

Relativistic radial expansion: do we need dark energy?

N. Ben-Amots

P.O.Box 3193, Haifa 31032, Israel

E-mail: benamots@alumni.technion.ac.il

Abstract. The Hubble law, for which the radial expansion velocity is linear in the distance r , implies a visible Universe limited to a horizon at $r = c/H$. No information beyond this horizon can reach us.

Here we suggest a modified Hubble law for the *measured* radial expansion velocity that does not put a limit on the radius of the visible Universe. The modified Hubble law is based on Einstein's well known addition of relativistic velocities, and differs from the (conventional linear) Hubble law in orders of Hr/c that are higher than the first order. The modified Hubble law solves the Olbers's paradox (as the linear Hubble law does), and the existence of dark energy becomes unnecessary for explaining existing astronomical data on dependence of luminosity as a function of redshift for type Ia supernovas.

1. Introduction

The linear Hubble law (in some references referred to as the Hubble-Humason law) implies a horizon. See, for example, Rindler [1] and others [2-6]. Ellis and Rothman [5] named their paper on the subject of cosmological horizons: "*Lost horizon.*"

Here we present a modified Hubble's law so that no horizon exists or is necessary.

Our formulas indicate that Perlmutter's interpretation of data of luminosity versus redshift of type Ia supernovas as demanding an accelerating expanding Universe is not correct. Analyzing the data with the modified Hubble law, shows that no cosmic acceleration is present, so that invoking Dark Energy is superfluous.

2. Horizon in radial expansion

In an expanding Universe with the linear Hubble law

$$v = Hr \tag{1}$$

we get a velocity greater than the speed of light when

$$r > \frac{c}{H} \tag{2}$$

We may interpret $\frac{c}{H}$ as the horizon of the Universe, because if Hubble's law (1) is accurate, no communication is possible between us and regions beyond this limiting radius. This horizon is referred as the "Hubble radius."

3. Integration

Special relativity gives the formula for adding relativistic velocities

$$v = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2} \quad (3)$$

where v_2 is the velocity of a point 2 in an inertial system moving with a velocity v_1 relative to another inertial system. Velocity v is the velocity of the point 2 relative to the other system. This formula is symmetric in v_1, v_2 and is valid also if v_2 is the velocity of a point in an inertial system moving with a velocity v_1 relative to the other system.

This formula ensures that the velocity does not exceed the speed of light c , and is derived from this assumption.

Equation (3) may be written as

$$v_2 = \frac{v - v_1}{1 - v v_1 / c^2} \quad (4)$$

where v_2 is again the relative velocity between two systems 2 and 1 as defined above.

Using (4) and the radial expansion $v = Hr$ as in (5), the relative velocity between a point at a distance $r + \Delta r$ from us, and a point at a distance r from us on the same line of sight is:

$$H\Delta r = \frac{v(r + \Delta r) - v(r)}{1 - v(r + \Delta r)v(r)/c^2} \quad (5)$$

Going to infinitesimal $\Delta r \rightarrow dr$

$$H dr = \frac{dv}{1 - v^2/c^2} \quad (6)$$

that on integration leads to

$$\frac{v}{c} = \tanh \frac{Hr}{c} \quad (7)$$

or

$$v(r) = c \tanh \frac{Hr}{c} \quad (8)$$

which we suggest as a modified Hubble law for the *measured* radial expansion velocity.

Equation (8) can be written:

$$r(v) = \frac{c}{H} \operatorname{arctanh} \frac{v}{c} \quad (9)$$

4. Radial expansion

To enable $r > c/H$, without exceeding the speed of light, we present modified non-linear possibility for the linear Hubble law:

We showed above that applying the relativistic addition of velocities (3) on the linear radial expansion (1) leads to the relativistic formula of relative velocity between distant parts of the Universe (8).

Expansion of (8) results in:

$$v = Hr \left[1 - \frac{1}{3} \left(\frac{Hr}{c} \right)^2 + \frac{2}{15} \left(\frac{Hr}{c} \right)^4 - \dots \right] \quad (10)$$

Keeping the first term, we receive for equation (8) the linear Hubble law (1), which is valid in the view only for relatively small distances, before the correction terms in (10) become significant.

Yet for (8) when r approaches infinity, the *measured* velocity v approaches the velocity of light c , without exceeding it.

For (8) we can write a corresponding no-horizon line element:

$$ds^2 = \frac{c^2 dt^2}{\cosh^2 \frac{Hr}{c}} - dr^2 \cosh^2 \frac{Hr}{c} - r^2 (d\theta^2 + d\phi^2 \sin^2 \theta) \quad (11)$$

5. Distances and redshift

The formula for the redshift of special relativity is

$$z = \sqrt{\frac{c+v}{c-v}} - 1 \quad (12)$$

or

$$v = c \frac{z + \frac{1}{2}z^2}{1 + z + \frac{1}{2}z^2} \quad (13)$$

We see that each of the equations (12) and (13) imply that $v < c$ for any z .

Hubble's law (1) can be joined with (13) and we receive for the conventional linear Hubble law

$$r = \frac{c}{H} \frac{z + \frac{1}{2}z^2}{1 + z + \frac{1}{2}z^2} \quad (14)$$

that is, $r < \frac{c}{H}$ for any z .

We can substitute (13) in the modified Hubble's law (8) and obtain

$$r = \frac{c}{H} \operatorname{arctanh} \left(\frac{z + \frac{1}{2}z^2}{1 + z + \frac{1}{2}z^2} \right) \quad (15)$$

in which r is greater than in (14), thus for each z , each z star is further away from us compared with the distance given by the linear Hubble law (1). If the linear Hubble law (1) is an approximation of the modified Hubble law (8), then the conventional distance (14) is an approximation to our distance (15).

Substituting the linear Hubble law (1) in (12) gives:

$$z = \sqrt{\frac{c+Hr}{c-Hr}} - 1 \quad (16)$$

Substituting the modified Hubble law (8) in (12) gives:

$$z = \sqrt{\frac{1 + \tanh \frac{Hr}{c}}{1 - \tanh \frac{Hr}{c}}} - 1 \quad (17)$$

or:

$$z = \exp \left(\frac{Hr}{c} \right) - 1 \quad (18)$$

from which one gets:

$$r = \frac{c}{H} \ln(z + 1) \quad (19)$$

which is a simpler expression for (15).

6. Olbers’s paradox

Halley [7-8], Cheseaux [9] and Olbers [10] asked why the night sky is dark, although the light of the stars and galaxies should accumulate to a very bright sky even at night. This question is called Olbers’s paradox [11-20].

Citing Weinberg [12] pp. 611-612:

”... if the absorption is neglected, the apparent luminosity of a star of absolute luminosity B at a distance r in a naive cosmological model will be $\frac{B}{4\pi r^2}$. If the number density of such stars is a constant n , then the number of stars at distances between r and $r + dr$ is $4\pi nr^2 dr$, so the total radiant energy density due to all stars is

$$\int_0^\infty \frac{B}{4\pi r^2} 4\pi nr^2 dr = Bn \int_0^\infty dr = Bnr \Big|_0^\infty$$

Weinberg number : (16.1.1) our number :

(20)

The integral diverges, leading to an infinite energy density of starlight!”

Supposing that the universe is homogenous having a constant density n of stars of absolute luminosity B , the light reaching us from a spherical layer of the universe of thickness dr at a distance r from us is

$$4\pi r^2 n \frac{B}{4\pi r^2} dr$$
(21)

When integrated for an infinite universe, that is infinite r , the integral of (21) diverges, meaning infinitely bright sky, which is not observed. This is the well known Olbers’s paradox.

Today two accepted answers try to solve this paradox:

- A. The age of the Universe is finite, so that light from distant parts of the Universe more remote than a certain horizon has not yet reached us. Thus the light that we see is an integral of the light we see up to a certain distance.
- B. The Universe is expanding according to Hubble’s law, so that the light of distant galaxies is red-shifted. The integrated luminosity of the shifted light up to an infinite distance is finite.

We derive the following proof for the resolution of Olbers’s paradox based on B above only. The relativistic formula for the shift of frequencies is

$$\frac{f'}{f} = \sqrt{\frac{c - v}{c + v}}$$
(22)

Substituting the linear Hubble law (1) in (22) we obtain:

$$\frac{f'}{f} = \sqrt{\frac{c - Hr}{c + Hr}}$$
(23)

Checking for Olbers’s paradox integrated radiant energy density we receive the integral

$$\int_0^{\frac{c}{H}} Bn \sqrt{\frac{c - Hr}{c + Hr}} dr$$
(24)

which is finite.

Now we do the same for the modified Hubble law above. Substituting the modified Hubble law (8) in (22) we obtain

$$\frac{f'}{f} = \sqrt{\frac{1 - \tanh(Hr/c)}{1 + \tanh(Hr/c)}}$$
(25)

or

$$\frac{f'}{f} = \exp\left(\frac{-Hr}{c}\right) \tag{26}$$

The above energy density integral for this case is:

$$\int_0^\infty Bn \exp\left(\frac{-Hr}{c}\right) dr = \frac{c}{H} Bn \tag{27}$$

The integrals (24), (27), are finite. Thus the amount of light in the sky is finite for the modified Hubble law, so the night sky is not infinitely bright, and Olbers's paradox is solved for both the linear Hubble law (1), and for the modified Hubble law (8).

7. The "dark energy" hypothesis

Observations using Perlmutter et al.'s and Riess et al.'s method to measure type Ia supernova distances (Perlmutter et al. [21], and Riess et al. [22]) were interpreted as if the expansion of the universe is accelerating. See refs. 22-26.

An acceleration in the expansion of the universe came as a big surprise. It was interpreted as if some repelling force is acting against the gravitational attraction. This assumed force was associated with a further assumed "dark energy."

Perlmutter et al. [21] estimated distance according to the observed luminosity of remote type Ia supernovas, whose absolute peak luminosity is known. The corresponding redshift was the redshift of the galaxies that host these supernovas. Remote type Ia supernovas were found fainter than expected according to the value calculated with the linear Hubble's Law (1), using the corresponding redshift, thus Perlmutter et al. and others assumed that there is an acceleration in the universe expansion.

To explain the force necessary to accelerate the expansion of space dragging all the galaxies in the Universe, a "dark energy" in the Universe causing this acceleration was assumed.

Here we give a different interpretation to the observations, showing that if the observed redshift of remote galaxies is indeed originated in Doppler effect of radial velocity of expansion, then the acceleration of the expansion of the universe does not exist and is only a misinterpretation of data.

According to the modified Hubble law (Equation (8)), the measured expansion velocity deduced from z at a certain distance r is *less* than expansion velocity Hr according to the *linear* Hubble's Law (1). So, the distance r calculated from our equation (8) is *larger* than the distance r calculated from the linear Hubble law (1). This larger distance explains the observed fainter luminosity of type Ia supernovas. Thus, the expanding universe did not accelerate, and no dark energy is necessary to explain the observations.

We conclude that the expanding universe is not accelerated, hence there is no accelerating force, and no "dark energy" is needed to explain the (non existing) acceleration of the expansion.

7. Observed luminosity and magnitude

The observed luminosity L of a celestial object is proportional to the reciprocal of its distance r from us:

$$L = \frac{kB}{r^2} \tag{28}$$

Substituting r of (14) (derived from the linear Hubble Law (1)) in Eq. (28) results in observed luminosity:

$$L = kB \left(\frac{H}{c}\right)^2 \left(\frac{1+z+\frac{1}{2}z^2}{z+\frac{1}{2}z^2}\right)^2 \tag{29}$$

or:

$$L = kB \left(\frac{H}{c} \right)^2 \left(1 + \frac{1}{z + \frac{1}{2}z^2} \right)^2 \quad (30)$$

Substituting r of (19) (derived from the modified Hubble Law (8)) in (28) results in observed luminosity:

$$L = kB \left(\frac{H}{c \ln(1+z)} \right)^2 \quad (31)$$

that is fainter than the luminosity according (29) (or (30)).

The defined astronomical magnitude of a celestial body is larger when its luminosity is smaller. The magnitude of a celestial body whose luminosity is smaller hundred times than the luminosity of another celestial body, is defined to be *larger* by five.

In our case the relevant measurements are all of explosions of type Ia supernovas, which have the same absolute peak luminosity, thus the observed luminosity depends solely on their distance r from us. So, for a type Ia supernova distant ten times from another type Ia supernova, the observed luminosity will be hundred times weaker, or its magnitude will be larger by five:

$$\mu = \mu_0 + 5 \log_{10} \frac{r}{r_0} \quad (32)$$

where μ_0 and r_0 are the magnitude and the distance to a certain supernova of the type Ia, while μ and r are the magnitude and the distance to another supernova of type Ia.

The results of measurements are given by graphs plotting $\mu(z)$ of several type Ia supernovas. Substituting (14) in (32) gives the expected $\mu_{linear}(z)$ for the linear Hubble law (1) as:

$$\mu_{linear} = \mu_0 + 5 \log_{10} \left(\frac{c}{H r_0} \frac{z + \frac{1}{2}z^2}{1 + z + \frac{1}{2}z^2} \right) \quad (33)$$

Substituting (19) in (32) gives the expected $\mu(z)$ for the modified Hubble law (8) as:

$$\mu_{modified} = \mu_0 + 5 \log_{10} \left[\frac{c}{H r_0} \ln(z+1) \right] \quad (34)$$

Substituting (15) in (32) gives the expected $\mu_{modified}(z)$ for the modified Hubble law (8) as:

$$\mu_{modified} = \mu_0 + 5 \log_{10} \left[\frac{c}{H r_0} \operatorname{arctanh} \left(\frac{z + \frac{1}{2}z^2}{1 + z + \frac{1}{2}z^2} \right) \right] \quad (35)$$

which is equivalent to (34).

Comparing equation (35) with (33) we see that $\mu_{modified}(z)$ of (35) is larger than $\mu_{linear}(z)$ of (33) for any $z > 0$. Similarly, the accelerating expansion graph in Figure 1 of Perlmutter et al. [21] is higher than the non-accelerating expansion graph in Figure 1 of Perlmutter et al. [21]. Both Perlmutter's μ of accelerating expansion graph in his Figure 1, and our $\mu_{modified}(z)$ in our Figure 1 here, give better fitting to data being higher than non-accelerating expansion graph in Figure 1 of Perlmutter et al. [21].

Figures 1 and 2 present the expected magnitude μ of supernovas of type Ia versus \log_{10} of the redshift z , according the linear and the modified Hubble laws. Figure 1 presents the expected magnitude μ for redshift z in the range $z=0.01$ to $z=1$ (the range covered by refs. 21-22, 26), while Figure 2 shows the expected magnitude μ in the range $z=0.01$ to $z=10$ (covered partially by observations up to $z \approx 2$ in refs. 27-29). In both figures, the lower graph is the function

$$\mu = 25 + 5 \log_{10} \left(\frac{z + \frac{1}{2}z^2}{1 + z + \frac{1}{2}z^2} \right) \quad (36)$$

Figure 1: Magnitude μ versus \log_{10} of redshift z for z between 0.01 and 1

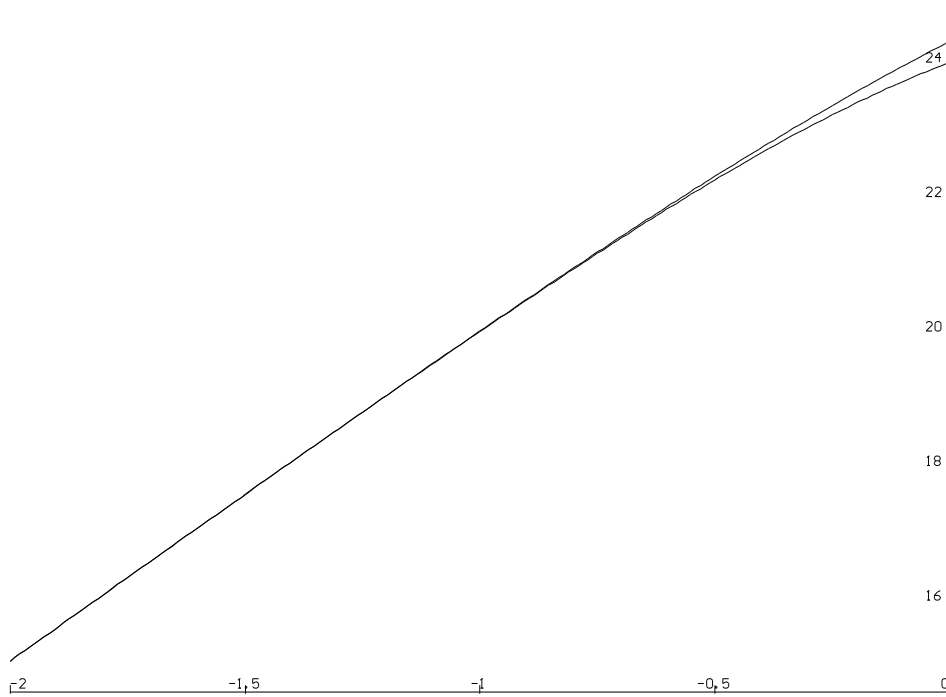
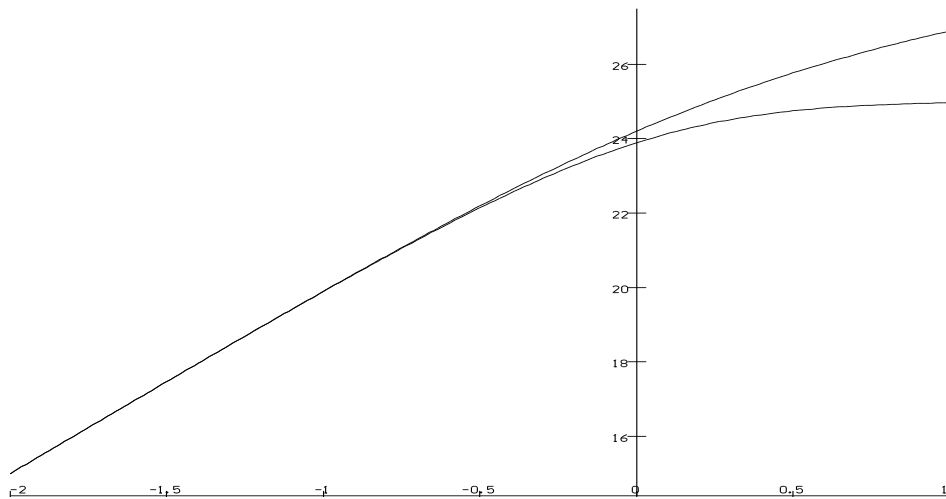


Figure 2: Magnitude μ versus \log_{10} of redshift z for z between 0.01 and 10



representing the expected magnitude according to the linear Hubble law (1), while the upper graph is the function

$$\mu = 25 + 5 \log_{10} [\ln(z + 1)] \tag{37}$$

representing the expected magnitude according to the modified Hubble law (8).

Comparing our Figure 1 to the graphs of $\mu(z)$ in Figure 1 by Perlmutter et al. [21] in their Figure 1 shows that the difference between our upper graph (μ for modified Hubble law (8)) and the lower graph (μ for linear Hubble law (1)) in Figure 1 here, is similar to the difference between the upper (μ for accelerating expansion) and the lower (μ for expansion without acceleration) graphs in Figure 1 of Perlmutter et al. [21].

Figure 2 shows significant difference of the predicted magnitude according the modified Hubble law (8) (the upper graph), comparing to the expected magnitude according to the linear Hubble law (1) (the lower graph), in the redshift range $z=0.01$ to $z=10$.

The modified Hubble law (8) implies that measurements of redshifts of $z=0.01$ to $z=10$ of galaxies hosting type Ia supernovas will fit the upper graph of Figure 2, which significantly goes downwards for large z compared to the lower graph calculated for the case of no-acceleration linear Hubble law (1). Bertram Schwarzschild [27] and Riess et al. [28] pointed out that apparent magnitude of supernova SN1997ff whose host galaxy (HDF 4-403.0) redshift z was measured as 1.7, is significantly downwards compared to the extrapolation of the graph of accelerating universe up to $z=1$. See their graphs in Figure 2 of Schwarzschild [27] and figures 11 and 12 by Riess et al. [28]. Later Riess et al. [29] observed downwards trend for experimental data curves for more type Ia supernovas at larger z . They explained [27-29] the downward trend by assuming that once the Universe decelerated, then started to accelerate. Yet our Figure 2 shows that this significant downwards departure is part of the solution for both the linear Hubble law (1) and the modified Hubble law (8), without needing decelerating and accelerating Universe. The reason for this downwards departure is that according to (12) the redshift z is not linearly proportional to the expansion velocity v .

Just in case that our considerations above are not able to explain the experimental measurements sufficiently accurate, so that acceleration of the expansion is still necessary for an explanation, below in Section 9 we derive formulas that consider *constant* acceleration *and* the modified Hubble law (8).

8. Radial acceleration

As can be shown (See for example Adler and Brehme [30]), for linear motion with *constant* acceleration a :

$$\frac{v}{c} = \tanh \frac{aT}{c} \tag{38}$$

where T is the proper time of the accelerated body in linear motion.

Integration gives (Adler and Brehme [30]):

$$X(t) = \frac{c^2}{a} \left[\cosh \left(\frac{aT}{c} \right) - 1 \right] \tag{39}$$

In (39) the velocity v is measured with the proper time T of a "traveler," yet X represents "our" measured distance.

Applying a similar method for *constant* acceleration a and expansion Hr similarly gives

$$\frac{v}{c} = \tanh \frac{aT + Hr}{c} \tag{40}$$

that represents modified Hubble Law that also considers possible *constant* linear acceleration a in the \vec{v} direction.

Formula (40) incorporates possible constant acceleration a as a free parameter. Substituting (19) in (40) gives:

$$v(z) = c \tanh \left[\frac{aT}{c} + \ln(z + 1) \right] \quad (41)$$

Formulas (38)-(41) are not for variable acceleration.

Taking advantage of these formulas that consider *constant* radial acceleration a , we may compare them to observations' results (Perlmutter et al. [21], Riess et al. [22, 28-29], Schwarzschild [27]) for finding the *constant* acceleration a that best fits the observations.

9. Summary

The linear Hubble law leads to a visible Universe limited in radius.

Here we suggested a modified Hubble law that does not limit the visible radius of the Universe, thus enabling a Universe without an horizon. The linear Hubble law may be viewed as an approximation of the modified Hubble law for small expansion velocity of relatively close galaxies.

We showed that the modified Hubble law, like the linear Hubble law, solves Olbers's paradox.

This paper shows that if the origin of Hubble's redshift of galaxies is the Doppler effect, (caused by the receding velocity of the galaxies), there exists a modified Hubble law that fits the present observations for small Hr/c – yet without exhibiting a horizon at large Hr/c . Further, we suggested explanation for Perlmutter et al.'s [21] and others' observations by the non-linear, modified Hubble's law (8), without needing dark energy.

The originally suggested Hubble law (1) is arbitrary, while the modified Hubble law (8) suggested above obeys the correct relativistic addition formula (3):

$$v = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2} \quad (3)$$

Thus we derived a new formula for the Hubble law, which contains the existing as a special case for small expansion velocity of relatively close galaxies.

So, using our formula (8)

- a) No horizon is needed,
- b) Olbers's paradox is explained by both Hubble formulas.
- c) We find that Perlmutter et al.'s and others' interpretation of accelerating expanding universe may be incorrect. We suggest that analyzing the phenomenon with the modified Hubble law, will find that no cosmic acceleration is present, so that also Dark Energy may be superfluous.

Acknowledgments

The author is indebted to Dan Igner, who helped find the appropriate wording for a clearer presentation. Thanks to Professor Larry Horwitz who read the manuscript, for his guiding questions.

References

- [1] Rindler N 1956 *Mon. Not. R. Astron. Soc.* **116** 662
- [2] Ellis G F R and Stoeger W 1988 *Class. Quant. Grav.* **5** 207
- [3] Berry M V 1989 *Principles of cosmology and gravitation*, (Bristol: Institute of Physics) pp 107-111
- [4] Harrison E 1991 *Astroph. J* **383** 60
- [5] Ellis G F R and Rothman T 1993 *Am. J. Phys.* **61** 883

- [6] Allday J 1998 *Quarks, leptons and the big bang*, (Bristol: Institute of Physics Publishing) pp 257-259
- [7] Halley E 1720 *Phil. Trans. Roy. Soc. (London)* **31** 22
- [8] Halley E 1720 *Phil. Trans. Roy. Soc. (London)* **31(364)**
- [9] de Cheseaux J P L 1744 *Traite de la comete*, (Lausanne:MM Bousquet) pp 223 ff; Reprinted in Dickson [11] Appendix II
- [10] Olbers H W M 1826 *Über die Durchsichtigkeit des Weltraums*, in: *Bode's Astronomisches Jahrbuch* **111** (Berlin: CFE Spaethen) pp 110-121. First English translation: *On the transparency of space*, *Edinburgh New Phil. J.* **1** 141 (1826). Reprinted in Dickson [11] Appendix I
- [11] Dickson F P 1968 *The bowl of night* (Cambridge, Mass.: M.I.T. Press) Appendixes I, II
- [12] Weinberg S 1972 *Gravitation and cosmology*, (New York: Wiley) See Chapter 16.1, pp 611-613
- [13] Harrison E R 1964 *Nature* **204** 271
- [14] Harrison E R 1974 *Phys. Today* **27** 30
- [15] Harrison E R 1977 *Am. J. Phys.* **45** 119
- [16] Harrison E R 1981 *Cosmology. The science of the Universe* (Cambridge, Mass.: Cambridge Univ. Press)
- [17] Harrison E 1984 *Science* **226** 941
- [18] Harrison E R 1986 *Nature* **322** 417
- [19] Harrison E 1987 *Darkness at night – a riddle of the Universe*, (Cambridge, Mass.: Harvard Univ. Press)
- [20] Arpino M and Sardigli F 2003 *Eur. J. Phys.* **24** 39
- [21] Perlmutter S et al. 1999 *Astrophys. J.* **517** 565
- [22] Riess A G et al. 1998 *Astron. J.* **116(3)** 1009
- [23] Glanz J 1998 *Science* **282** 2156
- [24] Hoggan C J et al. 1.1999 *Sci. Am.* **280** 46
- [25] Krauss L M 2.1999 *Sci. Am.* **280** 52
- [26] Perlmutter S 4.2003 *Phys. Today* **56(4)**
- [27] Schwarzschild B 6.2001 *Phys. Today* **54(6)** 17
- [28] Riess A 2001 *Astrophys. J.* **560** 49 (*Preprint astro-ph/0104455*)
- [29] Riess A G et al. 2004 *Astrophys. J.* **607** 665 (*Preprint astro-ph/0402512*)
- [30] Adler C G and Brehme R W 1991 *Am. J. Phys.* **59** 209. See Equation 7b.