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OBSERVATIONAL ASPECTS OF CRITICAL QCD*

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A systematic search for observables associated with the critical sector of QCD is attempted in view of the progress made recently in lattice QCD regarding the existence and the location of a critical endpoint in the phase diagram of the theory. Our search is necessarily oriented towards multiparticle dynamics in collisions of nuclei and in particular towards the σ-mode in multipion production where critical fluctuations are expected to occur. The predictions of critical QCD are incorporated in a Monte Carlo simulation of critical events and the domain in the vicinity of the endpoint, where critical fluctuations prevail, is examined on the basis of the Ginzburg criterion. The relevance of our results for measurements in the experiments with nuclei is also discussed.

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1. Introduction

The existence of a critical point of second order in strongly interacting matter at high temperatures, is a fundamental property of QCD in close association with chiral phase transition of the vacuum [1]. The latest results of lattice QCD give the location of the critical point in the phase diagram, close to the area accessible by experiments with nuclei at the CERN/SPS, namely: $T_{\rm c} \approx 162 \pm 2\,{\rm MeV}$, $\mu_{\rm c} \approx 360 \pm 40\,{\rm MeV}$ [2]. This result is the improved version in a series of solutions in lattice QCD [2,3] and corresponds to realistic values of the quark masses (m_u, m_d). Increasing the masses of the light quarks, the chemical potential $\mu_{\rm c}$ increases as well in these solutions but the temperature $T_{\rm c}$ remains almost constant. We may, therefore, conjecture that in the final solution, which requires the continuum limit, the critical temperature is not going to change substantially even if the critical chemical potential ends up to a smaller value. In what follows we fix, for simplicity,

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the critical temperature taking the upper bound of the three lattice QCD solutions, $T_c = 165\,\mathrm{MeV}$ and restrict, therefore, the problem of localisation of the critical point along the direction of fixed temperature, $T = T_c$, in the phase diagram.

The basic ingredients in our approach are (a) the effective action of the sigma field in the vicinity of the critical point, (b) the mass of the sigma field $m_\sigma(T, \mu_c)$ in thermal environment, along the direction $\mu = \mu_c$ in the phase diagram and (c) the Ginzburg criterion [4,5] applied to critical QCD. With these ingredients one may build up a theory of critical fluctuations for the sigma field, $(\delta\sigma)^2 \approx \langle\sigma^2\rangle$, translate it to a theory of density fluctuations of isoscalars in momentum space and determine the region in the neighbourhood of the critical point beyond which these fluctuations die out (critical region). Our main objective, in what follows, is to extract the observational aspects of this scheme in the framework of heavy-ion physics.

In Sec. 2 we formulate the theory of critical fluctuations on the basis of the effective action of the σ-field with the constraints of the universality class of the critical point ($3d$ Ising system). With arguments inspired by the Ginzburg criterion for critical systems the domain of the phase diagram which may accommodate critical fluctuations is examined in association with the location of freeze-out points of a number of processes in experiments at the CERN/SPS or AGS [6].

In Sec. 3 a class of observables associated with the development of critical fluctuations in the sigma mode, is discussed. We have restricted ourselves to the reconstruction of sigmas in the environment of pions ($\sigma \to \pi^+\pi^-$) created in collisions of nuclei. The ideal environment for studying fluctuations of a zero mass σ-field would be the system of pairs of collinear photons, produced in the electromagnetic mode $\sigma \to \gamma\gamma$, but taking into account the degree of difficulty of the experimental measurements in these two cases, certainly the study of $\pi^+\pi^-$ pairs near the two-pion threshold is of higher feasibility and priority.

2. The effective action and the Ginzburg domain

Critical QCD as a thermal theory near a critical point is consistently described by an effective action written in terms of a scalar field (σ-field) in medium, with mass m_σ dependent on the temperature and baryon density (or chemical potential) of the environment created in the extreme conditions of central collisions of nuclei at high energies. At the critical point ($T = T_c$, $\mu = \mu_c$) the mass of sigma vanishes, $m_\sigma = 0$, and increases towards its physical value as the system expands and cools. Critical QCD belongs to the $3d$-Ising universality class [7] and the corresponding effective action $\Gamma[\sigma]$ is written, in the vicinity of the critical point, as follows:

$$\begin{aligned}
\Gamma[\sigma] &= \Gamma_{\rm c}[\sigma] + \Gamma_0[\sigma]\,, \\
\Gamma_{\rm c}[\sigma] &= T_{\rm c}^{-1}\int d^3\vec{x}\left[\tfrac{1}{2}(\nabla\sigma)^2 + G\,T_{\rm c}^4(T_{\rm c}^{-1}\sigma)^{\delta+1}\right]\,, \\
\Gamma_0[\sigma] &= T_{\rm c}^{-1}\int d^3\vec{x}\left[m_\sigma^2\sigma^2 + g_4 m_\sigma T_{\rm c}^{-1}\sigma^4\right]\,. \qquad (1)
\end{aligned}$$

For simplicity, we have omitted in Eqs. (1) a small term proportional to the quark mass which plays the role of a weak magnetic field in the analogue Ising system. The first component $\Gamma_{\rm c}$ is a singular term which survives at the critical point ($m_\sigma = 0$) whereas the second, Γ_0, is a smooth correction as we depart from the critical point ($m_\sigma \neq 0$). The exponent δ is the isotherm critical exponent of the Ising universality class ($\delta \approx 5$) and the dimensionless constants (G, g_4) are also fixed by a universal effective potential [8] in this class ($G \approx 2,\ g_4 \approx 1$).

An important observation in the theory (1) is that it introduces two independent, characteristic length scales in the description of the system: $T_{\rm c}^{-1}$ and $m_\sigma^{-1}(T,\mu)$. The first is a conventional scale of strong interactions ($T_{\rm c}^{-1}$ is of the order of 1 fm); it defines a finite correlation length, independent of temperature or density, and has no association whatsoever with the critical fluctuations of the system. On the contrary, the second scale, m_σ^{-1}, depends on the temperature and chemical potential, and defines a correlation length which diverges at the critical point of the system. Comparing these two scales (Ginzburg criterion) one may define a region in the phase diagram where critical fluctuations prevail. This important area (Ginzburg domain) is specified by the inequality: $m_\sigma(T,\mu) \leq T_{\rm c}$, which guarantees that inside the Ginzburg domain the critical correlation length m_σ^{-1} dominates and creates fluctuations at all scales when the system comes close to the critical point ($m_\sigma \to 0$). Outside this domain ($m_\sigma > T_{\rm c}$) the system develops conventional correlations with a finite correlation length of the order of $T_{\rm c}^{-1}$.

In order to determine the critical region in the phase diagram and ask for phenomenological implications one needs a theory for the σ-field in medium and in particular a knowledge of its mass as a function of temperature and chemical potential. In the absence of such a detailed theory we shall follow a simplified approach restricting our discussion along the line of constant chemical potential, $\mu = \mu_{\rm c}$, where scaling arguments lead to the following parametrisation [9]:

$$m_\sigma(T,\mu_{\rm c}) \approx m_\sigma^*\left[1 - \left(\frac{T}{T_{\rm c}}\right)^2\right]^{\nu/\beta\delta}\,, \qquad (2)$$

where (ν, β, δ) are the Ising critical exponents $(\nu \approx 2/3, \beta \approx 1/3, \delta \approx 5)$ and m^*_σ the physical mass of sigma at zero temperature $(m^*_\sigma \geq 400\,\mathrm{MeV})$. In this case the Ginzburg criterion leads to a critical domain $\delta T = T_\mathrm{c} - T$

$$\frac{\delta T}{T_\mathrm{c}} = 1 - \left[1 - \left(\frac{T_\mathrm{c}}{m^*_\sigma}\right)^{\frac{\beta\delta}{\nu}}\right]^{1/2}, \tag{3}$$

which, with the conservative choice $m^*_\sigma \approx 400\,\mathrm{MeV}$ and the upper bound of the lattice values, $T_\mathrm{c} \approx 165\,\mathrm{MeV}$, defines a narrow strip in the phase diagram (Fig. 1) the width of which is $\delta T \approx 8\,\mathrm{MeV}$. Inside this domain, critical fluctuations are expected to get developed, in small areas close to the critical point, the location of which is not fixed but it is restricted along the upper boundary of the strip $(T = T_\mathrm{c})$. In the same figure the three lattice solutions for the critical point are shown together with the freeze-out points of a number of processes, extracted recently from measurements at the SPS and AGS in a systematic study of chemical equilibrium in nucleus–nucleus collisions [6]. We observe that the cluster of systems of different size (PbPb, SiSi, CC) and approximately the same chemical potential, $(\mu_b \approx 245\,\mathrm{MeV})$ corresponding to collision energy $158\,\mathrm{GeV}/c$, obeys the Ginzburg criterion. It is also suggested that the critical point is likely to be located in the neighbourhood of the smaller systems (CC, SiSi) in the cluster, namely near the point: $T_\mathrm{c} \approx 165\,\mathrm{MeV}$, $\mu_\mathrm{c} \approx 245\,\mathrm{MeV}$ in the phase diagram. We also expect that the strength of critical fluctuations diminishes as we move from the smaller systems (CC, SiSi) to the largest one (PbPb). In particular the freeze-out point of the system PbPb lies close to the borderline of the Ginzburg domain and, therefore, the critical fluctuations in the process Pb+Pb at $158\,\mathrm{GeV}/c$ (CERN/SPS) must be rather weak.

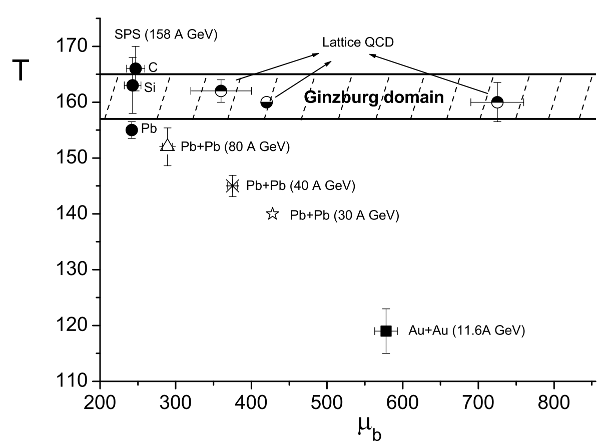

Fig. 1. QCD phase diagram.

Given the limitations of this approach and the simplifications in the construction of the Ginzburg domain, the above predictions cannot be conclusive. They provide us, however, with further guidance in the search for the QCD critical point, suggesting in particular that the final answer may come after systematic measurements of particle density fluctuations in the σ-mode within the framework of an observational theory of critical fluctuations. In the next section we briefly describe how to build up such a theory and which are the prospects to reveal, with its help, critical phenomena in collisions of nuclei and verify the formation of a Ginzburg domain in the phase diagram of QCD matter.

3. Critical fluctuations of QCD matter

When QCD matter becomes critical its properties are specified by the term $\Gamma_{\rm c}[\sigma]$ in Eqs. (1). The system undergoes a second-order phase transition and the prevailing phenomenon in the critical state is the development of self similar density fluctuations in the σ-mode. In what follows we consider cylindrical geometry in the collision and restrict ourselves in boost-invariant configurations of the σ-field along the rapidity axis. We end up with a two-dimensional system described by the corresponding effective action [10]:

$$\Gamma_{\rm c}^{(2)}[\sigma] = \Delta \frac{\tau_{\rm c}}{T_{\rm c}} \int d^2\vec{x}_\perp \left[\tfrac{1}{2} |\nabla_\perp \sigma|^2 + G T_{\rm c}^4 \left(T_{\rm c}^{-1} \sigma \right)^{\delta+1} \right] , \qquad (4)$$

where Δ is the total rapidity gap available in the collision and $\tau_{\rm c}$ the proper time scale of the critical system. The fluctuations of the σ-field, $(\delta\sigma)^2 \approx \langle \sigma^2 \rangle$, at the critical point, are measured by the density–density correlations of σ-particles with respect to an arbitrary origin, $\langle \sigma^2(\vec{x}_\perp) \rangle = \langle \rho(\vec{x}_\perp)\rho(0) \rangle$. Therefore, dynamical fluctuations of the σ-field at large scales (critical fluctuations) manifest themselves as density fluctuations of sigma mesons at small scales in momentum space, leading to a measurable intermittency effect [11]. This physical picture can be formulated by solving approximately the theory, namely, summing over the dominant saddle points of the effective action (4) and writing the partition function as follows:

$$Z = \sum_\sigma e^{-\Gamma_{\rm c}^{(2)}[\sigma]} , \qquad \delta \Gamma_{\rm c}^{(2)}[\sigma] = 0 . \qquad (5)$$

The solution of Eqs. (5) leads to fractal structures in transverse space [10]

$$\langle \rho(\vec{x}_\perp)\rho(0) \rangle \sim |\vec{x}_\perp|^{-\frac{4}{\delta+1}} , \qquad d_{\rm F}^{(2)} = \frac{2(\delta-1)}{\delta+1} , \qquad (6)$$

within σ-clusters of maximal size: $\xi_\perp \approx \frac{\pi^{3/2}}{32} R_\perp$ (correlation length) where $R_\perp$ is the transverse radius of the critical system, formed by the excitation of the vacuum in nucleus–nucleus collisions at high energies. In momentum space, the solution (6) leads to a similar fractal geometry but with different dimension:

$$\langle \rho(\vec{p}_\perp)\rho(0)\rangle \sim |\vec{p}_\perp|^{-\frac{2(\delta-1)}{\delta+1}}, \qquad \tilde{d}_{\mathrm{F}}^{(2)} = \frac{4}{\delta+1}. \tag{7}$$

Eq. (7) implies a strong intermittency effect in the σ-mode, revealed by a power-law behaviour of the factorial moments: $F_2(M) \sim (M^2)^{\frac{\delta-1}{\delta+1}}$, where M^{-2} measures the size of a $2d$ cell in momentum space over which the correlation function (7) is integrated [10].

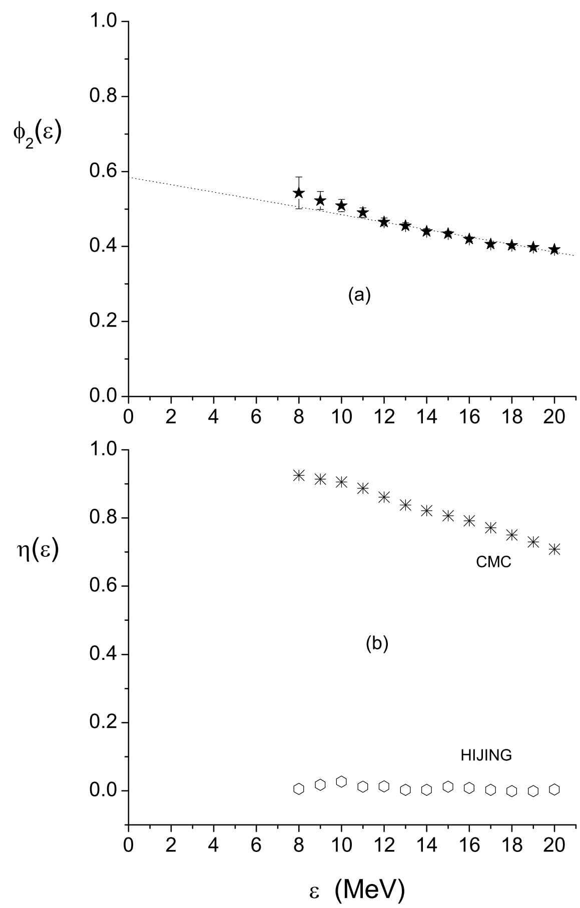

Fig. 2. The functions (a) $\phi_2(\varepsilon)$ and (b) $\eta(\varepsilon)$ for CMC events.

With these ingredients it is possible to develop a Monte Carlo generator (CMC) simulating the critical σ-sector as well as the pions produced by the decay of the sigmas during the freeze-out. The properties of the CMC events can be used as a guide for the formulation of a self-consistent algorithm revealing the critical geometry through the reconstruction of the sigma's momenta from the measurable pion momenta [10]. All possible pairs of pions of opposite charge are formed for each event and subsequently filtered out only those with invariant mass lying within a very small kinematical window ε above $2m_\pi$. The background introduced by combinatorics leading to fake sigmas can be simulated through mixed events and subtracted out using the correlator G_2 instead of the factorial moment F_2 [12]. It is straightforward to show that the fractal pattern in the sigma momenta leads to a power-law behaviour of the correlator as a function of the resolution scale M: $G_2(M,\varepsilon) \sim M^{2\phi_2(\varepsilon)}$ with the condition $\phi_2(0) = 1 - \tilde{d}_{\rm F}^{(2)}/2$. In a similar way it is possible to calculate the percentage of real sigmas $\eta(\varepsilon)$ in the reconstructed sector. In Figs. 2(a), (b) we present the functions $\phi_2(\varepsilon)$ and $\eta(\varepsilon)$ for a set of 10^5 CMC events with parameters adapted to the SPS $C+C$-system. These two quantities offer the most suitable observables to characterise and reveal the formation of a critical sigma sector in $A+A$ collisions.

4. Conclusions

We have discussed the characteristic features of an observational theory of critical fluctuations in QCD matter and have examined its applicability to heavy-ion physics. We have argued that severe restrictions on the critical region, in the phase diagram of QCD, are imposed on general grounds (Ginzburg criterion, universality class of the critical point, critical temperature in lattice QCD, lower bound of the physical mass of sigma) providing us with an extra guideline in the search for the QCD critical point. The theory of critical fluctuations is incorporated in a Monte Carlo code (CMC) and a method how to single out, in the reconstructed σ-mode, the genuine critical effect, in the form of a power law, is proposed.

Preliminary studies of chemical equilibrium in nucleus–nucleus collisions [6] shows that the freeze-out points of the collisions Pb+Pb, C+C, Si+Si at 158 GeV/c are located, within errors, inside the Ginzburg domain, at the same chemical potential, suggesting for the critical point, the physical values: $T_{\rm c} \approx 165\,{\rm MeV}$, $\mu_{\rm c} \approx 245\,{\rm MeV}$. From our point of view, in order to verify this very preliminary suggestion, one has to pursue systematically the search for critical fluctuations, in the spirit of the proposed theory, analysing the data of NA49 experiment for PbPb, CC, SiSi collisions at 158 GeV/c. Work along this direction is now in progress.

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