Bulk viscosities and r-mode of massive neutron stars

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The r-modes are oscillatory modes that exist in neutron stars. They are characterized by Coriolis force acting as the restoring force and are particularly important because they have been shown to drive gravitational radiation (GR) unstable for all rotational frequencies of perfect fluid stars [1]. Dissipative phenomena such as shear and bulk viscosities suppress the growth of GR driven instabilities.

Bulk viscosity arises when a system is driven out of equilibrium due to a change in the volume of a fluid element. The equilibrium is then restored by various internal processes, each of which act on a characteristic timescale known as the relaxation timescale 'τ'. It has been shown that non-leptonic processes involving hyperons contribute very significantly towards bulk viscosity at temperatures relevant to neutron stars. The real part of the coefficient of bulk viscosity ζ gives the damping due to bulk viscosity and an expression for it within a relaxation time approximation is [2]

$$\mathrm{Re}[\zeta] = \frac{p\,(\gamma_\infty - \gamma_0)\,\tau}{1 + (\omega\tau)^2}, \tag{1}$$

where ω denotes angular frequency of the perturbation in a co-rotating frame and τ denotes net microscopic relaxation time. The term $\gamma_\infty - \gamma_0$ is a thermodynamic factor involving the logarithmic derivative of pressure with respect to different baryon densities. The expression for relaxation timescale at temperature T in the presence of both Σ^- and Λ^0 hyperons is given by [2],

$$\frac{1}{\tau} = \frac{(k_B T)^2}{192\pi^3} \left(k_\Sigma \left\langle |\mathcal{M}_\Sigma^2| \right\rangle + k_\Lambda \left\langle |\mathcal{M}_\Lambda^2| \right\rangle\right) \frac{\delta\mu}{\rho_B \delta x_n}.$$

Here k_B is Boltzmann's constant, k_Λ and k_Σ denote Fermi momenta of Λ and Σ hyperons, $\delta\mu$ is the chemical potential imbalance, δx_n is the difference between the perturbed and equilibrium values of neutron fraction, $\left\langle |\mathcal{M}^2| \right\rangle$ are the angle averaged, squared and summed over initial spinors matrix elements of the reactions calculated from Feynman diagrams and $\delta\mu/\rho_B\delta x_n$ is calculated using charge neutrality and baryon number conservation [3].

In the present work, we study the damping of r-modes due to dissipation in a hyperonic neutron star. For the hyperon matter we consider an effective chiral model invoking $\sigma - \rho$ cross-coupling within a relativistic mean field approximation [4]. The dissipation mechanisms considered are hyperonic bulk viscosity (H), shear viscosity (η) and viscosity due to Urca processes (U). These effects can damp out GR driven instabilities if they act on timescales comparable to the timescale of GR. To study the evolution of r-modes, we define an overall r-mode timescale τ_r in terms of the timescales of each process under consideration

$$\frac{1}{\tau_r(\Omega, T)} = \frac{1}{\tau_{GR}} + \frac{1}{\tau_H} + \frac{1}{\tau_U} + \frac{1}{\tau_\eta}. \tag{2}$$

The critical angular velocity Ω_C of a star with core temperature T is defined by the equation $1/\tau_r(\Omega_C, T) = 0$. If a star rotates at a velocity higher than Ω_C, it will be unstable to GR emission. The equation of state (EoS) of the model under consideration is given in Ref. [4]. We use the Hartle-Thorne approximation to calculate the density distribution

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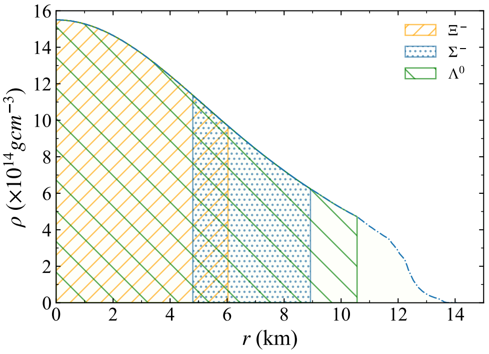

FIG. 1: Density distribution of the rotating star. Also indicated are the radii at which the hyperons appear.

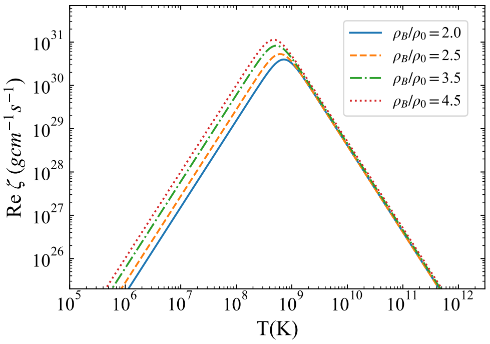

FIG. 2: Hyperonic bulk viscosity coefficient Re[ζ] plotted as a function of temperature T for various baryon number densities ρ_B/ρ_0.

$\rho(r)$ of the star rotating at Kepler frequency Ω_K and it is plotted in Fig. 1. This serves as an input to calculate the various dissipative timescales. The hyperonic bulk viscosity coefficient is obtained from Eq. 1 and the values are plotted against temperature for different baryon densities in Fig. 2. We find that the bulk viscosity is maximum for core temperatures of a few times 10^8K.

By also calculating the timescales of each dissipative phenomena, we solve for the equation $1/\tau_r(\Omega_C, T) = 0$. The instability window is plotted in Fig. 3. To estimate the extent to which a neutron star could be spun down due to r-mode instability, we calculate the spin evolution of a neutron star with core temperature 10^{11}K rotating initially at Kepler frequency, assuming that it cools through modified Urca processes [4]. The evolution curve is also plotted in Fig. 3. We report that there are two separate instability windows for the model considered. It is seen that between temperatures of 1.8×10^8K and 2.67×10^9K, the instability is completely suppressed due to hyperon bulk viscosity. The minima of the two windows occur at $\sim 0.15\Omega_K$. We also find that the instability can reduce the angular velocity up to $0.3\Omega_K$ as it crosses the first window, and it doesn't lose any angular momentum as it crosses the second window. [4].

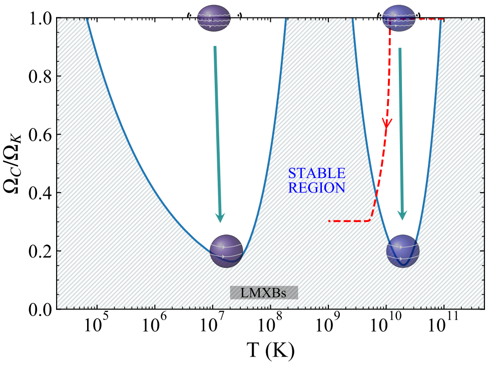

FIG. 3: Critical angular velocities in units of Kepler frequency as a function of the core temperature T. The dashed line shows the evolution of a neutron star for an initial r-mode amplitude of 10^{-5}. The shaded box shows the observed Low Mass X-ray Binaries (LMXBs).

References

[1] Andersson N., 1998, The Astrophysical Journal, 502, 708–713

[2] L. Lindblom and B. J. Owen, Phys. Rev. D **65**, 063006 (2002).

[3] T.K. Jha, H. Mishra and V. Sreekanth, Phys.Rev.C **82** (2010) 025803

[4] O. P. Jyothilakshmi, P. E. Sravan Krishnan, Prashant Thakur, V. Sreekanth and T. K. Jha, DOI: `https://doi.org/10.1093/mnras/stac2360` (to appear in MNRAS)