Eur. Phys. J. C (2024) 84:855
https://doi.org/10.1140/epjc/s10052-024-13165-y

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Z boson emission by an electron and the decays of Z boson into fermions in a de Sitter universe

Cosmin Crucean[a], **Diana Dumitrele**[b]

Faculty of Physics, West University of Timişoara, V. Parvan Avenue 4, 300223 Timişoara, Romania

Received: 16 May 2024 / Accepted: 25 July 2024

Abstract In this paper we develop a method to obtain the rates for the decay of a Z boson into fermions in de Sitter geometry. Our general results obtained in de Sitter space-time allow us to obtain the Minkowski limits for the transition rates when the expansion parameter vanishes. Another important result reported in the present study is related to the emission of Z bosons by electrons and positrons. All the processes were studied by implementing perturbative methods that allow us to define the transition amplitudes in the first order of perturbation theory. The variation of probabilities and rates in terms of expansion parameter and particles masses is also given, pointing out that the processes that generate particle production are possible only in the early universe. For computing the transition rates and probabilities we use the dimensional regularization method and the minimal substraction method.

1 Introduction

The problem of the Proca fields in Minkowski space-time was studied in [1–4], while the Proca particles in the field theory on curved space-times is less studied in literature, and the most relevant studies can be found in [5,6]. The fundamental importance of the Proca equation in the electro-weak Minkowski theory is well known [1–4,7,8] and in the present paper we want to extend the study of the Proca fields on de Sitter space-time. The problems related to the mechanisms of matter–anti-matter generation in early universe can be approached by using the perturbative methods that are close to the formalism of the quantum field theory from Minkowski space-time. The first results related to the problem of particle production due to the space expansion can be found in [9–13]. Recently the problem of massive bosons production from vacuum in a de Sitter space-time was discussed [14–18], and the results prove that these processes are possible only in the early universe where the expansion parameter is significantly larger than the particle mass. The Feynman propagators for the Dirac and Proca fields on de Sitter space-time were also obtained in [6,19–22]. The propagator of the Dirac field on the de Sitter space-time was constructed by Candelas and Reine [21]. The propagator of the Dirac field was also obtained as a mode sum in the Friedmann–Lemaître–Robertson–Walker space-times of arbitrary dimensions [20]. Recently a integral representation in momentum space of the Feynman propagator for Dirac field was obtained in [22]. One of the regularization methods that seems to work in the de Sitter case is the dimensional regularization method, which was used for obtaining the Proca propagator in coordinate representation [6]. A fundamental problem of the field theory in curved space-time is related to the regularization methods that need to be used in order to obtain finite quantities. Recently it was proven that a combination between the dimensional regularization and the minimal substraction method allows one to obtain further results in what concerns the calculations of the total probabilities and transition rates for the elementary processes in the first order of perturbation theory [17,18].

It is a well established fact that the particle production could occur in pure gravitational fields [9,13,23,24], as well as in the case of field interactions where one considers the electromagnetic interactions [10,14–18,25–28]. In the quantum field theory on curved space-time one of the quantities of interest is the density number of particles, which can be computed using perturbative and non-perturbative methods [23,24]. But with both methods the stringent issues are related to the divergent results that are obtained in many cases of interest. For these reasons it is important to use regularization methods that give finite results for the transition rates of elementary processes which generate particle production,

[a] e-mail: cosmin.crucean@e-uvt.ro (corresponding author)
[b] e-mail: diana.dumitrele97@e-uvt.ro

Published online: 26 August 2024

and one may think to use the same methods as in Minkowski theory [7,29–34]. The density number of particles could then, in principle, be defined as the ratio between the production rate and the decay rate, both obtained by using regularization methods in de Sitter case.

In the present paper we approach the problem of computing the rates for the decay of the Z boson into fermions in de Sitter space-time. The Z boson decay rates in de Sitter space-time will be obtained by using the dimensional regularization together with the minimal substraction method [31–33]. These results are fundamental, since they also have Minkowski limits that need to be obtained from our general results [8,35,36]. In order to complete our previous results in what concerns the neutral currents interactions in de Sitter space-time [15,17], we will study the process where a Z boson is produced by an electron or positron.

The paper is organized as follows: in the second section we discuss the transition amplitude for the process of Z boson emission by an electron and we compute the total transition rate and probability. In the third section the decay of a Z boson into a neutrino–antineutrino pair is analysed and the decay rate is established in de Sitter space-time. The Minkowski limit is obtained by considering the ultra-relativistic case, when we connect the de Sitter and Minkowski amplitudes, and we obtain the transition rates from flat space-time. The fourth section is dedicated to the decay of the Z boson into an electron-positron pair. In this section the decay rates in de Sitter space-time are computed and we obtain the Minkowski limits by considering the ultra-relativistic case. In the fifth section we present our conclusions and in the Appendix we give the essential equations for our calculations. In this paper we consider the natural units such that $\hbar = c = 1$.

2 Z boson emission by an electron

In this section we study the problem of the emission of a Z boson by an electron in a de Sitter expanding universe. The transition amplitude corresponding to the first order perturbation theory is computed, and we prove that this is a quantity dependant on the particles momenta and the ratio between the particles masses and the expansion parameter. The process that we present here is not allowed as a perturbative process in Minkowski theory because of the energy and momentum conservation. In the de Sitter case this restriction is no longer valid, since the translational invariance with respect to time is lost. We propose a new mechanism for Z bosons generation in the emission processes by electrons and positrons.

2.1 Transition amplitude with transversal modes

This subsection is dedicated to the perturbative method for investigating the production of Z bosons in the emission processes by fermions in the early universe. We mention that the production of Z bosons in emission processes by neutrinos was studied in [37]. Our method uses the generalization of the Minkowski transition amplitude to de Sitter geometry that was investigated in [14–16,25–28]. This method allows one to obtain the dependence of the transition rate on the expansion parameter, and we mention that the transition amplitude used here were defined in [15]. Let us begin with the de Sitter line element [38]:

$$ds^2 = dt^2 - e^{2\omega t} d\vec{x}^2 = \frac{1}{(\omega t_c)^2}(dt_c^2 - d\vec{x}^2), \tag{1}$$

where we introduce the conformal time that is related to the proper time by $t_c = \frac{-e^{-\omega t}}{\omega}$, where ω is the expansion factor or Hubble parameter ($\omega > 0$). Our computations are done in the conformal chart with conformal time $t_c \in (-\infty, 0)$, which covers the expanding portion of de Sitter space-time [38]. For the line element (1) in the Cartesian gauge we have the non-vanishing tetrad components:

$$e^0_{\hat{0}} = -\omega t_c; \quad e^i_{\hat{j}} = -\delta^i_{\hat{j}}\omega t_c. \tag{2}$$

The three leg transition amplitude that corresponds to the process of Z boson emission by an electron $e^- \to e^- + Z$, is defined in the first order of perturbation theory as [15]:

$$\begin{aligned}
\mathcal{A}_{Ze\bar{e}}(\lambda = \pm 1) &= \int d^4x\sqrt{-g}\left\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)\right.\\
&\times \overline{U}_{\vec{p}\,',\sigma'}(x)\gamma^{\hat{i}}\, e^j_{\hat{i}}\left(\frac{1-\gamma^5}{2}\right)U_{\vec{p},\sigma}(x) f^*_{j\,\vec{P},\lambda=\pm1}(x)\\
&-e_0\tan(\theta_W)\overline{U}_{\vec{p}\,',\sigma'}(x)\gamma^{\hat{i}}\, e^j_{\hat{i}}\left(\frac{1+\gamma^5}{2}\right)\\
&\left. U_{\vec{p},\sigma}(x) f^*_{j\,\vec{P},\lambda=\pm1}(x)\right\}
\end{aligned} \tag{3}$$

The momentum-helicity solutions of the free field equations in de Sitter geometry are well studied in literature. Then the positive frequency solution for the Dirac equation with a well defined momentum and helicity is [39]:

$$\begin{aligned}
U_{\vec{p},\sigma}(t,\vec{x}) &= i\sqrt{\frac{\pi p'}{\omega}}\left(\frac{1}{2\pi}\right)^{3/2}\\
&\times \begin{pmatrix} \frac{1}{2}e^{\pi k/2}H^{(1)}_{\nu_-}(q\,e^{-\omega t})\xi_\sigma(\vec{p}\,)\\ \sigma e^{-\pi k/2}H^{(1)}_{\nu_+}(q\,e^{-\omega t})\xi_\sigma(\vec{p}\,)\end{pmatrix} e^{i\vec{p}\cdot\vec{x}-2\omega t},
\end{aligned} \tag{4}$$

where $H^{(1)}_\nu(z)$ are Hankel functions of first kind that contain the time modulation, $q = \frac{p}{\omega}$, $\nu_\pm = \frac{1}{2} \pm iK$, with $K = \frac{m}{\omega}$, m is the electron mass and $\xi_\sigma(\vec{p}\,)$ are helicity bispinors. The negative frequency solutions can be obtained using the charge conjugation $V_{\vec{p},\sigma}(x) = i\gamma^2\gamma^0(\bar{U}_{\vec{p},\sigma}(x))^T$, [39]. In the case of Proca equation the transversal modes with a well defined momentum and polarization $\lambda = \pm 1$ in de Sitter geometry, reads [5]:

$$\vec{f}_{\vec{P},\lambda}(x)(\lambda=\pm 1) = \frac{\sqrt{\pi}\, e^{-\pi k/2}}{2(2\pi)^{3/2}} \sqrt{-t_c}\, H^{(1)}_{ik}(-Pt_c) e^{i\vec{P}\cdot\vec{x}}\, \vec{\epsilon}(\vec{n}_P,\lambda). \tag{5}$$

where $k=\sqrt{\frac{M}{\omega}-\frac{1}{4}}$ and M is the Z boson mass. The temporal part of the solution give no contribution if we consider only the modes with $\lambda=\pm 1$, because $f^0_{\vec{P},\lambda}(x)(\lambda=\pm 1)=0$, [5].

Computing the amplitude (3) with the help of the solutions (4), (5) we obtain the momentum conservation from spatial integral, while for the temporal integral we change the variable to $z=-t_c$

$$\begin{aligned}\mathcal{A}_{Ze\bar{e}}(\lambda=\pm 1) &= \frac{e_0\,\pi^{3/2}\sqrt{pp'}\, e^{-\pi k/2}}{8(2\pi)^{3/2}}\, \delta^3(\vec{p}\,'+\vec{P}-\vec{p}\,)\\ &\times\Bigg\{\frac{\cos(2\theta_W)}{\sin(2\theta_W)}\, e^{\pi K}\int_0^\infty dz\\ &\times\left[z^{3/2} H^{(1)}_{\nu_-}(pz) H^{(2)}_{\nu_+}(p'z) H^{(2)}_{-ik}(Pz)\right]\\ &+sgn(\sigma\sigma')\tan(\theta_W) e^{-\pi K}\int_0^\infty dz\\ &\times\left[z^{3/2} H^{(1)}_{\nu_+}(pz) H^{(2)}_{\nu_-}(p'z) H^{(2)}_{-ik}(Pz)\right]\Bigg\}\\ &\times \xi^+_{\sigma'}(\vec{p}\,')\,\vec{\sigma}\cdot\vec{\epsilon}^{\,*}(\vec{n}_P,\lambda=\pm 1)\xi_\sigma(\vec{p}\,).\end{aligned} \tag{6}$$

The spatial integrals have the same result as in the flat space case giving the momentum conservation, while the temporal integrals contain the dependence of gravity with the expansion parameter ω.

The amplitude can be rewritten as:

$$\mathcal{A}_{Ze\bar{e}}(\lambda=\pm 1)=\delta^3(\vec{p}\,'+\vec{P}-\vec{p}\,)\, M_{if}\, I_{i\to f} \tag{7}$$

where we introduce the notation

$$M_{if} = \frac{e_0\,\pi^{3/2}\sqrt{pp'}}{8(2\pi)^{3/2}}\, \xi^+_{\sigma'}(\vec{p}\,')\,\vec{\sigma}\cdot\vec{\epsilon}^{\,*}(\vec{n}_P,\lambda=\pm 1)\,\xi_\sigma(\vec{p}\,), \tag{8}$$

and the temporal integrals were denoted by:

$$\begin{aligned}I_{i\to f} &= e^{-\pi k/2}\Bigg[\frac{\cos(2\theta_W)}{\sin(2\theta_W)}\, e^{\pi K}\\ &\times\int_0^\infty dz\cdot z^{3/2} H^{(1)}_{\nu_-}(pz) H^{(2)}_{\nu_+}(p'z) H^{(2)}_{-ik}(Pz)\\ &+sgn(\sigma\sigma')\tan(\theta_W) e^{-\pi K}\\ &\times\int_0^\infty dz\cdot z^{3/2} H^{(1)}_{\nu_+}(pz) H^{(2)}_{\nu_-}(p'z) H^{(2)}_{-ik}(Pz)\Bigg].\end{aligned} \tag{9}$$

For solving the integrals in z we use the relations between Hankel functions and Bessel K functions [40,41]

$$H^{(1,2)}_\nu(z) = \mp\frac{2i}{\pi}\, e^{\mp i\pi\nu/2} K_\nu(\mp iz). \tag{10}$$

and obtain the new form for $I_{i\to f}$

$$\begin{aligned}I_{i\to f} &= \frac{8i}{\pi^3}\Bigg[\frac{\cos(2\theta_W)}{\sin(2\theta_W)}\int_0^\infty dz\cdot z^{3/2}\\ &K_{\nu_-}(-ipz)K_{\nu_+}(ip'z)K_{-ik}(iPz)\\ &+sgn(\sigma\sigma')\tan(\theta_W)\int_0^\infty dz\cdot z^{3/2}\\ &K_{\nu_+}(-ipz)K_{\nu_-}(ip'z)K_{-ik}(iPz)\Bigg]\end{aligned} \tag{11}$$

The above integrals can be rewritten by using the connection between Bessel K functions and Bessel J functions as given in Eq. (223) from Appendix [40,41]. Finally the temporal integrals can be solved by using Eqs. (224), (225) and (82) from Appendix and we obtain:

$$I_{i\to f} = \frac{2i}{\pi}\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)} A_1 + sgn(\sigma\sigma')\tan(\theta_W) A_2\right) \tag{12}$$

where the functions $A_1,\ A_2$ are defined bellow

$$A_1 = -pB_{Kk} - B_{1Kk} + B_{2Kk} - p'B_{-Kk} \tag{13}$$

$$A_2 = -p'B_{Kk} - B_{1-Kk} + B_{2-Kk} - pB_{-Kk} \tag{14}$$

The B functions are written in terms of Gamma Euler functions and Appel functions F_4 of double argument as follows:

$$\begin{aligned}B_{Kk}(pp'P) &= \frac{e^{\pi K}\sqrt{2}\,(-pp')^{-\frac{1}{2}-iK}(iP)^{-\frac{5}{2}+2iK}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{3}{2}-iK\right)}\\ &\times\Gamma\left(\frac{5-4iK+2ik}{4}\right)\Gamma\left(\frac{5-4iK-2ik}{4}\right)\\ &\times F_4\left(\frac{5-4iK+2ik}{4}, \frac{5-4iK-2ik}{4},\right.\\ &\left.\frac{3}{2}-iK, \frac{1}{2}-iK; \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right),\end{aligned} \tag{15}$$

$$\begin{aligned}B_{1Kk}(pp'P) &= \frac{i\sqrt{2}\,(-p)^{-\frac{1}{2}+iK}(p')^{-\frac{1}{2}-iK}(iP)^{-3/2}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{1}{2}+iK\right)}\\ &\times\Gamma\left(\frac{3+2ik}{4}\right)\Gamma\left(\frac{3-2ik}{4}\right)\\ &\times F_4\left(\frac{3+2ik}{4}, \frac{3-2ik}{4}, \frac{1}{2}-iK,\right.\\ &\left.\frac{1}{2}+iK; \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right),\end{aligned} \tag{16}$$

$$\begin{aligned}B_{2Kk}(pp'P) &= \frac{i\sqrt{2}\,(-p)^{\frac{1}{2}-iK}(p')^{\frac{1}{2}+iK}(iP)^{-7/2}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}+iK\right)\Gamma\left(\frac{3}{2}-iK\right)}\\ &\times\Gamma\left(\frac{7+2ik}{4}\right)\Gamma\left(\frac{7-2ik}{4}\right)\\ &\times F_4\left(\frac{7+2ik}{4}, \frac{7-2ik}{4}, \frac{3}{2}+iK,\right.\\ &\left.\frac{3}{2}-iK; \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right).\end{aligned} \tag{17}$$

The final result for the transition amplitude is obtained by collecting all the results from the spatial and temporal integrals:

$$\mathcal{A}_{Ze\bar{e}}(\lambda=\pm 1) = \delta^3(\vec{p}\,'+\vec{P}-\vec{p}\,)\frac{e_0\, i\,\pi^{1/2}\sqrt{pp'}}{4(2\pi)^{3/2}\cosh^2(\pi K)} \times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}A_1 + sgn(\sigma\sigma')\tan(\theta_W)A_2\right) \times\xi^+_{\sigma'}(\vec{p}\,')\,\vec{\sigma}\cdot\vec{\epsilon}^{\,*}(\vec{n}_P,\lambda=\pm 1)\,\xi_\sigma(\vec{p}\,). \tag{18}$$

In de Sitter geometry the translational invariance with respect to time is lost and the first order processes that are forbidden in Minkowski theory have nonvanishing probabilities and rates. The important quantity that needs to be establish here is the transition rate that allows one to define all fundamental quantities of interest, like the density number of particles.

2.2 Amplitude obtained with longitudinal modes

For the case where $\lambda = 0$, the amplitude will have both a temporal and a spacial part, since the temporal part of the Proca equation for this polarization is no longer vanishing [5]:

$$\begin{aligned}\vec{f}_{\vec{P},\lambda}(x) &= \frac{i\sqrt{\pi}\omega P e^{-\pi k/2}}{2M_Z(2\pi)^{3/2}} \times\left[\left(\frac{1}{2}+ik\right)\frac{\sqrt{-t_c}}{P}H^{(1)}_{ik}(-Pt_c) -(-t_c)^{3/2}H^{(1)}_{1+ik}(-Pt_c)\right]e^{i\vec{P}\vec{x}}\vec{\epsilon}\,(\vec{n}_P,\lambda);\\ f_{0\vec{P},\lambda}(x) &= \frac{\sqrt{\pi}\omega P e^{-\pi k/2}}{2M_Z(2\pi)^{3/2}}(-t_c)^{3/2}H^{(1)}_{ik}(-Pt_c)\,e^{i\vec{P}\vec{x}}.\end{aligned} \tag{19}$$

The transition amplitude will therefore be defined with the above longitudinal modes of the Proca equation [15,17]:

$$\begin{aligned}\mathcal{A}_{Ze\bar{e}}(\lambda=0) &= \int d^4x\cdot\sqrt{-g}\left\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)\right. \times\bar{U}_{\vec{p}',\sigma'}(x)\gamma^{\hat{0}}e^0_{\hat{0}}\left(\frac{1-\gamma^5}{2}\right)U_{\vec{p},\sigma}(x)f^*_{0,\vec{P},\lambda=0}(x)\\ &-e_0\tan(\theta_W)\bar{U}_{\vec{p}\,',\sigma'}(x)\gamma^{\hat{0}}e^0_{\hat{0}}\left(\frac{1+\gamma^5}{2}\right) \left.U_{\vec{p},\sigma}(x)f^*_{0,\vec{P},\lambda=0}(x)\right\} + \int d^4x\cdot\sqrt{-g}\left\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)\right.\\ &\times\bar{U}_{\vec{p}\,',\sigma'}(x)\gamma^{\hat{i}}e^j_{\hat{i}}\left(\frac{1-\gamma^5}{2}\right)U_{\vec{p},\sigma}(x)\vec{f}^*_{\vec{P},\lambda=0}(x) -e_0\tan(\theta_W)\bar{U}_{\vec{p}',\sigma'}(x)\gamma^{\hat{i}}e^j_{\hat{i}}\left(\frac{1+\gamma^5}{2}\right)\\ &\left.\times U_{\vec{p},\sigma}(x)\vec{f}^*_{\vec{P},\lambda=0}(x)\right\}.\end{aligned} \tag{20}$$

The amplitude is a sum of terms that contain the temporal and spacial parts of the solutions for Proca equations, which can be computed separately. We shall start with the contribution given by the temporal part of Proca equation, which will be denoted by T_1. Repeating the computations from the case of $\lambda = \pm 1$, and keeping the same variable change of $z = -t_c$, we get the temporal integral of the amplitude which includes the Hankel functions:

$$\begin{aligned}T_{1\,(Ze\bar{e})} &= \int dz\cdot z^{5/2}\frac{\pi^{3/2}\omega P\sqrt{pp'}e^{-\frac{\pi k}{2}}\delta^3(\vec{p}\,'+\vec{P}-\vec{p})}{8M_Z(2\pi)^{3/2}} \times\left[\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}e^{\pi K}H^{(2)}_{\nu^+}(p'z)H^{(1)}_{\nu^-}(pz)H^{(2)}_{-ik}(Pz)\right.\\ &+e_0\tan(\theta_W)sgn(\sigma)sgn(\sigma')e^{-\pi K}H^{(2)}_{\nu^-}(p'z)H^{(1)}_{\nu^+}(pz) \left.H^{(2)}_{-ik}(Pz)\right]\xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot\vec{\epsilon}^*(\vec{n}_P,\lambda)\xi_\sigma(\vec{p}).\end{aligned} \tag{21}$$

Using the Eq. (229), which gives the connection between the Hankel functions and the Bessel K functions [40,41], we get the temporal integral containing Bessel K functions. This change will help in getting rid of the terms $e^{\pi K}, e^{-\pi K}$, which would otherwise cause problems in the computation process.

$$\begin{aligned}T_{1\,(Ze\bar{e})} &= \int dz\cdot z^{5/2}\delta^3(\vec{p}\,'+\vec{P}-\vec{p})\frac{\pi^{3/2}i\omega P\sqrt{pp'}}{M_Z\pi^3(2\pi)^{3/2}} \times\left[\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}K_{\nu^+}(ip'z)K_{\nu^-}(-ipz)K_{-ik}(iPz)\right.\\ &+e_0\tan(\theta_W)sgn(\sigma)sgn(\sigma')K_{\nu^-}(ip'z) \left.K_{\nu^+}(-ipz)K_{-ik}(iPz)\right]\xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot\vec{\epsilon}^*(\vec{n}_P,\lambda)\xi_\sigma(\vec{p}).\end{aligned} \tag{22}$$

The integral containing three Bessel K functions can be solved by further rewriting two of the functions into Bessel J functions [41], using the Eq. (223), and then computing the resulting integrals with the relations (230). Then the amplitude obtained with the temporal part of Proca solutions is:

$$\begin{aligned}T_{1\,(Ze\bar{e})} &= \frac{\pi^{3/2}i\omega P\sqrt{pp'}}{4M_Z\pi(2\pi)^{3/2}}\delta^3(\vec{p}\,'+\vec{P}-\vec{p})\left[\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}D_1\right.\\ &\left.+e_0\tan(\theta_W)sgn(\sigma)sgn(\sigma')D_2\right]\xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot \times\vec{\epsilon}^*(\vec{n}_P,\lambda)\xi_\sigma(\vec{p}),\end{aligned} \tag{23}$$

where D_1 and D_2 represent:

$$D_1 = -C_{1Kk} - \left(-\frac{p}{p'}\right)^{\frac{1}{2}}C_{Kk} - \left(-\frac{p'}{p}\right)^{\frac{1}{2}}C_{-Kk} +C_{2Kk}; \tag{24}$$

$$D_2 = -C_{1-Kk} - \left(-\frac{p}{p'}\right)^{\frac{1}{2}}C_{-Kk} - \left(-\frac{p'}{p}\right)^{\frac{1}{2}}C_{Kk} +C_{2-Kk}, \tag{25}$$

and the Gamma Euler functions, as well as the hyperbolic functions F_4 are contained within the functions:

$$C_{Kk} = \frac{e^{\pi K} 2^{\frac{3}{2}} (p')^{-iK} (-p)^{-iK} (iP)^{-\frac{7}{2}+2iK}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{1}{2}-iK\right)} \times \Gamma\left(\frac{7-4iK+2ik}{4}\right)\Gamma\left(\frac{7-4iK-2ik}{4}\right) \times F_4\left(\frac{7-4iK+2ik}{4}, \frac{7-4iK-2ik}{4}, \frac{3}{2}-iK, \frac{1}{2}-iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right); \quad (26)$$

$$C_{1Kk} = \frac{i2^{\frac{3}{2}}(-p)^{-\frac{1}{2}+iK}(p')^{-\frac{1}{2}-iK}(iP)^{-\frac{5}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \times \Gamma\left(\frac{5-2ik}{4}\right)\Gamma\left(\frac{5+2ik}{4}\right) \times F_4\left(\frac{5-2ik}{4}, \frac{5+2ik}{4}, \frac{1}{2}-iK, \frac{1}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right); \quad (27)$$

$$C_{2Kk} = \frac{i2^{\frac{3}{2}}(-p)^{\frac{1}{2}-iK}(p')^{\frac{1}{2}+iK}(iP)^{-\frac{9}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{3}{2}+iK\right)} \times \Gamma\left(\frac{9-2ik}{4}\right)\Gamma\left(\frac{9+2ik}{4}\right) \times F_4\left(\frac{9-2ik}{4}, \frac{9+2ik}{4}, \frac{3}{2}-iK, \frac{3}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right). \quad (28)$$

The amplitude obtained with the spacial part of the Proca solutions will be denoted by T_2, and have the following form:

$$T_{2\,(Ze\bar{e})} = \int dz \cdot \delta^3(\vec{p}\,' + \vec{P} - \vec{p}) \frac{i\pi^{3/2}\omega P\sqrt{pp'}e^{-\frac{\pi k}{2}}}{8M_Z(2\pi)^{3/2}} \times \left\{ \frac{e_0 \cos(2\theta_W)}{\sin(2\theta_W)} e^{\pi K} H^{(2)}_{\nu^+}(p'z) H^{(1)}_{\nu^-}(pz) \times \left[\left(\frac{1}{2}-ik\right)\frac{z^{\frac{3}{2}}}{P} H^{(2)}_{-ik}(Pz) - z^{\frac{5}{2}} H^{(2)}_{1-ik}(Pz)\right] - e_0 \tan(\theta_W) sgn(\sigma) sgn(\sigma') e^{-\pi K} H^{(2)}_{\nu^-}(p'z) H^{(1)}_{\nu^+}(pz) \times \left[\left(\frac{1}{2}-ik\right)\frac{z^{\frac{3}{2}}}{P} H^{(2)}_{-ik}(Pz) - z^{\frac{5}{2}} H^{(1)}_{1-ik}(Pz)\right]\right\} \times \xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma} \cdot \vec{\epsilon}^*(\vec{n}_P, \lambda)\xi_\sigma(\vec{p}). \quad (29)$$

We shall repeat the computations done for T_1, starting by rewriting the Hankel functions in terms of Bessel K functions [40,41], as per the relation (229).

$$T_{2\,(Ze\bar{e})} = \int dz \cdot \delta^3(\vec{p}\,' + \vec{P} - \vec{p}) \frac{i^2\pi^{3/2}\omega P\sqrt{pp'}}{M_Z\pi^3(2\pi)^{3/2}} \times \left\{ \frac{e_0 \cos(2\theta_W)}{\sin(2\theta_W)} \left[\left(\frac{1}{2}-ik\right)\frac{z^{\frac{3}{2}}}{P} K_{\nu^+}(ip'z) \times K_{\nu^-}(-ipz)K_{-ik}(iPz) + z^{\frac{5}{2}} K_{\nu^+}(ip'z) \times K_{\nu^-}(-ipz)K_{1-ik}(iPz)\right] - e_0 \tan(\theta_W) sgn(\sigma) sgn(\sigma') \times \left[\left(\frac{1}{2}-ik\right)\frac{z^{\frac{3}{2}}}{P} K_{\nu^-}(ip'z)K_{\nu^+}(-ipz)K_{-ik}(iPz) + z^{\frac{5}{2}} K_{\nu^-}(ip'z)K_{\nu^+}(-ipz)K_{1-ik}(iPz)\right]\right\} \times \xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma} \cdot \vec{\epsilon}^*(\vec{n}_P, \lambda)\xi_\sigma(\vec{p}). \quad (30)$$

The temporal integrals contained within T_2 can be solved with the help of (230), for which we will need to write the Bessel K functions into Bessel J functions with (223). Finally, the spacial part of the amplitude T_2 can be expressed as:

$$T_{2\,(Ze\bar{e})} = \frac{i^2\omega P\sqrt{\pi pp'}}{4M_Z(2\pi)^{3/2}} \delta^3(\vec{p}\,' + \vec{P} - \vec{p}) \times \left\{ \frac{e_0 \cos(2\theta_W)}{\sin(2\theta_W)} \left[\left(\frac{1}{2}-ik\right)\frac{1}{P}A_1 + F_1\right] - e_0 \tan(\theta_W) sgn(\sigma) sgn(\sigma') \left[\left(\frac{1}{2}-ik\right)\frac{1}{P}A_2 + F_2\right]\right\} \xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma} \cdot \vec{\epsilon}^*(\vec{n}_P, \lambda)\xi_\sigma(\vec{p}), \quad (31)$$

where A_1 and A_2 have already been defined in the case for $\lambda = \pm 1$, (13)–(14), and will retain the same form in this case. F_1 and F_2 will be denoted by:

$$F_1 = -G_{1Kk} - \left(-\frac{p}{p'}\right)^{\frac{1}{2}} G_{Kk} - \left(-\frac{p'}{p}\right)^{\frac{1}{2}} G_{-Kk} + G_{2Kk}; \quad (32)$$

$$F_2 = -G_{1-Kk} - \left(\frac{-p}{p'}\right)^{\frac{1}{2}} G_{-Kk} - \left(\frac{p'}{-p}\right)^{\frac{1}{2}} G_{Kk} + G_{2-Kk}, \quad (33)$$

and the G functions in this case will contain the Gamma Euler and hyperbolic functions F_4:

$$G_{Kk} = \frac{e^{\pi K} 2^{\frac{3}{2}} (p')^{-iK} (-p)^{-iK} (iP)^{-\frac{7}{2}+2iK}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{1}{2}-iK\right)} \times \Gamma\left(\frac{9-4iK-2ik}{4}\right)\Gamma\left(\frac{5-4iK+2ik}{4}\right) \times F_4\left(\frac{9-4iK-2ik}{4}, \frac{5-4iK+2ik}{4}, \frac{3}{2}-iK, \frac{1}{2}-iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right); \quad (34)$$

$$G_{1Kk} = \frac{i2^{\frac{3}{2}}(-p)^{-\frac{1}{2}-iK}(p')^{-\frac{1}{2}+iK}(iP)^{-\frac{5}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \times \Gamma\left(\frac{7-2ik}{4}\right)\Gamma\left(\frac{3+2ik}{4}\right)$$

$$\times F_4\left(\frac{7-2ik}{4}, \frac{3+2ik}{4}, \frac{1}{2}-iK, \frac{1}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right); \quad (35)$$

$$G_{2Kk} = \frac{i2^{\frac{3}{2}}(-p)^{\frac{1}{2}+iK}(p')^{\frac{1}{2}-iK}(iP)^{-\frac{9}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{3}{2}+iK\right)} \times\Gamma\left(\frac{11-2ik}{4}\right)\Gamma\left(\frac{3+2ik}{4}\right) \times F_4\left(\frac{11-2ik}{4}, \frac{3+2ik}{4}, \frac{3}{2}-iK, \frac{3}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\right). \quad (36)$$

Finally, the amplitude for $\lambda = 0$ for the emission process of a Z boson from an electron will be written as $\mathcal{A}_{Ze\bar{e}}(\lambda = 0) = T_1 + T_2$:

$$\mathcal{A}_{Ze\bar{e}}(\lambda=0) = \frac{i\omega P\sqrt{\pi pp'}}{4M_Z(2\pi)^{3/2}}\frac{\delta^3(\vec{p}\,'+\vec{P}-\vec{p})}{\cosh^2(\pi K)} \times\left\{\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\left[D_1+\left(\frac{1}{2}-ik\right)\frac{i}{P}A_1 +iF_1\right]+e_0\tan(\theta_W)sgn(\sigma)sgn(\sigma') \left[D_2-\left(\frac{1}{2}-ik\right)\frac{i}{P}A_2-iF_2\right]\right\} \times\xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot\vec{\epsilon}^*(\vec{n}_P,\lambda)\xi_\sigma(\vec{p}). \quad (37)$$

We observe that in the case of longitudinal modes the amplitude expression is far more complicated and also observe the presence of the factor $\frac{\omega}{M_Z}$, that prove that in the Minkowski limit this amplitude is vanishing.

In the end of this section we want to bring into attention the Goldstone equivalence theorem that is well known in Minkowski field theory. This is a very interesting issue and the theorem states the amplitude for emission of a longitudinally polarized Z boson is equal to the amplitude for emission of the corresponding Goldstone boson at high energy. In our case the solutions have a well determined momentum and helicity and the energy is not specified, but still one may think to adapt this theorem to the de Sitter case by considering the case of large momenta in our amplitudes. This is an interesting issue and would require to study the behaviour of our amplitudes of Z boson emission in the case of large momenta, which has not been done so far. One of the problem that may appear is related to the amplitude complicated dependence on momenta given in the Appel hypergeometric functions and this is a problem that deserve further study and debate.

2.3 Transition rate

In this section we will compute the transition rate in the general case, meaning that the dependence on the parameters k, K will enter in our final result. This will permit us to study the influence of gravity in the transition rate. One important aspect is related to the fact that the energy is no longer conserved in de Sitter space-time, and we not longer have the usual delta function of energy as in the Minkowski theory. The energy and momentum operators do not commute, and consequently they determine separate basis for solutions such as the energy basis and the momentum basis. We use here only the solutions with a defined momentum, so that the energy is not specified. The rate of transition in de Sitter geometry, cab be obtained as in [23]. Consider the transition amplitude between the initial and final state in a de Sitter geometry:

$$A_{if} = \delta^3(\vec{p}_f-\vec{p}_i)M_{if}I_{if}, \quad (38)$$

where the delta functions assure the momentum conservation in the process. In the Eq. (38), the usual delta function of energy $\delta(E_f - E_i)$ is missing, because the de Sitter amplitudes are obtained using the momentum-helicity solutions. This result comes from the temporal integrals denoted by I_{if}, which do not longer depend on energy:

$$I_{if} = \int_0^\infty dt\mathcal{K}_{if}, \quad (39)$$

and do not give the usual $\delta(E_f - E_i)$ as in Minkowski space-time. In Minkowski space-time the transition rates is defined by using the fact that the squared four delta Dirac functions $|\delta^4(p)|^2$ give the usual $\delta(0)\delta^3(0) = \frac{1}{(2\pi)^4}VT$, where V is the volume and T is the interaction time. Then, the rate is obtained dividing the probability by VT, which is the rate definition as the probability derivative with respect to time. Since in de Sitter geometry these arguments are no longer valid, we adopt the definition given in [23], where the derivative with respect to time is applied on the integrals I_{if}. For this reason we recall the fact that the de Sitter metric is conformal with the Minkowski metric, and one expects that in the conformal chart the definition of the transition rate is similar to the rate defined in the Minkowski theory [23]:

$$R_{if} = \lim_{t_c\to 0}\frac{1}{2V}\frac{d}{dt_c}|A_{if}|^2 = \lim_{t\to\infty}\frac{e^{\omega t}}{2V}\frac{d}{dt}|A_{if}|^2, \quad (40)$$

that can be seen as the derivative of the probability with respect to the conformal time. By making use of Eq. (7) the rate of the transition for a Z boson emission by an electron can be written in terms of the above quantities as:

$$R_{if} = \frac{1}{4}\sum_{\sigma'\lambda}\frac{1}{(2\pi)^3}\delta^3(\vec{p}\,'+\vec{P}-\vec{p})|M_{i\to f}|^2|I_{i\to f}| \times\lim_{t\to\infty}|e^{\omega t}\mathcal{K}_{if}|, \quad (41)$$

where we include the summation after the final helicities and polarizations, and $I_{i\to f}$ is defined in Eq. (12). For extracting the limit from the definition of the transition rate we consider that the transition from *in* to *out* state takes place after a sufficiently large time denoted by t_∞, and that the limits from the rate equation (41) will be evaluated for this time. The factor $\mathcal{K}_{i\to f}$ from the rate equation is defined bellow and represents the integrand from the temporal integrals:

$$\mathcal{K}_{i\to f} = e^{-\omega t}\left(\frac{e^{-\omega t}}{\omega}\right)^{3/2} \times\left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)}K_{\nu_-}(-ipz)K_{\nu_+}(ip'z)K_{-ik}(iPz) +sgn(\sigma\sigma')\tan(\theta_W)K_{\nu_+}(-ipz)K_{\nu_-}(ip'z)K_{-ik}(iPz)\right] \tag{42}$$

To extract an analytical result one can use the expansion of the Bessel K functions for small arguments, since for $t\to\infty$ the argument $z=e^{-\omega t}$ become very small:

$$K_\nu(z)\simeq\frac{2^{\nu-1}\Gamma(\nu)}{z^\nu}. \tag{43}$$

Moreover, in the case of the Proca solutions we use the approximation where $M_Z/\omega << 1/2$, and the index of Bessel K functions becomes $-ik\to\frac{1}{2}$. The limit that enters in the definition of the transition rate then gives:

$$\begin{aligned}\lim_{t\to t_\infty}\left|e^{\omega t}\mathcal{K}_{i\to f}\right| &= \lim_{t\to t_\infty}\left|\frac{\cos(2\theta_W)}{\sin(2\theta_W)}\left(\frac{e^{-\omega t}}{\omega}\right)^{3/2}\right.\\ &\times\frac{2^{-\frac12-iK}\Gamma\left(\frac12-iK\right)}{\left(\frac{-ip\,e^{-\omega t}}{\omega}\right)^{\frac12-iK}}\,\frac{2^{-\frac12+iK}\Gamma\left(\frac12+iK\right)}{\left(\frac{ip'\,e^{-\omega t}}{\omega}\right)^{\frac12+iK}}\\ &\times\frac{2^{-\frac12}\Gamma\left(\frac12\right)}{\left(\frac{iPe^{-\omega t}}{\omega}\right)^{\frac12-iK}}+sgn(\sigma\sigma')\tan(\theta_W)\left(\frac{e^{-\omega t}}{\omega}\right)^{3/2}\\ &\times\frac{2^{-\frac12+iK}\Gamma\left(\frac12+iK\right)}{\left(\frac{-ip\,e^{-\omega t}}{\omega}\right)^{\frac12+iK}}\\ &\left.\times\frac{2^{-\frac12-iK}\Gamma\left(\frac12-iK\right)}{\left(\frac{ip'\,e^{-\omega t}}{\omega}\right)^{\frac12-iK}}\,\frac{2^{-\frac12}\Gamma\left(\frac12\right)}{\left(\frac{iPe^{-\omega t}}{\omega}\right)^{\frac12}}\right|\\ &=\frac{8}{\pi^3}\frac{\pi^{3/2}}{2^{3/2}\sqrt{pp'P}\cosh(\pi K)}\left[\frac{\cos^2(2\theta_W)}{\sin^2(2\theta_W)}+\tan^2(\theta_W)\right.\\ &\left.+\frac{\cos(2\theta_W)}{\sin(2\theta_W)}\tan(\theta_W)sgn(\sigma\sigma')2\cos\left(2K\ln\left(\frac{p'}{p}\right)\right)\right]^{1/2}\end{aligned} \tag{44}$$

The final result for the transition rate is obtained by collecting the above results:

$$\begin{aligned}R_{if} &= \frac{\delta^3(\vec{p}\,'+\vec{P}-\vec{p}\,)}{(2\pi)^3}\frac{e^2pp'}{64\cdot(2\pi)^3\sqrt{pp'P}\cosh(\pi K)}\\ &\sum_{\sigma\sigma'}\left|\xi^+_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot\vec{\epsilon}\,^*\xi_\sigma(\vec{p}\,)\right|^2\sqrt{I_{i\to f}\cdot I^*_{i\to f}}\\ &\times\left[\frac{\cos^2(2\theta_W)}{\sin^2(2\theta_W)}+\tan^2(\theta_W)+\frac{\cos(2\theta_W)}{\sin(2\theta_W)}\tan(\theta_W)\right.\\ &\left.\times sgn(\sigma\sigma')2\cos\left(2K\ln\left(\frac{p'}{p}\right)\right)\right]^{1/2},\end{aligned} \tag{45}$$

where we mention that the dependence in the parameters k, K is also preserved in the expression for $I_{i\to f}$.

A graphical analysis of the transition rate in terms of the parameters k, K reveals that the rate is nonvanishing only in the inflationary regime of the early universe when the expansion parameter was larger or approximatively to the same order with the mass of the particles.

In our plots the momenta for the particles are fixed, and their values can be found in the captions, so that the momenta ratios are subunitary. We also mention that the definition of the Appel hypergeometric functions is given in Eq. (226) from Appendix, and we restrict to the first six terms from the infinite sums in our plots. The infinite sums are very convergent for subunitary arguments, so for that reason we keep only the first relevant terms since the rest of them become very small. We observe that the case where the helicity is not conserved is also present in the right panel from Fig. 1, where the helicities σ and σ' are of opposite signs. The breaking of the helicity conservation can only happen for massive particles, so the mass breaks the helicity conservation. As the ratio between the particles' masses and expansion parameter becomes larger, the rates are vanishing. Indeed we prove that the result from Minkowski theory [10,35,36] is recovered from our perturbative calculation. In this section we prove the rate dependence on the expansion parameter, and that the process of massive boson generation from fermions could take place along the entire inflation (Fig. 2).

2.4 Total rate in the limit of large expansion

Integration after the final momenta in Eq. (45) defines the total transition rate. In the general case, this is a complicated task due to the momenta dependence in Appel hypergeometric functions, which are less studied in literature. A solution for computing the total transition rate is to consider the limits where the expansion factor is larger than the particles' masses so that $\frac{m}{\omega}\simeq\frac{M}{\omega}=0$ or $\omega>>m, M_Z$. This is also the relevant case for study, since our graphical analysis reveals that in this limit the transition rates are finite and do not vanish. Because there are no relations between the Appel hypergeometric functions and other well studied special functions we will compute this limit beginning with the transition amplitude, and then we will translate our result in transition rate given in Eq. (41). Then the temporal integrals defined in rela-

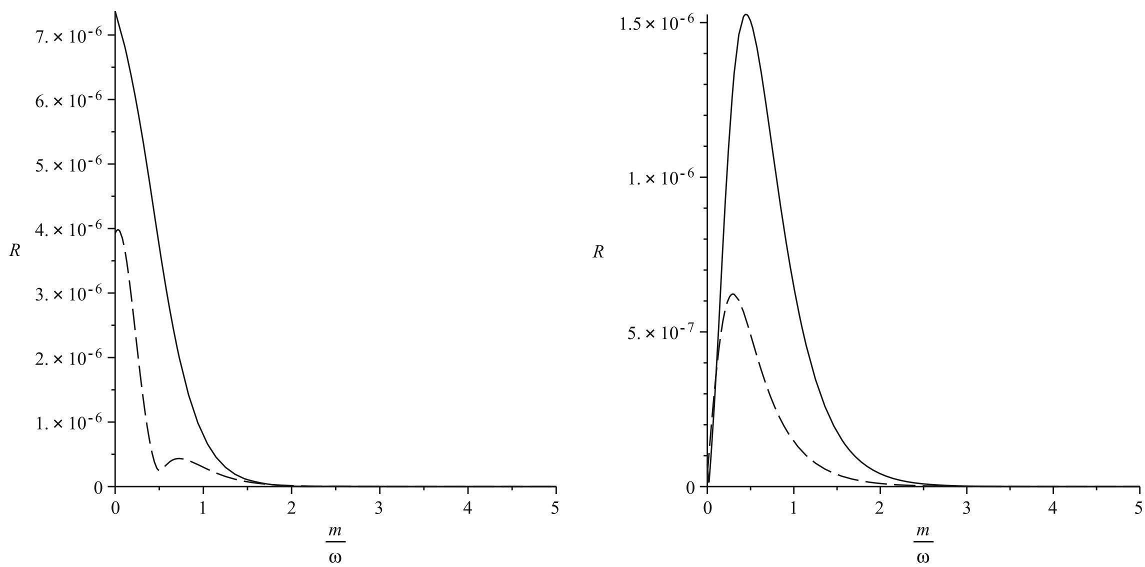

Fig. 1 Transition rate as a function of m/ω for $\lambda = \pm 1$ with $M_Z/\omega = 0.9$, σ and σ' having the same sign and opposite signs respectively. The solid line is for $p/P = 0.1$, $p'/P = 0.2$, and the dotted line is for $p/P = 0.3$, $p'/P = 0.4$

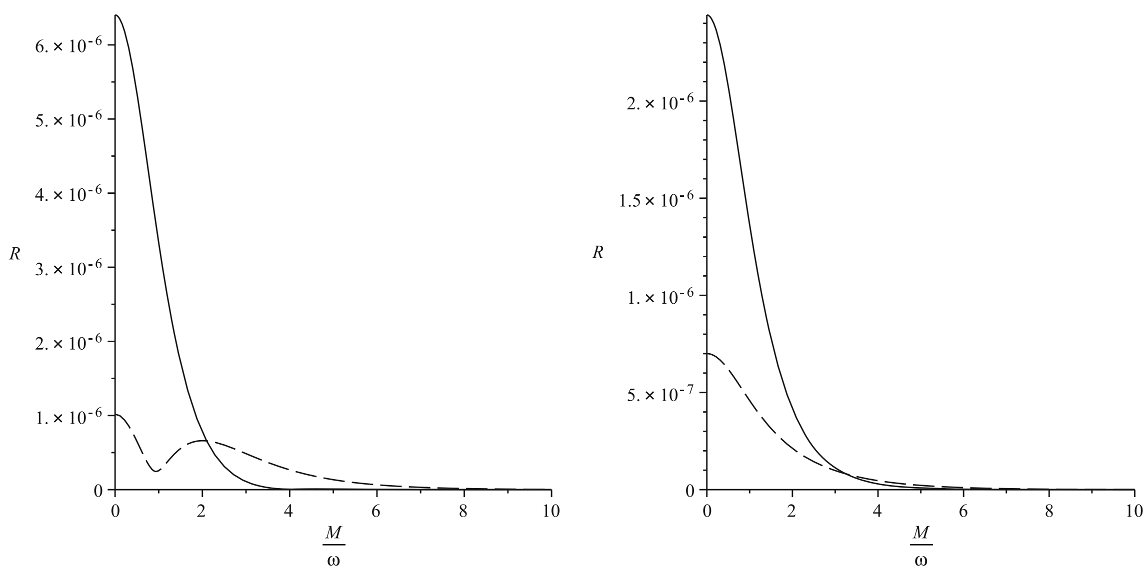

Fig. 2 Transition rate as a function of M_Z/ω for $\lambda = \pm 1$ with $m/\omega = 0.5$, σ and σ' having the same sign and opposite signs respectively. The solid line is for $p/P = 0.1$, $p'/P = 0.2$, and the dotted line is for $p/P = 0.3$, $p'/P = 0.4$

tion (9) for $\frac{m}{\omega} = \frac{M}{\omega} = 0$ give:

$$I_{i\to f}\left(\frac{m}{\omega} = \frac{M}{\omega} = 0\right) = e^{-\frac{i\pi}{4}} \times \left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} + sgn(\sigma\sigma')\tan(\theta_W)\right] \times \int_0^\infty dz \cdot z^{3/2} H^{(1)}_{\frac{1}{2}}(pz) H^{(2)}_{\frac{1}{2}}(p'z) H^{(2)}_{\frac{1}{2}}(Pz). \tag{46}$$

By using the relation of Hankel functions for non-integer index [40]

$$H^{(1,2)}_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}} \cdot \frac{e^{\pm iz}}{\pm i}, \tag{47}$$

we arrive at the following equation for the transition amplitude:

$$A_{i\to f} = \frac{e_0 \cdot e^{-\frac{i\pi}{4}}}{8\pi^{3/2}} \frac{\delta^3(\vec{p}\,' + \vec{P} - \vec{p}\,)}{\sqrt{P}(p' + P - p)} \times \left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} + sgn(\sigma\sigma')\tan(\theta_W)\right] \xi^+_{\sigma'}(\vec{p}\,')\,\vec{\sigma} \cdot \vec{\epsilon}^{\,*}\,\xi_\sigma(\vec{p}\,). \tag{48}$$

Then the integrand of the temporal integrals becomes:

$$\mathcal{K}_{i\to f} = e^{-\omega t} z^{3/2} H^{(1)}_{\frac{1}{2}}(pz) H^{(2)}_{\frac{1}{2}}(p'z) H^{(2)}_{\frac{1}{2}}(Pz) \times \left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} + sgn(\sigma\sigma')\tan(\theta_W)\right], \tag{49}$$

and we use the expansion for the Hankel functions in the case of small arguments $H^{(1,2)}_{\nu}(x) \simeq \mp i \left(\frac{2}{x}\right)^{\nu} \frac{\Gamma(\nu)}{\pi}$, [40,41]. Then the limit that needs to be computed for the rate equation gives:

$$\lim_{t\to t_\infty} \left|e^{\omega t}\mathcal{K}_{i\to f}\right| = \frac{2^{3/2}\,\Gamma^3(1/2)}{\sqrt{pp'P}\,\pi^3} \times \left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} + sgn(\sigma\sigma')\tan(\theta_W)\right], \tag{50}$$

Remarkably, this is the limit obtained from Eq. (44) in the case for $\frac{m}{\omega} = \frac{M}{\omega} = 0$. The final result for the transition rate is defined in Eq. (41), and in the limit for large expansion factors it becomes:

$$R_{if} = \frac{e_0^2\,\delta^3(\vec{p}\,' + \vec{P} - \vec{p}\,)}{64(2\pi)^3 P(p' + P - p)\,(2\pi)^3}\frac{1}{2}\sum_{\sigma'\lambda} \times \left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} + sgn(\sigma\sigma')\tan(\theta_W)\right]^2 \left|\xi^+_{\sigma'}(\vec{p}\,') \times \vec{\sigma}\cdot\vec{\epsilon}^{\,*}\xi_\sigma(\vec{p}\,)\right|^2. \tag{51}$$

The total rate is then defined by the final momenta integration:

$$R_{e^-\to e^- + Z} = \int R_{i\to f}\, d^3p'\, d^3P. \tag{52}$$

The computations of momenta integrals are done by considering that the electron momentum is aligned with the third axis such that $\vec{p} = p\cdot\vec{e}_3$, $\vec{p}\,' = p'\vec{e}_3$, and for the Z boson moneta we consider $\vec{P} = -P\vec{e}_3$, which will facilitate to perform the integral with delta Dirac function.

The sum with the bispinors is simple since the in/out electrons are on the third axis and the sum is reduced to a numerical factor. The helicity bispinors in this particular case are a column matrix with elements $0, \pm 1$ i.e:

$$\xi_{-\frac{1}{2}}(\vec{p}\,) = \begin{pmatrix}0\\1\end{pmatrix};\ \xi_{\frac{1}{2}}(\vec{p}\,) = \begin{pmatrix}1\\0\end{pmatrix} \tag{53}$$

In the summation term the factors with same helicities $\sigma = \sigma'$ will be cancelled due to the above structure of the bispinors, and only the terms with the opposite helicities $\sigma = -\sigma'$, will give contribution. We also mention that we use the circular polarizations for the Z boson, $\vec{\epsilon}_{\pm 1} = \frac{1}{\sqrt{2}}(\pm\vec{e}_1 + i\vec{e}_2)$.

As expected, in the limit of large momenta the momenta integrals are divergent, and after we perform the P integration with delta function their form is:

$$I = \frac{1}{(2\pi)^3}\int d^3p' \int d^3P \frac{\delta^3(\vec{p}\,' + \vec{P} - \vec{p}\,)}{P(p' + P - p)} = \frac{1}{(2\pi)^3}\int d^3p' \frac{1}{2(p' - p)^2}. \tag{54}$$

To extract a finite result from the above integral we will apply the dimensional regularization [29–33] and rewrite the integral in D dimensions:

$$I(D) = \frac{1}{(2\pi)^D}\int \frac{d^Dp'}{(p' - p)^2} \tag{55}$$

An exact and finite result can be obtained if we consider that the momenta modulus are close in values, and approximate $(p' - p)^2 \simeq p'^2 - p^2$. The p' integral can be written as:

$$I(D) = \frac{1}{(2\pi)^D}\int \frac{d^Dp'}{p'^2 - p^2} = \frac{1}{(2\pi)^D}\frac{2\pi^{D/2}}{\Gamma(D/2)} \times \int_0^\infty dp' \frac{p'^{D-1}}{p'^2 - p^2} \tag{56}$$

where the factors $\frac{2\pi^{D/2}}{\Gamma(D/2)}$ are the result of the angular integrals in D dimensions. Performing the variable change $p' = ip\sqrt{y}$ we arrive at the following integral:

$$I(D) = \frac{1}{(2\pi)^D}\frac{\pi^{D/2}}{\Gamma(D/2)}(ip)^{D-2} \times \int_0^\infty \frac{dy}{(1+y)}\, y^{\frac{D}{2}-1} \tag{57}$$

which is just the integral that defines the Beta Euler function [40,41]

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^\infty dy \frac{y^{a-1}}{(1+y)^{a+b}}, \tag{58}$$

for $a = \frac{D}{2}$, $b = 1 - \frac{D}{2}$. The final result for the D dimensional integral can be written with the help of the above integral

$$I(D) = \frac{\pi^{D/2}(ip)^{D-2}}{(2\pi)^D\,\Gamma(D/2)}\frac{\Gamma(D/2)\Gamma\left(1-\frac{D}{2}\right)}{\Gamma(1)} = \frac{(ip)^{D-2}\,\Gamma\left(1-\frac{D}{2}\right)}{(4\pi)^{D/2}}. \tag{59}$$

In general, the poles of the gamma functions could be extracted by introducing an arbitrary mass parameter denoted by μ and an arbitrary coupling dimensionless constant denoted by g so that our initial integral is replaced by the regularized integral $I(D)_r = i\lambda I(D)$ where,

$$g = \lambda\mu^{D-3} = \lambda\mu^{-\varepsilon} \tag{60}$$

and we can write the result of the integral (59) in terms of g and ε for $D = 3 - \varepsilon$.

$$I(D)_r = \frac{ig\mu^{\varepsilon}(ip)^{1-\varepsilon}\,\Gamma\left(\frac{\varepsilon}{2} - \frac{1}{2}\right)}{(4\pi)^{\frac{3-\varepsilon}{2}}} = -\frac{g(i)^{-\varepsilon}p}{(4\pi)^{3/2}}\left(\frac{4\pi\mu^2}{p^2}\right)^{\varepsilon/2}\Gamma\left(\frac{\varepsilon}{2} - \frac{1}{2}\right). \tag{61}$$

In order to write down the final result we use the expansion in powers of ε

$$\left(\frac{4\pi\mu^2}{M_Z^2}\right)^{\varepsilon/2} = 1 + \frac{\varepsilon}{2}\ln\left(\frac{4\pi\mu^2}{M_Z^2}\right) + o(\varepsilon^2) \tag{62}$$

while the gamma Euler function expands as:

$$\Gamma\left(\frac{\varepsilon}{2} - \frac{1}{2}\right) = -2\sqrt{\pi} - \varepsilon\sqrt{\pi}\psi(-1/2) + o(\varepsilon^2), \tag{63}$$

where $\psi(x)$ is the digamma Euler function [40]. Replacing the above expansions in Eq. (61) and by taking the limit $\varepsilon \to 0$ we obtain a finite result:

$$\lim_{\varepsilon\to 0} I(D)_r = \frac{gp}{4\pi}. \tag{64}$$

We prove that the regularized integral is finite in the case where the fermions momenta are on the same direction. Our result proves that the perturbative results for the transition rates can be regularized by using the same methods as in the flat space field theory. However, for the regularization of the momentum integrals for any momentum configuration it may be necessary to introduce counter-terms in the lagrangian density of the theory that describe the fermions interactions with neutral massive bosons. It is also worth mentioning that the dimensional regularization was also used to study propagators in curved space [6,19], including the Proca propagator.

The final expression for the total transition rate is:

$$\begin{aligned} R_{e^-\to e^-+Z} &= \frac{e^2 gp}{64\pi(2\pi)^3}\left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} - \tan(\theta_W)\right]^2 \\ &= \frac{pM_W^2\sin^2(\theta_W)G_F g}{8\sqrt{2}\pi(2\pi)^3} \\ &\quad\times\left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} - \tan(\theta_W)\right]^2. \end{aligned} \tag{65}$$

where the second equality is obtained after we introduce the Fermi constant [42], $G_F = \frac{e^2}{4\sqrt{2}M_W^2\sin(2\theta_W)}$ and M_W is the mass of the W boson. The transition rate computed is finite and constant if we consider the case where the momenta of the incident electron is fixed so that it is of the order of the electron mass $p \simeq m$.

The density number of Z bosons in the emission processes by electrons is proportional with the ratio between the rate computed perturbatively $R_{e^-\to e^-+Z}$, and the decay rate R_{decay} for the Z boson. In the Minkowski space-time the decay rate for the Z boson is well established [24]:

$$\begin{aligned} R_{decay} &= \frac{\alpha M}{12\sin^2(\theta_W)\cos^2(\theta_W)}\left((g_v^f)^2 + (g_A^f)^2\right) \\ &= 2.5\,\text{GeV}, \end{aligned} \tag{66}$$

for various fermions f. In our computations it will be of interest to use the decay rates computed in de Sitter geometry to define the density numbers, which should have a correction due to gravity influence.

The density number of Z bosons produced in emission processes by electrons is the ratio between the production rate and the decay rate multiplied by the density number of electrons:

$$n_Z = \frac{R_{e^-\to e^-+Z}}{R_{decay}}\, n_{e^-}, \tag{67}$$

where n_{e^-} is the density number of electrons. The total transition rate given in Eq. (65) can be estimated by taking $p \simeq m = 0.000511\,\text{GeV}$, the masses of Z and W bosons $M_Z = 91.2\,\text{GeV}$, $M_W = 80.33\,\text{GeV}$ and Fermi constant $G_F = 1.16\cdot 10^{-5}\,\text{GeV}^{-2}$, together with the values for the Weinberg angles $\sin^2(\theta_W) = 1 - \frac{M_W^2}{M^2} = 0.2229$; $\cos(\theta_W) = \frac{M_W}{M} = 0.88$ [24], and we obtain $R_{e^-\to e^-+Z} = 1.64\cdot 10^{-11}\,\text{GeV}$. By using the production rate and the decay rate one can obtain the ratio between the density numbers of electrons and Z bosons:

$$\frac{n_Z}{n_{e^-}} = \frac{R_{e^-\to e^-+Z}}{R_{decay}}. \tag{68}$$

To establish the quantitative results in our equations for the density number the total decay rates for Z bosons into fermions must be computed in de Sitter geometry. This will be the topic of the next two sections. We also must mention that the results obtained here are valid for massive bosons generation only due to space expansion, and we do not take into account other effects.

3 Z boson decay into neutrino–antineutrino pair in de Sitter universe

The decays of a Z boson into fermions is analysed, and the Minkowski limits are discussed in this section. The decay of Z bosons into neutrinos can be studied as a perturbative process in a non-stationary background. To obtain the Minkowski limit when the Hubble constant becomes equal with zero we need to use only the transversal modes with $\lambda = \pm 1$, since for the longitudinal modes the probabilities vanish in this limit, $\omega = 0$. Also we need to take into account that the solutions in de Sitter background have just a determined momentum and the energy is not specified for these modes since the momentum operator and the Hamiltonian not longer commute [5]. This situation could also pose a challenge when we try to discuss the well known Minkowski limits where the decay rates for the three families of neutrinos are equal.

3.1 Transition amplitude

In order to compute the transition amplitude for the decay process on a Z boson into a neutrino and antineutrino, we will need to utilise the solutions for the Proca equations, as well as the solutions for the Dirac equations for a zero mass field. We will begin with the general form of the transition amplitude, which reads as [15]:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}} = \int d^4x\cdot\sqrt{-g}\Bigg\{\frac{e_0}{\sin(2\theta_W)}\big(\bar{U}_{\vec{p},\sigma}(x)\big)_\nu\gamma^{\hat{\mu}}e^{\nu}_{\hat{\mu}} \times\left(\frac{1-\gamma^5}{2}\right)\big(V_{\vec{p}\,',\sigma'}(x)\big)_\nu f_{\vec{P},\lambda\,\nu}(x) -e_0\tan(\theta_W)\big(\bar{U}_{\vec{p},\sigma}(x)\big)_\nu\gamma^{\hat{\mu}}e^{\nu}_{\hat{\mu}} \times\left(\frac{1+\gamma^5}{2}\right)\big(V_{\vec{p}\,',\sigma'}(x)\big)_\nu f_{\vec{P},\lambda\,\nu}(x)\Bigg\}, \tag{69}$$

where $\big(U_{\vec{p}\,',\sigma'}(x)\big)_\nu$; $\big(V_{\vec{p}\,',\sigma'}(x)\big)_\nu$ stands for the Dirac spinors for zero mass [39].

Similarly to the previous case, we will first start with the evaluation of the amplitude for $\lambda = \pm 1$, which will only include the spacial components of the solutions for Proca equation. This will result in an easier computation process.

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=\pm1) = \int d^4x\cdot\sqrt{-g} \times\Bigg\{\frac{e_0}{\sin(2\theta_W)}\big(\bar{U}_{\vec{p},\sigma}(x)\big)_\nu\gamma^{\hat{i}}e^{j}_{\hat{i}} \times\left(\frac{1-\gamma^5}{2}\right)\big(V_{\vec{p}\,',\sigma'}(x)\big)_\nu f_{\vec{P},\lambda\,j}(x) -e_0\tan(\theta_W)\big(\bar{U}_{\vec{p},\sigma}(x)\big)_\nu\gamma^{\hat{i}}e^{j}_{\hat{i}} \times\left(\frac{1+\gamma^5}{2}\right)\big(V_{\vec{p}\,',\sigma'}(x)\big)_\nu f_{\vec{P},\lambda\,j}(x)\Bigg\}. \tag{70}$$

The U, V solutions that define the amplitude describe a neutrino and antineutrino field in a de Sitter geometry, and have the forms [39]:

$$\big(U_{\vec{p},\sigma}(x)\big)_\nu = \left(-\frac{\omega t_c}{2\pi}\right)^{3/2}\begin{pmatrix}(\frac{1}{2}-\sigma)\xi_\sigma(\vec{p}\,)\\ 0\end{pmatrix}e^{i\vec{p}\cdot\vec{x}-ipt_c} \tag{71}$$

$$\big(V_{\vec{p},\sigma}(x)\big)_\nu = \left(-\frac{\omega t_c}{2\pi}\right)^{3/2}\begin{pmatrix}(\frac{1}{2}+\sigma)\eta_\sigma(\vec{p}\,)\\ 0\end{pmatrix}e^{-i\vec{p}\cdot\vec{x}+ipt_c}. \tag{72}$$

The Proca solutions remain the same as in the previous computations (5). Therefore, for $\lambda = \pm 1$, the amplitude takes the form:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=\pm1) = \int dz\cdot z^{1/2}e^{-iz(p+p')}\delta^3(\vec{P}-\vec{p}-\vec{p}\,') \times\frac{\sqrt{\pi}e^{-\frac{\pi k}{2}}}{2(2\pi)^{3/2}}\frac{e_0}{\sin(2\theta_W)} \times H^{(1)}_{ik}(Pz)\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \times\xi^+_\sigma(\vec{p}\,)\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'), \tag{73}$$

which is an integral dependant on the term $z^{1/2}$, an exponential function and a Hankel function. In order to solve it, we will first need to transform the Hankel function into a Bessel K function, using the expression (229). Afterwards, we will be able to compute the integral which now contains the exponential function $e^{-iz(p+p')}$ and a Bessel K function of the form $K_{ik}(-iPz)$ using the formula (231) [40,41]. This integral will be written as:

$$\int dz z^{1/2}e^{-iz(p+p')}K_{ik}(iPz) = \frac{\sqrt{\pi}(-2iP)^{ik}}{[i(p+p')-iP]^{\frac{3}{2}+ik}} \times\frac{\Gamma\left(\frac{3}{2}+ik\right)\Gamma\left(\frac{3}{2}-ik\right)}{\Gamma(2)} \times{}_2F_1\left(\frac{3}{2}+ik,\frac{1}{2}+ik;2;\frac{p+p'+P}{p+p'-P}\right) \tag{74}$$

With the solution to the integral, we can now write the amplitude in a more comprehensive manner.

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=\pm1) = -\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\frac{e_0}{\sin(2\theta_W)} \times\frac{i\sqrt{\pi}}{(2\pi)^{3/2}}\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \times\mathcal{B}'_k\,\xi^+_\sigma(\vec{p}\,)\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'), \tag{75}$$

where the function $\mathcal{B}'_k$ contains the Gauss hypergeometric function ${}_2F_1$, as well as the Gamma Euler functions:

$$\mathcal{B}'_k = \frac{(-2iP)^{ik}}{[i(p+p')-iP]^{\frac{3}{2}+ik}}\Gamma\left(\frac{3}{2}+ik\right)\Gamma\left(\frac{3}{2}-ik\right) \times{}_2F_1\left(\frac{3}{2}+ik,\frac{1}{2}+ik;2;\frac{p+p'+P}{p+p'-P}\right) \tag{76}$$

We will calculate the probability of the decay process using the form of the amplitude (75), by taking the square modulus of the amplitude and sum after the helicities.

$$P_{Z\to\nu\tilde{\nu}}(\lambda=\pm1) = \frac{1}{4}\sum_{\sigma\sigma'}\left|\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=\pm1)\right|^2 = \frac{\pi e_0^2\,\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{(2\pi)^3\sin^2(2\theta_W)}\frac{1}{4}\sum_{\sigma\sigma'}\left(\frac{1}{2}-\sigma\right)^2\left(\frac{1}{2}+\sigma'\right)^2 \times\mathcal{B}'_k\cdot\mathcal{B}'^*_k\left|\xi^+_\sigma(\vec{p}\,)\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')\right|^2 \tag{77}$$

where $\mathcal{B}'^*_k$ represents the complex conjugate of the $\mathcal{B}'_k$ function.

Let us further study the properties of our temporal integrals, since the Minkowski limit should be contained in our result given in the Eqs. (75) and (76). The temporal integral from the amplitude:

$$\int_0^\infty dz\sqrt{z}\,e^{-i(p'+p)z}K_{ik}(iPz), \tag{78}$$

can be transformed by using the following relations [40,41]:

$$e^{-i(p'+p)z} = \sqrt{\frac{2i(p'+p)z}{\pi}} K_{\frac{1}{2}}(i(p'+p)z)$$
$$K_{\frac{1}{2}}(i(p'+p)z) = -\frac{i\pi}{2} e^{-i\pi/4} H_{\frac{1}{2}}((p'+p)z) \quad (79)$$

Then we transform the Hankel function into two Bessel J functions, and we obtain the following temporal integral [41]:

$$I = C(p,p') \int_0^\infty dz\, z K_{ik}(iPz)(J_{\frac{1}{2}}((p'+p)z) + iJ_{-\frac{1}{2}}((p'+p)z)) = I_1 + I_2. \quad (80)$$

where we denote the constants from the integral with:

$$C(p,p') = \sqrt{\frac{2i(p'+p)}{\pi}} \frac{\pi}{2i} e^{-i\pi/4} \quad (81)$$

These integrals can be evaluated by using the formula [40, 41]:

$$\int_0^\infty dz z^{-\lambda} J_\nu(\alpha z) K_\mu(\beta z) = \frac{\alpha^\nu}{2^{\lambda+1}\beta^{\nu-\lambda+1}} \times \frac{\Gamma\left(\frac{\mu+\nu-\lambda+1}{2}\right)\Gamma\left(\frac{\nu-\mu-\lambda+1}{2}\right)}{\Gamma(1+\nu)} \times {}_2F_1\left(\frac{\mu+\nu-\lambda+1}{2}, \frac{\nu-\mu-\lambda+1}{2}; \nu+1; -\frac{\alpha^2}{\beta^2}\right),$$
$$Re(\alpha \pm i\beta) > 0, |Re(\nu-\lambda+1)| > |Re(\mu)|. \quad (82)$$

In our case we add a small imaginary part to the Z boson momenta in order to assure the convergence of our integral $P \to P + i\epsilon$, and at the end of our computations we will take the limit $\epsilon \to 0$. The result for the first integral I_1 from Eq. (80) reads as:

$$I_1(\epsilon) = C(p,p') \frac{\sqrt{p+p'}\,\Gamma\left(\frac{5}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{5}{4}-\frac{ik}{2}\right)}{\Gamma\left(\frac{3}{2}\right)(\epsilon - iP)^{5/2}} \times {}_2F_1\left(\frac{5}{4}-\frac{ik}{2}, \frac{5}{4}+\frac{ik}{2}; \frac{3}{2}; \frac{(p+p')^2}{(P+i\epsilon)^2}\right). \quad (83)$$

The above result for the integral must contain the cases where $P = p+p'$ and $P \neq p+p'$. Let us first study the case where $P = p+p'$, and we transform the Gauss hypergeometric function according to the formula [41] ${}_2F_1(a,b;c;z) = (1-z)^{c-a-b}\,{}_2F_1(c-a, c-b; c; z)$:

$${}_2F_1\left(\frac{5}{4}-\frac{ik}{2}, \frac{5}{4}+\frac{ik}{2}; \frac{3}{2}; \frac{(p+p')^2}{(P+i\epsilon)^2}\right) = \left(1 - \frac{(p+p')^2}{(P+i\epsilon)^2}\right)^{-1} \times {}_2F_1\left(\frac{1}{4}-\frac{ik}{2}, \frac{1}{4}+\frac{ik}{2}; \frac{3}{2}; \frac{(p+p')^2}{(P+i\epsilon)^2}\right), \quad (84)$$

We rewrite the factor in front of hypergeometric function as:

$$\left(1 - \frac{(p+p')^2}{(P+i\epsilon)^2}\right)^{-1} = -\frac{(P+i\epsilon)^2}{P\left(\frac{(p+p')^2+\epsilon^2}{P} - P - 2i\epsilon\right)},$$
$$(\epsilon - iP)^{-5/2} = -e^{i\pi/4}(i\epsilon + P)^{-5/2}. \quad (85)$$

The $I_1(\epsilon)$ integral can be put into the final form:

$$I_1(\epsilon) = \frac{\sqrt{\pi}}{\sqrt{2i}} \frac{(p+p')}{4P\left(\frac{(p+p')^2+\epsilon^2}{2P} - \frac{P}{2} - i\epsilon\right)} \times \frac{\Gamma\left(\frac{5}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{5}{4}-\frac{ik}{2}\right)}{\Gamma\left(\frac{3}{2}\right)(i\epsilon+P)^{1/2}} \times {}_2F_1\left(\frac{1}{4}-\frac{ik}{2}, \frac{1}{4}+\frac{ik}{2}; \frac{3}{2}; \frac{(p+p')^2}{(P+i\epsilon)^2}\right) \quad (86)$$

To obtain the final result of the integral we will take the limit $\epsilon = 0$. This limit can be evaluated by using the well known principal part prescription [41]:

$$\lim_{\epsilon\to 0} \frac{1}{x - x_0 \pm i\epsilon} = Pp\left(\frac{1}{x-x_0}\right) \mp i\pi\delta(x-x_0), \quad (87)$$

where the first part of the result is given in Eq. (83) in terms of gamma Euler functions and hypergeometric function. The delta Dirac function is obtained for the case where the momenta modulus are conserved in this process. In the present case the limit that has as result the delta Dirac distribution is:

$$\lim_{\epsilon\to 0} \frac{1}{\left(\frac{(p+p')^2+\epsilon^2}{2P} - \frac{P^2}{2P} - i\epsilon\right)} = i\pi\delta\left(\frac{(p+p')^2 - P^2}{2P}\right) + Pp\left(\frac{1}{\left(\frac{(p+p')^2}{2P} - \frac{P^2}{2P}\right)}\right). \quad (88)$$

The final result for I_1 in the limit $\epsilon = 0$, and when $P = p+p'$ is:

$$\lim_{\epsilon\to 0} I_1|_{P=p+p'} = \sqrt{\frac{i\pi^3}{2}} \frac{(p+p')}{4P^{3/2}} \frac{\Gamma\left(\frac{5}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{5}{4}-\frac{ik}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} \times {}_2F_1\left(\frac{5}{4}-\frac{ik}{2}, \frac{5}{4}+\frac{ik}{2}; \frac{3}{2}; 1\right)\delta\left(\frac{(p+p')^2-P^2}{2P}\right)$$
$$= \sqrt{\frac{i\pi^3}{2}} \frac{(p+p')}{4P^{3/2}} \delta(p+p'-P), \quad (89)$$

where we use ${}_2F_1(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$ [41].

Following the same steps, the final result for the second integral I_2 is:

$$I_2 = C(p,p') \frac{\Gamma\left(\frac{3}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{3}{4}-\frac{ik}{2}\right)}{\sqrt{p+p'}\,\Gamma\left(\frac{1}{2}\right)(\epsilon - iP)^{3/2}}$$

$$\times {}_2F_1\left(\frac{3}{4}-\frac{ik}{2},\frac{3}{4}+\frac{ik}{2};\frac{1}{2};\frac{(p+p')^2}{(P+i\epsilon)^2}\right) \tag{90}$$

Then in the limit $\epsilon = 0$ we can obtain the result for $P = p + p'$ in the form:

$$\lim_{\epsilon\to 0} I_2|_{P=p+p'} = \sqrt{\frac{i\pi^3}{2}}\frac{(p+p')}{4P^{1/2}}\delta(p+p'-P) \tag{91}$$

By collecting all the above results we arrive at the final transition amplitude for the decay of Z bosons into neutrino–antineutrino pairs:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=\pm 1) = \frac{2ie_0}{\sin(2\theta_W)}\delta^3(\vec{P}-\vec{p}\,'-\vec{p}\,)$$
$$\frac{\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right)}{\sqrt{\pi}(2\pi)^{3/2}}\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_{\mathcal{P}},\lambda)\eta_{\sigma'}(\vec{p}\,')I(p,p') \tag{92}$$

where

$$I(p,p') = \sqrt{\frac{i\pi^3}{2}}\frac{\delta(p+p'-P)}{4\sqrt{P}}\left(\frac{p+p'}{P}+1\right)$$
$$+\theta(P-p+p')\left[f_1\left(\frac{p+p'}{P}\right)+f_2\left(\frac{p+p'}{P}\right)\right]$$
$$+\theta(p+p'-P)\left[f_1'\left(\frac{p}{p+p'}\right)+f_2'\left(\frac{P}{p+p'}\right)\right] \tag{93}$$

where the functions $f_1\left(\frac{p+p'}{P}\right)$, $f_2\left(\frac{p+p'}{P}\right)$ are defined as:

$$f_1\left(\frac{p+p'}{P}\right) = \sqrt{\frac{\pi}{2i}}e^{-i\pi/4}\frac{(p+p')\,\Gamma\left(\frac{5}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{5}{4}-\frac{ik}{2}\right)}{\Gamma\left(\frac{3}{2}\right)P^{5/2}}$$
$$\times {}_2F_1\left(\frac{5}{4}-\frac{ik}{2},\frac{5}{4}+\frac{ik}{2};\frac{3}{2};\frac{(p+p')^2}{P^2}-i0\right);$$
$$f_2\left(\frac{p+p'}{P}\right) = \sqrt{\frac{\pi}{2i}}e^{-i\pi/4}\frac{\Gamma\left(\frac{3}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{3}{4}-\frac{ik}{2}\right)}{\Gamma\left(\frac{1}{2}\right)P^{3/2}}$$
$$\times {}_2F_1\left(\frac{3}{4}-\frac{ik}{2},\frac{3}{4}+\frac{ik}{2};\frac{1}{2};\frac{(p+p')^2}{P^2}-i0\right);$$
$$f_1'\left(\frac{P}{p+p'}\right) = \sqrt{\frac{\pi(p+p')}{2i}}\frac{P^{ik}e^{\pi k/2}}{\cosh\pi k}\frac{\Gamma\left(\frac{5}{4}+\frac{ik}{2}\right)\Gamma\left(\frac{3}{4}+\frac{ik}{2}\right)}{\Gamma(1+ik)(p+p')^{ik+2}}$$
$$\times {}_2F_1\left(\frac{5}{4}+\frac{ik}{2},\frac{3}{4}+\frac{ik}{2};1+ik;\frac{P^2}{(p+p')^2}-i0\right);$$
$$f_2'\left(\frac{P}{p+p'}\right) = -\sqrt{\frac{\pi(p+p')}{2i}}\frac{P^{-ik}}{\cosh\pi k}\frac{\Gamma\left(\frac{5}{4}-\frac{ik}{2}\right)\Gamma\left(\frac{3}{4}-\frac{ik}{2}\right)}{\Gamma(1-ik)(p+p')^{-ik+2}}$$
$$\times {}_2F_1\left(\frac{5}{4}-\frac{ik}{2},\frac{3}{4}-\frac{ik}{2};1-ik;\frac{P^2}{(p+p')^2}-i0\right). \tag{94}$$

The above results prove that our integrals contain a delta Dirac of momenta module function. This result can be also obtained in processes that generate particles from vacuum and in emission processes studied in the first section of our paper, with the observations that the delta Dirac terms get cancelled between themselves in this cases. For the present decay process that has a Minkowski limit the delta terms do not cancel in the calculations, and we obtain a more general result of our integral that contains the case with momentum conservation and the case where the momentum is not conserved.

3.2 Amplitude for Proca modes with $\lambda = 0$

We shall compute the amplitude of the decay process for $\lambda = 0$, which will include both spacial and temporal components of the solutions for Proca equation, as given in Eq. (19), [5]:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=0) = \int d^4x\cdot\sqrt{-g}$$
$$\times\left\{\frac{e_0}{\sin(2\theta_W)}\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{0}}e^0_{\hat{0}}\right.$$
$$\times\left(\frac{1-\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{0,\vec{P},\lambda=0}(x) - e_0\tan(\theta_W)$$
$$\left.\times\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{0}}e^0_{\hat{0}}\left(\frac{1+\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{0,\vec{P},\lambda=0}(x)\right\}$$
$$+\int d^4x\cdot\sqrt{-g}\left\{\frac{e_0}{\sin(2\theta_W)}\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e^j_{\hat{i}}\left(\frac{1-\gamma^5}{2}\right)\right.$$
$$\times V_{\vec{p}\,',\sigma'}(x)\vec{f}_{\vec{P},\lambda=0}(x) - e_0\tan(\theta_W)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e^j_{\hat{i}}$$
$$\left.\times\left(\frac{1+\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)\vec{f}_{\vec{P},\lambda=0}(x)\right\} \tag{95}$$

As we have already done in the previous section, we will split the amplitude for $\lambda = 0$ into two components, T_1 which will represent the amplitude obtained with temporal part of Proca modes and T_2 which will contain the amplitude obtained with the spacial part of the Proca modes. Then for $\lambda = \pm 1$, the term T_1 will have the form:

$$T_{1\,(Z\to\nu\tilde{\nu})} = \int dz\cdot z^{3/2}e^{-iz(p+p')}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')$$
$$\times\frac{\sqrt{\pi}\omega Pe^{-\frac{\pi k}{2}}}{2M_Z(2\pi)^{3/2}}\frac{e_0}{\sin(2\theta_W)}$$
$$\times H^{(1)}_{ik}(Pz)\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right)\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot$$
$$\times\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,') \tag{96}$$

By transforming the Hankel function into a Bessel K function, as dictated by the formula (229), we will obtain an integral containing $z^{3/2}$, an exponential function $e^{-iz(p+p')}$, and the Bessel K function $K_{ik}(iPz)$. The result of the integral, as indicated by the formula (231), is:

$$\int dz\cdot z^{3/2}e^{-iz(p+p')}K_{ik}(iPz)$$
$$=\frac{\sqrt{\pi}(-2iP)^{ik}}{[i(p+p')-iP]^{\frac{5}{2}+ik}}\frac{\Gamma\left(\frac{5}{2}+ik\right)\Gamma\left(\frac{5}{2}-ik\right)}{\Gamma(3)}$$

$$\times {}_2F_1\left(\frac{5}{2}+ik, \frac{1}{2}+ik; 3; \frac{p+p'+P}{p+p'-P}\right) \quad (97)$$

This allows us to evaluate the term T_1, and write it under the form:

$$T_{1\,(Z\to\nu\tilde{\nu})} = -\delta^3(\vec{P}-\vec{p}-\vec{p}\,') \times \frac{e_0}{\sin(2\theta_W)} \frac{i\omega P}{M_Z(2\pi)^{3/2}} \left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \times \mathcal{C}'_k\, \xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'), \quad (98)$$

where $\mathcal{C}'_k$ is a function that encapsulates the Gamma Euler functions, and the Gauss hypergeometric function:

$$\mathcal{C}'_k = \frac{(-2iP)^{ik}}{[i(p+p')-iP]^{\frac{5}{2}+ik}} \frac{\Gamma\left(\frac{5}{2}+ik\right)\Gamma\left(\frac{5}{2}-ik\right)}{\Gamma(3)} \times {}_2F_1\left(\frac{5}{2}+ik, \frac{1}{2}+ik; 3; \frac{p+p'+P}{p+p'-P}\right). \quad (99)$$

The second term that contribute to the amplitude, T_2 will be evaluated in the same manner. This one, however, will consist of two integrals which contain Hankel functions:

$$T_{2\,(Z\to\nu\tilde{\nu})} = -\int dz\cdot e^{-iz(p+p')}\delta^3(\vec{P}-\vec{p}-\vec{p}\,') \times \frac{i\sqrt{\pi}\omega P e^{-\frac{\pi k}{2}}}{2M_Z(2\pi)^{3/2}} \left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right)\frac{e_0}{\sin(2\theta_W)} \times\left[\left(\frac{1}{2}+ik\right)\frac{z^{1/2}}{P}H^{(1)}_{ik}(Pz) - z^{3/2}H^{(1)}_{1+ik}(Pz)\right]\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \quad (100)$$

We observe that the integral containing $H^{(1)}_{ik}(Pz)$ has already been evaluated for $\lambda = \pm 1$ in Eq. (74). We will transform the second Hankel function using the same formula (229), $H^{(1)}_{1+ik}(Pz) = -\frac{2i}{\pi}e^{-\frac{i\pi}{2}+\frac{\pi k}{2}}K_{1+ik}(-iPz)$ [40,41]. The temporal integral containing this function can then be evaluated using (231), and its result will be:

$$\int dz\cdot z^{3/2}e^{-iz(p+p')}K_{1+ik}(iPz) = \frac{\sqrt{\pi}(-2iP)^{1+ik}}{[i(p+p')-iP]^{\frac{7}{2}+ik}} \frac{\Gamma\left(\frac{7}{2}+ik\right)\Gamma\left(\frac{3}{2}-ik\right)}{\Gamma(3)} \times {}_2F_1\left(\frac{7}{2}+ik, \frac{3}{2}+ik; 3; \frac{p+p'+P}{p+p'-P}\right) \quad (101)$$

This will allow us to write the T_2 under the form:

$$T_{2\,(Z\to\nu\tilde{\nu})} = -\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \times\frac{e_0}{\sin(2\theta_W)}\frac{i^2\omega P}{M_Z\pi(2\pi)^{3/2}} \times\left[\left(\frac{1}{2}+ik\right)\frac{1}{P}\mathcal{B}'_k + i\mathcal{D}'_k\right] \times\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'), \quad (102)$$

where $\mathcal{B}'_k$ is given by (76), and the notation $\mathcal{D}'_k$ represents:

$$\mathcal{D}'_k = \frac{(-2iP)^{1+ik}}{[i(p+p')-iP]^{\frac{7}{2}+ik}} \frac{\Gamma\left(\frac{7}{2}+ik\right)\Gamma\left(\frac{3}{2}-ik\right)}{\Gamma(3)} \times {}_2F_1\left(\frac{7}{2}+ik, \frac{3}{2}+ik; 3; \frac{p+p'+P}{p+p'-P}\right). \quad (103)$$

Finally, summing up both the T_1 (98) and T_2 (102), we arrive at the definitive form of the transition amplitude for the decay process of a Z boson into a neutrino–antineutrino pair for $\lambda = 0$:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}(\lambda=0) = -\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\left(\frac{1}{2}-\sigma\right) \times\left(\frac{1}{2}+\sigma'\right)\frac{e_0}{\sin(2\theta_W)}\frac{i\omega P}{M_Z(2\pi)^{3/2}} \times\left[\mathcal{C}'_k + \frac{i}{\pi P}\left(\frac{1}{2}+ik\right)\mathcal{B}'_k - \frac{1}{\pi}\mathcal{D}'_k\right] \times\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \quad (104)$$

3.3 Probability in the limit $\frac{M_z}{\omega} \to 0$

As we prove by graphical results it is important to analyze the decay process in the limit where the expansion factor ω is much larger than the Z boson mass M_Z. We will therefore consider the case where $\frac{M_z}{\omega} \to 0$. Looking at the amplitude in both cases $\lambda = \pm 1$ and $\lambda = 0$, one can notice that the computations in this limit can only be properly done for $\lambda = \pm 1$ (75), since for $\lambda = 0$ in (104) the amplitude contains the term ω/M_Z, which would result in an indetermination in the limit $\frac{M_z}{\omega} \to 0$.

Analyzing the amplitude (75) in this limit, we get the approximation $ik = -1/2$. The hypergeometric function ${}_2F_1$ is reduced to 1, and we get:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}\left(\frac{M_Z}{\omega}\to 0\right) = -\frac{i^{1/2}e_0\sqrt{\pi}}{(2\pi)^{3/2}\sin(2\theta_W)}\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{\sqrt{2P}(p+p'-P)} \times\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \times\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \quad (105)$$

The probability of transition in this limit will still be written as the square modulus of the amplitude $\mathcal{A}_{Z\to\nu\tilde{\nu}}\left(\frac{M_Z}{\omega}\to 0\right)$:

$$P = \frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')e_0^2\left|S_{\sigma\sigma'}\right|^2}{(2\pi)^3\sin^2(2\theta_W)2P(p+p'-P)^2}, \quad (106)$$

where we have introduced the notation:

$$S_{\sigma\sigma'} = \left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right)\xi^{+}_{\sigma}(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \tag{107}$$

The total probability will be obtained from the integral after the final momenta:

$$P_{Z\to\nu\tilde{\nu}} = \int d^3p\int d^3p' \times\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')e_0^2\left|S_{\sigma\sigma'}\right|^2}{(2\pi)^3\sin^2(2\theta_W)2P(p+p'-P)^2}. \tag{108}$$

In order to solve the integrals, we must first establish the movement of the particles. We will use the same estimation as for the decay process of a Z boson into an electron-positron pair. Therefore, the Z boson, as well as the neutrino and antineutrino will have their momenta on the third axis, as such $\vec{P} = P\cdot\vec{e}_3$, $\vec{p} = p\cdot\vec{e}_3$, $\vec{p'} = -p'\cdot\vec{p}_3$. Then the spherical coordinates of the neutrinos momenta are $\vec{p} = (p,0,0)$ and $\vec{p}\,' = (p',\pi,0)$, and we deduce the form of the helicity spinors for this particular case to be [8,36]:

$$\xi_{-\frac{1}{2}}(\vec{p}) = \begin{pmatrix}0\\1\end{pmatrix};\ \eta_{\frac{1}{2}}(\vec{p}\,') = \begin{pmatrix}1\\0\end{pmatrix} \tag{109}$$

which helps to compute $S_{\sigma\sigma'} = -\sqrt{2}$ by also using the circular polarizations defined in the previous section.

With the help of the Delta function we will write the modulus of the antineutrino p' in terms of the modulus of the other two particles $|p'| = p - P$. The integrals containing the momenta can now be written as:

$$I = \frac{1}{(2\pi)^3}\int d^3p\int d^3p'\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{P(p+p'-P)^2} = \frac{1}{(2\pi)^3}\int d^3p\frac{1}{4P(p-P)^2}. \tag{110}$$

Further we can use the dimensional regularization method for solving the momenta integrals. The integral after the p momenta can be solved by considering the D dimensional integral [31–33]:

$$I(D) = \frac{1}{P(2\pi)^D}\int\frac{d^Dp}{(p-P)^2} = \frac{2\pi^{D/2}}{P(2\pi)^D\Gamma(D/2)} \times\int_0^\infty dp\frac{p^{D-1}}{(p-P)^2} \tag{111}$$

By performing the variable change $p = -P\cdot y \to dp = -P\cdot dy$, the integral will become:

$$I(D) = \frac{2\pi^{D/2}}{(2\pi)^D\Gamma(D/2)P}\int_0^\infty\frac{(-P\cdot dy)(-P)^{D-1}y^{D-1}}{P^2(1+y)^2}. \tag{112}$$

This change allows us to use the properties of the Beta Euler function in order to further solve the integral.

$$I(D) = \frac{2\pi^{D/2}(-P)^D}{(2\pi)^D\Gamma(D/2)P^3}\int_0^\infty\frac{y^{D-1}}{(1+y)^2}dy = \frac{2\pi^{D/2}(-P)^D\Gamma(D)\Gamma(2-D)}{(2\pi)^D\Gamma(D/2)P^3}. \tag{113}$$

Since we cannot properly estimate the Gamma function $\Gamma(2-D)$ if we consider $D=3$, we will need to rewrite it utilizing the formula $\Gamma(z+1) = z\cdot\Gamma(z)$, [40,41].

$$\Gamma(2-D) = \frac{\Gamma(3-D)}{2-D} = \frac{\Gamma(4-D)}{(2-D)(3-D)}. \tag{114}$$

With this, the D dimensional integral takes the form:

$$I(D) = \frac{2\pi^{D/2}(-P)^D\Gamma(D)\Gamma(4-D)}{P^3(2\pi)^D(2-D)(3-D)\Gamma(D/2)}. \tag{115}$$

The integral is still divergent in $D=3$ and for removing all the divergences we use the minimal substraction method [33]. Then we compute the residue of the dimensional integral in the limit $D=3$:

$$ResI(D) = \lim_{D\to3}(3-D)I(D) = \frac{1}{\pi^2}. \tag{116}$$

Finally, the regularized dimensional integral can be written as [17,33]:

$$I(D)_r = I(D) - \frac{\mu^{D-3}}{\pi^2(3-D)}; \tag{117}$$

$$I(D)_r = \frac{1}{3-D}\left(\frac{2\pi^{D/2}(-P)^D\Gamma(D)\Gamma(4-D)}{P^3(2\pi)^D(2-D)\Gamma(D/2)} - \frac{\mu^{D-3}}{\pi^2}\right). \tag{118}$$

If we expand the term contained between the parenthesis around the values $D=3$ we will get:

$$\frac{2\pi^{D/2}(-P)^D\Gamma(D)\Gamma(4-D)}{P^3(2\pi)^D(2-D)\Gamma(D/2)} - \frac{\mu^{D-3}}{\pi^2} = \frac{D-3}{2\pi^3}\left[\psi\left(\frac{3}{2}\right) - 1 + \ln\left(\frac{4\pi\mu^2}{p^2}\right)\right] + \mathcal{O}\big((D-3)^2\big), \tag{119}$$

where $\mathcal{O}\big((D-3)^2\big)$ is a function that encapsulates all the terms that contain $(D-3)^2$. In our computations we will disregard these terms for the limit $D=3$.

With this, the regularized D dimensional integral will have the finite form:

$$I(D)_r = \frac{1}{2\pi^2}\left[1 - \psi\left(\frac{3}{2}\right) - \ln\left(\frac{4\pi\mu^2}{p^2}\right)\right]. \tag{120}$$

The final equation for total probability of this decay process will have the form:

$$P_{Z\to\nu\tilde{\nu}} = \frac{e_0^2}{2\pi^2\sin^2(2\theta_W)}\left[1 - \psi\left(\frac{3}{2}\right) - \ln\left(\frac{4\pi\mu^2}{p^2}\right)\right].$$

(121)

3.4 Transition rate in the limit $\frac{M_z}{\omega} \to 0$

For this particular decay process, we cannot accurately calculate the transition rate in the general case, since we will end up encountering an indetermination when trying to compute the limit $\lim_{t\to\infty} |e^{\omega t} K_{if}|$. Therefore, we can only analyze the case for the transition rate under the limit $\frac{M_z}{\omega} \to 0$. For this, we will begin by writing the amplitude in this limit, that require computing the integral containing the Bessel K function:

$$\mathcal{A}_{Z\to\nu\tilde{\nu}}\left(\frac{M_Z}{\omega} \to 0\right) = \frac{ie_0\sqrt{\pi}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{(2\pi)^{3/2}\sin(2\theta_W)} \times\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \times\int_0^\infty dz\cdot z^{1/2}e^{-i(p+p')z}K_{-\frac{1}{2}}(-iPz)\cdot\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot \times\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \tag{122}$$

The transition rate will then be computed from the formula:

$$R = \frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{(2\pi)^3}\sum_{\sigma\sigma'}|M_{\sigma\sigma'}|^2|I_{if}|\lim_{t\to\infty}|e^{\omega t}K_{if}| \tag{123}$$

where we have introduced several notations. The term $M_{\sigma\sigma'}$ contains all the constants and the bispinors:

$$M_{\sigma\sigma'} = \frac{e_0\sqrt{\pi}}{(2\pi)^{3/2}\sin(2\theta_W)}\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right) \xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,') \tag{124}$$

The integral containing the Bessel K function, the exponential function $e^{-i(p+p')z}$, and the term $z^{1/2}$ is denoted by I_{if}. We estimate the Bessel K function in this limit using the expression:

$$K_{-\frac{1}{2}}(-iPz) = e^{iPz}\sqrt{\frac{\pi}{-2iPz}} \tag{125}$$

With this, the integral will then have the result:

$$I_{if} = i\int_0^\infty dz\cdot z^{1/2}e^{-i(p+p')z}K_{-\frac{1}{2}}(-iPz) = \frac{\sqrt{\pi}}{\sqrt{(-2iP)}(p+p'-P)} \tag{126}$$

Finally, the term K_{if} represents the integrand of I_{if}. After computing its limit for $t\to\infty$, we get:

$$\lim_{t\to\infty}|e^{\omega t}K_{if}| = \sqrt{\frac{\pi}{2P}} \tag{127}$$

Utilising all of these notations, we can write the transition rate for the decay process as:

$$R = \frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')e_0^2\pi^2}{(2\pi)^6\sin^2(2\theta_W)2P(p+p'-P)}, \tag{128}$$

with the total transition rate being the integral after the final momenta:

$$R_{Z\to\nu\tilde{\nu}} = \int d^3p\int d^3p'\cdot R = \int d^3p\int d^3p'\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\,e_0^2\pi^2}{(2\pi)^6\sin^2(2\theta_W)2P(p+p'-P)}. \tag{129}$$

We keep the same framework when setting the momenta of the particles as we have when computing the total probability in the limit. As such, the movement of all particles will be set on the third axis, with $\vec{P} = P\cdot\vec{e}_3$, $\vec{p'} = -p'\vec{p}_3$, and $\vec{p} = p\vec{e}_3$. We express the modulus of the antineutrino particle in terms of the momentum of the other two particles, with the help of Delta Dirac function, as previously done $|p'| = p - P$. This will help us simplify the integrals that need computing:

$$I = \frac{1}{(2\pi)^3}\int d^3p\int d^3p'\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{P(p+p'-P)} = \frac{1}{(2\pi)^3}\int d^3p\frac{1}{2P(p-P)} \tag{130}$$

To solve the integral after the p momentum, we will use the dimensional regularization [31,32]. The D dimensional integral can then be written as:

$$I(D) = \frac{1}{(2\pi)^3}\int\frac{d^3p}{2P(p-P)} = \frac{1}{2P(2\pi)^D}\int\frac{d^Dp}{p-P} = \frac{2\pi^{D/2}}{2P(2\pi)^D\Gamma(D/2)}\int_0^\infty dp\frac{p^{D-1}}{p-P} \tag{131}$$

By performing the following substitution $p = -P\cdot y$, $dp = -p\cdot dy$, we can rewrite the integral to obtain the Beta Euler function:

$$I(D) = \frac{2\pi^{D/2}}{2P(2\pi)^D\Gamma(D/2)}\int_0^\infty\frac{dy(-P)^{D-1}y^{D-1}}{1+y} \tag{132}$$

$$= \frac{2\pi^{D/2}(-P)^{D-1}}{2P(2\pi)^D\Gamma(D/2)}\int_0^\infty\frac{y^{D-1}}{1+y}dy = \frac{2\pi^{D/2}(-P)^{D-1}\Gamma(D)\Gamma(1-D)}{(2\pi)^D 2P\Gamma(D/2)} \tag{133}$$

For $D=3$, the Gamma function $\Gamma(1-D)$ cannot be properly estimated. As such, we will rewrite it using the formula $z\cdot\Gamma(z) = \Gamma(z+1)$:

$$\Gamma(1-D) = \frac{\Gamma(2-D)}{1-D} = \frac{\Gamma(3-D)}{(1-D)(2-D)} = \frac{\Gamma(4-D)}{(1-D)(2-D)(3-D)} \tag{134}$$

By replacing the Gamma function with the new estimation, the D dimensional integral now reads:

$$I(D)=\frac{2\pi^{D/2}(-P)^{D-1}\Gamma(D)\Gamma(4-D)}{(2\pi)^{D}2P(1-D)(2-D)(3-D)\Gamma(D/2)} \tag{135}$$

In order to obtain the result for the regularized integral, we have to use the minimal substraction method and calculate the residue for the dimensional integral (135) in the limit $D=3$:

$$Res I(D)=\lim_{D\to 3}(3-D)I(D)=-\frac{P}{4\pi^2} \tag{136}$$

This will leave us with the following form for the result of the regularized integral:

$$I(D)_r=I(D)-\frac{P\mu^{D-3}}{4\pi^2(3-D)} \tag{137}$$

$$=\frac{1}{3-D}\left(\frac{2\pi^{D/2}(-P)^{D-1}\Gamma(D)\Gamma(4-D)}{2P(2\pi)^{D}(1-D)(2-D)\Gamma(D/2)}-\frac{P\mu^{D-3}}{4\pi^2}\right) \tag{138}$$

The terms contained between the parentheses can be expanded around the value $D=3$ as follows.

$$\frac{2\pi^{D/2}(-P)^{D-1}\Gamma(D)\Gamma(4-D)}{2P(2\pi)^{D}(1-D)(2-D)\Gamma(D/2)}-\frac{P\mu^{D-3}}{4\pi^2}=\frac{P(3-D)}{8\pi^2}\left[\ln\left(\frac{4\pi\mu^2}{P^2}\right)+\psi\left(\frac{3}{2}\right)\right]+\mathcal{O}\left((D-3)^2\right) \tag{139}$$

where $\mathcal{O}\left((D-3)^2\right)$ contains all the terms with $(D-3)^2$, which can be disregarded when we consider the limit $D=3$. With this, the final form for the result of the regularized integral $I(D)_r$ will read as:

$$I(D)_r=\frac{P}{8\pi^2}\left[\ln\left(\frac{4\pi\mu^2}{P^2}\right)+\psi\left(\frac{3}{2}\right)\right] \tag{140}$$

The total transition rate for Z boson decay into neutrinos in de Sitter space-time will have the form:

$$R_{Z\to\nu\widetilde{\nu}}=\frac{e_0^2\,P}{8(2\pi)^3\sin^2(2\theta_W)}\left[\ln\left(\frac{4\pi\mu^2}{P^2}\right)+\psi\left(\frac{3}{2}\right)\right]=\frac{\sqrt{2}G_F M_Z^2\,P}{8(2\pi)^3}\left[\ln\left(\frac{4\pi\mu^2}{P^2}\right)+\psi\left(\frac{3}{2}\right)\right]. \tag{141}$$

This result is finite and depend on the parameters of the initial particle, P. Taking $P=M_Z$ one can obtain a numerical value for the above decay rate into neutrinos in the limit $\omega >> M_Z$, $R_{Z\to\nu\widetilde{\nu}}=0.015598$ GeV.

3.5 Transition rate in Minkowski limit

In the Minkowski theory the energy and momentum are conserved, and because the neutrinos do not have mass then the delta Dirac function of momenta modulus can be turned into a delta of energy function. The terms that are not proportional with the delta Dirac function vanish in the Minkowski limit and the amplitude becomes:

$$\mathcal{A}_{Z\to\nu\widetilde{\nu}}(\lambda=\pm1)=\frac{2ie_0}{\sin(2\theta_W)}\,\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\times\frac{\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}+\sigma'\right)}{\sqrt{\pi}(2\pi)^{3/2}}\xi^{+}_{\sigma}(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_{\mathcal{P}},\lambda)\eta_{\sigma'}(\vec{p}\,')\times\sqrt{\frac{i\pi^3}{2}}\frac{\delta(\mathcal{E}-E-E')}{4\sqrt{|\vec{P}|}}\left(\frac{|\vec{p}|+|\vec{p}\,'|}{|\vec{P}|}+1\right) \tag{142}$$

where E, E' are the energies of the neutrinos, and $\mathcal{E}=\sqrt{M_Z^2+\vec{P}^2}$ is the energy of the Z boson in Minkowski space [8,9]. In addition we denote the momenta modulus by $|\vec{p}|$ as to not confuse it with the four momentum notations from Minkowski space-time. We specify that the limit is taken in the ultra-relativistic case where $\mathcal{E}=|\vec{P}|$, because only in this situation the delta function of momenta modulus transforms into a delta function of energy conservation that permits us to make the connection with the results from Minkowski theory.

The permitted transitions are only those with $\sigma=-\frac{1}{2}$ and $\sigma'=\frac{1}{2}$, and we consider the case where the neutrino and antineutrino are emitted on the same direction, but their momenta are pointing opposite. In this case the momenta vectors have the components $\vec{p}(p,0,0)$, $\vec{p}\,'(p',\pi,0)$. By taking into account the circular polarizations $\vec{\epsilon}_{\pm1}=\frac{1}{\sqrt{2}}(\pm\vec{e}_1+i\vec{e}_2)$ and the helicity bispinors for this particular situation, $\xi_{-1/2}=(0,1)^T$ and $\eta_{1/2}=(1,0)^T$, we find that the bispinor term is reduced to:

$$\xi^{+}_{-1/2}(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}_{-1}(\vec{n}_{\mathcal{P}},\lambda)\eta_{1/2}(\vec{p}\,')=-\sqrt{2};\quad \xi^{+}_{-1/2}(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}_{1}(\vec{n}_{\mathcal{P}},\lambda)\eta_{1/2}(\vec{p}\,')=0. \tag{143}$$

The decay rate will be defined as the probability derivative with respect to time, and the total rate will be obtained by an integration after the final momenta of the neutrinos. This is the well known method for defining the transition rate in Minkowski space-time, because of the delta Dirac of energy term that is present in Eq. (216). The probability and decay rate then are [8,35,36,42]:

$$\mathcal{P}_{Z\to\nu\widetilde{\nu}}=|\mathcal{A}_{Z\to\nu\widetilde{\nu}}|^2;\quad R=\frac{d\mathcal{P}_{Z\to\nu\widetilde{\nu}}}{dt};\ R_{Z\to\nu\widetilde{\nu}}=\int d^3p\int d^3p'R. \tag{144}$$

By making use of the well known equations $|(2\pi)\delta(E)|^2=(2\pi)t\delta(E)$ and $|(2\pi)^3\delta^3(\vec{p})|^2=(2\pi)^3V\delta^3(\vec{p})$ we will define the total decay rate in volume unit:

$$R_{Z\to\nu\tilde{\nu}} = \frac{e^2}{32(2\pi)^2\sin^2(2\theta_W)}\int d^3p\int d^3p'\frac{1}{|\vec{P}|}\times\left(\frac{|\vec{p}|+|\vec{p}\,'|}{|\vec{P}|}+1\right)^2\delta(E+E'-\mathcal{E})\delta^3(\vec{p}\,'+\vec{p}-\vec{P}). \tag{145}$$

In order to solve the momenta integrals we take into account the four delta function $\delta^4(p+p'-P)=\delta(E+E'-\mathcal{E})\delta^3(\vec{p}\,'+\vec{p}-\vec{P})$ where p, p', P are four momenta vectors, and we restrict to write only the integrals of interest [8,35,36,42]:

$$\begin{aligned} I(p,p') &= \frac{1}{|\vec{P}|}\int d^3p\int d^3p'\frac{E'}{E'}\delta^4(p+p'-P)\times\left(\frac{|\vec{p}|+|\vec{p}\,'|}{|\vec{P}|}+1\right)^2\\ &= \frac{1}{|\vec{P}|}\int d^3p\int d^4p'\,E'\delta^4(p+p'-P)\times\left(\frac{|\vec{p}|+|\vec{p}\,'|}{|\vec{P}|}+1\right)^2\delta(p'^2)\theta(p^{0'})\\ &= \frac{1}{|\vec{P}|}\int d^3p(\mathcal{E}-E)\delta((P-p)^2)\theta(\mathcal{E}-E)\times\left(\frac{|\vec{p}|+|\vec{P}-\vec{p}|}{|\vec{P}|}+1\right)^2. \end{aligned} \tag{146}$$

The factor of E'^{-1} was introduced in the first row of the above equation to facilitate the transformation of the three momenta integral into a four momenta integral. Taking into account the momenta conservation and the fact that the momenta of the Z boson and neutrino are aligned on the third axis on the same direction $|\vec{P}-\vec{p}|=|\vec{P}|-|\vec{p}|$, with $p^2=p_\mu p^\mu=0$; $|\vec{p}|=E$ for neutrino, the delta term can be simplified to:

$$\begin{aligned}\delta((P-p)^2) &= \delta(P^2+p^2-2pP)\\ &= \delta(2\mathcal{E}E-2\vec{p}\vec{P}) = \frac{\delta\,(E-|\vec{p}|)}{2\mathcal{E}}.\end{aligned} \tag{147}$$

The momenta integral can then be reduced, if we take into account that $dpp^2=dEE^2$, to:

$$\begin{aligned} I(p,p') &= \frac{2}{\mathcal{E}^2}\int d\Omega_p\int_0^{\mathcal{E}} dE\,E^2(\mathcal{E}-E)\delta\,(E-|\vec{p}|)\\ &= \frac{8\pi}{\mathcal{E}^2}|\vec{p}|^2(\mathcal{E}-|\vec{p}|) = \frac{8\pi}{\mathcal{E}^2}E^2(\mathcal{E}-E). \end{aligned} \tag{148}$$

Let us recall the decay from Minkowski space where, for example, one could take the Z boson at rest and in this particular situation $\mathcal{E}=2E=M_Z$, then the result of the integral becomes:

$$I(p,p') = 4\pi\mathcal{E} = 4\pi M_Z. \tag{149}$$

To establish the final formula for the decay rate we express the constants in terms of the Fermi constant and the mass of the Z boson, $\frac{e^2}{\sin^2(2\theta_W)}=\sqrt{2}G_FM_Z^2$ [8,42]. The final result is:

$$R_{Z\to\nu\tilde{\nu}} = \frac{G_FM_Z^3}{16\pi\sqrt{2}}. \tag{150}$$

This result can now be turned in the well known result from Minkowski space-time [42], $R_{Minkowski}=\frac{G_FM_Z^3}{12\pi\sqrt{2}}=R(Z\to\nu_\mu\nu_\mu)=R(Z\to\nu_e\nu_e)=R(Z\to\nu_\tau\nu_\tau)=165.9MeV$ [42], and knowing the fact that the decay rate for all the three neutrino families are equal. Then the decay rate for one of the neutrino families obtained in the limit $\omega=0$ from our computations in de Sitter geometry is:

$$R_1 = \frac{1}{3}R_{Z\to\nu\tilde{\nu}} = \frac{G_FM_Z^3}{48\pi\sqrt{2}} = \frac{1}{4}R_{Minkowski} \tag{151}$$

This is precisely the decay rate from Minkowski field theory up to a factor of $\frac{1}{4}$, which could be considered the result of the fact that we work only in the expanding part of the de Sitter space-time, or in other words that we integrate on a non-compact domain. This result is confirmed by the Minkowski limits of the transition amplitudes that are recovered up to a factor of $\frac{1}{2}$ [43]. Thus the remarkable result of this section is related to the Minkowski limit of the decay rate, which is recovered from our general result obtained in de Sitter metric. An extended discussion about the Minkowski limits of the modes and of the transition probabilities corresponding to processes in which particles are produced can be also found in [6,14–16,19,26].

4 Z boson decay into electron–positron pair in de Sitter universe

In this section we will discuss the Z boson decay into an electron-positron pair in de Sitter geometry. Our results will complete the computations from the previous section where we discuss the decay of a Z boson into neutrinos, and help us establish the total decay rate in the conditions of the early universe. At early stages of the universe one could expect that a large number of Z bosons were produced due the various mechanisms where one can also include the space expansion, and for this reason it is important to establish a formula for the decay rates in these conditions. The Z decay into an electron-positron pair will complete the first order processes for the Weinberg-Salam theory in de Sitter geometry.

4.1 Transition amplitude

We shall compute the transition amplitude for the decay process of a Z boson into an electron and a positron, with the aid of the solutions of the Dirac and Proca equations. The form of the amplitude reads as [17]:

$$\mathcal{A}_{Z\to e^-e^+} = \int d^4x\sqrt{-g}\Bigg\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right) \times\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{\mu}}e^{\nu}_{\hat{\mu}}\left(\frac{1-\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda\,\nu}(x) -e_0\tan(\theta_W)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{\mu}}e^{\nu}_{\hat{\mu}}\left(\frac{1+\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda\,\nu}(x)\Bigg\}. \tag{152}$$

We first evaluate the amplitude for $\lambda=\pm1$, since in this case the solution for the Proca equation $f_{\vec{P},\lambda}(x)$ only contains spacial components. Our amplitude in this case is:

$$\mathcal{A}_{Z\to e^-e^+}(\lambda=\pm1) = \int d^4x\sqrt{-g} \times\Bigg\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e^{j}_{\hat{i}}\left(\frac{1-\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x) \times f_{\vec{P},\lambda\,j}(x) - e_0\tan(\theta_W)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e^{j}_{\hat{i}} \times\left(\frac{1+\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda\,j}(x)\Bigg\}. \tag{153}$$

By replacing the solutions of the Proca equation (5) and the Dirac equations (4) in the amplitude, we get a version of the transition amplitude that includes the Hankel functions. We make the variable change $z=-t_c$, and the amplitude becomes:

$$\mathcal{A}_{Z\to e^-e^+}(\lambda=\pm1) = -\int dz\cdot z^{3/2}\delta^3(\vec{P}-\vec{p}-\vec{p}\,') \times\frac{\pi^{3/2}\sqrt{pp'}e^{-\frac{\pi k}{2}}}{8(2\pi)^{3/2}}\Bigg[\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma') \times H^{(2)}_{\nu^+}(pz)H^{(2)}_{\nu^-}(p'z)H^{(1)}_{ik}(Pz) -e_0\tan(\theta_W)sgn(\sigma)H^{(2)}_{\nu^-}(pz)H^{(2)}_{\nu^+}(p'z)H^{(1)}_{ik}(Pz)\Bigg] \times\xi^+_{\sigma}(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \tag{154}$$

We will evaluate the products of the three Hankel functions first. To do so, we will need to rewrite them into easier forms, which will include Bessel functions J and modified Bessel functions K, as shown in the formulas (228) and (229). After doing the proper computations, the two products will now read as [40,41]:

$$H^{(2)}_{\nu^+}(pz)H^{(2)}_{\nu^-}(p'z)H^{(1)}_{ik}(Pz) = \left(-\frac{2i}{\pi}\right)e^{\frac{\pi k}{2}} \times\Bigg[e^{i\pi}J_{\frac{1}{2}+iK}(pz)J_{\frac{1}{2}-iK}(p'z) -e^{i\pi(\frac{1}{2}+iK)}J_{\frac{1}{2}+iK}(pz)J_{-\frac{1}{2}+iK}(p'z) -e^{i\pi(\frac{1}{2}-iK)}J_{-\frac{1}{2}-iK}(pz)J_{\frac{1}{2}-iK}(p'z) +J_{-\frac{1}{2}-iK}(pz)J_{-\frac{1}{2}+iK}(p'z)\Bigg]K_{ik}(-iPz); \tag{155}$$

$$H^{(2)}_{\nu^+}(p'z)H^{(2)}_{\nu^-}(pz)H^{(1)}_{ik}(Pz) = \left(-\frac{2i}{\pi}\right)e^{\frac{\pi k}{2}}\Bigg[e^{i\pi}J_{\frac{1}{2}-iK}(pz)J_{\frac{1}{2}+iK}(p'z) -e^{i\pi(\frac{1}{2}-iK)}J_{\frac{1}{2}-iK}(pz)J_{-\frac{1}{2}-iK}(p'z) -e^{i\pi(\frac{1}{2}+iK)}J_{-\frac{1}{2}+iK}(pz)J_{\frac{1}{2}+iK}(p'z) +J_{-\frac{1}{2}+iK}(pz)J_{-\frac{1}{2}-iK}(p'z)\Bigg]K_{ik}(-iPz). \tag{156}$$

With the new forms of the Hankel functions, we can calculate the temporal integrals with the formula (230). After more computations, the transition amplitude for $\lambda=\pm1$ will have the form:

$$\mathcal{A}_{Z\to e^-e^+}(\lambda=\pm1) = \frac{i\sqrt{2\pi pp'}}{4(2\pi)^{3/2}}\delta^3(\vec{P}-\vec{p}-\vec{p}\,') \times\Bigg[\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma')\mathcal{A}_1 -e_0\tan(\theta_W)sgn(\sigma)\mathcal{A}_2\Bigg] \times\xi^+_{\sigma}(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \tag{157}$$

where the $\mathcal{A}_1$ and $\mathcal{A}_2$ functions are defined by:

$$\mathcal{A}_1 = \mathcal{B}_{1Kk} + \left(\frac{p}{p'}\right)^{\frac{1}{2}}\mathcal{B}_{Kk} + \left(\frac{p'}{p}\right)^{\frac{1}{2}}\mathcal{B}_{-Kk} + \mathcal{B}_{2Kk}; \tag{158}$$

$$\mathcal{A}_2 = \mathcal{B}_{1-Kk} + \left(\frac{p}{p'}\right)^{\frac{1}{2}}\mathcal{B}_{-Kk} + \left(\frac{p'}{p}\right)^{\frac{1}{2}}\mathcal{B}_{Kk} + \mathcal{B}_{2-Kk}. \tag{159}$$

and the $\mathcal{B}$ functions are defined in terms of the Gamma Euler functions and the hypergeometric functions F_4:

$$\mathcal{B}_{Kk} = \frac{-ie^{-\pi K}(pp')^{iK}(-iP)^{-\frac{5}{2}-2iK}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}+iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \times\Gamma\left(\frac{5+4iK-2ik}{4}\right)\Gamma\left(\frac{5+4iK+2ik}{4}\right) \times F_4\Bigg(\frac{5+4iK-2ik}{4},\frac{5+4iK+2ik}{4}, \frac{3}{2}+iK,\frac{1}{2}+iK,\left(\frac{p}{P}\right)^2,\left(\frac{p'}{P}\right)^2\Bigg); \tag{160}$$

$$\mathcal{B}_{1Kk} = -\frac{p^{\frac{1}{2}+iK}(p')^{\frac{1}{2}-iK}(-iP)^{-\frac{7}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{3}{2}+iK\right)} \times\Gamma\left(\frac{7-2ik}{4}\right)\Gamma\left(\frac{7+2ik}{4}\right) \times F_4\Bigg(\frac{7-2ik}{4},\frac{7+2ik}{4},\frac{3}{2}-iK,$$

$$\frac{3}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\Big); \quad (161)$$

$$\mathcal{B}_{2Kk} = \frac{p^{-\frac{1}{2}-iK}(p')^{-\frac{1}{2}+iK}(-iP)^{-\frac{3}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \times\Gamma\left(\frac{3-2ik}{4}\right)\Gamma\left(\frac{3+2ik}{4}\right) \times F_4\Big(\frac{3-2ik}{4}, \frac{3+2ik}{4}, \frac{1}{2}-iK, \frac{1}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\Big). \quad (162)$$

We note that the functions $\mathcal{B}_{Kk}$, $\mathcal{B}_{1Kk}$, and $\mathcal{B}_{2Kk}$ are different form the previously defined functions B_{Kk}, B_{1Kk}, and B_{2Kk}. Likewise for the functions $\mathcal{A}_1$ and $\mathcal{A}_2$, which are different from A_1 and A_2 previously defined for the emission case.

The probability of the decay process of a Z boson into an electron and a positron can be calculated and studied by taking the square modulus of the amplitude, as well as the sum after the helicities. Using the final form of the transition amplitude (157), the probability of transition in volume unit is given by:

$$P_{i\to f}(\lambda=\pm1) = \frac{1}{4}\sum_{\sigma\sigma'}\left|\mathcal{A}_{Z\to e^-e^+}(\lambda=\pm1)\right|^2 = \frac{pp'e_0^2\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{64\pi^2}\frac{1}{4}\sum_{\sigma\sigma'}\Big[\frac{\cos^2(2\theta_W)}{\sin^2(2\theta_W)}|\mathcal{A}_1|^2 + \tan^2(\theta_W)|\mathcal{A}_2|^2 - \frac{\cos(2\theta_W)}{\sin(2\theta_W)}\tan(\theta_W)sgn(\sigma)sgn(\sigma') \times\left(\mathcal{A}_1\cdot\mathcal{A}_2^*+A_1^*\cdot\mathcal{A}_2\right)\Big] \times\left|\xi_\sigma^+(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')\right|^2, \quad (163)$$

where $\mathcal{A}_1^*$ and $\mathcal{A}_2^*$ represent the complex conjugate of the $\mathcal{A}_1$ and $\mathcal{A}_2$ functions respectively.

The electron charge found in the probability equation e_0^2 can be expressed with the help of the Fermi constant G_F, as well as the mass of the W boson M_W as follows: $e_0^2 = 4\sqrt{2}G_F M_W^2\sin^2(2\theta_W)$.

The transition amplitude for the decay process in the case of $\lambda=0$ has a more complicated form because the temporal part of the solution for the Proca equation is not vanishing [5]. The transition amplitude in this case will be:

$$\mathcal{A}_{Z\to e^-e^+}(\lambda=0) = \int d^4x\cdot\sqrt{-g} \times\Big\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{0}}e_{\hat{0}}^0\left(\frac{1-\gamma^5}{2}\right) \times V_{\vec{p}\,',\sigma'}(x)f_{0,\vec{P},\lambda=0}(x) - e_0\tan(\theta_W)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{0}}e_{\hat{0}}^0\left(\frac{1+\gamma^5}{2}\right) \times V_{\vec{p}\,',\sigma'}(x)f_{0,\vec{P},\lambda=0}(x)\Big\} + \int d^4x\cdot\sqrt{-g}\Big\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right) \times\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e_{\hat{i}}^j\left(\frac{1-\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda=0,j}(x) - e_0\tan(\theta_W)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e_{\hat{i}}^j\left(\frac{1+\gamma^5}{2}\right) \times V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda=0,j}(x)\Big\}. \quad (164)$$

In order to more easily compute the amplitude, we will split it into two terms, T_1 denoting the contribution with spacial part of the Proca modes, and T_2 denoting the contribution of the temporal part of the Proca modes. Using the solutions for the Proca and Dirac equations, as done in the previous sections the first term T_1, will read as:

$$T_{1\,(Z\to e^-e^+)} = -\int dz\cdot z\frac{\pi^{3/2}i\sqrt{pp'}Pe^{-\frac{\pi k}{2}}}{8(2\pi)^{3/2}} \frac{\omega\cdot\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{M_Z}\Big\{\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)} \times sgn(\sigma')H_{\nu^+}^{(2)}(pz)H_{\nu^-}^{(2)}(p'z) \times\left[\left(\frac{1}{2}+ik\right)\frac{z^{\frac{1}{2}}}{P}H_{ik}^{(1)}(Pz) - z^{\frac{3}{2}}H_{1+ik}^{(1)}(Pz)\right] - e_0\tan(\theta_W)sgn(\sigma)H_{\nu^-}^{(2)}(pz)H_{\nu^+}^{(2)}(p'z) \times\left[\left(\frac{1}{2}+ik\right)\frac{z^{\frac{1}{2}}}{P}H_{ik}^{(1)}(Pz) - z^{\frac{3}{2}}H_{1+ik}^{(1)}(Pz)\right]\Big\} \xi_\sigma^+(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'), \quad (165)$$

where we have kept the change of variable from the previous cases of $z=-t_c$.

Similarly to the case of $\lambda=\pm1$, we will rewrite the Hankel functions into Bessel functions J and modified Bessel functions K. The products of the three Hankel functions will have the following forms:

$$H_{\nu^+}^{(2)}(pz)H_{\nu^-}^{(2)}(p'z)H_{1+ik}^{(1)}(Pz) = \left(-\frac{2i}{\pi}\right)\frac{e^{-\frac{i\pi}{2}}e^{\frac{\pi k}{2}}}{\cosh^2(\pi K)} \times\Big[e^{i\pi}J_{\frac{1}{2}+iK}(pz)J_{\frac{1}{2}-iK}(p'z) - e^{i\pi(\frac{1}{2}+iK)}J_{\frac{1}{2}+iK}(pz)J_{-\frac{1}{2}+iK}(p'z) - e^{i\pi(\frac{1}{2}-iK)}J_{-\frac{1}{2}-iK}(pz)J_{\frac{1}{2}-iK}(p'z) + J_{-\frac{1}{2}-iK}(pz)J_{-\frac{1}{2}+iK}(p'z)\Big]K_{1+ik}(-iPz); \quad (166)$$

$$H^{(2)}_{\nu^-}(pz)H^{(2)}_{\nu^+}(p'z)H^{(1)}_{1+ik}(Pz) = \left(-\frac{2i}{\pi}\right)\frac{e^{-\frac{i\pi}{2}}e^{\frac{\pi k}{2}}}{\cosh^2(\pi K)} \times\Big[e^{i\pi}J_{\frac{1}{2}+iK}(p'z)J_{\frac{1}{2}-iK}(pz) -e^{i\pi(\frac{1}{2}+iK)}J_{\frac{1}{2}+iK}(p'z)J_{-\frac{1}{2}+iK}(pz) -e^{i\pi(\frac{1}{2}-iK)}J_{-\frac{1}{2}-iK}(p'z)J_{\frac{1}{2}-iK}(pz) +J_{-\frac{1}{2}-iK}(p'z)J_{-\frac{1}{2}+iK}(pz)\Big]K_{1+ik}(-iPz). \tag{167}$$

We will once again use the formula (230) to solve the temporal integrals that include two Bessel J functions and one Bessel K function. Doing the necessary computations will result into the following form of the term T_1 that contribute to the amplitude:

$$T_{1\,(Z\to e^-e^+)} = \frac{\pi^{3/2}i\sqrt{pp'}P}{8(2\pi)^{3/2}}\frac{2i}{\pi}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\frac{\omega}{M_Z} \times\left\{\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}sgn(\sigma')\left[\left(\frac{1}{2}+ik\right)\frac{\sqrt{2}}{P}\mathcal{A}_1 -2^{\frac{3}{2}}e^{-\frac{i\pi}{2}}\mathcal{D}_1\right]-e_0\tan(\theta_W)sgn(\sigma) \times\left[\left(\frac{1}{2}+ik\right)\frac{\sqrt{2}}{P}\mathcal{A}_2-2^{\frac{3}{2}}e^{-\frac{i\pi}{2}}\mathcal{D}_2\right] \xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}\eta_{\sigma'}(\vec{p}\,'), \tag{168}$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are the same functions as stated in (158)–(159), while the $\mathcal{D}_1$ and $\mathcal{D}_2$ functions are defined as:

$$\mathcal{D}_1 = \mathcal{C}_{1Kk}-\left(\frac{p}{p'}\right)^{\frac{1}{2}}\mathcal{C}_{Kk}-\left(\frac{p'}{p}\right)^{\frac{1}{2}}\mathcal{C}_{-Kk}+\mathcal{C}_{2Kk}; \tag{169}$$

$$\mathcal{D}_2 = \mathcal{C}_{1-Kk}-\left(\frac{p}{p'}\right)^{\frac{1}{2}}\mathcal{C}_{-Kk}-\left(\frac{p'}{p}\right)^{\frac{1}{2}}\mathcal{C}_{Kk}+\mathcal{C}_{2-Kk}, \tag{170}$$

with the remaining $\mathcal{C}$ functions being dependant on the Gamma Euler functions, as well as the hypergeometric functions:

$$\mathcal{C}_{Kk} = \frac{ie^{-\pi K}(pp')^{iK}(-iP)^{-\frac{7}{2}-2iK}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}+iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \Gamma\left(\frac{5+4iK-2ik}{4}\right)\Gamma\left(\frac{9+4iK+2ik}{4}\right) \times F_4\left(\frac{5+4iK-2ik}{4},\frac{9+4iK+2ik}{4}, \frac{3}{2}+iK,\frac{1}{2}+iK,\left(\frac{p}{P}\right)^2,\left(\frac{p'}{P}\right)^2\right); \tag{171}$$

$$\mathcal{C}_{1Kk} = -\frac{p^{\frac{1}{2}+iK}(p')^{\frac{1}{2}-iK}(-iP)^{-\frac{9}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{3}{2}+iK\right)} \Gamma\left(\frac{7-2ik}{4}\right)\Gamma\left(\frac{11+2ik}{4}\right) \times F_4\left(\frac{7-2ik}{4},\frac{11+2ik}{4},\frac{3}{2}-iK, \frac{3}{2}+iK,\left(\frac{p}{P}\right)^2,\left(\frac{p'}{P}\right)^2\right); \tag{172}$$

$$\mathcal{C}_{2Kk} = \frac{p^{-\frac{1}{2}-iK}(p')^{-\frac{1}{2}+iK}(-iP)^{-\frac{5}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \Gamma\left(\frac{3-2ik}{4}\right)\Gamma\left(\frac{7+2ik}{4}\right) \times F_4\left(\frac{3-2ik}{4},\frac{7+2ik}{4},\frac{1}{2}-iK, \frac{1}{2}+iK,\left(\frac{p}{P}\right)^2,\left(\frac{p'}{P}\right)^2\right). \tag{173}$$

The second term T_2 that contribute to the amplitude will be:

$$T_{2\,(Z\to e^-e^+)} = -\int dz\cdot z^{\frac{5}{2}}\frac{\pi^{3/2}P\sqrt{pp'}\omega e^{-\frac{\pi k}{2}}}{8M_Z(2\pi)^{3/2}} \times\left[\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}sgn(\sigma')H^{(2)}_{\nu^+}(pz)H^{(2)}_{\nu^-}(p'z)H^{(1)}_{ik}(Pz) -e_0\tan(\theta_W)sgn(\sigma)H^{(2)}_{\nu^-}(pz)H^{(2)}_{\nu^+}(p'z)H^{(1)}_{ik}(Pz)\right] \xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')\delta^3(\vec{P}-\vec{p}-\vec{p}\,'). \tag{174}$$

As previously, we will analyze the products of the Hankel functions first, which will have a similar form as in the case for $\lambda=\pm1$, and then compute the resulting temporal integrals. The final form of the term T_2 is:

$$T_{2\,(Z\to e^-e^+)} = \left(\frac{2i}{\pi}\right)\frac{P\sqrt{pp'}}{8}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\frac{\omega}{M_Z} \times\left[\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma')\mathcal{F}_1 -e_0\tan(\theta_W)sgn(\sigma)\mathcal{F}_2\right]\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,') \tag{175}$$

where $\mathcal{F}_1$ and $\mathcal{F}_2$ are functions denoted by:

$$\mathcal{F}_1 = \mathcal{G}_{1Kk}-\left(\frac{p}{p'}\right)^{\frac{1}{2}}\mathcal{G}_{Kk}-\left(\frac{p'}{p}\right)^{\frac{1}{2}}\mathcal{G}_{-Kk}+\mathcal{G}_{2Kk}; \tag{176}$$

$$\mathcal{F}_2 = \mathcal{G}_{1-Kk}-\left(\frac{p}{p'}\right)^{\frac{1}{2}}\mathcal{G}_{-Kk}-\left(\frac{p'}{p}\right)^{\frac{1}{2}}\mathcal{G}_{Kk}+\mathcal{G}_{2-Kk}, \tag{177}$$

and the $\mathcal{G}$ functions, which are dependant on the Gamma Euler functions, as well as the hypergeometric functions, are:

$$\mathcal{G}_{Kk} = \frac{ie^{-\pi K}(pp')^{iK}(-iP)^{-\frac{7}{2}-2iK}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}+iK\right)\Gamma\left(\frac{1}{2}+iK\right)} \times\Gamma\left(\frac{7+4iK-2ik}{4}\right)\Gamma\left(\frac{7+4iK+2ik}{4}\right) \times F_4\left(\frac{7+4iK-2ik}{4},\frac{7+4iK+2ik}{4},\frac{3}{2}+iK,\frac{1}{2}+iK,\right.$$

$$\left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\Big); \tag{178}$$

$$\begin{aligned}\mathcal{G}_{1Kk} &= -\frac{p^{\frac{1}{2}+iK}(p')^{\frac{1}{2}-iK}(-iP)^{-\frac{9}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{3}{2}-iK\right)\Gamma\left(\frac{3}{2}+iK\right)}\\ &\times\Gamma\left(\frac{9-2ik}{4}\right)\Gamma\left(\frac{9+2ik}{4}\right)\\ &\times F_4\Big(\frac{9-2ik}{4}, \frac{9+2ik}{4}, \frac{3}{2}-iK,\\ &\frac{3}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\Big);\end{aligned} \tag{179}$$

$$\begin{aligned}\mathcal{G}_{2Kk} &= \frac{p^{-\frac{1}{2}-iK}(p')^{-\frac{1}{2}+iK}(-iP)^{-\frac{5}{2}}}{\cosh^2(\pi K)\Gamma\left(\frac{1}{2}-iK\right)\Gamma\left(\frac{1}{2}+iK\right)}\\ &\times\Gamma\left(\frac{5-2ik}{4}\right)\Gamma\left(\frac{5+2ik}{4}\right)\\ &\times F_4\Big(\frac{5-ik}{4}, \frac{5+2ik}{4}, \frac{1}{2}-iK,\\ &\frac{1}{2}+iK, \left(\frac{p}{P}\right)^2, \left(\frac{p'}{P}\right)^2\Big).\end{aligned} \tag{180}$$

We combine T_1 (168) and T_2 (175) in order to obtain the complete form of the transition amplitude for $\lambda = 0$.

$$\begin{aligned}&\mathcal{A}_{Z\to e^-e^+}(\lambda=0) = \frac{\pi^{3/2}P\sqrt{pp'}}{8(2\pi)^{3/2}}\frac{2i}{\pi}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\\ &\times\frac{\omega}{M_Z}\Big\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma')\Big[\left(\frac{1}{2}+ik\right)\\ &\times\frac{i\sqrt{2}}{P}\mathcal{A}_1 - 2^{\frac{3}{2}}ie^{-\frac{i\pi}{2}}\mathcal{D}_1 + 2^{\frac{3}{2}}\mathcal{F}_1\Big]\\ &-e_0\tan(\theta_W)sgn(\sigma)\Big[\left(\frac{1}{2}+ik\right)\frac{i\sqrt{2}}{P}\mathcal{A}_2 - 2^{\frac{3}{2}}ie^{-\frac{i\pi}{2}}\mathcal{D}_2\\ &+2^{\frac{3}{2}}\mathcal{F}_2\Big]\Big\}\,\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,').\end{aligned} \tag{181}$$

4.2 Probability in the limit $\frac{M_Z}{\omega} = \frac{m}{\omega} = 0$

It is important to consider the case where the expansion factor is much larger than the masses of the particles, and therefore will give us the limit where $\frac{M_Z}{\omega} = \frac{m}{\omega} \to 0$. To compute the probability in such a limit, we will consider the case for $\lambda = \pm 1$, since it has a much simpler form, and the amplitude in the case of $\lambda = 0$ contains the term $\frac{\omega}{M_Z}$ (181), which makes it impossible for us to determine the limit. In the Eq. (163) for the transition probability we can see that in our chosen limit, the functions $\mathcal{A}_1$ and $\mathcal{A}_2$ have the same form. This will make it easier for us to calculate the probability of the decay process in our chosen limit. The probability will have the form:

$$\begin{aligned}P_{i\to f} &= \frac{e_0^2\,\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{16(2\pi)^3P(P-p-p')^2}\frac{1}{4}\\ &\sum_{\sigma\sigma'}\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}sgn(\sigma') - \tan(\theta_W)sgn(\sigma)\right)^2\\ &\times\left|\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')\right|^2.\end{aligned} \tag{182}$$

To obtain the total probability, we will need to integrate after the final momenta:

$$P_{Z\to e^-e^+} = \int d^3p\int d^3p'\,P_{i\to f} \tag{183}$$

To solve such integrals, we need to establish the momentum of the particles. For a more general computation, we will choose the momentum to be on the third axis for the Z boson, as well as the positron and electron particles, as such $\vec{P} = P\cdot\vec{e}_3$, $\vec{p} = -p\cdot\vec{e}_3$, $\vec{p'} = p'\cdot\vec{e}_3$. The Delta function allows us to express the modulus of the positron particle p' in relation to the other two, as $|p'| = |P-p| = p-P$. The integrals after the momenta can now be written as:

$$\begin{aligned}I &= \frac{1}{(2\pi)^3}\int d^3p\int d^3p'\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{P(p+p'-P)^2}\\ &= \frac{1}{(2\pi)^3}\int d^3p\frac{1}{4P(p-P)^2}\end{aligned} \tag{184}$$

By using the dimensional regularization method [29–32], we arrive to the same integrals as those discussed in the previous section that can be found on equations (111)–(119).

By using these results from Eqs. (111)–(119) the form for the total probability of the decay process become (183):

$$\begin{aligned}P_{Z\to e^-e^+} &= \frac{e_0^2}{32\pi^2}\left[1-\psi\left(\frac{3}{2}\right)-\ln\left(\frac{4\pi\mu^2}{P^2}\right)\right]\\ &\times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}+\tan(\theta_W)\right)^2\\ &= \frac{\sqrt{2}G_FM_Z^2\sin^2(2\theta_W)}{8\pi^2}\left[1-\psi\left(\frac{3}{2}\right)-\ln\left(\frac{4\pi\mu^2}{P^2}\right)\right]\\ &\times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}+\tan(\theta_W)\right)^2\end{aligned} \tag{185}$$

where we use the relation $e_0^2 = \sqrt{2}G_FM_Z^2\sin^2(2\theta_W)$.

4.3 Transition rate

The transition rate for the decay process of a Z boson into an electron-positron pair can be written as:

$$\begin{aligned}R_{i\to f} &= \frac{1}{(2\pi)^3}\sum_{\sigma\sigma'}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\left|M_{i\to f}\right|^2\\ &\times\sqrt{I_{i\to f}\cdot I^*_{i\to f}}\lim_{t\to\infty}\left|e^{\omega t}K_{if}\right|\end{aligned} \tag{186}$$

where the constants and the bispinors are included in the term $M_{i\to f}$, while $I_{i\to f}$ contains the temporal integrals:

$$M_{i\to f} = \frac{e_0\pi^{3/2}\sqrt{pp'}}{8(2\pi)^{3/2}}|\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')| \tag{187}$$

$$I_{i\to f} = \frac{\cos(2\theta_W)}{\sin(2\theta_W)} sgn(\sigma') \int_0^\infty dz \cdot z^{3/2} \times H^{(2)}_{\nu^+}(pz) H^{(2)}_{\nu^-}(p'z) H^{(1)}_{ik}(Pz) - \tan(\theta_W) sgn(\sigma) \int_0^\infty dz \cdot z^{3/2} H^{(2)}_{\nu^-}(pz) \times H^{(2)}_{\nu^+}(p'z) H^{(1)}_{ik}(Pz) \tag{188}$$

We can further write the temporal integrals as $I_{i\to f} = \int_0^\infty K_{if} dz = \int_0^\infty e^{\omega t} K_{if} dt$, where K_{if} denotes the integrand:

$$K_{if} = e^{-\omega t}\left(\frac{e^{\omega t}}{\omega}\right)^{3/2}\left[\frac{\cos(2\theta_W)}{\sin(2\theta_W)} sgn(\sigma') \times H^{(2)}_{\nu^+}(pz) H^{(2)}_{\nu^-}(p'z) H^{(1)}_{ik}(Pz) - \tan(\theta_W) sgn(\sigma) H^{(2)}_{\nu^-}(pz) H^{(2)}_{\nu^+}(p'z) H^{(1)}_{ik}(Pz)\right] \tag{189}$$

If we consider that the transition takes place after a sufficiently long time, which will be denoted by t_∞, then we will need to evaluate the limit in Eq. (189) for that specific time. In the limit of $t \to \infty$, the argument of the Hankel functions $z = e^{-\omega t}/\omega$ becomes very small, and therefore we will use the expansion for Hankel functions:

$$H^{(2)}_{\nu^\pm}(pz) \simeq i\left(\frac{2}{pz}\right)^{\nu^\pm} \frac{\Gamma(\nu^\pm)}{\pi} \tag{190}$$

Our functions will then become:

$$H^{(2)}_{\frac{1}{2}+iK}(pz) \simeq i\left(\frac{2}{pz}\right)^{\frac{1}{2}+iK} \frac{\Gamma(\frac{1}{2}+iK)}{\pi} \tag{191}$$

$$H^{(2)}_{\frac{1}{2}-iK}(pz) \simeq i\left(\frac{2}{pz}\right)^{\frac{1}{2}-iK} \frac{\Gamma(\frac{1}{2}-iK)}{\pi} \tag{192}$$

For the Z boson, we consider the approximation when $M_Z \to 1/2$, and therefore the index of the Hankel function will become $ik \to -1/2$:

$$H^{(1)}_{-\frac{1}{2}}(Pz) = e^{\frac{i\pi}{2}} \sqrt{\frac{2}{\pi P z}} \frac{e^{iPz}}{i} \tag{193}$$

By replacing the Hnakel functions with our expansions, the limit from the rate transition (186), which contains the integrand K_{if}, will be:

$$\lim_{t\to\infty} \left|e^{\omega t} K_{if}\right| = \lim_{t\to\infty} \left| \frac{\cos(2\theta_W)}{\sin(2\theta_W)} sgn(\sigma') \left(\frac{e^{-\omega t}}{\omega}\right)^{3/2} \times \left(\frac{2}{p\frac{e^{-\omega t}}{\omega}}\right)^{\frac{1}{2}+iK} \left(\frac{2}{p'\frac{e^{-\omega t}}{\omega}}\right)^{\frac{1}{2}-iK} \times \left(\frac{2}{\pi P\frac{e^{-\omega t}}{\omega}}\right)^{\frac{1}{2}} \frac{e^{\frac{i\pi}{2}} e^{iP\frac{e^{-\omega t}}{\omega}} \Gamma(\frac{1}{2}-iK)\Gamma(\frac{1}{2}+iK)}{i\pi^2} - \tan(\theta_W) sgn(\sigma) \left(\frac{e^{-\omega t}}{\omega}\right)^{3/2} \times \left(\frac{2}{p\frac{e^{-\omega t}}{\omega}}\right)^{\frac{1}{2}-iK} \left(\frac{2}{p'\frac{e^{-\omega t}}{\omega}}\right)^{\frac{1}{2}+iK} \left(\frac{2}{\pi P\frac{e^{-\omega t}}{\omega}}\right)^{\frac{1}{2}} \times \frac{e^{\frac{i\pi}{2}} e^{iP\frac{e^{-\omega t}}{\omega}} \Gamma(\frac{1}{2}-iK)\Gamma(\frac{1}{2}+iK)}{i\pi^2} \right| \tag{194}$$

If we extract the modulus and rewrite the imaginary part, we can obtain a more comprehensible expression for the limit:

$$\lim_{t\to\infty} \left|e^{\omega t} K_{if}\right| = \frac{2^{3/2}}{\pi^2 \cosh(\pi K)\sqrt{\pi p p' P}} \times \left[\frac{\cos^2(2\theta_W)}{\sin^2(2\theta_W)} + \tan^2(\theta_W) - sgn(\sigma) sgn(\sigma') \frac{\cos(2\theta_W)}{\sin(2\theta_W)} \tan(\theta_W) 2\cos \times \left(2K \ln\left(\frac{p'}{p}\right)\right)\right]^{1/2} \tag{195}$$

Finally, the transition rate gives:

$$R_{i\to f} = \sum_{\sigma\sigma'} \frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{64(2\pi)^6} \frac{e_0^2 \pi 2^{3/2} p p'}{\sqrt{\pi p p' P}} \times \sqrt{I_{i\to f} \cdot I^*_{i\to f}} \left|\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')\right|^2 \times \left[\frac{\cos^2(2\theta_W)}{\sin^2(2\theta_W)} + \tan^2(\theta_W) - sgn(\sigma) sgn(\sigma') \times \frac{\cos(2\theta_W)}{\sin(2\theta_W)} \tan(\theta_W) 2\cos\left(2K \ln\left(\frac{p'}{p}\right)\right)\right]^{1/2} \tag{196}$$

This expression is to complicated to be integrated after the final momenta, since the momenta dependence is contained in hypergeometric functions. For this reason it is important to discuss the limit $omega >> M_Z$ which is relevant for our computations and corresponds to the conditions from the early universe.

It is known from the Minkowski field theory that optical theorem have as consequence the fact that the imaginary part of the Z boson self-energy diagram corresponds to the width of the Z boson and give the decay rate. One may ask if this result can be recovered in de Sitter case and in what follows we will give the main points that need to be studied to obtain this result. The Z boson self-energy diagram in de Sitter space-time need to be computed by using the solutions of the Proca equation and the Dirac propagator for fermions. The computations for the Z boson self-energy diagram should use the Feynmann propagator representation in momentum space for Dirac field, that was obtained in [22]. However there are technical difficulties in completing the computations in the general case that depends on the ratio between the particle mass and expansion parameter and we hope to approach this matter in a future study. The fundamental result this kind of computation could bring is related to the presence of the expansion parameter (Hubble constant) in the loop corrections, that may be one of the indications that the gravitation may be quantized.

4.4 Transition rate in the limit $\frac{M_Z}{\omega}=\frac{m}{\omega}=0$

If we consider the case where the expansion factor is larger than the masses of the particles $\omega\gg M_Z$, $\omega\gg m$, then we can calculate the transition rate in the limits $\frac{M_Z}{\omega}\to 0$, $\frac{m}{\omega}\to 0$. We will consider the case for $\lambda=\pm1$, and the Hankel functions will be reduced to a simpler form in this particular case [40,41]. Then the transition amplitude written in this limit will be:

$$\mathcal{A}_{\lambda=\pm1}\left(\frac{M_Z}{\omega}=\frac{m}{\omega}\to 0\right)$$
$$=-\int dz\cdot z^{3/2}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\frac{\pi^{3/2}\sqrt{pp'}}{8(2\pi)^{3/2}}$$
$$\times\left[\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma')\right.$$
$$\left.-e_0\tan(\theta_W)sgn(\sigma)\right]H^{(2)}_{\frac{1}{2}}(pz)H^{(2)}_{\frac{1}{2}}(p'z)$$
$$H^{(1)}_{-\frac{1}{2}}(Pz)\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \tag{197}$$

We use the following expressions for the Hankel functions [40,41]:

$$H^{(2)}_{\frac{1}{2}}(pz)=-\sqrt{\frac{2}{\pi pz}}\frac{e^{-ipz}}{i} \tag{198}$$

$$H^{(1)}_{-\frac{1}{2}}(Pz)=e^{\frac{i\pi}{2}}H^{(1)}_{\frac{1}{2}}(Pz)=e^{\frac{i\pi}{2}}\sqrt{\frac{2}{\pi Pz}}\frac{e^{iPz}}{i}. \tag{199}$$

Therefore, the transition amplitude becomes:

$$A_{\lambda=\pm1}\left(\frac{M_Z}{\omega}=\frac{m}{\omega}\to 0\right)=\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{\sqrt{P}(P-p-p')}\frac{i\cdot 2^{3/2}}{8(2\pi)^{3/2}}$$
$$\times\left[\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma')\right.$$
$$\left.-e_0\tan(\theta_W)sgn(\sigma)\right]\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'). \tag{200}$$

From this amplitude, we can calculate the transition rate using the following expression:

$$R_{i\to f}=\frac{1}{(2\pi)^3}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\sum_{\sigma\sigma'}|M_{if}|^2|I_{if}|$$
$$\times\lim_{t\to\infty}|e^{\omega t}K_{if}|. \tag{201}$$

Where the M_{if} component contains the all constants:

$$M_{if}=\frac{e_0\pi^{3/2}\sqrt{pp'}}{8(2\pi)^{3/2}}\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}sgn(\sigma')\right.$$
$$\left.-\tan(\theta_W)sgn(\sigma)\right)\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,'), \tag{202}$$

while I_{if} and K_{if} represent the integral and integrand respectively, and have the following forms:

$$I_{if}=\left(\frac{2}{\pi}\right)^{3/2}\frac{1}{\sqrt{pp'P}(p+p'-P)}; \tag{203}$$

$$\lim_{t\to\infty}|e^{\omega t}K_{if}|=\left(\frac{2}{\pi}\right)^{3/2}\frac{1}{\sqrt{pp'P}}. \tag{204}$$

If we replace all the values in the expression (201) for the transition rate, we get:

$$R_{i\to f}=\sum_{\sigma\sigma'}\frac{e_0^2}{32(2\pi)^6}\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{P(p+p'-P)}$$
$$\times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}sgn(\sigma')-\tan(\theta_W)sgn(\sigma)\right)^2\left|\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\right.$$
$$\left.\times\vec{\epsilon}(\vec{n}_P,\lambda)\eta_{\sigma'}(\vec{p}\,')\right|^2. \tag{205}$$

In order to obtain the total transition rate for this process, we need to integrate after the final momenta.

$$R_{Z\to e^-e^+}=\int d^3p\int d^3p'\cdot R_{i\to f}. \tag{206}$$

We choose the momentum for each of the particles to be on the third axis, in such a way that $\vec{p}=p\cdot\vec{e_3}$, $\vec{p'}=p'\cdot\vec{e_3}$, and $\vec{P}=P\cdot\vec{e_3}$. From the delta function $\delta^3(\vec{P}-\vec{p}-\vec{p}')$ we can find out the modulus of the momenta of the positron p', which will be $|p'|=p-P$. With this, we can compute the integrals after the positron momentum.

$$I=\frac{1}{(2\pi)^3}\int d^3p\int d^3p'\frac{\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{P(p+p'-P)}=\frac{1}{(2\pi)^3}$$
$$\times\int d^3p\frac{1}{2P(p-P)} \tag{207}$$

We notice that this integral is identical to the one we obtained when computing the transition rate in this limit for the decay process of a Z boson into an neutrino–antineutrino pair (207). Therefore, we can use the previous calculations from that case (131)–(139).

Finally, the total transition rate in the limit $\frac{M_Z}{\omega}=\frac{m}{\omega}=0$ is:

$$R_{Z\to e^-e^+}=\frac{e_0^2}{32(2\pi)^3}\frac{P}{8\pi^2}\left[\ln\left(\frac{4\pi\mu^2}{P^2}\right)+\psi\left(\frac{3}{2}\right)\right]$$
$$\times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}+\tan(\theta_W)\right)^2$$
$$=\frac{\sqrt{2}G_FM_Z^2\sin^2(2\theta_W)}{32(2\pi)^3}\frac{P}{8\pi^2}\left[\ln\left(\frac{4\pi\mu^2}{P^2}\right)+\psi\left(\frac{3}{2}\right)\right]$$
$$\times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}+\tan(\theta_W)\right)^2. \tag{208}$$

An estimation of the density numbers of Z bosons produced by electrons can be now given considering Eqs. (65), (68), (141) and (208) and obtain that the ratio of the rates give $\frac{R_{e^-\rightarrow e^-+Z}}{R_{decay}} \simeq 10^{-9}$, then

$$n_Z \simeq 10^{-9} n_{e^-}, \tag{209}$$

which proves that one need a large number of electrons for obtaining a considerable amount of Z bosons by this perturbative process. This is because one need 10^9 transitions for one single Z boson to be produced.

4.5 Decay rate in Minkowski limit

In this section we study the problem of the Z boson decay into an electron and positron in de Sitter geometry. Our scope is to establish the decay rate in the general case on de Sitter expanding patch, and then to discuss the Minkowski limit of our computations. Since in the Minkowski limit the longitudinal modes contribution vanish we will analyse the contribution of the transversal modes to the transition amplitudes i.e.

$$\begin{aligned}\mathcal{A}_{Z\rightarrow e^-e^+} = &\int d^4x \cdot \sqrt{-g}\Bigg\{\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e^k_{\hat{i}}\\ &\times\left(\frac{1-\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda_k}(x)\\ &-e_0\tan(\theta_W)\bar{U}_{\vec{p},\sigma}(x)\gamma^{\hat{i}}e^k_{\hat{i}}\left(\frac{1+\gamma^5}{2}\right)V_{\vec{p}\,',\sigma'}(x)f_{\vec{P},\lambda_k}(x)\Bigg\}.\end{aligned} \tag{210}$$

The amplitude can be brought to the following form by using the free fields solutions on de Sitter space and by solving the spatial integrals:

$$\begin{aligned}\mathcal{A}_{Z\rightarrow e^-e^+} = &\frac{\pi^{3/2}\sqrt{pp'}e^{-\frac{\pi k}{2}}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')}{8(2\pi)^{3/2}}\\ &\times\Bigg[\left(\frac{e_0\cos(2\theta_W)}{\sin(2\theta_W)}\right)sgn(\sigma')\int_0^\infty dz\cdot z^{3/2}\\ &\times H^{(2)}_{\nu^+}(pz)H^{(2)}_{\nu^-}(p'z)\\ &\cdot H^{(1)}_{ik}(Pz) - e_0\tan(\theta_W)sgn(\sigma)\\ &\times\int_0^\infty dz\cdot z^{3/2}H^{(2)}_{\nu^-}(pz)H^{(2)}_{\nu^+}(p'z)H^{(1)}_{ik}(Pz)\Bigg]\\ &\times\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}\,\eta_{\sigma'}(\vec{p}\,').\end{aligned} \tag{211}$$

The above integrals cannot be brought to the integrals that contain a power and two Bessel J functions as in the case of Z boson decay into neutrinos. The present integrals contain the product of three Bessel J functions and a power, and their result is expressed in terms of Appel F_4 functions. Since the Appel functions are not very well studied in the literature it is impossible to repeat the same steps as those used in the case of the integrals with two Bessel functions from section three. The integrals with three Bessel functions and a power were studied in [44], where the indexes of the Bessel functions are integer numbers. In this situation the result of the integral is obtained as a semi-empirical approximate formula in terms of the Dirac delta function [44]. In our case it is impossible to repeat these arguments since the index of the Hankel functions are imaginary. The only way to study the Minkowski limit of our integrals is to consider the case where the fermions' masses are neglected directly in the Hankel functions, corresponding to the ultra-relativistic limit:

$$\begin{aligned}\int_0^\infty dz\cdot z^{3/2}H^{(2)}_{\frac{1}{2}}(pz)H^{(2)}_{\frac{1}{2}}(p'z)H^{(1)}_{ik}(Pz) = \frac{-2}{\pi\sqrt{pp'}}\\ \times\int_0^\infty dz\cdot\sqrt{z}\,e^{-i(p+p')z}H^{(1)}_{ik}(Pz)\end{aligned} \tag{212}$$

In this particular case the above integrals from Eq. (211) become equal and their form is:

$$\begin{aligned}I = &\frac{2i}{\pi\sqrt{pp'}}\sqrt{\frac{\pi(p+p')}{2}}\int_0^\infty dz\cdot z\\ &\times H^{(2)}_{\frac{1}{2}}\big((p+p')z\big)H^{(1)}_{ik}(Pz),\end{aligned} \tag{213}$$

from where we can write:

$$\begin{aligned}I = &\sqrt{\frac{p+p'}{pp'}}\left(\sqrt{\frac{2}{\pi}}\right)^3 e^{\pi k/2}\int_0^\infty dz\cdot z\\ &\times\Big(J_{\frac{1}{2}}\big((p+p')z\big)+iJ_{-\frac{1}{2}}\big((p+p')z\big)\Big)K_{ik}(-iPz)\end{aligned} \tag{214}$$

The above integral can be solved using the method presented in Eqs. (83), (86), (89) from section three, and the final result for $p+p'=P$ is:

$$\begin{aligned}\lim_{\epsilon\rightarrow 0} I = &\left(\frac{2}{\pi}\right)^{3/2}\frac{i\pi e^{\pi k/2}}{2}\left(\frac{p+p'}{P\sqrt{P}}\delta(p+p'-P)\right.\\ &\left.+\frac{1}{\sqrt{P}}\delta(p+p'-P)\right),\end{aligned} \tag{215}$$

while for $p+p'\neq P$ the result can be found in Eqs. (93), (94).

When we take the Minkowski limit of the above result we consider the ultrarelativistic situation where the momenta modulus are equal to the particles energies, and we define the rate as the derivative of the probability with respect to time. The resulted transition amplitude in the Minkowski limit reads as:

$$\begin{aligned}\mathcal{A}_{Z\rightarrow e^-e^+}(\lambda=\pm 1) = &\frac{i\pi e_0\sqrt{2}}{8(2\pi)^{3/2}\sqrt{|\vec{P}|}}\delta^3(\vec{P}-\vec{p}-\vec{p}\,')\\ &\times\delta(\mathcal{E}-E-E')\left(\frac{|\vec{p}|+|\vec{p}\,'|}{|\vec{P}|}+1\right)\end{aligned}$$

$$\times\left(\frac{\cos(2\theta_W)}{\sin(2\theta_W)}sgn(\sigma') - \tan(\theta_W)sgn(\sigma)\right)$$
$$\times\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}(\vec{n}_{\mathcal{P}},\lambda)\eta_{\sigma'}(\vec{p}\,') \tag{216}$$

Then the transition rate is:

$$R_{Z\to e^-e^+}=\frac{1}{4}\sum_{\sigma\sigma'}\frac{e_0^2}{\sin^2(2\theta_W)}\int d^3p\,d^3p'\cdot\delta^3(\vec{p}+\vec{p}\,'-\vec{P})$$
$$\frac{2\pi\delta(E+E'-\mathcal{E})}{64P(2\pi)^3}\left(\frac{p+p'}{P}+1\right)^2\left|\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}\,\eta_{\sigma'}(\vec{p}\,')\right|^2$$
$$\left(\cos(2\theta_W)sgn(\sigma')-\tan(\theta_W)\sin(2\theta_W)sgn(\sigma)\right)^2, \tag{217}$$

where E, E' are the electron and positron Minkowski energies, and $\mathcal{E}$ is the Z boson energy. The rate will be computed by considering that the particles momenta are aligned on the third axis where we take into account that the electron and positron momenta could have the same orientation, or could be oriented in opposite orientations. In this way the helicity bispinor sum and the term that contains the Weinberg angles are reduced to:

$$\sum_{\sigma\sigma'}\left|\xi^+_\sigma(\vec{p})\vec{\sigma}\cdot\vec{\epsilon}\,\eta_{\sigma'}(\vec{p}\,')\right|^2\left(\cos(2\theta_W)sgn(\sigma')\right.$$
$$\left.-\tan(\theta_W)\sin(2\theta_W)sgn(\sigma)\right)^2$$
$$=8\left[(1-2\sin^2(\theta_W))^2+4\sin^4(\theta_W)\right] \tag{218}$$

The integrals that need to be solved are exactly the same as those from section three, and we give only the final result:

$$I(p,p')=\frac{1}{|\vec{P}|}\int d^3p\int d^3p'\,\frac{E'}{E'}\delta^4(p+p'-P)$$
$$\times\left(\frac{|\vec{p}|+|\vec{p}\,'|}{|\vec{P}|}+1\right)^2=\frac{8\pi}{\mathcal{E}^2}E^2(\mathcal{E}-E). \tag{219}$$

In the Minkowski case one could take the Z boson at rest and in this particular situation $\mathcal{E}=2E=M_Z$ [42], and the result of the integral is:

$$I(p,p')=4\pi\mathcal{E}=4\pi M_Z. \tag{220}$$

Then the final equation for the transition rate give:

$$R_{Z\to e^-e^+}=\frac{G_F\cdot M_Z^3}{16\sqrt{2}\pi}\left[(1-2\sin^2(\theta_W))^2+4\sin^4(\theta_W)\right]. \tag{221}$$

It is known that the decay rates into charged lepton–antilepton pairs e^-e^+, $\mu^-\mu^+$, $\tau^-\tau^+$, are approximately equal [42], and for that reason we will define the rate for the decay into any lepton–antilepton pair as:

$$R_1'=\frac{R_{Z\to e^-e^+}}{3}=\frac{G_F\cdot M_Z^3}{48\sqrt{2}\pi}$$
$$\times\left[(1-2\sin^2(\theta_W))^2+4\sin^4(\theta_W)\right]=\frac{R_{Minkowski}}{4} \tag{222}$$

We mention that in Minkowski theory the decay rate for a Z boson into charged leptons is obtained in the lowest order and neglecting the terms $\frac{m^2_{lepton}}{M_Z^2}$, which is just the limit obtained above up to a factor of 1/4.

5 Conclusions

In this paper we investigate three elementary processes in the first order of perturbation theory that imply the neutral current interactions. We obtain the rates for the processes in which the Z boson can be produced by a charged lepton, and the decay rates of the Z bosons into neutrinos and charged leptons in the conditions of the early universe. These results are exact since in the limit of large expansion the Hubble parameter becomes much larger than the particle masses, and this facilitate our computations. In order to obtain the transition rates in the strong gravitational regime we employ the dimensional regularization method combined with the minimal substraction method, which allows us to obtain finite results for the final momenta integrals. Here we must point out that the amplitudes in the general case depend on the ratios between particles masses and the expansion parameter, and when one try to perform the momenta integrals for obtaining the probabilities the results seems also to be divergent. This is the result of the fact that the algebraic arguments of the hypergeometric functions depend on the momenta ratios and the main contributions that give the form of the momenta integrals come from the factors outside hypergeometric functions. This can be seen very well in the case of Z boson decay into neutrinos where the factor that determines the amplitude $[p+p'-P]^{-3/2-ik}$, is also obtained in the limit $M_Z/\omega=0$. In the general case the momenta integrals that will determine the total probabilities and transition rates will contain ratios between sums of momenta at imaginary powers and also products of two hypergeometric functions. This kind of integrals are not well studied in the literature so far and the fact that there are imaginary powers in momenta could lead also to undetermined cases. As far as we know there are no results that could help one to understand what kind of regularization method to apply in these complicated momenta integrals. Still only looking at the real powers from the momenta integrals one can see the presence of a logarithmical divergence in these integrals that also was obtained in the limit $M_Z/\omega=0$. This seems to lead to the conclusion that the limit $M_Z/\omega=0$ does not induce the divergence since it is already present in the general case with an additional dependence of momenta at imaginary powers that further complicate the problem. For these reasons we investigate the relevant physical case of the early universe, when the computations could be done by using well established regularization methods.

We prove that the processes that generate spontaneous emission of a Z boson by a charged lepton are possible only in the early universe when the expansion parameter is considerably larger than the particle mass, and their rates vanish in the Minkowski limit. This result confirm the previously results obtained by both perturbative [10,14–18,26] and non-perturbative methods [13,23,24],that the particle generation is possible only in the early universe. Other important limits that we manage to study are the Minkowski limits of our quantities of interest, like the transition rates. An important result is related to the recovery of the Minkowski decay rates of the Z boson up to some numerical factors that could be explained because we work only on the expanding portion of de Sitter space. To obtain a complete picture about the production of massive bosons in the early universe one needs to investigate the processes with the massive charged bosons $W^{\pm}$ and their decays in strong gravitational fields of the early universe.

In the end of our summary we want to comment on the relevance of our result in the context of the actual effort to understand the mechanisms that were responsible for particle generation in early universe. First the exact perturbative computations give finite results when one apply regularization methods, but as it is very well established in the literature one need to take into account other mechanisms that could also produce particle. One is related to the non-perturbative production in pure gravitational field. However the literature is poor in what concerns the production of Proca particles, but recent studies could bring some new interesting ideas related to what one may obtain by using different vacua states between "in-out" transitions. In our study we use the Bunch-Davies vacuum [5], but recently the rest frame vacuum was defined [45], and this result was extended to the Proca field. Then it may be possible to make a study in which the in and out vacua are not the same and establish the Bogoliubov coefficients for the Proca field in de Sitter space-time. Once this study is completed, one could in principle compare the non-perturbative results with our results obtained by using the traditional perturbative mechanism that was employed in Minkowski space-time. We hope to approach this problem in a future study and also hope that our results to be the first steps that would open a more detailed study of the massive boson fields in the context of General Relativity.

Acknowledgements This work is supported by the European Union - NextGenerationEU through the Grant no. 760079/23.05.2023, funded by the Romanian ministry of research, innovation and digitalization through Romania's National Recovery and Resilience Plan, call no. PNRR-III-C9-2022-I8.

Data Availability Statement Data will be made available on reasonable request. [Authors' comment: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.]

Code Availability Statement Code/software will be made available on reasonable request. [Authors' comment: The code/software generated during and/or analysed during the current study is available from the corresponding author on reasonable request.]

6 Appendix

To obtain the transition amplitude for Z bososn emission by an electron we use the relation between the Bessel K functions and Bessel J functions according to formula [40, 41]:

$$K_{\nu}(iz) = \frac{\pi}{2\sin(\pi\nu)}\left[i^{-\nu}J_{-\nu}(z) - i^{\nu}J_{\nu}(z)\right]. \tag{223}$$

The two integrands from Eq. (11) transform according to the above equation as [40,41]:

$$\begin{aligned}
K_{\nu_-}(-ipz)K_{\nu_+}(ip'z)K_{-ik}(iPz) &= \frac{\pi^2}{4\cosh^2(\pi K)} \\
&\times\Big[-iJ_{-\frac{1}{2}+iK}(-pz)J_{-\frac{1}{2}-iK}(p'z) \\
&-e^{-\pi k}J_{-\frac{1}{2}+iK}(-pz)J_{\frac{1}{2}+iK}(p'z) - e^{\pi k} \\
&\times J_{\frac{1}{2}-iK}(-pz)J_{-\frac{1}{2}-iK}(p'z) \\
&+iJ_{\frac{1}{2}-iK}(-pz)J_{\frac{1}{2}+iK}(p'z)\Big]K_{-ik}(iPz),
\end{aligned} \tag{224}$$

$$\begin{aligned}
K_{\nu_+}(-ipz)K_{\nu_-}(ip'z)K_{-ik}(iPz) &= \frac{\pi^2}{4\cosh^2(\pi K)} \\
&\times\Big[-iJ_{-\frac{1}{2}-iK}(-pz)J_{-\frac{1}{2}+iK}(p'z) \\
&-e^{\pi k}J_{-\frac{1}{2}-iK}(-pz)J_{\frac{1}{2}-iK}(p'z) - e^{-\pi k} \\
&\times J_{\frac{1}{2}+iK}(-pz)J_{-\frac{1}{2}+iK}(p'z) \\
&+iJ_{\frac{1}{2}+iK}(-pz)J_{\frac{1}{2}-iK}(p'z)\Big]K_{-ik}(iPz).
\end{aligned} \tag{225}$$

The definition of Appel hypergeometric function is given bellow [41]:

$$F_4(a,b,c,d;x,y) = \sum_{m,n=0}^{\infty} \times \frac{\Gamma(a+m+n)\Gamma(b+m+n)\Gamma(c)\Gamma(d)x^m \cdot y^n}{\Gamma(a)\Gamma(b)\Gamma(c+m)\Gamma(d+n)m!\cdot n!}. \tag{226}$$

In our computation we use the following relations that connect the Hankel functions with Bessel J, K functions:

$$H_\mu^{(1)}(z) = \frac{J_{-\mu}(z) - e^{-i\pi\mu}J_\mu(z)}{i\sin(\pi\mu)} \tag{227}$$

$$H_\mu^{(2)}(z) = \frac{e^{i\pi\mu}J_\mu(z) - J_{-\mu}(z)}{i\sin(\pi\mu)}. \tag{228}$$

$$H_\nu^{(1,2)}(z) = \mp\left(\frac{2i}{\pi}\right)e^{\mp i\pi\nu/2}K_\nu(\mp iz), \tag{229}$$

The integrals that help us to compute the transition amplitudes are:

$$\int_0^\infty dz\cdot z^{\lambda-1}J_\mu(az)J_\nu(bz)K_\rho(cz) = \frac{2^{\lambda-2}a^\mu b^\nu c^{-\lambda-\mu-\nu}}{\Gamma(1+\mu)\Gamma(1+\nu)} \times\Gamma\left(\frac{\lambda+\mu+\nu-\rho}{2}\right)\Gamma\left(\frac{\lambda+\mu+\nu+\rho}{2}\right) \times F_4\left(\frac{\lambda+\mu+\nu-\rho}{2}, \frac{\lambda+\mu+\nu+\rho}{2}; 1+\mu, 1+\nu; -\frac{a^2}{c^2}, -\frac{b^2}{c^2}\right),$$
$$Re(\lambda+\mu+\nu) > Re(\rho),\ Re(c) > |Im(a)| + |Im(b)|. \tag{230}$$

$$\int_0^\infty dz z^{\mu-1}e^{-\alpha z}K_\nu(\beta z) = \frac{\sqrt{\pi}(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma\left(\mu+\frac{1}{2}\right)} \times {}_2F_1\left(\mu+\nu, \nu+\frac{1}{2}; \mu+\frac{1}{2}; \frac{\alpha-\beta}{\alpha+\beta}\right),$$
$$Re(\alpha+\beta) > 0, |Re(\mu)| > |Re(\nu)|. \tag{231}$$

References

1. A. Proca, J. Phys. Radium **7**, 347–353 (1936)
2. A. Proca, C. R. Acad. Sci. Paris **202**, 1366 (1936)
3. A. Proca, C. R. Acad. Sci. Paris **202**, 1490 (1936)
4. A. Proca, J. Phys. Radium **9**, 61 (1938)
5. I.I. Cotăescu, Gen. Relativ. Gravit. **42**, 861–876 (2010)
6. N.C. Tsamis, R.P. Woodard, J. Math. Phys. **48**, 052306 (2007)
7. C. Rubbia, Rev. Mod. Phys. **57**, 699 (1985)
8. S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995)
9. N.D. Birrel, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982)
10. N.D. Birrel, P.C.W. Davies, L.H. Ford, J. Phys. A **13**, 961 (1980)
11. E. Schrödinger, Physica **6**, 899 (1939)
12. L. Parker, Phys. Rev. Lett. **21**, 562 (1968)
13. L. Parker, Phys. Rev. **183**, 1057 (1969)
14. C. Cosmin, Phys. Rev. D **85**, 084036 (2012)
15. C. Crucean, Eur. Phys. J. C **79**, 483 (2019)
16. C. Crucean, M.A. Băloi, Phys. Rev. D **93**, 044070 (2016)
17. D. Dumitrele, M.A. Băloi, C. Crucean, Eur. Phys. J. C **83**, 738 (2023)
18. C. Crucean, A.D. Fodor, Eur. Phys. J. C **83**, 929 (2023)
19. T. Prokopec, N.C. Tsamis, R.P. Woodard, Ann. Phys. **323**, 1324–1360 (2008)
20. J.F. Koksma, T. Prokopec, Class. Quantum Gravity **26**, 125003 (2009)
21. P. Candelas, D.J. Raine, Phys. Rev. D **12**, 965 (1975)
22. I.I. Cotăescu, Eur. Phys. J. C **78**, 769 (2018)
23. I.I. Cotăescu, D. Popescu, Chin. Phys. C **44** (2020)
24. M. Tanabashi et al. (Particle Data Group), Rev. Part. Phys. Phys. Rev. D **98**, 030001 (2018)
25. J. Lankinen, I. Vilja, Phys. Rev. D **96**, 105026–1 (2017)
26. I.I. Cotăescu, C. Crucean, Phys. Rev. D **87**, 044016 (2013)
27. M.A. Băloi, C. Crucean, Int. J. Mod. Phys. A **32**, 1750208 (2017)
28. M.A. Băloi, C. Crucean, D. Popescu, Eur. Phys. J. C **78**, 395 (2018)
29. M.E. Fisher, K.G. Wilson, Phys. Rev. Lett. **28**, 240 (1972)
30. K.G. Wilson, Phys. Rev. D **7**, 2911 (1973)
31. G. ' t Hooft, M. Veltman, Nucl. Phys. B **44**, 189 (1972)
32. C.G. Bollini, J.J. Giambiagi, Nuovo Cimento B **12**, 20 (1972)
33. G. Hooft, Nucl. Phys. B **61**, 455 (1973)
34. W. Pauli, F. Villars, Rev. Mod. Phys. **21**, 434 (1949)
35. S. Drell, J.D. Bjorken, *Relativistic Quantum Fields* (Mc Graw-Hill Book Co., New York, 1965)
36. L. Landau, E.M. Lifsit, *Theorie Quantique Relativiste* (Mir Moscou, 1972)
37. M.A. Băloi, C. Crucean, D. Dumitrele, Particles **7**, 275 (2024)
38. C.W. Misner, K.S. Thorne, J.A. Wheleer, *Gravitation* (W. H. Freeman and Company, New York, 1973)
39. I.I. Cotăescu, Phys. Rev. D **65**, 084008 (2002)
40. M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1964)
41. I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 2007)
42. W.N. Cottingham, D.A. Greenwood, *An Introduction to the Standard Model of Particle Physics* (Cambridge University Press, Cambridge, 2007)
43. C. Cosmin, Mod. Phys. Lett. A **25**, 1679 (2010)
44. S.K.H. Auluck, Math. J. **14** (2012)
45. I.I. Cotăescu, Eur. Phys. J. C **80**, 535 (2020)