GRAVITATIONAL WAVE DETECTORS ARE DRIVEN AWAY FROM THERMODYNAMIC EQUILIBRIUM, WHY SHOULD WE CARE

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Ground based gravitational wave detectors show extremely high displacement sensitivity which approaches the level set by the quantum limit. However a detection will likely be achieved at a low signal-to-noise ratio, making it mandatory to know the noise budget and statistics. The RareNoise project has pointed out a few mechanisms that cause the instruments to operate at non-equilibrium states. We argue that this aspect has not been given appropriate consideration and that it could alter the overall predicted performance of the detector. The large fluctuations of a nonequilibrium object often differ statistically from those studied at thermodynamic equilibrium. We present experimental and theoretical activity devised to further investigate this question.

1 Introduction

Ground-based Gravitational Wave (GW) detectors are so sensitive low-loss macroscopic objects that managing their thermal fluctuations is a challenging necessity for experimentalists. In fact the intensity of a typical GW of astrophysical origin does not excite the apparatus well above its intrinsic noise threshold. Concurrently, the fact that the intrinsic thermal fluctuations of such low-loss macroscopic objects can be measured is, in and of itself, an impressive achievement, needing further reflection.

One problem that has so far attracted very little attention is the question as to whether the detectors' performance can be hampered by non-equilibrium thermodynamic effects, due to their peculiar architecture [1]. One example is that of interferometers. Test masses are prone to heating due to the absorbed laser power, which in turn causes mechanical deformation of the mirrors, further requiring feedback heating to restore the design optimal geometry [2,3]. By this point, a typical mirror develops a thermal gradient of several degrees, possibly altering its elastic or thermodynamic features. Furthermore, the power must be dissipated through the other parts of the apparatus. This situation is very neatly illustrated in the case of a cryogenic interferometer, in which mirror suspension fibers will mediate a gradient of 10 to 20 K between the mirror and the cryogenic thermal bath[4,5]. Another example, which we shall discuss, concerns extra power exchanged with some electronic feedback mechanism[6,7,8].

Situations in which thermal equilibrium is not attained are very peculiar and hotly debated in statistical mechanics. It is almost a general principle that the fluctuations of observables, far away from the mean, can be very different from those of equilibrium, one example being the rate of energy dissipation. If GW detectors are not in equilibrium, then it is crucial that we distinguish an 'event' from a mere rare nonequilibrium fluctuation.

One of the few results demonstrated to be applicable with some generality deals with the probability that the time-average of an observable $\mathcal{O}$ of positive mean, say $\mathcal{O}_\tau$, assumes values around $+x$, over the probability that it assumes values around $-x$, with τ the duration of the observation. Loosely speaking, relations have been shown to hold[9], of the kind,

$$\frac{P(\mathcal{O}_\tau \approx x)}{P(\mathcal{O}_\tau \approx -x)} \propto e^{\tau x} \tag{1}$$

provided that $\mathcal{O}$ satisfies certain criteria. One instance is the case of a harmonic oscillator, i.e. a precision torsion pendulum[10], which is excited by an electrical field and dissipates energy through the fluid it is immersed into, the rate of energy dissipation playing the role of $\mathcal{O}$. As GW detectors are monitored for long enough time scales, the rare events characterized by Eq. 1 may become observable.

2 The electro-mechanical feedback in AURIGA

A first striking conclusion has been drawn by studying the feedback cooling system that has been developed recently, in the AURIGA detector[6]. One useful and intuitive mathematical scheme is to consider the fundamental mode of vibration of the electro-mechanical oscillator modeled by an instantaneous current $I(T)$ satisfying a Langevin equation[11], which in the absence of feedback would read,

$$L\dot{I}(t) + RI(t) + \frac{1}{C}q(t) = V_T(t) \tag{2}$$

where L, R and C are circuital parameters explicitly related to the mechanical and circuital features of the apparatus, while $V_T(t)$ is the exciting force due to the thermal cryogenic bath. It satisfies $\langle V_T(t)V_T(t')\rangle = 2Rk_BT\delta(t-t')$, T being the temperature and k_B Boltzmann's constant. Via a feedback apparatus that recycles the current with an appropriate phase shift, to a 'quasi-harmonic' approximation Eq. (2) turns into[11],

$$L\dot{I}(t) + \tilde{R}I(t) + \frac{1}{C}q(t) = V_T(t) \tag{3}$$

Here, $\tilde{R}$ is an effective resistance which can be expressed in terms of the feedback parameters The ratio $\tilde{R}/R > 1$ quantifies the extra damping, and therefore the effective 'cooling'. The current around the resonance preserves its approximate Lorentzian shape, with a modified quality factor decreased precisely by the mentioned ratio.

While this paradigm which describes the feedback to have the effect of 'cooling' the system is suggestive and useful for some purposes, it may be misleading if taken too literally. The thermodynamic balance[11] is completely different from that at a mere lowered T. Take for instance the heat absorbed by the oscillator averaged over a time interval of duration τ, Q_τ, or the power injected by the (stochastic) thermal force, P_τ, which would both have zero mean without the feedback. Now we have,

$$Q_\tau = \Delta U_\tau + W_\tau; \qquad P_\tau = Q_\tau + Q_\tau^{(\to \mathrm{bath})} \ \ (\simeq Q_\tau \text{ if } \tilde{R} \gg R) \tag{4}$$

ΔU_τ is the stored energy, $Q_\tau^{(\to \mathrm{bath})}$ the heat dissipated toward the bath. The key is W_τ, the work done on the feedback by the oscillator, which is an entirely new factor in the thermodynamic balance. It is strictly positive in the quasi harmonic approximation of Eq. 3. More surprisingly, P_τ satisfies relations other than Eq. 1. Indeed, writing $\mathcal{O}_\tau = P_\tau L/(k_B T R)$, we now have,

$$\frac{P(\mathcal{O} \approx x)}{P(\mathcal{O} = -x)} \propto e^{a\tau x} \ \ (\text{small } x); \qquad \frac{P(\mathcal{O} \approx x)}{P(\mathcal{O} = -x)} \propto e^{b\tau x} \ \ (\text{large } x) \tag{5}$$

with $a/b = 16/7$ [12], a and b dependent on $\tilde{R}$.

A step further in the characterization of the feedback effect can be obtained by abandoning the quasi harmonic approximation and writing a Langevin equation with explicit memory terms, a more correct expression for the dynamical evolution of the current. The formalism is far from trivial and is treated elsewhere[13]. One obtains an improved prediction for the power spectrum of the current $I(t)$, $S_I(\omega)$, which reveals the possibility that the resonance frequency be fine tuned, by adjusting the cut-off frequency of the low-pass filter, Ω. Figure 1 illustrates one example.

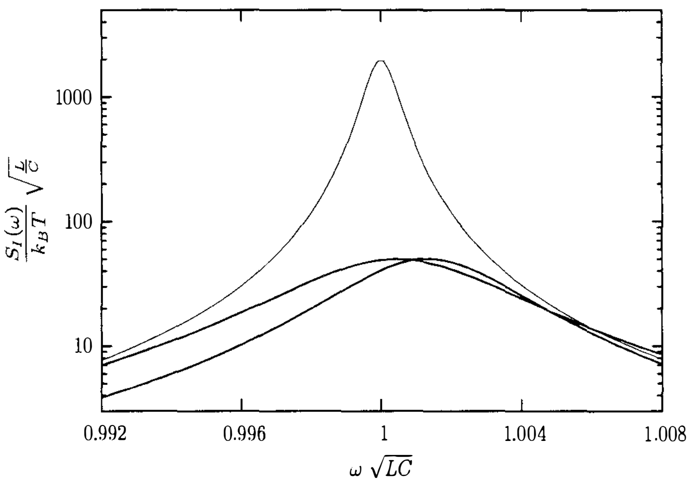

Figure 1: Thin solid line is a Lorenzian curve representing the power spectrum of the current in the absence of feedback. The two thick solid lines approximate Lorenzians and represent two instances in which the control frequency Ω (the low-pass cut-off) is varied, to illustrate the shift of the resonance frequency. The effective resistance $\tilde{R}$ is kept fixed. The damping effect is visible in both cases.

3 Oscillators with gradients - the RareNoise project

The RareNoise project [1] deals with the systematic study of fluctuations of oscillators of high quality factors, which are subject to thermal gradients. As mentioned, this is a situation more reminiscent of interferometric detectors. Other than the implementation of a thermal gradient, crucial aspects are the possibility to control the effect of the bath temperature and of the quality factor of the material. One dimensional models of molecular dynamics have also been devised to mimic the thermo-elastic properties of solids. These models are very simple, hence their length vibrations and thermal fluctuations are more easily controllable and measurable than in more realistic, but more complicated, models. Indeed, they provide qualitative agreement with rea solids, for example the behavior of their elastic constant E with temperature, see Figure 2 More advanced, 3 dimensional models of molecular dynamics are also being developed. Together witl the ongoing experiment, they will provide the groundwork for advancing our knowledge in botl GW detectors, and general nonequilibrium problems.

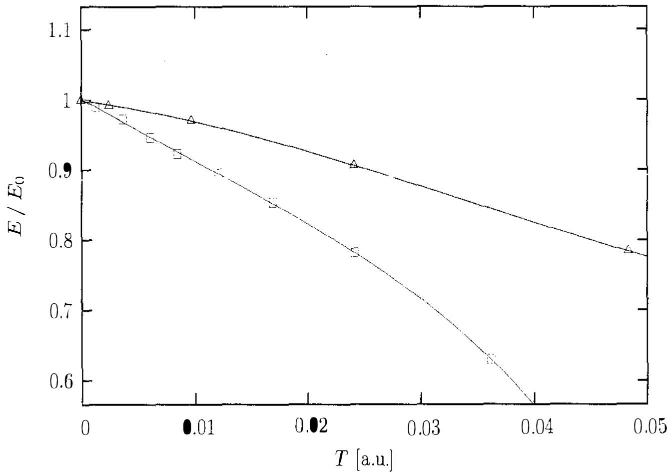

Figure 2: Elastic constants E for classical MD simulations of one-dimensional models with two different interatomic potentials, referred to the extrapolated constant at zero temperature (harmonic oscillators).

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II.
Experimental Gravity

6.
Short Range Gravity