

# Polyakov loop susceptibilities in pure gauge system

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**Abstract.** We present new lattice results on Polyakov loop susceptibilities in the SU(3) pure gauge system. Ratios of these susceptibilities are useful probes for deconfinement. Progress toward understanding these ratios in the presence of quarks are briefly discussed.

## 1. Introduction

Deconfinement can be described by the spontaneous breaking of  $\mathcal{Z}_3$  center symmetry [1, 2, 3, 4, 5]. This symmetry, however, is explicitly broken by dynamical quarks. The result is the flattening of the static quark potential at long distances, even when the temperature is below criticality. If one aims at studying the confining part of the heavy quark potential, and probe the related symmetry breaking phase transition, it is useful to study QCD in the limit of exact  $\mathcal{Z}_3$  symmetry — SU(3) pure gauge theory.

The relevant observables to study deconfinement are the Polyakov loop and its susceptibilities. The Polyakov loop, which measures the free energy of a static quark immersed in a hot gluonic medium [5, 6], defines an order parameter for the deconfinement transition. At low temperatures its thermal expectation value vanishes, signaling color confinement, while at high temperatures it is nonzero, resulting in a finite energy of a static quark and consequently the deconfinement of color. The Polyakov loop susceptibility, on the other hand, represents fluctuations of the order parameter. It features a peak at the transition temperature, and a width that signals the temperature window in which phase transition takes place.

While the basic thermodynamic functions of the SU(3) pure gauge theory, such as pressure and entropy, are well established within the lattice approach [7, 8], it is less clear for the temperature dependence of the renormalized Polyakov loop and its susceptibilities. Careful study of these quantities can enhance our understanding of the QCD phase structure.

## 2. Polyakov loop and its susceptibilities on the lattice

On an  $N_\sigma^3 \times N_\tau$  lattice, the Polyakov loop is defined as the trace of the product over temporal gauge links,

$$L_{\vec{x}}^{\text{bare}} = \frac{1}{3} \text{Tr} \prod_{\tau=1}^{N_\tau} U_{(\vec{x},\tau),4}, \quad (1)$$

$$L^{\text{bare}} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_{\vec{x}}^{\text{bare}}. \quad (2)$$



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The bare Polyakov loop needs to be renormalized to give a physical,  $N_\tau$ -independent result. We perform the following multiplicative renormalization [9]:

$$L^{\text{ren}} = (Z(g^2))^{N_\tau} L^{\text{bare}}. \quad (3)$$

and introduce the ensemble average of its modulus,  $\langle |L^{\text{ren}}| \rangle$ . This quantity is well defined in the continuum and thermodynamic limits and is an order parameter for the spontaneous breaking of  $\mathcal{Z}_3$  center symmetry.

We now define the corresponding susceptibilities. In the SU(3) gauge theory, the Polyakov loop operator is complex. One can therefore explore its fluctuations along the longitudinal and transverse directions, as well as that of its absolute value,

$$T^3 \chi_L = \frac{N_\tau^3}{N_\tau^3} [\langle (L_L^{\text{ren}})^2 \rangle - \langle L_L^{\text{ren}} \rangle^2], \quad (4)$$

$$T^3 \chi_T = \frac{N_\tau^3}{N_\tau^3} [\langle (L_T^{\text{ren}})^2 \rangle - \langle L_T^{\text{ren}} \rangle^2], \quad (5)$$

$$T^3 \chi_A = \frac{N_\tau^3}{N_\tau^3} (\langle |L^{\text{ren}}|^2 \rangle - \langle |L^{\text{ren}}| \rangle^2), \quad (6)$$

where  $L_L = \text{Re}(\tilde{L})$  and  $L_T = \text{Im}(\tilde{L})$ . Here we have introduced the  $\mathcal{Z}_3$ -transformed Polyakov loop,  $\tilde{L} = L e^{2\pi n i/3}$ , with  $n = 0, \pm 1$ . The phase of the transformation is chosen such that the transformed Polyakov loop is located in the main sector, defined by  $-\pi/3 < \arg(\tilde{L}) < \pi/3$ .

Our lattice results [10] for these quantities are collected in Fig. 1.

### 3. The ratios of susceptibilities

The renormalization of gluonic correlation functions in general, Polyakov loop susceptibility in particular, is still an unsolved problem. One way to get around this is by considering ratios of these susceptibilities.

Fig. 2 shows the temperature dependence of the ratios  $R_A = \chi_A/\chi_L$  and  $R_T = \chi_T/\chi_L$ , obtained in the SU(3) pure gauge theory. Also shown in the figure are the analogous results extracted from simulations of (2+1)-flavor QCD by the HotQCD collaboration, using the highly improved staggered quark (HISQ) action with almost physical quark masses on a  $32^3 \times 8$  lattice [11].

In the pure gauge limit ( $N_f = 0$ ), ratios  $R_A$  and  $R_T$  exhibit a  $\theta$ -like discontinuity at  $T_c$ , and change only weakly with temperature on either side of the transition. These features make them ideal for probing deconfinement.

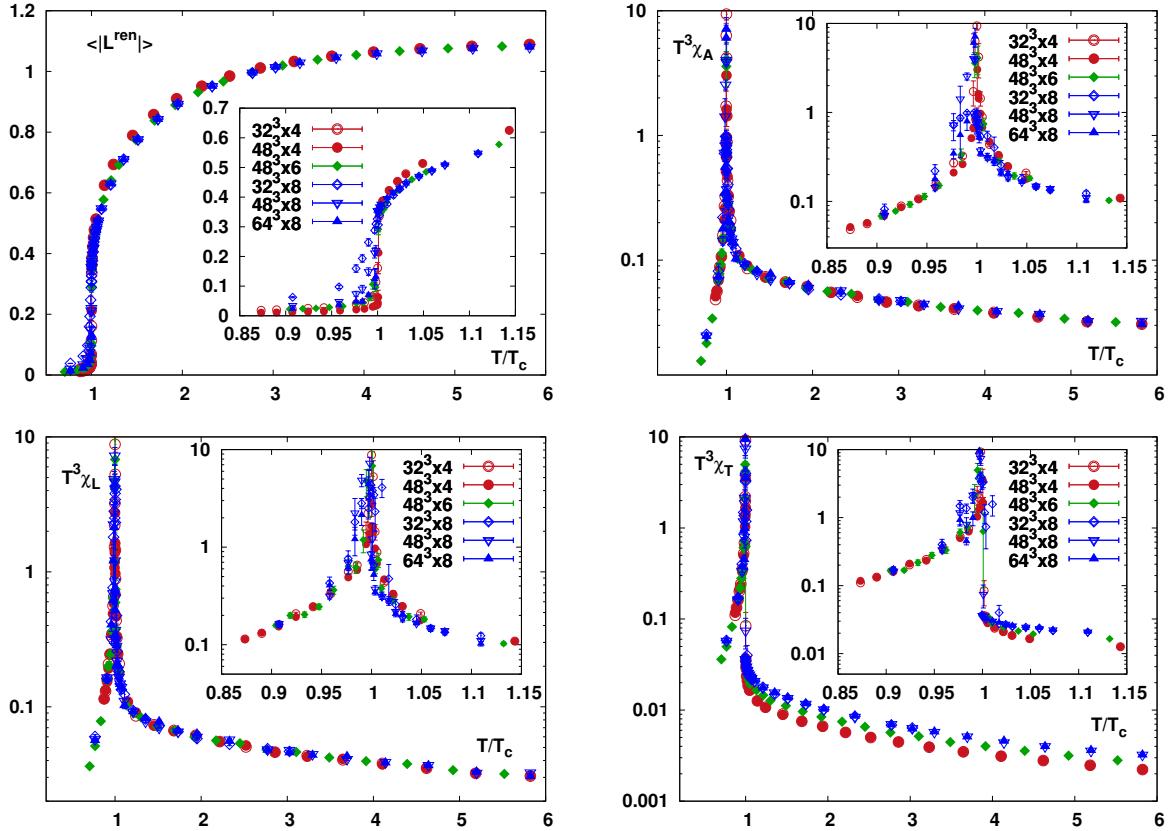
We now interpret their limiting values, deep in the confining and deconfining phases, based on general symmetry arguments and the characteristics of Polyakov loop distribution function. Consider first the confining phase at low temperatures. Due to the  $\mathcal{Z}_3$ -symmetric vacuum, the expectation value of any symmetry breaking operator must vanish. In particular,

$$V(\langle \tilde{L}^2 \rangle - \langle \tilde{L} \rangle^2) = \chi_L - \chi_T = 0. \quad (7)$$

It follows that  $R_T = 1$  in the confining phase.

In the same temperature range,  $R_A$  closely follows the Gaussian distribution result of  $(2 - \pi/2) \simeq 0.429$  [12]. This suggests that the quadratic term in the effective action contributes dominantly to the susceptibilities deep in the confining phase. However, we expect non-Gaussian terms to be crucial in determining higher-order cumulants.

At high temperatures,  $\mathcal{Z}_3$  symmetry is spontaneously broken. This has been shown to yield  $R_A \simeq 1$  [12]. On the other hand,  $R_T$  is not restricted by symmetries. Our result indicates a



**Figure 1.** The temperature dependence of the modulus of renormalized Polyakov loop and its susceptibilities in the SU(3) pure gauge theory.

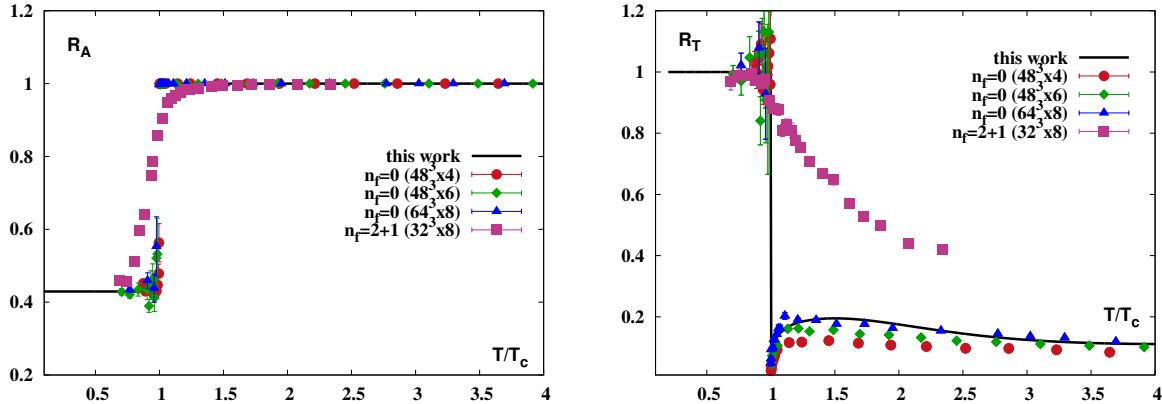
small value for this ratio. In the language of an effective Polyakov loop potential, this finding suggests that, around the global minimum associated with the symmetry-broken vacuum, the local curvature along the transverse (imaginary) direction is much steeper than that along the longitudinal (real) direction. We note, however, that a residual  $N_\tau$  dependence remains in our results. Therefore we cannot yet draw any firm conclusions about the continuum extrapolation of this quantity.

In the presence of light quarks, the Polyakov loop is no longer an order parameter for deconfinement. Due to the explicit breaking of  $\mathcal{Z}_3$  symmetry, the ratios are smoothed and vary continuously across the pseudocritical temperature.  $R_A$  interpolates between the two limits set by the pure gauge theory, joined by a crossover region. We expect the width of this crossover to depend on the number of flavors and the values of quark masses. We also observe that the value of  $R_T$  at high temperatures deviate substantially from the pure gauge limit. For this there is as yet no good theoretical understanding.

Further work is needed for the physical interpretations of these results, and to analyze in detail the systematic uncertainties involved in the extractions of these quantities.

#### 4. Conclusions

Polyakov loop susceptibilities and their ratios provide excellent signals for deconfinement phase transition in the pure gauge system. Most of their features can be related to  $\mathcal{Z}_3$  center symmetry and its spontaneous breaking. In this sense, they are the natural composite operators to study deconfinement.



**Figure 2.** Lattice results on the ratios of Polyakov loop susceptibilities,  $R_A = \chi_A/\chi_L$  and  $R_T = \chi_T/\chi_L$ , for the pure gauge system and (2+1)-flavor QCD. The temperature is normalized to its (pseudo)critical value for the respective lattice. The lines are results from the Polyakov loop model proposed in Ref. [10].

Many effective potentials have their model parameters tuned to match the lattice data of thermodynamic pressure and Polyakov loop [13, 14, 15, 16]. Fluctuation effects, on the other hand, have yet to be included. The new susceptibility results presented here can help to better constrain these seemingly arbitrary model parameters [10].

Our preliminary results for (2+1)-flavor QCD suggest that ratios of susceptibilities are considerably smoothed in the presence of light quarks. Also, the value of  $R_T$  in the high temperature phase deviates substantially from the pure gauge limit. To understand these issues, theoretical inputs are essential. In particular, it is important to understand the quark mass and flavor dependence of the Polyakov loop and the susceptibilities. An exploratory first step would be to include a small explicit breaking term in the pure glue potential and study how the picture changes from the pure gauge theory.

For the lattice calculations, more work is needed for the robust extractions of these gluonic correlation functions, as well as for the detailed understanding of their systematic uncertainties.

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