim \langle\overline{Q}Q\rangle$ breaks the global symmetry $SU(4)$ spontaneously.

As it is seen from the form of the covariant derivative (19), the local electroweak group is embedded into global $Sp(4)$ and breaks it as well as its chiral subgroup explicitly. The covariant derivative of the (pseudo)scalars follows from the transformation properties of M:

$$D_\mu M = \partial_\mu M - \sqrt{2}ig_W W_\mu^k\left(\Sigma_k M + M\Sigma_k^T\right). \tag{27}$$

Using the above derivative, the scalar sector of the model can be written as follows:

$$\begin{aligned}
L = {} & D_\mu\mathscr{H}^\dagger\cdot D^\mu\mathscr{H} + \operatorname{Tr}D_\mu M^\dagger\cdot D^\mu M - U = \frac{1}{2}\Big(D_\mu h\\
& \cdot D^\mu h + D_\mu h_k\cdot D^\mu h_k + \partial_\mu\tilde\sigma\cdot\partial^\mu\tilde\sigma + D_\mu\tilde\pi_k\cdot D^\mu\tilde\pi_k\\
& + \partial_\mu\tilde\eta\cdot\partial^\mu\tilde\eta + D_\mu\tilde a_k\cdot D^\mu\tilde a_k\Big) + \partial_\mu\overline{A}^0\cdot\partial^\mu A^0 + \partial_\mu\overline{B}^0\\
& \cdot\partial^\mu B^0 - U,
\end{aligned} \tag{28}$$

where the covariant derivatives of the H-meson fields read

$$\begin{aligned}
D_\mu\tilde\pi_k &= \partial_\mu\tilde\pi_k + g_W e_{klm}W_\mu^l\tilde\pi_m,\\
D_\mu\tilde a_k &= \partial_\mu\tilde a_k + g_W e_{klm}W_\mu^l\tilde a_m.
\end{aligned} \tag{29}$$

In (28) it is assumed that the Higgs doublet $\mathscr{H}$ of SM is fundamental, not composite. Its transformation properties are defined as usual in SM—the covariant derivative of $\mathscr{H}$ is

$$D_\mu\mathscr{H} = \partial_\mu\mathscr{H} + \frac{i}{2}g_B B_\mu\mathscr{H} - \frac{i}{2}g_W W_\mu^k\tau_k\mathscr{H}, \tag{30}$$

or equivalently

$$\begin{aligned}
\mathscr{H} &= \frac{1}{\sqrt{2}}\begin{pmatrix}h_2 + ih_1\\ h - ih_3\end{pmatrix} = \frac{1}{\sqrt{2}}\left(h + ih_k\tau_k\right)\begin{pmatrix}0\\ 1\end{pmatrix},\\
D_\mu h &= \partial_\mu h + \frac{1}{2}\left(g_B\delta_3^k B_\mu + g_W W_\mu^k\right)h_k,\\
D_\mu h_k &= \partial_\mu h_k - \frac{1}{2}\left(g_B\delta_3^k B_\mu + g_W W_\mu^k\right)h\\
&\quad - \frac{1}{2}e_{klm}\left(g_B\delta_3^l B_\mu - g_W W_\mu^l\right)h_m.
\end{aligned} \tag{31}$$

In Lagrangian (28) the potential term U consists of self-interactions of the scalar fields:

$$U = -\sum_{i=0}^{3}\mu_i^2 I_i + \sum_{i\le j=0}^{3}\lambda_{ij}I_iI_j, \tag{32}$$

where I_0 is the $SU(2)_L \otimes U(1)_Y$ invariant of the SM Higgs doublet and I_i, $i = 1,2,3$, are three independent $SU(4)$ invariants of the field M:

$$\begin{aligned} I_0 &= \mathscr{H}^\dagger \mathscr{H} = \frac{1}{2}(v+h)^2, \\ I_1 &= \operatorname{Tr} M^\dagger M - 4 \operatorname{Re} \operatorname{Pf} M \\ &= \frac{1}{2}\left[\left(u+\sigma'\right)^2 + \tilde{\pi}_k \tilde{\pi}_k + 2\overline{B}^0 B^0\right], \\ I_2 &= \operatorname{Tr} M^\dagger M + 4 \operatorname{Re} \operatorname{Pf} M \\ &= \frac{1}{2}\left[\tilde{\eta}^2 + \tilde{a}_k \tilde{a}_k + 2\overline{A}^0 A^0\right], \\ I_3 &= 4 \operatorname{Im} \operatorname{Pf} M \\ &= -\left(u+\sigma'\right)\tilde{\eta} + \tilde{a}_k \tilde{\pi}_k + \overline{B}^0 A^0 + \overline{A}^0 B^0. \end{aligned} \tag{33}$$

Here $\operatorname{Pf} M = -(1/4)\operatorname{Tr} M\widetilde{M} = (1/8)\epsilon_{prst} M_{pr} M_{st}$ is the Pfaffian of M; ϵ_{prst} is the 4-dimensional Levi-Civita symbol ($\epsilon_{1234} = +1$); $v = \langle h \rangle$ is the Higgs-field v.e.v. We consider only renormalizable self-interactions of the scalar fields, although renormalizability in general has nothing to do with effective field theories. The invariant I_3 is odd under CP conjugation. CP invariance implies that $\lambda_{03} = \lambda_{13} = \lambda_{23} = 0$ and $\mu_3 = 0$.

Tadpole equations for $v, u \neq 0$ are

$$\begin{aligned} \mu_0^2 &= \lambda_{00} v^2 + \frac{1}{2}\lambda_{01} u^2, \\ \mu_1^2 &= \lambda_{11} u^2 + \frac{1}{2}\lambda_{01} v^2 + \frac{\zeta \left\langle \overline{Q}Q \right\rangle}{u}. \end{aligned} \tag{34}$$

Vacuum stability is ensured by the following inequalities:

$$\begin{aligned} \Lambda_{11} &= \lambda_{11} - \frac{\zeta \left\langle \overline{Q}Q \right\rangle}{2u^3} > 0, \\ \lambda_{00} &> 0, \\ 4\lambda_{00}\Lambda_{11} - \lambda_{01}^2 &> 0. \end{aligned} \tag{35}$$

Deriving (34) and (35) we have taken into account a tadpole-like source term $L_{\text{SB}} = -\zeta\langle\overline{Q}Q\rangle(u+\sigma')$, where ζ is a parameter proportional to the current mass m_Q of the H-quarks. Such term in phenomenological fashion communicates effects of explicit breaking of the $SU(4)$ global symmetry to the vacuum parameters and the H-hadron spectrum. This resembles QCD—the chiral symmetry is broken both dynamically (with the quark condensate $\langle\overline{q}q\rangle$ as an order parameter) and explicitly (by the quark masses). In the sigma models with linear realization of the chiral symmetry, the spontaneous breaking is induced by v.e.v. of σ meson field. The effects of the explicit breaking can be mimicked by different chirally noninvariant terms [64–66], but the most common one, which is sometimes referred to as "standard breaking," is a tadpole-like σ term (see [67, 68], for example).

The masses of the (pseudo)scalar fields read

$$\begin{aligned} m_{\tilde{\sigma},H}^2 &= \lambda_{00} v^2 + \Lambda_{11} u^2 \\ &\quad \pm \sqrt{\left(\lambda_{00} v^2 - \Lambda_{11} u^2\right)^2 + \lambda_{01}^2 v^2 u^2}, \\ m_{\tilde{\pi}}^2 &= m_B^2 = -\frac{\zeta \left\langle \overline{Q}Q \right\rangle}{u}, \\ m_{\tilde{\eta}}^2 &= m_{\tilde{a}}^2 + 2\lambda_{33} u^2, \\ m_{\tilde{a}}^2 &= m_A^2 = -\mu_2^2 + \frac{1}{2}\lambda_{02} v^2 + \frac{1}{2}\lambda_{12} u^2. \end{aligned} \tag{36}$$

The physical Higgs boson becomes partially composite receiving a tiny admixture of the scalar field σ':

$$\begin{aligned} h &= \cos\theta_s H - \sin\theta_s \tilde{\sigma}, \\ \sigma' &= \sin\theta_s H + \cos\theta_s \tilde{\sigma}, \\ \tan 2\theta_s &= \frac{\lambda_{01} v u}{\lambda_{00} v^2 - \Lambda_{11} u^2}, \\ \operatorname{sgn}\sin\theta_s &= -\operatorname{sgn}\lambda_{01}, \end{aligned} \tag{37}$$

where h and σ' are the fields being mixed, while H and $\tilde{\sigma}$ are physical ones.

Finally, the self-interactions of scalar fields take the form

$$\begin{aligned} L = &-\lambda_{00} h^3 \left(v + \frac{1}{4}h\right) \\ &- \frac{1}{4}\lambda_{11}\left(B_{\dot{\alpha}} B_{\dot{\alpha}} + \sigma'^2\right)\left(B_{\dot{\alpha}} B_{\dot{\alpha}} + \sigma'^2 + 4u\sigma'\right) \\ &- \frac{1}{4}\lambda_{01} h\left[(2v+h)\left(B_{\dot{\alpha}} B_{\dot{\alpha}} + \sigma'^2\right) + 2u\sigma' h\right] \\ &- \frac{1}{4}\lambda_{02} h(2v+h)\left(A_{\dot{\alpha}} A_{\dot{\alpha}} + \tilde{\eta}^2\right) \\ &- \frac{1}{4}\lambda_{12}\left(B_{\dot{\alpha}} B_{\dot{\alpha}} + \sigma'^2 + 2u\sigma'\right)\left(A_{\dot{\alpha}} A_{\dot{\alpha}} + \tilde{\eta}^2\right) \\ &- \frac{1}{4}\lambda_{22}\left(A_{\dot{\alpha}} A_{\dot{\alpha}} + \tilde{\eta}^2\right)^2 \\ &- \lambda_{33}\left[-\left(u+\sigma'\right)\tilde{\eta} + \tilde{a}_k \tilde{\pi}_k + \overline{B}^0 A^0 + \overline{A}^0 B^0\right]^2, \end{aligned} \tag{38}$$

where $A_{\dot{\alpha}} A_{\dot{\alpha}} = 2\tilde{a}^+\tilde{a}^- + \tilde{a}^0\tilde{a}^0 + 2\overline{A}^0 A^0$ and $B_{\dot{\alpha}} B_{\dot{\alpha}} = 2\tilde{\pi}^+\tilde{\pi}^- + \tilde{\pi}^0\tilde{\pi}^0 + 2\overline{B}^0 B^0$.

The complete set of the lightest spin-0 H-hadrons in the model includes pNG states (pseudoscalar H-pions $\tilde{\pi}_k$ and scalar complex H-diquarks/H-baryons B^0), their opposite-parity chiral partners $\tilde{a}_k$ and A^0, and singlet H-mesons $\tilde{\sigma}$ and $\tilde{\eta}$. These H-hadrons are listed in Table 1 along with their quantum numbers and associated H-quark currents. Note

Table 1: Quantum numbers of the lightest (pseudo)scalar H-hadrons and the corresponding H-quark currents in $SU(2)_{\rm HC}$ model. $\widetilde{G}$ denotes hyper-G-parity of a state (see Section 4). $\widetilde{B}$ is the H-baryon number. $Q_{\rm em}$ is the electric charge. T is the weak isospin. Hyperbaryons do not carry intrinsic C- and HG-parities, since the charge conjugation reverses the sign of the H-baryon number.

State	H-quark current	$T^{\widetilde{G}}(J^{PC})$	$\widetilde{B}$	$Q_{\rm em}$
$\tilde{\sigma}$	$\overline{Q}Q$	$0^+(0^{++})$	0	0
$\tilde{\eta}$	$i\overline{Q}\gamma_5 Q$	$0^+(0^{-+})$	0	0
$\tilde{a}_k$	$\overline{Q}\tau_k Q$	$1^-(0^{++})$	0	±1, 0
$\tilde{\pi}_k$	$i\overline{Q}\gamma_5\tau_k Q$	$1^-(0^{-+})$	0	±1, 0
A^0	$\overline{Q}^C_{a\underline{a}}\epsilon_{ab}\epsilon_{\underline{a}\underline{b}}Q_{b\underline{b}}$	$0\ (0^-\ )$	1	0
B^0	$i\overline{Q}^C_{a\underline{a}}\epsilon_{ab}\epsilon_{\underline{a}\underline{b}}\gamma_5 Q_{b\underline{b}}$	$0\ (0^+\ )$	1	0

that the total Lagrangian of the model given by (24), (25), (28), and (32) is invariant under a global transformation

$$Q' = e^{(i/2)\xi}Q,\quad \left(A^0\right)' = e^{i\xi}A^0,\quad \left(B^0\right)' = e^{i\xi}B^0 \tag{39}$$

or equivalently the Lagrangian given by (18), (28), and (32) in terms of the quartet field P_L and the antisymmetric field M is invariant under a transformation

$$P_L' = e^{(i/2)\xi\Sigma_4}P_L,\quad M' = e^{(i/2)\xi\Sigma_4}Me^{(i/2)\xi\Sigma_4^T},\quad \Sigma_4 = \frac{1}{2\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{40}$$

where Σ_4 is a generator of $Sp(4)\subset SU(4)$. The EW symmetry, which is spanned by the generators Σ_k, $k=1,2,3$, defined by (14), does not break the symmetry (40), since the generator Σ_4 commutes with Σ_k. This additional global $U(1)_{\rm HB}$ symmetry (40) allows us to introduce a conserved H-baryon number, which makes the lightest H-diquark stable. We remind of the fact that the model contains the elementary Higgs field which is not a pNG state. There is, however, a scenario with a composite Higgs boson having also a new strongly coupled sector with the symmetry breaking pattern $SU(4)\longrightarrow Sp(4)$ [69].

4. Physical Lagrangian of the Minimal Model

Now, we represent the part of physical Lagrangian which is relevant for further analysis of the most interesting case with zero hypercharge (stable H-pion scenario). The H-quark interactions with the EW bosons are vectorlike, and the corresponding Lagrangian follows from (25):

$$L(Q,G) = \frac{1}{\sqrt{2}}g_W\overline{U}\gamma^\mu DW^+_\mu + \frac{1}{\sqrt{2}}g_W\overline{D}\gamma^\mu UW^-_\mu + \frac{1}{2}g_W\left(\overline{U}\gamma^\mu U - \overline{D}\gamma^\mu D\right)\left(c_W Z_\mu + s_W A_\mu\right). \tag{41}$$

Here c_W and s_W denote cosine and sine of the Weinberg angle. Interactions of (pseudo)scalars with photons and intermediate bosons are described by the following Lagrangians:

$$\begin{aligned} L(\tilde{\sigma},H,G) &= \frac{1}{8}\left[2g_W^2W^+_\mu W^\mu_- + \left(g_B^2+g_W^2\right)Z_\mu Z^\mu\right]\cdot\left(\cos\theta_s H - \sin\theta_s\sigma\right)^2,\\ L(\tilde{\pi},\tilde{a},G) &= \left[ig_W W^\mu_+\left(\tilde{\pi}^0\tilde{\pi}^-_{,\mu} - \tilde{\pi}^-\tilde{\pi}^0_{,\mu}\right) + \text{h.c.}\right] + ig_W\left(c_W Z^\mu - s_W A^\mu\right)\left(\tilde{\pi}^-\tilde{\pi}^+_{,\mu} - \tilde{\pi}^+\tilde{\pi}^-_{,\mu}\right)\\ &+ g_W^2\tilde{\pi}^+\tilde{\pi}^-\left(c_W Z^\mu - s_W A^\mu\right)^2 - g_W^2\tilde{\pi}^0\left(c_W Z^\mu - s_W A^\mu\right)\left(\tilde{\pi}^+W^-_\mu + \tilde{\pi}^-W^+_\mu\right) - \frac{1}{2}\\ &\cdot g_W^2\left(\tilde{\pi}_+^2W^-_\mu W^\mu_- + \tilde{\pi}_-^2W^+_\mu W^\mu_+\right) + g_W^2\left(\tilde{\pi}_0^2 + \tilde{\pi}^-\tilde{\pi}^+\right)\\ &\cdot W^+_\mu W^\mu_- + \left(\tilde{\pi}\longrightarrow\tilde{a}\right). \end{aligned} \tag{42}$$

In the above Lagrangian $L(\tilde{\pi}/\tilde{a},G)$ the last term means that the interactions of the triplet scalar H-mesons $\tilde{a}$ have the same couplings and vertices as the interactions of the H-pions.

The scalar and pseudoscalar fields $\tilde{\sigma}$, $\tilde{\pi}$, and H interaction with the H-quarks is described by the Lagrangian which directly follows from (24):

$$\begin{aligned} L(Q,\tilde{\sigma},H) &= -\kappa\left(c_\theta\tilde{\sigma} + s_\theta H\right)\left(\overline{U}U + \overline{D}D\right)\\ &+ i\sqrt{2}\kappa\tilde{\pi}^+\overline{U}\gamma_5 D + i\sqrt{2}\kappa\tilde{\pi}^-\overline{D}\gamma_5 U\\ &+ i\kappa\tilde{\pi}^0\left(\overline{U}\gamma_5 U - \overline{D}\gamma_5 D\right), \end{aligned} \tag{43}$$

where $c_\theta = \cos\theta_s$ and $s_\theta = \sin\theta_s$. There is a specific symmetry of the minimal hypercolor model leading to some phenomenological consequences. At the fundamental level, the Lagrangian of the current H-quarks (10) is invariant under modified charge conjugation of the H-quark fields (hyper-G-parity, HG-parity) which is defined as follows:

$$\left(Q_{a\underline{a}}\right)^{HG} = \epsilon_{ab}\epsilon_{\underline{a}\underline{b}}Q^C_{b\underline{b}}, \tag{44}$$

where C is the charge conjugation, a, b are isotopic indices, and $\underline{a}$, $\underline{b}$ are hypercolor indices (it is the same notation as in the Section 2). To prove the statement, we use (6) and the properties of bilinear forms with respect to the ordinary charge conjugation

$$\begin{aligned} \overline{Q}^C_{a\underline{a}} Q^C_{b\underline{b}} &= \overline{Q}_{b\underline{b}} Q_{a\underline{a}}, \\ \overline{Q}^C_{a\underline{a}} \gamma_5 Q^C_{b\underline{b}} &= \overline{Q}_{b\underline{b}} \gamma_5 Q_{a\underline{a}}, \\ \overline{Q}^C_{a\underline{a}} \gamma_\mu Q^C_{b\underline{b}} &= -\overline{Q}_{b\underline{b}} \gamma_\mu Q_{a\underline{a}}. \end{aligned} \quad (45)$$

By straightforward calculations one can check that Lagrangian (10) is invariant under the transformation (44), since the H-gluon T_μ and the SM fields are not transformed. To analyze transformation properties of the $\tilde{\pi}\overline{Q}Q$ effective vertex in more detail, we use (44) and (45) and have

$$\begin{aligned} \left(\overline{Q}_{a\underline{a}} \gamma_5 \tau^k_{ab} \tilde{\pi}_k Q_{b\underline{a}}\right)^{HG} &= \overline{Q}^C_{a\underline{a}} \epsilon_{ab} \epsilon_{\underline{a}\,\underline{b}} \gamma_5 \tau^k_{bc} \tilde{\pi}^{HG}_k \epsilon_{cd} \epsilon_{\underline{b}\,\underline{c}} Q^C_{d\underline{c}} \\ &= -\overline{Q}^C_{a\underline{a}} \gamma_5 \tau^{*k}_{ad} \tilde{\pi}^{HG}_k Q^C_{d\underline{a}} \\ &= -\overline{Q}_{a\underline{a}} \gamma_5 \tau^k_{ab} \tilde{\pi}^{HG}_k Q_{b\underline{a}}. \end{aligned} \quad (46)$$

So, the invariance condition results in the transformation $\tilde{\pi}^{HG}_k = -\tilde{\pi}_k$, that is, $\tilde{\pi}$ is odd, while the SM fields are even under modified charge conjugation (44). This is a special case of the treatment of general vectorlike HC models in [70, 71]. It is observed in [70] that HG-parity is a good quantum number of the theory and all SM particles are HG-even. Thus, HG-odd $\tilde{\pi}$ has no decay modes with only SM particles in the final states. In the model under consideration decay channels of type $\tilde{\pi}^\pm \to \tilde{\pi}^0 X^\pm$ are allowed due to HG-parity conservation.

It is important that all restrictions on the oblique corrections are fulfilled in this variant of hypercolor. If the hypercharge is zero and h-$\tilde{\sigma}$ mixing is absent, then T-parameter is equal to zero. If, however, we consider a HC scenario with a nonzero hypercharge and mixing, a constraint for the T parameter value emerges (see [35, 43]). Then the h-$\tilde{\sigma}$ mixing angle should be sufficiently small to avoid problems with the PT parameters and the measured properties of the SM Higgs boson.

5. Low-Energy Signature of the Model

In this section, we consider briefly main phenomenological consequences of the minimal model for the case of zero hypercharge. In spite of a simple structure and minimal particle content, the model can manifest a rich phenomenology and interesting signature in collider physics. Here we consider processes with the H-sigma ($\tilde{\sigma}$) and H-pions ($\tilde{\pi}$). It is supposed that these states are the lowest ones in the model (see, however, the results of lattice calculations in [72]). Indeed, the claim that pNG states are the lightest in the mass spectrum is based on the hypothesis of a hierarchy of H-physics scales. In other words, we suppose that other (not pNG) possible H-hadrons including vector H-mesons are heavier than the pNG bosons. Namely, the explicit $SU(4)$ symmetry breaking is considered as a small perturbation in comparison with the dynamical symmetry breaking in analogy with the orthodox QCD, where the scale of chiral symmetry breaking is much larger than the light quark masses. From our previous analysis of the parameter space, it follows that the masses of H-mesons are of the order of 10^2–10^3 GeV. Thus, the low-energy pNG states of the minimal model can be accessed at the LHC and future linear collider.

Channels of H-pion production and decay are described by the model Lagrangian (see the previous section). At the LHC these pNG states most effectively occur in two ways: in vector boson fusion (VBF) reaction $pp \to V^* V'^* \to \tilde{\pi}\tilde{\pi}'$, where $V = W, Z, \gamma$, or in the s-channel of $q\bar{q}'$- or $q\bar{q}$-fusion—Drell–Yan type (DYT) process, $pp \to V'^* \to \tilde{\pi}\tilde{\pi}$. Corresponding Feynman diagrams can be found in [35]. There is also an analog of usual associated production where H-pion pair is produced together with vector boson, $pp \to V'^* \to V\tilde{\pi}\tilde{\pi}$. Its cross section, is somewhat suppressed compared with the DYT reaction by extra factor g^2_W. The channel, however, has a specific set of final states (see below).

As to VBF and DYT mechanisms, their contributions to the cross section of H-pion pair production strongly depend on the invariant H-pion mass, kinematic cuts for final states, quark pdf's, combinatorial factors, and $q \to Vq'$ splitting functions at high energies. Of course, NLO and NNLO corrections for these channels should be different and can be important—as is the case for Higgs production at the LHC [73–76]. A detailed analysis of LO cross sections and NLO corrections is beyond the scope of the paper.

It seems that the VBF production of H-pions is suppressed, in particular, by an additional g^4_W, and Drell–Yan type process dominates (see [36]). The situation is, however, more complicated due to the above-mentioned factors, and in the TeV region VV'-fusion cross section is very close to DYT or even larger (see, for example, [76]). Moreover, due to suitable p_T cuts, it is possible that, as it happens for the high mass (~TeV) scalar boson production [74], s-channel qq'-fusion cross section should be comparatively small. Of course, it is not the same process; nevertheless, enhancing factors for the VBF are analogous—a lot of integrated partons with low x and p_T when vector boson splits off. Namely, due to integration with quark splitting functions in the region of low partonic p_T, VBF cross section can be increased by $\log^2(M/M_W)$; M is an invariant mass of H-pion pair. Note also that large resonance s-channel contribution to VBF production with intermediate $\tilde{\sigma}$ is possible if $m_{\tilde{\sigma}}$ is close to $2m_{\tilde{\pi}}$. This point should be considered separately.

VBF cross section of H-pion pair production, as function of qq' center-of-mass energy and H-pion mass, was calculated in our paper [35] and $\sigma_{\rm VBF}(pp \to qq'\tilde{\pi}\tilde{\pi}) \approx (0.01$–$0.02)$ pb when $E_{cm} \sim 1$ TeV and H-pion mass is 200–300 GeV. We also estimate the DYT cross section in this region as approximately 0.03–0.05 pb. Both of these cross sections decrease of about one order of magnitude with the mass of $\tilde{\pi}$ increasing up to ~500–700 GeV.

Almost the same situation is observed for the hierarchy of Higgs production mechanisms [77]—associated Higgs production dominates at $\sqrt{s} = 2$ TeV—but at higher energies, $\sqrt{s} \geq 4$ TeV, the situation is reversed and VBF cross section exceeds associated production by almost a half. In other words, behavior of these cross sections at high energies should be studied more carefully and it will be done in the next paper.

Estimated cross sections of H-pion production are small, so, to detect a signal, large statistics and the background suppression are necessary. From this point of view, VBF reactions are more promising due to the presence of the two hard tagging jets. Adding some reasonable cuts, for the rapidity to highlight the central region of the reaction, $|\eta| \leq 2.5$, and for final leptons, $p_T \geq 100\,\text{GeV}$, it is possible to separate leptons from $\tilde{\pi}^{\pm}$ decay. These decays are also marked by large missed p_T due to heavy stable neutral H-pions and neutrino.

H-pion production in the process of annihilation $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow 2\tilde{\pi}; 4\tilde{\pi}$ is also possible through the reactions of the type $Z^* \rightarrow \tilde{\pi}^+\tilde{\pi}^-$ and $W^{\pm *} \rightarrow \tilde{\pi}^{\pm}\tilde{\pi}^0$. These processes have transparent signature and can be studied at future linear colliders. Note that some interesting features should be observed: production of H-pions in associated process, $e^+e^- \rightarrow Z^* \rightarrow Z\tilde{\pi}^{\pm}\tilde{\pi}^{\mp}$, $e^+e^- \rightarrow Z^* \rightarrow W\tilde{\pi}^{\pm}\tilde{\pi}^0$, or via VV'-fusion. At the ILC Higgs boson production cross sections demonstrate evolution with energy [78] which is analogous to predicted for the LHC. In the H-pion production we expect the same behavior of cross sections.

To analyze a final signature in the reactions above, note that due to HG-parity conservation (see the previous section) H-pions have no tree-level decay modes having in the final states the SM particles only. The lowest order amplitudes which govern decays of the type $\tilde{\pi} \rightarrow V_1V_2, V_1V_2V_3$, where $V_a = \gamma, Z, W$, are described by triangle and box diagrams with H-quarks loops. It can be easily checked that interference contributions for the transition $\tilde{\pi} \rightarrow V_1V_2$ with U and D hyperquark loops cancel out each other. Since $M_U = M_D$, this compensation is obvious due to the opposite charges, $q_D = -q_U$, when $Y_1 = 0$. Analysis of the decay $\tilde{\pi} \rightarrow V_1V_2V_3$ reveals compensation of box contributions. More exactly, the diagrams with loop momenta circulating in opposite directions cancel out each other. It is easy to prove that the compensation results from generalized Furry's theorem [79]. To this end, we should use the following properties of the Dirac and Pauli matrices:

$$\begin{aligned} \text{Tr}\left\{\gamma_{\mu_1}\gamma_{\mu_2}\cdots\gamma_{\mu_n}\right\} &= \text{Tr}\left\{\gamma_{\mu_n}\cdots\gamma_{\mu_2}\gamma_{\mu_1}\right\}, \\ \text{Tr}\left\{\tau_{a_1}\tau_{a_2}\cdots\tau_{a_n}\right\} &= (-1)^n\,\text{Tr}\left\{\tau_{a_n}\cdots\tau_{a_2}\tau_{a_1}\right\}. \end{aligned} \tag{47}$$

The cancellation of amplitudes in the case of an even number of final bosons is inherently isotopic—it results from the zero H-quark hypercharge. If the number of final bosons is odd, such cancellation follows from the charge parity conservation along with the vectorlike structure of the H-quark EW interaction. As a result, the H-pion fields are stable in the framework of the vectorlike hypercolor model with zero hypercharge and degenerate masses in the H-quark doublet Q and triplet $\tilde{\pi}$. In the previous section, it was demonstrated that this stability follows from the presence of the discrete symmetry in the model.

Note that the H-quark masses remain degenerate, $M_U = M_D$, at the one-loop level. It can be easily checked that the self-energy contributions into the mass renormalization are defined by electroweak and H-pion loops. These terms are exactly the same for the U and D quarks. However, this effect does not take place in the case of the H-pion masses. The mass-splitting value of the H-pion can be calculated by summing over self-energy diagrams. Detailed analysis of the relevant amplitudes reveals that only EW diagrams contribute to the mass-splitting $\Delta m_{\tilde{\pi}} = m_{\tilde{\pi}^{\pm}} - m_{\tilde{\pi}^0}$; all strong (H-quark) loops are cancelled out. As a result we get

$$\begin{aligned} \Delta m_{\tilde{\pi}} = \frac{G_F M_W^4}{2\sqrt{2}\pi^2 m_{\tilde{\pi}}} \Bigg[& \ln\frac{M_Z^2}{M_W^2} - \beta_Z^2 \ln\mu_Z + \beta_W^2 \ln\mu_W \\ & - \frac{4\beta_Z^3}{\sqrt{\mu_Z}}\left(\arctan\frac{2-\mu_Z}{2\sqrt{\mu_Z}\beta_Z} + \arctan\frac{\sqrt{\mu_Z}}{2\beta_Z}\right) \\ & + \frac{4\beta_W^3}{\sqrt{\mu_W}}\left(\arctan\frac{2-\mu_W}{2\sqrt{\mu_W}\beta_W} + \arctan\frac{\sqrt{\mu_W}}{2\beta_W}\right)\Bigg], \end{aligned} \tag{48}$$

where $\mu_V = M_V^2/m_{\tilde{\pi}}^2$, $\beta_V = \sqrt{1-\mu_V/4}$, and G_F is Fermi's constant. For the H-pion masses in the interval 200–800 GeV from (48) it follows that $\Delta m_{\tilde{\pi}} \approx 0.170$–$0.162\,\text{GeV}$. Nonzero mass-splitting in the H-pion triplet violates isotopic invariance. However, HG-parity remains a conserved quantum number since it is induced by a discrete symmetry rather than a continuous transformation in the space of H-pion states. Thus, an account of higher order corrections does not lead to destabilization of the neutral H-pion.

So, the analysis performed leads to the conclusion that the model involves stable weakly interacting neutral H-pion. Then, production of the H-pions at colliders manifests itself with some unique signature of the final state—charge leptons and large missing energy. The stable H-pion $\tilde{\pi}^0$ can be also considered as a component of Dark Matter.

For the width of the charged H-pion decay in the strong channel we get

$$\begin{aligned} & \Gamma\left(\tilde{\pi}^{\pm} \longrightarrow \tilde{\pi}^0\pi^{\pm}\right) \\ & \quad = \frac{G_F^2}{\pi} f_\pi^2 \left|U_{ud}\right|^2 m_{\tilde{\pi}}^{\pm} \left(\Delta m_{\tilde{\pi}}\right)^2 \overline{\lambda}\left(m_{\pi^{\pm}}^2, m_{\tilde{\pi}^0}^2; m_{\tilde{\pi}^{\pm}}^2\right). \end{aligned} \tag{49}$$

Here $f_\pi = 132\,\text{MeV}$, $\pi^{\pm}$ is a standard pion, and

$$\overline{\lambda}(a,b;c) = \left[1 - 2\frac{a+b}{c} + \frac{(a-b)^2}{c^2}\right]^{1/2}. \tag{50}$$

The H-pion decay width in the lepton channel is

$$\begin{aligned} \Gamma\left(\tilde{\pi}^{\pm} \longrightarrow \tilde{\pi}^0 l^{\pm}\nu_l\right) = \frac{G_F^2 m_{\tilde{\pi}^{\pm}}^3}{24\pi^3} \int_{q_1^2}^{q_2^2} & \overline{\lambda}\left(q^2, m_{\tilde{\pi}^0}^2; m_{\tilde{\pi}^{\pm}}^2\right)^{3/2} \\ & \cdot\left(1 - \frac{3m_l^2}{2q^2} + \frac{m_l^6}{2q^6}\right) dq^2, \end{aligned} \tag{51}$$

where $q_1^2 = m_l^2$, $q_2^2 = (\Delta m_{\tilde{\pi}})^2$, and m_l is a lepton mass.

Now, using (49), (51), and the value $\Delta m_{\tilde{\pi}}$ from (48), we estimate decay widths, lifetimes, and track lengths in these channels as follows:

$$\begin{aligned}\Gamma\left(\tilde{\pi}^{\pm} \longrightarrow \tilde{\pi}^{0}\pi^{\pm}\right) &= 6\cdot 10^{-17}\,\text{GeV},\\ \tau_{\pi} &= 1.1\cdot 10^{-8}\,\text{sec},\\ c\tau_{\pi} &\approx 330\,\text{cm};\\ \Gamma\left(\tilde{\pi}^{\pm} \longrightarrow \tilde{\pi}^{0}l^{\pm}\nu_{l}\right) &= 3\cdot 10^{-15}\,\text{GeV},\\ \tau_{l} &= 2.2\cdot 10^{-10}\,\text{sec},\\ c\tau_{l} &\approx 6.6\,\text{cm}.\end{aligned} \tag{52}$$

From these analysis it follows that main characteristic fingerprints of H-pions at TeV scale in the VBF, DYT, and associated production are

(1) $V^{*}V^{*} \rightarrow \tilde{\pi}\tilde{\pi} + jj$—two hard tagging jets, high $p_{T,mis}$ from two $\tilde{\pi}^{0}$, neutrino, and a lepton (or two charged leptons) from $\tilde{\pi}^{\pm}$;

(2) $V^{*} \rightarrow \tilde{\pi}\tilde{\pi}$—high $p_{T,mis}$ from two $\tilde{\pi}^{0}$ and ν_l, and final one lepton, $l\bar{l}$ or $\pi^{\pm}\pi^{\pm}$, from pair of $\tilde{\pi}^{\pm}$;

(3) $V^{*} \rightarrow V\tilde{\pi}\tilde{\pi}$—hadron jets (or $l\bar{l}$ or $l\nu_l$) from W or Z decays, high $p_{T,mis}$ from two $\tilde{\pi}^{0}$, and neutrino (from $\tilde{\pi}^{\pm}$ and/or $W^{\pm}$); $l^{+}l^{-}$ —if there are two final charged H-pions or one charged H-pion; and W, trilepton signal from $W\tilde{\pi}^{\pm}\tilde{\pi}^{\pm}$ final state.

As to the production of a single scalar H-sigma $\tilde{\sigma}$ at the LHC and ILC, it is strongly suppressed reaction at the tree-level due to the small $\tilde{\sigma}$-h mixing. More exactly, the tree-level $\tilde{\sigma}$ production is suppressed with respect to the Higgs production by $\sin^2\theta_s$, where θ_s is a mixing angle.

At the one-loop level both single and double H-sigma production occur in the processes of type $V^{*}V'^{*} \rightarrow \tilde{\sigma}, 2\tilde{\sigma}$ and/or $V^{*} \rightarrow \Delta \rightarrow V'\tilde{\sigma}, 2\tilde{\sigma}$, where V^{*} and V' are vector bosons in the intermediate and final states; Δ denotes a H-quark triangle loop.

Decays of the type $\tilde{\sigma} \rightarrow V_1V_2$, where $V_{1,2} = \gamma, Z, W$, proceed through H-quark and H-pion loops. Dominant decay channels of H-sigma are $\tilde{\sigma} \rightarrow \tilde{\pi}^{0}\tilde{\pi}^{0}, \tilde{\pi}^{+}\tilde{\pi}^{-}$, which take place at the tree-level and provide large decay width for $m_{\tilde{\sigma}} \geqslant 2m_{\tilde{\pi}}$. The width is mostly defined by the coupling λ_{11} in the limit of small mixing:

$$\Gamma\left(\tilde{\sigma} \longrightarrow \tilde{\pi}\tilde{\pi}\right) = \frac{3u^{2}\lambda_{11}^{2}}{8\pi m_{\tilde{\sigma}}}\left(1 - \frac{4m_{\tilde{\pi}}^{2}}{m_{\tilde{\sigma}}^{2}}\right). \tag{53}$$

Using the previous parametric analysis in [35] concerning the value λ_{11} (λ_{HC} in [35]) and u, from (53), one can get $\Gamma(\tilde{\sigma} \rightarrow \tilde{\pi}\tilde{\pi}) \gtrsim 10\,\text{GeV}$ when $m_{\tilde{\sigma}} \gtrsim 2m_{\tilde{\pi}}$.

As it was noted above, the small mixing h–$\tilde{\sigma}$ in conformal approximation leads to the relation $m_{\tilde{\sigma}} \approx \sqrt{3}m_{\tilde{\pi}}$ and all tree-level decay widths are proportional to the square value of the $\tilde{\sigma}$-h mixing angle θ_s. Corresponding decay widths are as follows:

$$\begin{aligned}\Gamma\left(\tilde{\sigma} \longrightarrow f\bar{f}\right) &= \frac{g_W^2\sin^2\theta_s}{32\pi}m_{\tilde{\sigma}}\frac{m_f^2}{M_W^2}\left(1 - 4\frac{m_f^2}{m_{\tilde{\sigma}}^2}\right)^{3/2},\\ \Gamma\left(\tilde{\sigma} \longrightarrow ZZ\right) &= \frac{g_W^2\sin^2\theta_s}{16\pi c_W^2}\frac{M_Z^2}{m_{\tilde{\sigma}}}\left(1 - 4\frac{m_Z^2}{m_{\tilde{\sigma}}^2}\right)^{1/2}\\ &\quad\cdot\left[1 + \frac{\left(m_{\tilde{\sigma}}^2 - 2M_Z^2\right)^2}{8M_Z^4}\right],\\ \Gamma\left(\tilde{\sigma} \longrightarrow W^{+}W^{-}\right) &= \frac{g_W^2\sin^2\theta_s}{8\pi}\frac{M_W^2}{m_{\tilde{\sigma}}}\left(1 - 4\frac{m_W^2}{m_{\tilde{\sigma}}^2}\right)^{1/2}\\ &\quad\cdot\left[1 + \frac{\left(m_{\tilde{\sigma}}^2 - 2M_W^2\right)^2}{8M_W^4}\right].\end{aligned} \tag{54}$$

In (54) m_f is a mass of standard fermion f and $c_W = \cos\theta_W$. In the limit of zero mixing we should consider the loop-level decay channels. Here, we consider the decay channel $\tilde{\sigma} \rightarrow \gamma\gamma$ which proceeds mainly through H-quark and H-pion loops. The width can be written in the form

$$\Gamma\left(\tilde{\sigma} \longrightarrow \gamma\gamma\right) = \frac{\alpha^2 m_{\tilde{\sigma}}}{16\pi^3}\left|F_{\text{Q}} + F_{\tilde{\pi}} + F_{\tilde{a}} + F_W + F_{\text{top}}\right|^2, \tag{55}$$

where the contributions of H-quarks, F_{Q}, H-pions, $F_{\tilde{\pi}}$, W-bosons, F_W, and top-quarks, F_{top}, are defined by the following expressions:

$$\begin{aligned}F_{\text{Q}} &= -2\kappa\frac{M_{\text{Q}}}{m_{\tilde{\sigma}}}\left[1 + \left(1 - \tau_{\text{Q}}^{-1}\right)f\left(\tau_{\text{Q}}\right)\right],\\ F_{\tilde{\pi}} &= \frac{g_{\tilde{\pi}\tilde{\sigma}}}{m_{\tilde{\sigma}}}\left[1 - \tau_{\tilde{\pi}}^{-1}f\left(\tau_{\tilde{\pi}}\right)\right], \quad g_{\tilde{\pi}\tilde{\sigma}} \approx u\lambda_{11},\\ F_{\tilde{a}} &= \frac{g_{\tilde{a}\tilde{\sigma}}}{m_{\tilde{\sigma}}}\left[1 - \tau_{\tilde{a}}^{-1}f\left(\tau_{\tilde{a}}\right)\right], \quad g_{\tilde{a}\tilde{\sigma}} \approx u\lambda_{12},\\ F_W &= -\frac{g_W\sin\theta_s m_{\tilde{\sigma}}}{8M_W}\left[2 + 3\tau_W^{-1}\right.\\ &\quad\left. + 3\tau_W^{-1}\left(2 - \tau_W^{-1}\right)f\left(\tau_W\right)\right],\\ F_{\text{top}} &= \frac{4}{3}\frac{g_W\sin\theta_s M_t^2}{m_{\tilde{\sigma}}M_W}\left[1 + \left(1 - \tau_t^{-1}\right)f\left(\tau_t\right)\right],\\ f\left(\tau\right) &= \arcsin^2\sqrt{\tau}, \quad \tau < 1,\\ f\left(\tau\right) &= -\frac{1}{4}\left[\ln\frac{1 + \sqrt{1 - \tau^{-1}}}{1 - \sqrt{1 - \tau^{-1}}} - i\pi\right]^2, \quad \tau > 1.\end{aligned} \tag{56}$$

Nonzero $\tilde{\sigma}$-h mixing influences the width via W- and t-quark loops, their amplitudes being proportional to $\sin\theta_s$. Using the analysis of the model parameter space in [35], we get an estimation $\Gamma(\tilde{\sigma} \rightarrow \gamma\gamma) \approx 5$–$10\,\text{MeV}$. To calculate the $\tilde{\sigma}$ production in full processes $p\bar{p} \rightarrow \tilde{\sigma} \rightarrow \text{all}$, a corresponding program which integrates partonic cross sections with the quark distribution functions should be used. Instead, we give an approximate evaluation of this cross section for the subprocess of vector boson fusion $VV \to \tilde{\sigma}(s) \to$ all, where $V = \gamma, Z, W$. Namely, the cross section can be calculated with sufficient accuracy using a simple formula in the framework of factorization method [80]:

$$\sigma(VV \longrightarrow \tilde{\sigma}(s)) = \frac{16\pi^2 \Gamma(\tilde{\sigma}(s) \longrightarrow VV)}{9\sqrt{s}\bar{\lambda}^2 (M_V^2, M_V^2; s)} \rho_{\tilde{\sigma}}(s), \quad (57)$$

where $\tilde{\sigma}(s)$ is $\tilde{\sigma}$ in the intermediate state with energy $\sqrt{s}$ and $\Gamma(\tilde{\sigma}(s) \to VV)$ is a partial width. The probability density $\rho_{\tilde{\sigma}}(s)$ is defined by the following expression:

$$\rho_{\tilde{\sigma}}(s) = \frac{1}{\pi} \frac{\sqrt{s}\Gamma_{\tilde{\sigma}}(s)}{\left(s - M_{\tilde{\sigma}}^2\right)^2 + s\Gamma_{\tilde{\sigma}}^2(s)}, \quad (58)$$

where $\Gamma_{\tilde{\sigma}}(s)$ is the total width of the $\tilde{\sigma}$ with a mass squared equal to s. Exclusive cross section at peak energy region $\sqrt{s} = M_{\tilde{\sigma}}$ can be found by the change in the numerator of the expression (58) $\Gamma_{\tilde{\sigma}} \to \Gamma(\tilde{\sigma} \to V'V') = \Gamma_{\tilde{\sigma}} \cdot \mathrm{Br}(\tilde{\sigma} \to V'V')$:

$$\begin{aligned} \sigma\left(VV \longrightarrow \tilde{\sigma} \longrightarrow V'V'\right) &= \frac{16\pi}{9} \frac{\mathrm{Br}(\tilde{\sigma} \longrightarrow VV)\,\mathrm{Br}\left(\tilde{\sigma} \longrightarrow V'V'\right)}{m_{\tilde{\sigma}}^2 \left(1 - 4M_V^2/m_{\tilde{\sigma}}^2\right)} \\ &\approx \frac{16\pi}{9m_{\tilde{\sigma}}^2} \cdot \mathrm{Br}(\tilde{\sigma} \longrightarrow VV)\,\mathrm{Br}\left(\tilde{\sigma} \longrightarrow V'V'\right). \end{aligned} \quad (59)$$

So, the cross section at $m_{\tilde{\sigma}}^2 \gg M_V^2$ is fully defined by branchings of sigma decay and $m_{\tilde{\sigma}}$. When $2m_{\tilde{\pi}} > m_{\tilde{\sigma}}$ dominant decay channels are $\tilde{\sigma} \to WW, ZZ$, which lead to a narrow peak ($\Gamma \lesssim$ 10–100 MeV). However, here we have the cross section of the subprocess and do not take into account the distribution function. Moreover, we should also average cross section over energy resolution. Both these factors reduce significantly the value of cross section. When $2m_{\tilde{\pi}} < m_{\tilde{\sigma}}$ dominant decay channel is $\tilde{\sigma} \to \tilde{\pi}\tilde{\pi}$ which leads to a wide peak ($\Gamma \sim 10$ GeV). In this case $\mathrm{Br}(\tilde{\sigma} \to VV)$ is small and we get very small cross section. Thus, the main signature of the H-sigma production and decay is a wide peak at $2m_{\tilde{\pi}} < m_{\tilde{\sigma}}$, mostly caused by the strong possible decay $\tilde{\sigma} \to 2\tilde{\pi}$ along with weak signals caused by two-photon, lepton, and quark-jet final states (from WW, ZZ, and standard $\pi^{\pm}$ channels). There is also specific decay mode with two stable neutral H-pions as products of $\tilde{\sigma}$ decay—this manifests itself in a large missing energy together with charged leptons in the final states. As it was shown in the end of Section 3, due to the global $U(1)_{\mathrm{HB}}$ symmetry, the lightest H-diquark is stable. Then, from the physical Lagrangian, it follows that the other H-diquark can decay to the stable one and something else. So, there is a possibility of constructing the Dark Matter from two types of particles: stable neutral H-pion and the lightest scalar (or pseudoscalar) H-diquark with conserved H-baryon number. Detailed consideration of the two-component scenario of the DM depends on the variety of model parameters, mass-splitting between the pNG states, and agreement with the data on the DM relic. We add that the suggested DM model does not contain (stable) H-baryon carrying the EW charge (see, for example, [81]), so there are no strong constraints for the DM relic in the case. The study is in progress now and results will be presented in the next paper.

As to A^0, B^0 production at the colliders, these particles can be produced only by intermediate pNG states, $\tilde{a}_a$, $\tilde{\eta}$, and the Higgs boson, h, or $\tilde{\sigma}$. At the tree-level these channels are suppressed by the mixing angle. They also can originate from loops with the participation of pNG.

6. Conclusion

The analysis performed demonstrates some unique features of the simplest minimal HC model with two generations of H-quarks and $SU(2)_{\mathrm{HC}}$ as the H-confinement group. This scenario makes it possible to construct vectorlike interaction, starting from chiral nonsymmetric H-quark set of fields. In the simplest case of two-flavor scenario the set of pNG bosons, (pseudo)scalar H-mesons and H-baryons (H-diquarks), arises, which provides the rich phenomenology. The neutral H-pion $\tilde{\pi}^0$ is stable when $Y_1 = 0$ due to hyper-G-parity conservation, so specific decay channels for $\tilde{\pi}^{\pm}$ and $\tilde{\sigma}$ with a large missing energy open. Moreover, analysis of the production and decays of (pseudo)scalar states, H-pion and H-sigma, allows distinguishing between scenarios with zero and nonzero H-quarks hypercharge [35]. At the same time, the model predicts a strong signal with large missing energy in the case $2m_{\tilde{\pi}} < m_{\tilde{\sigma}}$ or weak signal with two-vector final states in the opposite case. The presence of nonanomalous global symmetry $U(1)_{\mathrm{HB}}$ in the model leads to the conservation of H-baryon charge. This, in turn, manifests itself in the presence of the stable H-baryon complex field B^0. Note that the H-baryon state A^0 can be stable also when $M_{A^0} < M_{B^0}$. This possibility will be studied separately.

The minimal model under consideration has some phenomenological features which can be verified both at collider experiments and by astrophysical observations. An interesting consequence of the model structure is a possible interpretation of the stable neutral H-pions and H-baryons as particles of DM. So, the model with stable neutral fields gives the possibility of constructing two-component DM. In this work we concentrated mainly on the methodological aspects of the model. To make complete phenomenological analysis, we should consider astrophysical applications and take into account all experimental restrictions on new physics.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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