THE INFRARED PROBLEM IN COLOR DYNAMICS

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ABSTRACT

Calculations of infrared singularities in non-Abelian (Yang-Mills) gauge theories and in particular Quantum Chromodynamics are reported in the leading-logarithm approximation. Possible applications to hadron physics are discussed such as 1) an explanation of the quark counting rules for exclusive fixed-angle cross sections, 2) an indication of an infrared mechanism for quark confinement and 3) a picture of hadronic matter at large interquark distances resulting from the assumption of very strong long range forces between colored quanta.

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INTRODUCTION

In recent years gauge theories have dominated theoretical thinking in particle physics and with good reason: they provided us with unified, renormalizable models of weak and electromagnetic interactions[1]; and they also furnished us with a model of strong interactions such as Quantum Chromodynamics (QCD) in which interactions between quarks become vanishingly weak at short distances[2] - an explanation for (approximate) scaling in deep inelastic experiments. Furthermore, it seems that gauge theories are the only field theories capable of performing these two feats[3] - an indication that perhaps this time we are on the right track.

The dynamics of non-Abelian (Yang-Mills) gauge theories with unbroken symmetry such as QCD is at present far from well understood. Only recently, for instance, have we come to realize that the structure of the vacuum state in YM theories is considerably more complex[4] than people ever had reason to suspect on the basis of their experience with ordinary field theories. Another serious complication is that the problem of infrared singularities is much more difficult to handle in YM Theories than in Abelian gauge theories like QED. In these lectures, I will describe work on the following three areas in which the infrared behavior of QCD has been examined in order to understand the physics of hadronic matter:

(1) Quark-counting scaling laws for exclusive cross sections at high energy and wide angle and form factors at large momentum transfers.

(2) A signal for quark confinement from infrared singularities.

(3) Hadronic matter for large interquark distances.

Sections 1 and 2 are based on work[5] done in collaboration with J.M.Cornwall at UCLA. The part of section 3 dealing with gluon chains is based on ref. 6.

All calculations and arguments presented here were based on the Lagrangian density of QCD:

$$- \frac{1}{4} F^{i}_{\mu\nu} F^{i\mu\nu} + \bar{q}(i\partial\!\!\!/ - ig B\!\!\!/^{j} d^{j}) q$$

which couples a set of vector gauge fields B^i (gluon fields) to a multiplet of fermion fields q (quark fields). The matrices d^j are the group generators in quark-field space and $F^i_{\mu\nu}$ is the gauge-covariant curl of the gauge field B^i_μ :

$$F^{i}_{\mu\nu} = \partial_\mu B^i_\nu - \partial_\nu B^i_\mu + g c_{ijk} B^j_\mu B^k_\nu$$

Here c_{ijk} are the structure constants of the gauge group i.e. SU(3) of color and the matrices d^j are normalized according to

$$\{d^i, d^j\} = i c_{ijk} d^k$$

The Lagrangian can accomodate any number of quark flavors n,d,s,c,... each of which appears as a color triplet.

1. ASYMPTOTIC FREEDOM IN HIGH-ENERGY WIDE-ANGLE EXCLUSIVE SCATTERING

A few years ago Brodsky and Farrar[7] and independently Matveev, Muradyan and Tavkhelidze[8] pointed out that the experiments of exclusive high-energy wide-angle hadronic scattering are compactly summarized by scaling laws of the form

$$\frac{d\sigma}{dt} \approx s^{2-N} f(t/s), \quad s \to \infty, \ s/t \text{ fixed} \qquad (1)$$

where s and t denote the squared c.m. energy and momentum transfer respectively and

N = total number of quarks of the initial and final hadrons according to the usual quark model assignments (meson $\sim q\bar{q}$, baryon $\sim qqq$)

Moreover, the (spin-averaged) electromagnetic form factor of a hadron for large momentum transfer t behaves like

$$F(t) \sim t^{1-n} \qquad (2)$$

where n is the number of quarks in the hadron. Thus, for example,

$$\frac{d\sigma}{dt} \propto s^{-8} \quad \text{for} \quad \pi p \to \pi p$$
$$\propto s^{-10} \quad \text{for} \quad pp \to pp$$

$$F_\pi \propto t^{-1}$$
$$F_p \propto t^{-2}$$

If these remarkable "quark counting" rules are true asymptotic laws for hadronic reactions and not just a fluke of the present data, they probably reflect some fundamental feature of the interactions between quarks at short distances. Can asymptotic freedom be invoked here ? Brodsky and Farrar noted that Eq. (1) implies that the relevant amplitude behaves like $s^{2-N/2}$ and this happens to be the behavior of any connected tree with N quark legs, provided the basic interaction between quarks is renormalizable with dimensionless coupling constants. It could be, for instance, of the Yukawa type $\bar{q}q\phi$ with ϕ some set of "scalar gluons" or an Abelian vector-gluon theory $\bar{q}\gamma_\mu q B_\mu$ or, of course, it could be QCD. The connected tree graph of Fig. 1 is an example of a contribution to meson-meson scattering which, at wide angle, behaves like s^{-2} (dotted lines are gluons)

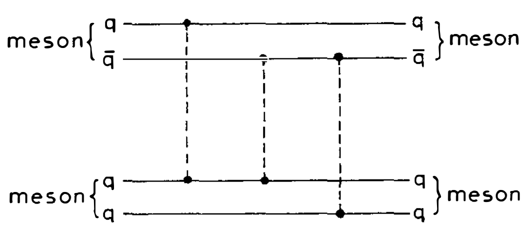

Fig. 1

To get the meson-meson amplitude this 8-quark amplitude must be appropriately convoluted by the Bethe-Salpeter wave functions for the mesons. But this will not change the s^{-2} behavior if the wave functions are sufficiently regular at short interquark distances. The same is true for all other cases: meson-baryon scattering, baryon-baryon scattering and form factors.

The question now arises: why isn't this tree-like behavior modified by loop diagrams ? What _is_ the behavior of loop diagrams ? The behavior of scattering amplitudes at high energy and wide angle in field theory has been discussed by a number of authors[9]. The upshot of these investigations may be briefly summarized as follows:

"In all renormalizable field theories involving _no elementary vector fields_ (this excludes gauge theories !) the asymptotic power of s associated with a connected tree graph for a given process is modified by higher-loop graphs simply by multiplication with some polynomial in $\ell n \frac{s}{\Lambda^2}$ where Λ is a renormalization-point mass which is taken to be much larger than all particle masses: $s \gg \Lambda^2 \gg m_i^2$".

I shall call these powers of $\ell n^s/\Lambda^2$ ultraviolet logs because they come from regions in the domain of integration of Feynman graphs where all internal momenta are of order at least Λ ; or, equivalently, all vertices are concentrated in a region of space-time of diameter Λ^{-1}. They happen to be the same logs one finds for a given graph at large Euclidean momenta(if taken proportional to $\sqrt{s}$). Thus, for this class of theories, the renormalization group[10] may be used to discuss the $s \to \infty$ behavior of the exclusive cross section. In general, the resulting power of s would be given by the anomalous dimensions of the quark fields at the relevant UV-stable point. There is, however, no _a priori_ reason to expect that these anomalous dimensions vanish so that we get asymptotically the same power of s as that of the tree graphs. Unless, of course, our theory is asymptotically free in which case all running coupling constants $g(\Lambda)$ go to zero as $\Lambda \to \infty$ and the trees would dominate. But, remember, for a field theory to be asymptotically free it must contain non-Abelian gauge (vector) mesons[3] and therefore it does not belong to the above category (i.e. the class of theories with only UV logs).

And so, to continue our search for an explanation of the quark counting rules within the framework of field theory, we turn to the study of the $s \to \infty$ limit in theories with vector mesons and, in particular, non-Abelian gauge theories.

It turns out that, in vector theories, in calculating the $s \to \infty$ behavior of individual graphs, one obtains powers of ℓns not only from the UV regions we encountered before but also from regions where certain virtual momenta of vector mesons are much smaller than Λ (or s). A new mass scale is thus introduced which can be taken to be of the order of the vector meson masses if the vector mesons are massive, or, as in QCD, it can measure the departure of the momenta of all external colored particles from mass-shell. Such a departure is necessary because of infrared divergences of the mass-shell amplitudes. So in QCD besides the UV logarithms of the type $\ell n(s/\Lambda^2)$, there appear also infrared-induced (IR) logarithms[11] of the type $\ell n(s/M^2)$. We may say that these IR logs come from domains where not all vertices of the graph are within a small distance of order Λ^{-1} from each other - some vector mesons are allowed to propagate by a finite distance of order M^{-1}.

Since we are interested in discussing hadron scattering we must look at "cluster" amplitudes in QCD (just like the one in Fig. 1) each cluster of external quarks representing a hadron. More precisely, we consider the $s \to \infty$ limit of connected cluster amplitudes (see Fig. 2) by taking the external momenta p_i as follows:

$$p_i p_j = O(M) \text{ if } i,j \text{ in the same cluster}$$
$$= \eta_{ij} s, \quad \eta_{ij} \text{ fixed} \neq 0 \text{ otherwise}$$
$$s \to \infty$$

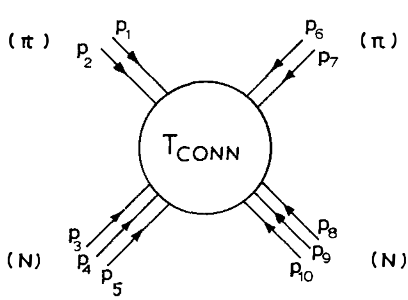

Fig. 2

In Fig. 2, p_1+p_2 is the 4-momentum of one of the pions and $p_1-p_2 = q$ is the relative quark momentum which will be integrated over after multiplication by the pion BS wave function $\psi(q)$ say, and similarly for all hadron clusters. In what follows we will just discuss the $s \to \infty$ of these cluster amplitudes with the p_i's at off-shell values. We do not worry at this point about the on-shell divergence - we presume it is taken care of by the subsequent q integration and the hadron wave function $\psi(q)$.

From a study of the asymptotic behavior of cluster amplitudes in perturbation theory there emerges, first of all, an important fact: <u>if all clusters are colorsinglets the IR logs cancel completely between graphs of the same order</u>[12]. This is quite analogous to the absence of infrared singularities in elastic scattering of electrically neutral atoms. And since our hadronic clusters are always color-singlet systems of quarks, it seems at this point that we can forget about IR logs altogether and that we have at least "explained" the quark-counting rules.

There is, however, one more complication. Right after the first suggestion of the power laws Eq. (1) and (2) it was pointed out by Landshoff[13] that the contributions of certain <u>disconnected</u> cluster amplitudes apparently <u>dominate</u> those of Eq. (1) and (2). The simplest case is the one depicted in Fig. 3 for meson-meson scattering.

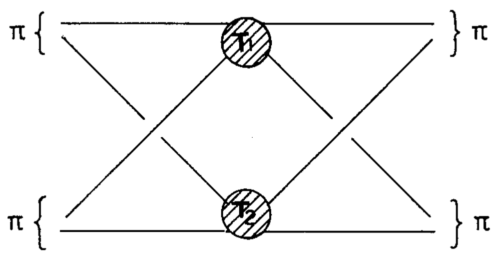

Fig. 3

This class of graphs (appropriately convoluted with hadron wave functions) gives a "pinch" contribution to the hadronic amplitude which behaves like

$$s^{-3/2}\, T_1 T_2 \qquad (3)$$

where T_1 and T_2 are the (connected) quark-quark scattering amplitudes shown in Fig. 3. What happens may be simply described in space-time language: quarks from different hadrons travel long (of order M^{-1}) distances and collide at two widely separated space-time regions (of size Λ^{-1}) with amplitudes T_1 and T_2 respectively. Since T_1, T_2 themselves represent wide-angle collisions, unless they vanish in the $s \to \infty$ limit we would be faced with an $s^{-3/2}$ behavior for the hadronic amplitude which would dominate over the s^{-2} behavior required by Eq. (1). Similar multiple-independent-scattering graphs exist for meson-baryon and baryon-baryon amplitudes.

So now we must investigate quark-quark scattering inthe $s \to \infty$ limit (at wide angle). Since we are no longer dealing with color-singlet clusters but with colored quarks the IR logs no longer cancel. Obviously, it would be interesting to know if they add up to something simple. Unfortunately, so far what has been done is a calculation of the <u>leading</u> IR logs in perturbation theory up to sixth order in the coupling constant g, for various processes. This is far from being complete or satisfactory. Nevertheless, something strikingly simple is suggested by these calculations: <u>the IR logs begin to exponentiate</u> just as in the more familiar case of QED. One may state the result [true at least to $O(g^6)$] for a general connected cluster amplitude T as follows:

$$T \equiv T_{tree} \exp -\{\frac{g^2}{32\pi^2}(\sum_\nu c_\nu)\ln^2(s/M^2)\} \qquad (4)$$

(only leading logs kept)

where T_{tree} is the tree approximation to T and c_ν is the eigenvalue of the quadratic Casimir operator for the representation of color SU(3) to which the ν-th cluster belongs. For example, for the quark-quark scattering amplitude $\sum_\nu c_\nu = 4c_F$ where c_F is the Casimir eigenvalue for the 3 representation of SU(3). Note, incidentally, that for color-singlet clusters, $c_\nu = 0$ so Eq. (4) is consistent with our previous finding that there are no IR logs whatsoever (leading <u>or</u> non-leading) if all the clusters are color-singlets.

So, finally, if we accept Eq. (4), we see that T_1 and T_2 in Eq. (3) vanish faster than any power of s and the Landshoff graph contributions are suppressed. We may then conclude that asymptotic freedom is indeed the explanation for the quark-counting rules[14] but only because infrared effects suppress the probability of scattering configurations which could require wide space-time separation between the constituent quarks. This looks like an intriguing hint - perhaps a related infrared mechanism is basically responsible for the strange fact that the quark constituents of a hadron never seem to fly apart to reveal themselves as free isolated particles. I shall pursue this idea and its possible relevance to quark confinement next.

2. INFRARED SINGULARITIES IN QCD — A SIGNAL FOR CONFINEMENT ?

In gauge theories like QED and QCD in which the gauge symmetry is not spontaneously broken, matrix elements of physical operators between asymptotic states containing charged or colored quanta are infrared-divergent when calculated in perturbation theory beyond the tree approximation level. Also exclusive cross-sections for gauge meson emission diverge at low momenta. In this section I shall briefly review the infrared problem in QED[15] and then point out certain apparent similarities and differences between it and QCD - insofar as infrared singularities in QCD can be organized and understood at present.

Begin with a simple process in QED like the one shown in Fig. 4. A (virtual) neutral particle (e.g. a hard photon) represented by the dotted line produces an e^+e^- pair. The graph represents the exchange of a photon between the outgoing charged particles. The "blob" amplitude T_o contains all the short range interactions.

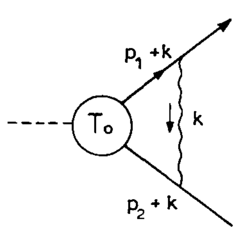

Fig. 4

The corresponding Feynman integral (assume scalar electrons for simplicity)

$$I_1 = \frac{e^2}{(2\pi)^4 i} \int \frac{d^4k}{k^2} \frac{(2p_1+k)(2p_2+k)}{\{(p_1+k)^2-m^2\}\{(p_1+k)-m^2\}} \times T_o$$

diverges at k=0 logarithmically. One may introduce an infrared cut-off mass by replacing the photon propagator by $(k^2-\mu^2)$. Then

$$I_1 \underset{\mu\to 0}{\sim} \alpha\, H(\frac{p_1p_2}{m^2})\ell n(\mu/M) \times T_o \tag{5}$$

where the mass M is related to the inverse range of interactions in T_o (and not necessarily to the electron mass m).

It turns out that if we add up all possible virtual photon exchanges (including the no photon exchange T_o) the result of Eq. (5) exponentiates:

$$T \equiv \Sigma\, I_n = \Sigma \frac{1}{n!} \{\alpha H\ell n(\mu/M)\}^n T_o = T_o \exp\{\alpha H \ell n(\mu/M)\} \tag{6}$$

where

$$I_n \sim \begin{cases} \text{sum of} \\ \text{all n-photon-exchange graphs (Fig. 5)} \end{cases}$$

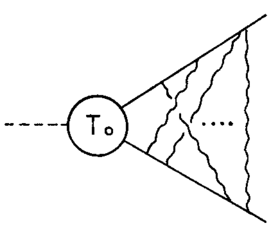

Fig. 5

From Eq. (6) we see that T vanishes as $\mu \to 0$. This is not suprising because it can be understood clasically: one cannot observe the charged particles separated from their long range electromagnetic field. In fact, the energy density $E(\omega)$ of classical radiation in this case approaches a nonzero constant as the frequency ω goes to zero, and so we expect the number of photons $E(\omega)/\omega$ to become infinite. Now, if we ask for the probability to observe the charged particles plus an indefinite number of photons we should get a non-vanishing answer. This expectation is borne out by a calculation of soft photon emission cross sections.

The graphs for the emission of one photon of momentum k_μ have a pole at $k_\mu=0$ (see fig. 6)

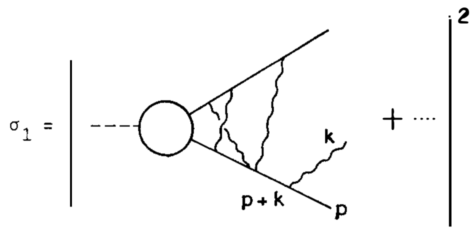

Fig. 6

This implies a logarithmic divergence in the cross-section σ_1 as $\mu \to 0$. We have

$$\sigma_1 \stackrel{\sim}{=} |T|^2 \cdot \alpha \int_0^Q \frac{d^3k}{\sqrt{\vec{k}^2+\mu^2}} \frac{1}{(2pk)^2} \underset{\mu\to 0}{\approx} |T|^2 (\alpha H \ln \frac{Q}{\mu}) \tag{7}$$

where Q is some fixed cut-off momentum (which defines what we mean by "soft" photon). Similarly, the n-photon amplitude has an n-ple pole at vanishing photon momenta, and it turns out that the corresponding cross-section σ_n is given by:

$$\sigma_n \underset{\mu\to 0}{\approx} |T|^2 \frac{1}{n!} (\alpha H \ln \frac{Q}{\mu})^n \tag{8}$$

Thus the <u>inclusive</u> cross section is infrared finite because:

$$\sigma = \sum_{n=0}^{\infty} \sigma_n = |T|^2 \sum_{n=0}^{\infty} \frac{1}{n!} (\alpha H \ln \frac{Q}{\mu})^n$$

$$= |T|^2 \exp\{\alpha H \ln \frac{Q}{\mu}\} = |T_0|^2 (\frac{Q}{M})^{\alpha H} \tag{9}$$

In fact, σ is <u>finite</u> <u>order-by-order</u> in perturbation theory since expression (9) is perfectly expandable in powers of α. This is a special case of a more general theorem: Lee and Nauenberg and independently Kinoshita[16] have shown that in theories with massless particles (or more generally in quantum systems with infinitely degenerate energy levels) inclusive cross-sections are free of infrared singularities order-by-order in perturbation theory.

Now let us turn to QCD. How much of what we said about QED is also true in QCD ? The evidence so far is based on leading-logarithm calculations in perturbation theory and even those are incomplete and scanty. Nevertheless, let us begin by a process like that of Fig. 4 where a time-like hard photon (dotted line) turns into a quark-antiquark pair and a gluon is exchanged. The integral is essentially the same as in QED; we just replace α in Eq. (5) by $c_F g^2/4\pi$. In higher orders of g, however, we encounter a much larger variety of graphs in QCD. If we decide to take into account only <u>skeleton</u> graphs (i.e. graphs with no vertex and propagator insertions) then it can be shown that the leading logs again exponentiate as in Eq. (6). The vertex and propagator insertions will almost certainly modify this (see next section) but we may assume here that anyway T will still vanish as $\mu \to 0$.

Next, we look at the one-gluon production cross section σ_1. The new feature of QCD is that the emitted gluon is not "neutral". Thus, it can exchange soft virtual gluons with all other colored quanta (e.g. the quarks) as illustrated in Fig. 7

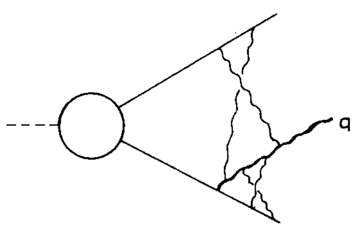

Fig. 7

It turns out that processes with external gluons "on the mass-shell" i.e. with gluon momenta q_i such that $q_i^2=\mu^2$, are more singular than those with just (massive) external quarks compared at the same order of perturbation theory. The leading logs are powers of $g^2 \ln^2\mu$ for gluon amplitudes (as compared to powers of $g^2 \ln\mu$ for quarks-only amplitudes). These "double logarithms" are intimately related to the ones we encountered in our discussion of high energy, wide angle limits (Section 1). Again, up to sixth order the double logs for a multigluon amplitude exponentiate as follows (compare with Eq. (4))

$$T \underset{|\vec{q}^{(\nu)}|>>\mu}{\approx} T_{tree} \exp\{-\frac{g^2}{32\pi^2} \sum_\nu c_\nu \ln^2(\frac{\mu}{|\vec{q}^{(\nu)}|})\} \tag{10}$$

The external gluons are grouped into clusters of one or more gluons each cluster having invariant mass of order μ. In Eq. (10) c_ν is the Casimir eigenvalue for the ν-th cluster and $\vec{q}^{(\nu)}$ is its 3-momentum. Going back to σ_1 now we see that, in contrast to the QED case, the exchanges of soft virtual gluons between the on-shell gluon of momentum $\vec{q}$ (see Fig. 7) and the other colored particles in the final state provide an exponential damping factor for the amplitude so that instead of Eq. (7) we now have:

$$\sigma_o = |T|^2$$

$$\sigma_1 \cong |T|^2 g^2 \int_0^Q \frac{d^2q}{\sqrt{\vec{q}^2+\mu^2}} \frac{1}{(2pq)^2} \exp\{-\frac{g^2}{16\pi^2} c_A \ln^2(\mu/|\vec{q}|)\} \tag{11}$$

The integral in Eq. (11) is finite for $\mu=0$ so that

$$\lim_{\mu\to 0} \sigma_1/\sigma_o = c_1 < \infty$$

Similarly, the n-ple poles of n-gluon-emission amplitudes do not cause logarithmic infinities in σ_n/σ_o since the exponential damping factor (10) (from soft gluon exchanges) effectively only allows the emission of gluons or gluon clusters with 3-momenta of order μ. Thus σ_n/σ_o approaches a finite value c_n as $\mu \to 0$. Assuming that the sum over n does not introduce a new kind of divergence, it seems that the inclusive cross section $\sigma=\Sigma\sigma_n$ vanishes as $\mu\to 0$ at the same rate as σ_o. We interpret this result as a signal that in QCD quarks and gluons are never produced in asymptotic states, i.e. as a signal for confinement.

Note that the suppression of gluons with three momenta much larger than μ occurs only after summation to all orders of perturbation theory for each σ_n and therefore there is no conflict with the Kinoshita-Lee-Nauenberg (KLN) theorem[16] which states that the infrared singularities cancel in the inclusive cross section order-by-order in perturbation theory. It is really a matter of when the $\mu \to 0$ limit of σ is taken: if it is taken before summation to all orders of g then the KLN applies, all $\ell n\mu$'s cancel and we get a certain finite answer up to each order of perturbation theory; but if μ is taken to zero after summation to all orders of g then σ vanishes because gluon emission is disallowed.

Note also that the $\mu=0$ value of the integral in Eq. (11) behaves as 1/g for small g indicating that it is not possible to recover the perturbation expansion for σ_1/σ_0 after μ is set equal to zero.

3. LONG RANGE FORCES AND GLUON CHAINS

In our discussion of the infrared singularities of amplitudes involving no external gluons we simplified our task drastically by admitting only skeleton graphs. By doing this we missed an important difference between QED and QCD referring to the character of the long range forces between quarks.

In QED the behavior of the electromagnetic force at long range is related to the transverse part of the photon propagator, which, for small values of the momentum q_μ, behaves like q^{-2} to all orders of perturbation theory provided all charged particles are massive. [This is because the vacuum polarization tensor is (i) gauge invariant and (ii) has no charged-particle-production threshold at $q^2 = 0$]. Furthermore the infrared singularities of charged-particle vertices and propagators cancel each other because of Ward-Takahashi identities. This is illustrated by Fig. 8 where (b), (c), (d), (e) and (f) are the propagator and vertex corrections to the second-order graph (a) for the process virtual photon $\to e^+e^-$. For the present discussion it is more appropriate to introduce the infrared cut-off μ as the departure of the momenta p and p' from mass-shell and not as a photon mass. Then as $\mu \to 0$, (a) is proportional to $\alpha \ell n\mu$ and (b) is proportional to $\alpha^2 \ell n\mu$ because the insertion of the particle loop in the photon propagator does not make it more singular. The leading singularity of both (c) and (d) is $\alpha^2 \ln^2 \mu$ but it cancels between them; similarly for (e) and (f). Thus the sum of (b), (c), (d), (e) and (f) does not modify the $\ln\mu$ behavior of (a) it just amounts to an infrared-regular charge and wave function renormalization. We can express this by saying that higher-order quantum corrections do not modify

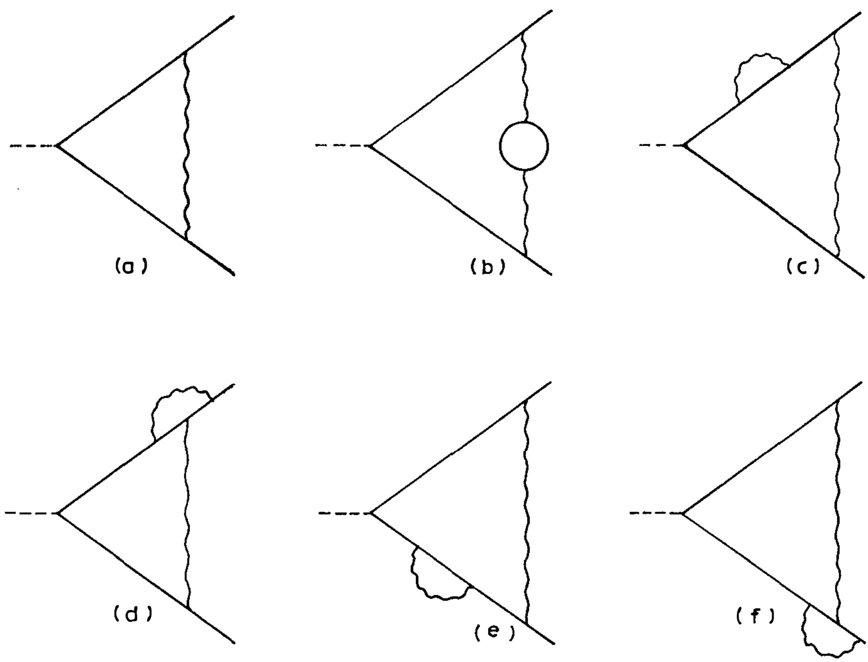

Fig. 8

the $1/r^2$ long-range character of the electromagnetic force.

The situation in QCD is different: the $1/r^2$ force associated with bare gluon exchange <u>is</u> modified by higher order corrections. Consider the process virtual photon→quark-antiquark pair (which is the analogue of $\gamma \to e^+e^-$ in QED). In addition to the graphs of Fig. 8 (where the wavy lines are now gluons) we have the graphs of Fig. 9. Note that the vacuum polarization now consists not only of the quark loop contribution of Fig. 8b but also of that of the gluon loop Fig. 9a and ghost loop Fig. 9b, both of which have threshold singularities at $q^2 = 0$.

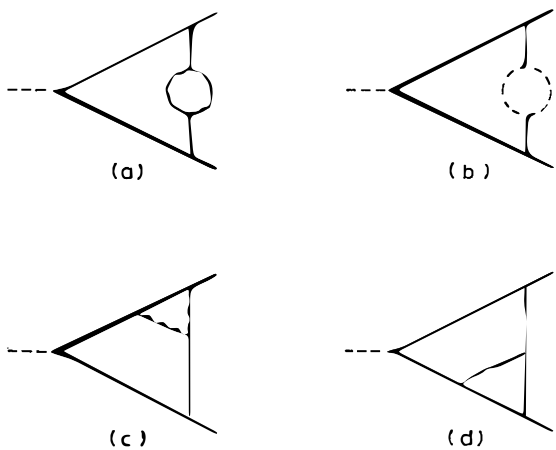

Fig. 9

Although the infrared singular contributions of individual graphs depends on the gauge, their sum (at least to this order of g) does not and it can be fully accounted for as a logarithmic modification of the small q^2 behavior of the transverse part of the bare gluon propagator[17] in the graph of Fig. 8a. In terms of the polarization tensor we have:

$$\Pi^{ab}_{\mu\nu}(q) = \delta^{ab}(\delta_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2)$$

$$\Pi(q^2) \approx c_1 g^2 \ln(q^2/m^2) + \ldots$$

for $q^2 \ll m^2$ where q is space-like and m is the renormalization-point mass (at which the running coupling constant is of order 1). From an inspection of higher order corrections it seems that one gets increasing powers of $\ln(q^2/m^2)$ for $\Pi(q^2)$. At present no one knows what these logs add up to and most likely perturbation theory estimates are not the way to explore the small q^2 behavior of the gluon propagator, since we know that $g=0$ is an infrared-unstable zero of the renormalization-group function $\beta(g)$. But these logarithms signal what could be a drastic departure from the q^{-2} of the gluon propagator $D_{\mu\nu}$. To see what may be hapenning, consider two colored quanta (quarks or gluons) r and s located at points x_r and x_s respectively. From the gluon propagator $D_{\mu\nu}$ we derive an instantaneous potential according to

$$U_{rs} = t^{(r)a}\, t^{(s)a}\, U(|\vec{x}_r - \vec{x}_s|) \tag{12}$$

$$U(|\vec{x}|) = ig^2 \int_{\infty}^{\infty} dx_o D_{oo}(\vec{x}, x_o)$$

where $t^{(r)a}$ ($a=1,2,\ldots,8$) are the matrix group generators for particle r. From the bare gluon propagator $D_{oo}(q^2) \approx q^{-2}$ we get the Coulombic $U \approx -|\vec{x}|^{-1}$, but if the full propagator behaves like $(q^2)^{-2}$, for example, we get $U \approx |\vec{x}|$. In this case the interaction energy of a quark-antiquark system in a color singlet configuration would grow linearly with their separation making it impossible to isolate a quark.

Gluon Chains[6]

Although we don't know what the small momentum behavior of the gluon propagator (more precisely: the <u>effective</u> gluon propagator, see above) is, it is interesting to speculate as to what would happen if indeed it led to a very strong confining force at long distances. So assume that as a color-singlet quark-antiquark system is pulled apart the energy required grows linearly or faster with distance. Then it can be shown that it is energetically favorable to expend some energy to create a chain of gluons $g_1, g_2, \ldots, g_n$ joining the quark (q) to the antiquark ($\bar{q}$) along some smooth curve:

$$g_o = q,\ g_1,\ h_2, \ldots, g_n,\ \bar{q} = g_{n+1}$$

The color-spins of the gluons must be coupled to the quarks so that the whole system is a color singlet (that's obvious) and so that the system $\{q, g_1, g_2, \ldots, g_k\}$

belongs to the 3 representation of color SU(3) for all k. It is straightforward to verify that this coupling in color space results in an exponential screening of the forces between q and $\bar{q}$. More explicitly, the color part of the wave function of the chains is

$$\Psi_{color} = (d^{a_1} d^{a_2} \ldots d^{a_n})_{ij}$$

where a_r, i, j are the color indices for the r-th gluon, the quark and the antiquark respectively and d^a is the a-th generator in the spinor representation. Using Eq. (12) we calculate the expectation value of the interaction energy between the r-th and s-th quantum in the chain

$$\langle U_{rs} \rangle \equiv \frac{\langle \Psi_{col.} | U_{rs} | \Psi_{col.} \rangle}{\langle \Psi_{col.} | \Psi_{col.} \rangle} \propto (-8)^{-|r-s|} U(|x_r - x_s|)$$

The factor $(-8)^{-|r-s|}$ due to the group theory of color is the reason for having chosen this particular coupling in color space. Thus if $|\vec{x}_{r+1} - \vec{x}_r| = O(m^{-1})$ then $|r-s| = O(m|\vec{x}_r - \vec{x}_s|)$ and we have exponential screening of the long range forces.

In a first estimate of the potential energy of the system we may neglect all but the nearest neighbor interactions, and check the stability of the chain by estimating its energy roughly by the relation:

$$E \approx nR^{-1} - ng^2(R)R^{-1} \qquad (13)$$

in terms of a length R representing the spacing between gluons. Then each gluon wave packet can have a spread of at most R and consequently an "uncertainty principle" kinetic-energy of at least R^{-1} which accounts for the first term in Eq.(13). The expression $g^2(R)R^{-1}$ is the interaction energy between two nearest neighbors. The assumptions made about the gluon propagator imply that $g^2(R)$ is large and negative for $R \gg m^{-1}$ and we also know that $g^2(R)$ is small and positive for $R \ll m^{-1}$ (asymptotic freedom). Then a stable minimum is attained for some value of R of order m [at which $g^2(R) \approx O(1)$]. The energy of the chain is therefore proportional to its length. Actually, the gluon spacing may vary along the chain so that the energy content will be proportional to the length locally (in a coarse-grained sense).

The chain is stable against splitting in two pieces $\{q,g_1,g_2,\ldots,g_k\}$ and $\{g_{k+1},\ldots,\bar{q}\}$ since the pieces, forming a 3 and a $\bar{3}$ color systems respectively, would be subject to a strong long range mutual attractive force like the one between q and $\bar{q}$ (if unscreened by the formation of a chain of gluons between them).

The chain may split in two when a new quark-antiquark pair is created somewhere along the chain and two color-singlet pieces $\{q,g_1,\ldots,g_k,\bar{q}\}$ and $\{q,g_{k+1},\ldots,\bar{q}\}$ are formed. These may be easily separated since the strong long range forces do not act between them. Clearly, the chain may be split into more than two pieces with the creation of more $q\bar{q}$ pairs at different points. We have here the picture of a meson decaying into mesons. Meson processes in general may be naturally visualized as events in which two chains combine into one (by the inverse process to the one described above) which subsequently is split in two or more color-singlet pieces. This description of hadronic matter is not much different from certain models of hadrons as relativistic strings with quarks at their ends[18]. What is different here is the coarse-grainedness of the chain and the specific role of color in its dynamics and stability.

Note that baryons may be described in an analogous way for large distances between the three constituent quarks: three gluon chains are tied at one point to form a Y with the quarks at the free ends. The color part of the wave function should be:

$$e_{i'j'k'}(d^{a_1}\ldots d^{a_r})_{i'i}(d^{a_{r+1}}\ldots d^{a_{r+s}})_{j'j}(d^{a_{r+s+1}}\ldots d^{a_{r+s+t}})_{k'k}$$

where the three chains consist of r,s at t gluons respectively with color indices $a_1,\ldots,a_{r+s+t}$; i,j,k are the color indices of the quarks. Again, this is the unique coupling in color space such that if the Y is cut at any point the two pieces have total color spins 3 and $\bar{3}$ respectively.

It is instructive to realize that were the fundamental fermions to belong to the adjoint representation of color (or any representation contained in a direct product of adjoint representations), it would be possible to screen the long range forces between say a fermion and an antifermion by surrounding each of them with a small number of gluons (i.e. a number independent of the distance between q and $\bar{q}$). A gluon chain would then be far from energetically favored. Such screened (or "bleached") fermions would probably behave like leptons toward each other or versus ordinary hadronic matter (i.e. color-triplet quarks held together by gluon chains). The possibility that physical leptons may fit this description should be kept in mind.

The whole concept of gluon chains may be criticized up to this point as based on the imprecise notion of an instantaneous color force and certain ad hoc assumptions about its strength at long range. However, it should be pointed out that the gluon chain emerges rather naturally when one considers states created out of the vacuum not by gauge-variant operators like $\bar{q}(x)q(y)$ but by their gauge-invariant (path-dependent) generalizations:

$$Q_C(x,y) = q(x)\exp\{ig \int_{x,C}^{y} dz_\mu B_\mu^a(z)d^a\}\bar{q}(y) \tag{14}$$

In Eq. (14) the line integral runs along the space-like curce C from x to y and the d matrices are ordered along C:

$$O_C(x,y) = \sum_n (ig)^n q_i(x)\sigma_{ij}^{(n)}(x,y)\bar{q}_j(y)$$

$$\sigma_{ij}^{(n)}(x,y) = (d^{a_1}d^{a_2}\ldots d^{a_n})_{ij} \int_x^y dz_{n\mu_n} \int_x^{z_n} dz_{(n-1)\mu_{n-1}}\cdots \times \int_x^{z_2} dz_{1}A_\mu^{a_1}(z_1)\ldots A_{\mu n}^{a_n}(z_n) \tag{15}$$

Since O_C creates meson-like physical states, a matrix element like $\langle O_C(x,y)J(o)\rangle_o$ where J is some local color-singlet current (e.g. the electromagnetic current) should be non zero. To simplify, take $x_o \approx y_o \approx z_{io}$ and consider $\langle O_C(x,y)J(o)\rangle_o$ for $|\vec{x}-\vec{y}|$ large i.e. $\gg m^{-1}$.

The n-gluon amplitude $<q\sigma^{(n)}\bar{q}J>$ calculated in perturbation theory will display powers of $g^2 \ln|\vec{x}-\vec{y}|$ which are the configuration space counterparts of the infrared logarithms we described in Section 2. We may assume that they exponentiate and make $<q\sigma^{n}\bar{q}J>_o$ vanish as $|\vec{x}-\vec{y}|\to\infty$ for any fixed n.

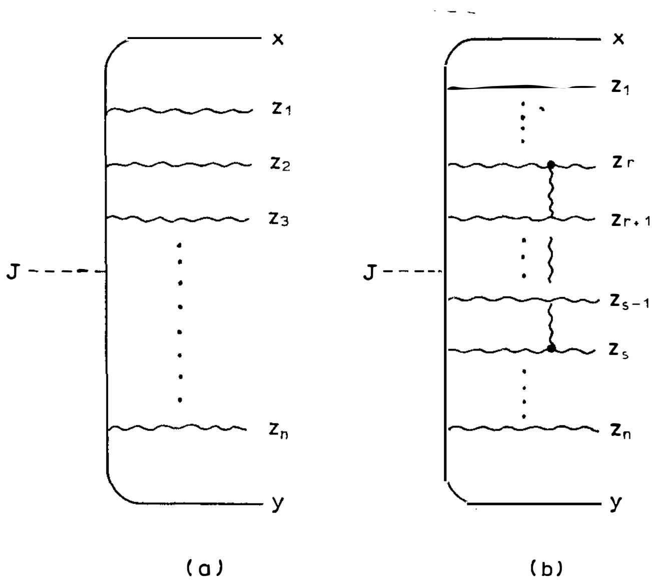

Fig. 10

However, for the band of terms in the expansion (15) with $n \propto m|\vec{x}-\vec{y}|$ the situation is different. At the tree approximation level, the planar graph of Fig. 10a has the color factor

$$(d^{a_n}\cdots d^{a_2}d^{a_1})_{ji} \qquad (16)$$

Consider next the one-loop graph of 10b obtained from that of Fig. 10a by exchanging a virtual gluon between the r-th and s-th gluon lines. If $|\vec{z}_r-\vec{z}_s|$ is large the loop integral behaves like $\ln|\vec{z}_r-\vec{z}_s|$, an infrared singular behavior. However, in calculating the contribution of this graph to $<q\sigma^{n}\bar{q}J>$ its color factor(17) combined with that of $\bar{\sigma}^{(n)}$ in Eq. (15) yields $(-8)^{-|r-s|}$ just as in the calculation of $\langle U_{rs} \rangle$ before. Thus, all one-loop infrared logarithms like $\ln|\vec{z}_r - \vec{z}_s|$ are suppressed provided $|r-s|$ is large whenever $|\vec{z}_r - \vec{z}_s|$ is large. This is achieved by a configuration in which the distances between successive points in the sequence $x, z_1, z_2, \ldots, z_n, y$ are kept bounded (below a common fixed bound) as $|\vec{x}-\vec{y}| \to \infty$. Such configurations are strikingly similar to gluon chains - look specifically, at the coupling of the color spins given by (16).

Tree graphs which can be obtained from the planar one by a small number ($\ll n$) of nearest neighbor interchanges still have their infrared singular loop corrections effectively suppressed by group factors and thus survive as contributions to the amplitude: these graphs have color factors representing minor local "disordering" in the coupling given by Eq. (16) which does not disturb either the screening of the color forces or the stability of the gluon chain. All other graphs are infrared-suppressed ! This is worth checking in higher orders of perturbation theory.

In conclusion, I should emphasize that most of the work I described has its roots in perturbation theory and in graph-by-graph asymptotic estimates, although the essential element behind most arguments is exponentiation of infrared logs which presumes a summation to all orders. Eventually, of course, we hope to be able to study the infrared problem in Yang-Mills theories directly without having to get our cues from perturbation theory.

FOOTNOTES

1. C.N. Yang and R. Mills, Phys.Rev. 96, 191 (1954); a standard reference work is the review paper by E.S. Abers and B.W. Lee, Phys.Rep. 9C, 1 (1973).
2. Asymptotic freedom for non-Abelian gauge theories was shown by H.D.Politzer, Phys.Rev.Lett. 30, 1346 (1973); D.J. Gross and F. Wilczek, ibid. 30, 1343 (1973); also G. t' Hooft, in Proceedings of the 1972 Marseille Conference on Yang-Mills Fields (unpublished). For a discussion of the advantages of Quantum Chromodynamics as a model of strong interactions see H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys.Lett. 47B, 365 (1973); D. Gross and F. Wilczek, Phys.Rev. D8; 3633 (1973); S. Weinberg, Phys.Rev.Lett. 31, 494 (1973).
3. For a discussion of the uniqueness of Y-M theories with respect to renormalizability see J.M. Cornwall, D.N. Levin and G. Tiktopoulos, Phys.Rev.Lett. 30, 1268 (1973); 31, 572 (E) (1973); C.H. Llewellyn Smith, Phys.Lett. 46B, 233 (1973); J.M. Cornwall, D.N. Levin, and G. Tiktopoulos, Phys.Rev. D10, 1145 (1974). Uniqueness with respect to asymptotic freedom is discussed by S. Coleman and D.J. Gross, Phys.Rev.Lett. 31, 851 (1973).
4. A.A. Belavin, A.M. Polyakov, A.S. Schwartz, Yu.S. Tyupkin, Phys.Lett. 59B, 85 (1975); G. t'Hooft, Phys.Rev.Lett. 37, 8 (1976); R. Jackiw and C.Rebbi, Phys.Rev.Lett. 37, 172 (1976). Instantons and/ tunnelling away from the zero-potential ($B^a_\mu=0$) vacuum are ignored throughout these lectures, although some authors have argued that they may be essential to the problem of quark confinement.
5. J.M. Cornwall and G. Tiktopoulos, Phys.Rev.Lett. 35, 338 (1975); Phys.Rev. D13, 3370 (1976).
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7. S.J. Brodsky and G.R. Farrar, Phys.Rev.Lett. 31, 1153 (1973); Phys.Rev. D11, 1309 (1975). For a recent review see D. Sivers, S.J. Brodsky and R. Blankenbecker, Phys.Rep. 23C, 1 (1976).
8. V. Matveev, R. Muradyan and A. Tavkhelidze, Lett.Nuovo Cimento 7, 719 (1973).

9. See G. Tiktopoulos, Phys.Rev. D11, 2252 (1975) for a rigorous discussion to all orders of perturbation theory for a Φ^4 interaction. The following authors have independently arrived at the same conclusions: M. Creutz and L.-L. Wang, Phys.Rev. 11, 3749 (1975); C.G. Callan and D.J. Gross, ibid., 11, 2905 (1975). The large momentum transfer limit of form factors in non-vector field theories has been discussed by G.C. Marques, Phys.Rev. D9, 386 (1974); S.-S. Shei, ibid. 11, 169 (1975); J.M. Borenstein, ibid. 11, 2900 (1975).

10. M. Gell-Mann and F.E. Low, Phys.Rev. 95, 1300 (1954); C.G. Callan, Jr., Phys.Rev. D2, 1541 (1970); K. Symanzik, Commun. Math.Phys. 23, 227 (1970).

11. The powers of $\ln(s/M^2)$ are called infrared-induced because they connect the on-mass-shell (M=0) divergence of the Feynman integral with its large s behavior.

12. The detailed proof will be included in a forthcoming publication. In ref. 5, only the cancellation of the leading logs had been checked.

13. P.V. Landshoff, Phys.Rev. D10, 1024 (1974); see also J.C. Polkinghorne, Phys.Lett. 49B, 277(1974) for the suggestion that the Landshoff-graph contribution might be suppressed by infrared effects in the context of an Abelian gluon model.

14. Strictly speaking, since the running coupling constant approaches zero, even the tree-graph contribution should eventually become negligible at high energy compared to the completely disconnected graphs, i.e. those without any large-momentum-transfer gluons whatsoever . This is the "constituent interchange model" of R. Blankenbecker, S.J. Brodsky and J.F. Gunion, Phys. Lett. 39B, 649 (1972); Phys.Rev. D8, 187 (1973).

15. The physics of the Abelian infrared problem (QED) was first correctly described by F. Block and A. Nordsieck, Phys.Rev. 52, 54 (1937). A comprehensive treatment of infrared logarithms is given by D.R. Yennie, S.C. Frantschi, and H. Suura, Ann.Phys. (N.Y.) 13, 379 (1961), and G. Grammar and D.R.Yennie, Phys.Rev. D8, 4332 (1973). For a highly readable account which also includes

infrared gravitons see S. Weinberg, Phys.Rev. 140, B516 (1965).

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17. J.M. Cornwall and G. Tiktopoulos, "Infrared Behavior of non-Abelian Gauge Theories II", UCLA preprint (1976).

18. I. Bars and A.J. Hanson, Phys.Rev. D13 (1976); W.A. Bardeen, I. Bars, A.J. Hanson and R.D. Peccei, Phys.Rev. D13 (1976).