A PARAMETRIZATION OF THE LATERAL PROFILE OF HADRONIC SHOWERS FROM 10 TO 300 GEV *

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1. Introduction

In order to simulate the calorimeter response to the showers generated by the traversing hadrons, we need to know the longitudinal and lateral development of the showers. For the purpose of fast simulation, we only have to examine the gross behaviour of the development. This is the reason why the parametrization of the shower development is of great importance. A parametrization of the longitudinal development of hadronic showers has been given by R. K. Bock et al [1]. Here we present a simple parametrization of the lateral profile of hadronic showers. This parametrization has only one adjustable parameter. We found that this parameter depends only on the incoming particle energy. We will give the expression of this dependence.

2. The data used

We have used the published data and curves on the measurements of the lateral profile of hadronic showers [2] - [6]. We also have used the Monte-Carlo data generated by a very elaborate program on the simulation of hadronic showers [7]. All these data were taken at different depths or in some spatial regions with different incoming particle energies so that the variation of the lateral profile with depth and incoming particle energy could be examined and a parametrization in this respect could be obtained. Since these experimental and Monte-Carlo results did not require the showers to start from the entry, we have convoluted the intrinsic shower profile with the fluctuation of the shower origin. In this work we are concerned with showers in *iron* only.

3. The fitting procedure

In general, the energy deposition density at depth z and radius r (the distance from shower axis) is

$$\mathrm{F}(r,z) = \int_0^z g(z_0) f_{\mathrm{L}}(z-z_0) f_{\mathrm{T}}(r,z-z_0) dz_0 \tag{1}$$

where $g(z_0)$ is the probability that the shower starts at z_0, $f_{\mathrm{L}}(z-z_0)$ is the longitudinal energy deposition per unit length at depth z when the shower starts at z_0 and $f_{\mathrm{T}}(r,z-z_0)$ is the lateral energy deposition fraction per unit area at radius r (which is also a function of $z-z_0$). We have used the parametrization of longitudinal profile given by R. K. Bock et al [1],

$$f_{\mathrm{L}}(z-z_0) = \mathrm{E}\,\{c\frac{s^{\alpha_{\mathrm{E}}-1}e^{-s}}{\Gamma(\alpha_{\mathrm{E}})}\frac{\beta_{\mathrm{E}}}{\mathrm{X}_0} + (1-c)\frac{t^{\alpha_{\mathrm{H}}-1}e^{-t}}{\Gamma(\alpha_{\mathrm{H}})}\frac{\beta_{\mathrm{H}}}{\lambda}\} \tag{2}$$

where

$s = \frac{\beta_{\mathrm{E}}}{\mathrm{X}_0}(z-z_0)$, ($\mathrm{X}_0$: radiation length)

$t = \frac{\beta_{\mathrm{H}}}{\lambda}(z-z_0)$, ($\lambda$: absorption length)

$c = .4634$

$\alpha_{\mathrm{E}} = .6165 + .3183 \ln \mathrm{E(GeV)}$

$\alpha_{\mathrm{H}} = \alpha_{\mathrm{E}}$

$\beta_{\mathrm{E}} = .2198$

$\beta_{\mathrm{H}} = .9099 - .0237 \ln \mathrm{E(GeV)}$

Γ : the Gamma Function

For the lateral profile, we have assumed a exponential form,

$$f_{\mathrm{T}}(r,z-z_0) = \frac{1}{2\pi\sigma^2}\exp(-\frac{r}{\sigma}) \tag{3}$$

The normalization has been so chosen that

$$\int_0^\infty f_{\mathrm{T}}(r,z-z_0)\, 2\pi r dr = 1 \tag{4}$$

where σ depends on $z-z_0$ and is the characteristic width of the lateral profile at depth $z-z_0$ from shower origin. We assumed σ is proportional to $z-z_0$,

$$\sigma = \mathrm{B}\,(z-z_0) \tag{5}$$

Later it will be shown that B depends on incoming particle energy. For $g(z_0)$, we have used

$$g(z_0) = \frac{1}{\lambda}\exp(-\frac{z_0}{\lambda}) \tag{6}$$

We therefore used equation (1), together with equations (2)-(6), to fit the data by adjusting the parameter B using the least-squares method. As the data are expressed in terms of equivalent particles or some arbitrary units, we have normalized the maximum value to 1 within a given set of data.

4. Result

Figures (1)-(9) show the comparison between the data(circles,solid lines) and the fitting results(triangles,dashed lines). In all graphs, z refers to the accumulated depth of iron in centimeters. All the figures except (3) and (5) are plots of the energy deposition along one transverse direction(the energy deposition along the other transverse direction has been integrated). The transverse positions are in centimeters. In figures (3) and (5) the energy deposition is plotted along radius(also in centimeters). In some cases, the energy deposition has been integrated over the depths. On closer look, the fitting profiles seem narrower than the fitted profiles in the former part of shower and wider in the latter part. This situation gets worse when energy gets higher. This is clearly seen in figures (8) and (5). However, the overall agreement is good. We then took the optimum value of B from each set of data and plotted them versus incoming particle energies. This is shown in figure (10). Also shown is the straight line which fits the data points using the least-squares method. The equation of this straight line is then our empirical expression of the dependence of B on incoming particle energy E:

$$B = a\,E^{b}(\text{E in GeV}) \qquad (7)$$

where $a = .872 \pm .084$ and $b = -.476 \pm .024$.

In summary, we have parametrized the lateral profile using an exponential form with one parameter. This parameter depends on incoming particle energy in a simple way. This parametrization does not seem to fit the data very well at all depths. However, it satisfactorily describes the core region of hadronic showers in the energy range from 10 to 300 GeV.

references

1. R. K. Bock et al, "Parametrization of the longitudinal development of hadronic showers in sampling calorimeters", *Nucl. Instr. and Meth.*, Vol. 186, 1981, pp. 533.

2. F. Binon et al, "Coordinate measurements with a hodoscope hadron calorimeter at energies up to 300 GeV", *Nucl. Instr. and Meth.*, Vol. 206, 1983, pp. 373.

3. D. Bollini et al, "Study of the structure of hadronic and electromagnetic showers with a segmented iron-scintillator calorimeter", *Nucl. Instr. and Meth.*, Vol. 171, 1980, pp. 237.

4. B. Friend et al, "Measurements of energy flow distributions of 10 GeV/c hadronic showers in iron and in aluminium", *Nucl. Instr. and Meth.*, Vol. 136, 1976, pp. 505.

5. M. Holder et al, "Performance of a magnetized total absorption calorimeter between 15 GeV and 140 GeV", *Nucl. Instr. and Meth.*, Vol. 151, 1978, pp. 69.

6. Y. Muraki et al, "Radial and longitudinal behaviour of hadronic cascade showers induced by 300 GeV protons in lead and iron absorbers", *Nucl. Instr. and Meth.*, Vol. A236, 1985, pp. 47.

7. T. A. Gabriel et al, "Calculated performance of iron-argon and iron-plastic calorimeters for incident hadrons with energies of 5 to 75 GeV", *Nucl. Instr. and Meth.*, Vol. 134, 1976, pp. 271.

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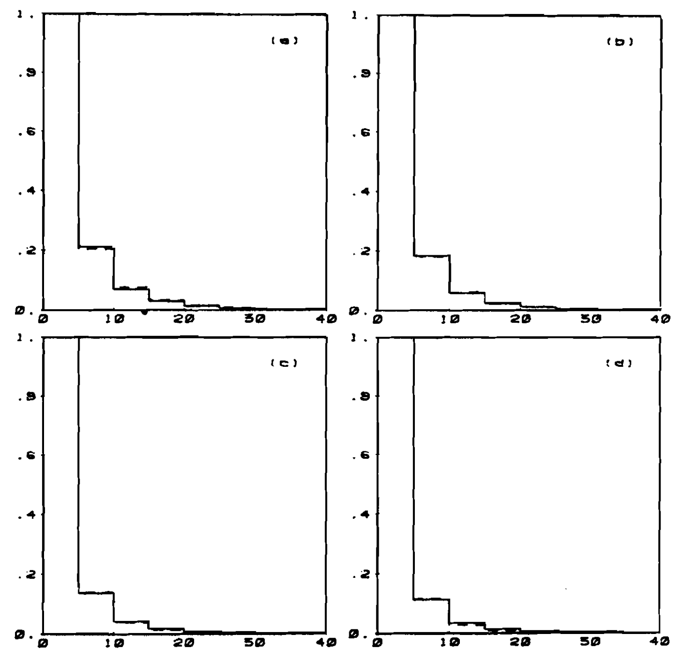

Figure 1:
Lateral shower profiles at different energies. Data adapted from Ref. [2]. $0 < z < 76$. (a) 25 GeV (b) 40 GeV (c) 100 GeV (d) 200 GeV

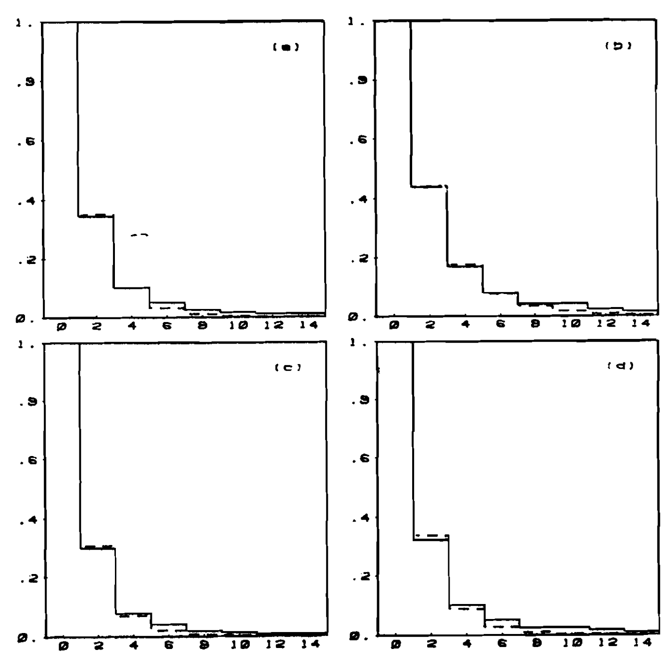

Figure 2:
Lateral shower profiles. Data adapted from Ref. [3]. (a) 40 GeV, $z = 15$ (b) 40 GeV, $z = 21$ (c) 86 GeV, $z = 15$ (d) 86 GeV, $z = 21$

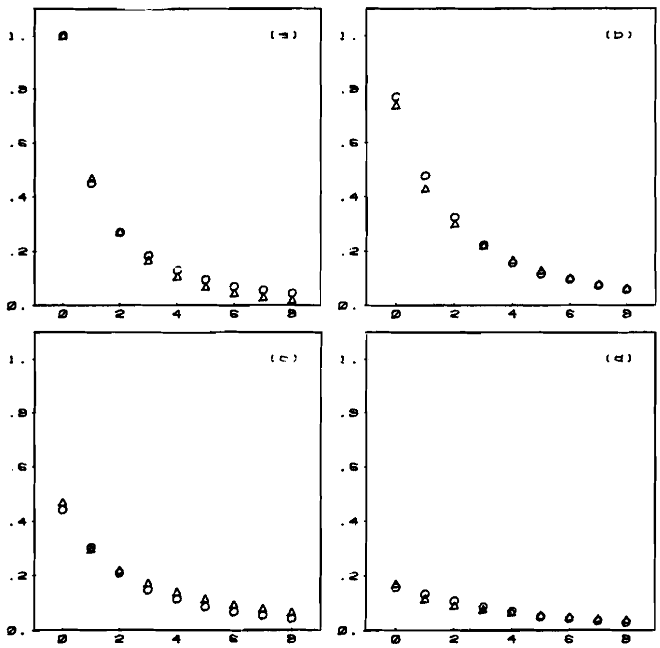

Figure 3:
Lateral shower profiles at different depths. Data adapted from Ref. [4]. The incoming particle energy is 10 GeV. (a) $z = 10$ (b) $z = 20$ (c) $z = 30$ (d) $z = 50$

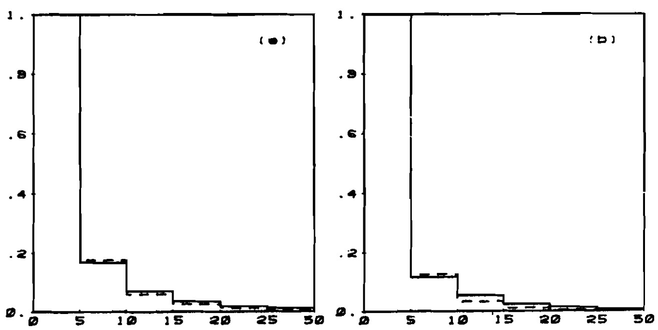

Figure 4:
Lateral shower profiles at different energies. Data adated from Ref. [5]. $0 < z < 90$. (a) 50 GeV (b) 140 GeV

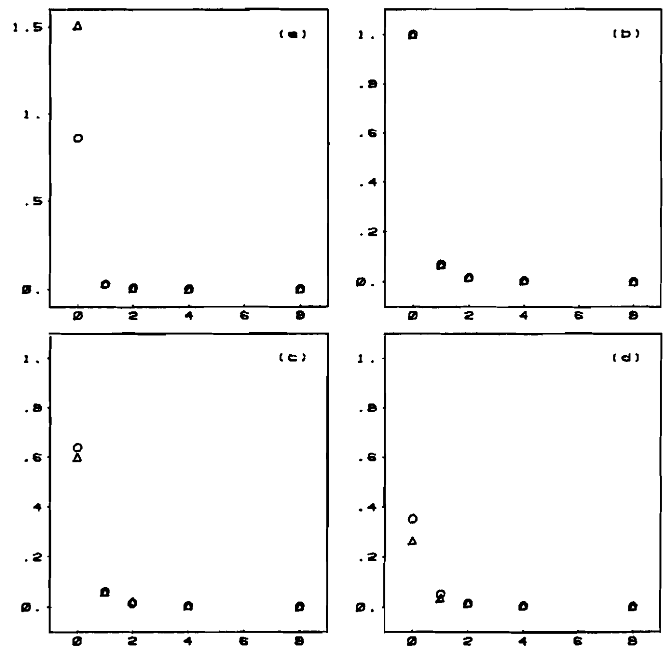

Figure 5:
Lateral shower profiles at different depths. Data adapted from Ref. [6]. The incoming particle energy is 300 GeV. (a) $z = 10$ (b) $z = 20$ (c) $z = 30$ (d) $z = 45$

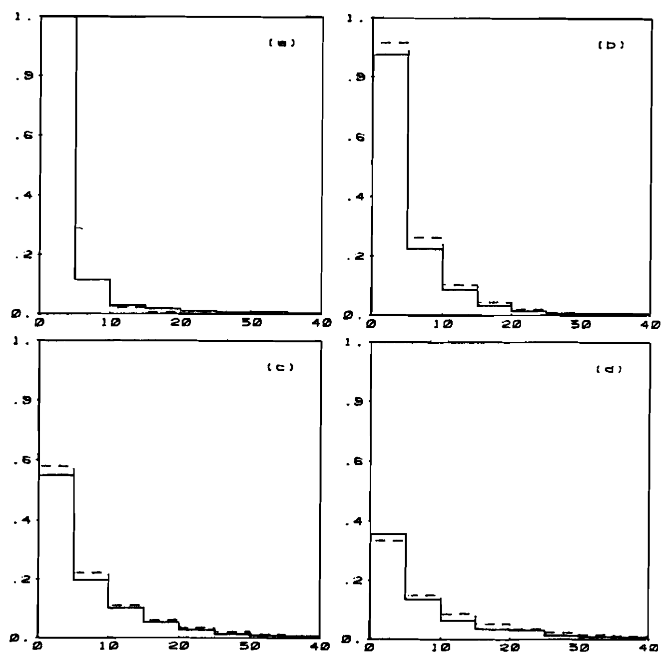

Figure 6:
Lateral shower profiles in different spatial regions. Data adapted from Ref. [7]. The incoming particle energy is 10 GeV. (a)$6 < z < 12$ (b)$18 < z < 24$ (c)$30 < z < 36$ (d)$42 < z < 48$

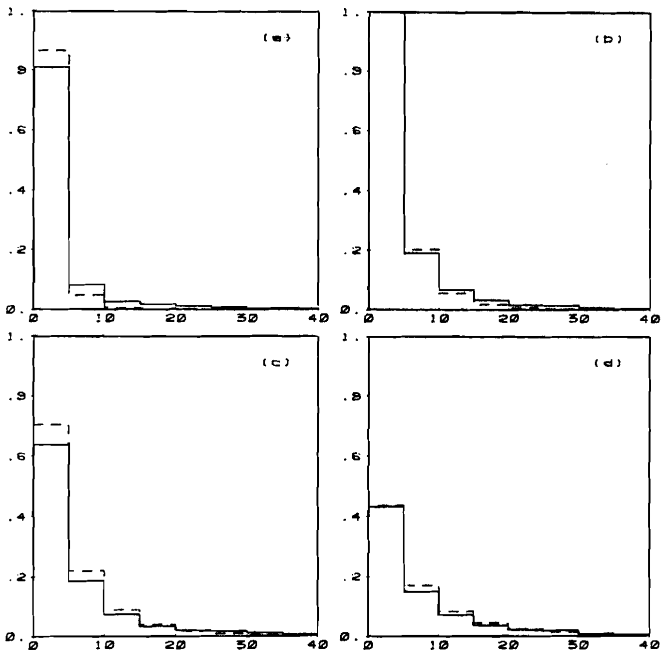

Figure 7:
Lateral shower profiles in different spatial regions. Data adapted from Ref. [7]. The incoming particle energy is 23 GeV. (a) $6 < z < 12$ (b) $18 < z < 24$ (c) $30 < z < 36$ (d) $42 < z < 48$

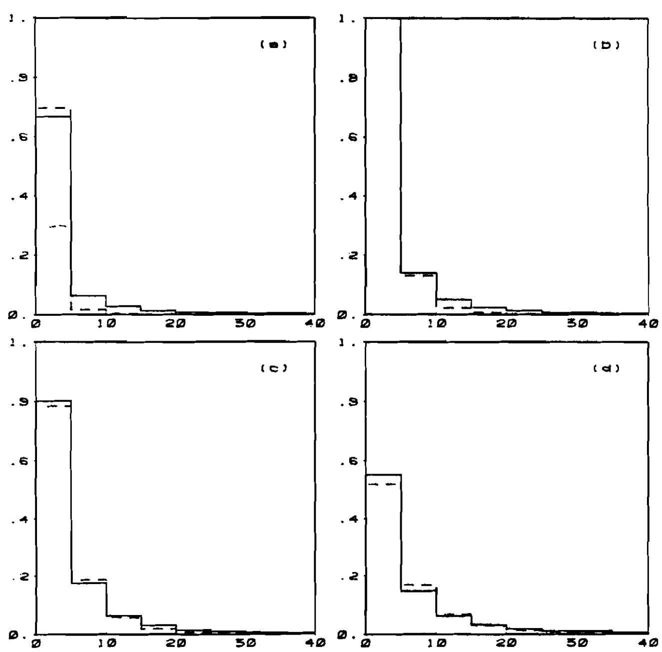

Figure 8:
Lateral shower profiles in different spatial regions. Data adapted from Ref. [7]. The incoming particle energy is 50 GeV. (a) $6 < z < 12$ (b) $18 < z < 24$ (c) $30 < z < 36$ (d) $42 < z < 48$

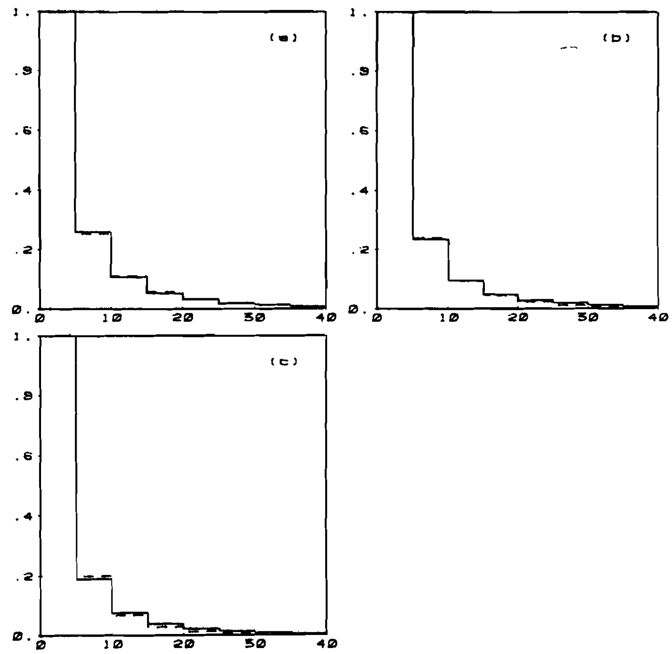

Figure 9:
lateral shower profiles at different energies. Data adapted from Ref. [7]. $0 < z < 60$. (a) 10 GeV (b) 23 GeV (c) 50 GeV

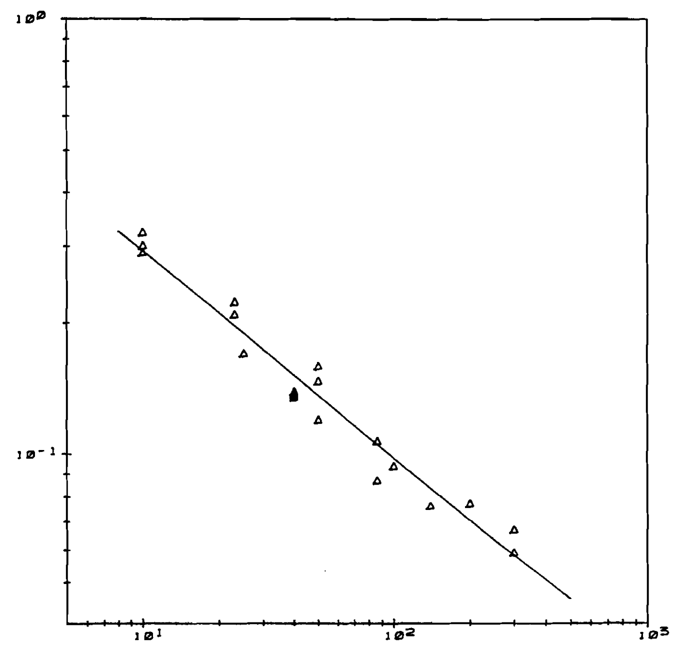

Figure 10:
The parameter B versus incoming particle energy in logarithmic scale.