LESSONS FROM WEAK DECAYS OF HEAVY QUARKS

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ABSTRACT: Recent experimental results on D-, F- and B-decays are interpreted on the basis of the valence quark model. Transition form factors of currents from heavy mesons to much lighter ones are calculated using relativistic wave functions. The semileptonic decay spectra are mainly determined by a few exclusive channels. The detection of the decay mode $B^- \rightarrow \rho^o e^- \bar{\nu}$ will be essential for obtaining $|V_{ub}/V_{cb}|$. For nonleptonic decays generally good results are obtained using factorization with little direct annihilation. Small bare amplitudes can be fed by stronger amplitudes through channel mixing caused by strong interaction, an effect which possibly simulates in some instances a direct weak annihilation process. To some extent this proposition can be tested in F^+-decays.

1. Introduction Recently, a wealth of new data on D-, F- and B-decays became available[1-3]. First theoretical interpretations of these results could be performed and gave new insights[4-6]. The valence quark picture appears to work in semileptonic as well as nonleptonic transitions. For detailed theoretical estimates, model assumptions for the bound state wave functions of mesons consisting of a heavy and a light quark have to be made. Exclusive semileptonic channels are best suited to test these assumptions and to learn more about the meson structure. Nonleptonic decays are notoriously difficult to calculate because of the influence of long-range QCD forces. It seems, however, that at least the energetic two-body transitions are describable by a factorized form of the amplitude[4,6-8] although final state interactions complicate the picture through phases and channel mixing. A channel, for which the bare amplitude is small or zero, can be fed through strong interaction by channels with a big amplitude. A detailed theoretical treatment is not possible in this latter case but the data suggest the presence of such effects, as we will see. Exclusive nonleptonic decays also provide information on short distance QCD coefficients and their relative signs and shed light on the origin of the lifetime difference between D^o and D^+. Applications to B-decays can be used to obtain more information on current matrix elements between very heavy and light systems, on the corresponding QCD coefficients and on the important Kobayashi-Maskawa matrix elements V_{ub} and V_{cb}. The present report is based on results obtained in collaboration with Manfred Bauer and Manfred Wirbel. A more detailed publication is in preparation[9].

2. Semileptonic Decays From the form of the inclusive lepton spectrum in D- and B-decays the V-A structure is qualitatively established. The finer details of the spectrum especially at the upper end can only be estimated considering exclusive channels[5]. In an exclusive calculation at least the correct spins, masses, Lorentz structure and phase space are used while an inclusive quark decay treatment does not account for the quark confinement in specific hadronic final states. The decisive input and testing ground is given by the form factors of the hadron matrix elements of the weak current which reflect the bound state structure of initial and final mesons. Unfortunately there is little knowledge about bound state wave functions of mesons consisting of a heavy and a light quark. A relativistic treatment is necessary since the average transverse momentum within the hadron is comparable to the constituent mass of the light quark. Bauer, Wirbel and the

author used therefore a relativistic oscillator wave function at infinite momentum[5)]:

$$\phi \sim \sqrt{x(1-x)}\ \exp(-p_T^2/2\omega^2)\ \exp(-\frac{m^2}{2\omega^2}\ (x - \frac{1}{2} - \frac{m_{q_1}^2 - m_{q_2}^2}{2m^2})^2) \tag{1}$$

$$\omega^2 = \langle p_T^2 \rangle$$

Here m is the meson mass, $m_{q_{1,2}}$ the consistent quark masses and x the longitudinal momentum fraction. For the parameter ω which fixes the average transverse momentum we took $\omega \approx 400$ MeV which fitted well the $D \to Ke\nu$ $D \to K^*e\nu$ Mark III results [1,5)]. The wave function at infinite momentum of the simple form (1) is applicable to calculate the form factors at zero momentum transfer $q^2 = 0$ and gives the overlap factors (h) between mesons of different quark masses. In table 1 a few of these overlap factors are tabulated.

Table 1 Overlap factors h_1 at $q^2 = 0$

$K \to \pi$	$D \to K$	$D \to \pi$	$F \to K$	$B \to D$	$B_s \to K$	$B \to \pi$
0.99	0.76	0.69	0.64	0.69	0.25	0.33

We note that the overlap between a B-meson and a π- and K-meson at $q^2 = 0$ is rather small. Their values are sensitive to the precise form of the wave functions near x = 1. These small values and the ones for $B \to \rho$ not shown in table 1 may therefore by subject to an error of about 30-40 %. For the q^2-dependence of the form factors single pole dominance has been used for simplicity. The model gives good results for $K \to \pi$, $D \to K$, $D \to K^*$, where it could be tested against experiments. I refer to the original publication for details[5)]. For semileptonic B-decays the model predicts

$$\frac{\Gamma_{s.l.}(B \to D^*)}{\Gamma_{s.l.}(B \to D) + \Gamma_{s.l.}(B \to D^*)} \approx 0.73$$

The experimental number[3)] has still large error bars, it is 0.85±0.32. An important exclusive channel in B-decays is $\bar{B} \to \rho^o e^- \bar{\nu}$ which we recommend for finding the Kobayashi-Maskawa matrix element $|V_{ub}|$. The lepton spectrum for $B \to \rho$ peaks rather sharply at 2.3 GeV. The calculation for the total width gives $\Gamma_{s.l.}(B \to \rho)/\Gamma_{s.l.}(B \to D, D^*) \approx 1 \cdot |V_{ub}/V_{cb}|^2$.

Another consequence of the use of the wave function (1) are estimates of the ratio of meson decay constants. We find

$$f_D/f_K \approx 0.75, \; f_B/f_K \approx 0.5 \quad \text{and} \; f_{B_S}/f_L \approx 0.5 \tag{2}$$

In two recent articles Grinstein, Isgur, and Wise[10] and Nussinov and Wetzel[11] also calculated exclusive semileptonic decays. In ref. (10) nonrelativistic mock wave functions are used to calculate the form factors for mesons with momenta near $q^2 = (M_i - M_f)^2$. Ref. 11 contains a more general discussion. The results of these calculations are in qualitative agreement with our earlier ones but differ from it in some details.

3. Nonleptonic D- and F-Decays The starting point for the treatment of nonleptonic decays is the short distance QCD corrected effective quark Hamiltonian. It contains charged and neutral current products multiplied by QCD coefficients $c_{1,2} = (c_+(\mu) \pm c_-(\mu))/2$. This Hamiltonian gives rise to a number of flavour flow diagrams corresponding to various quark decay and quark annihilation processes[12,13]. In general, however, this classification is not detailed enough to relate different decays. Even though QCD forces are flavour independent, the formation of the final mesons depends on the masses and spins involved, on different nonperturbative effects, and final state interactions.

To proceed, a working hypothesis is needed. We use - at least for energetic two-body decays - the factorization assumption[14]. It consists of replacing one of the quark currents in the current products by the asymptotic field of hadrons carrying the same quantum numbers as the current. The amplitudes are then determined by hadron matrix elements of single currents, i.e. by hadron currents which are in principle observable in leptonic and semileptonic decays. As a result, the weak interaction is approximated by a product of hadron currents. For instance, for D-decays one has[4]

$$\mathcal{H}_{eff} \simeq \frac{G}{\sqrt{2}} : \left(a_1 (\bar{u}d')_H (\bar{s}'c)_H + a_2 (\bar{s}'d')_H (\bar{u}c)_H \right) : \tag{3}$$

The index H stands for hadron currents and the real and scale-independent coefficients a_1, a_2 replace the scale-dependent QCD coefficients $c_1(\mu)$, $c_2(\mu)$. It has been pointed out by Buras, Gérard, and Rückl[6,15] that factorization as assumed here indeed follows to leading order in an 1/N expansion where N is the number of quark colours. Thus, there is some theoretical justification for the factorization approach. On the other hand, leading order of 1/N predicts no final state interaction among the outgoing hadrons while we know that final state interaction cannot be neglected. In $D \to K\pi$-decays the phase difference between I = 1/2 and I = 3/2 amplitudes is $\approx 80^o$ [7,8]. In the following factorization is used but we have to be aware of final state interaction effects. We take it into account in cases where the phases are known.

Factorization predicts only very small contributions from annihilation-type diagrams for energetic two-body decays, since here form factors at momentum transfer much higher than the particle masses are involved. We neglect "annihilation" for the results given in this section and return to it in section 4.

The two parameters a_1 and a_2 in eq. (3) can be fitted from $D \to K\pi$ decays using the Mark III results[1]. The above mentioned phase difference between isospin amplitudes is taken into account, but no inelasticity. The result is

$$a_1 = 1.28 \pm 0.1 \qquad a_2 = -0.5 \pm 0.1 \quad \text{and} \quad a_2/a_1 = -0.4 \pm 0.1 \tag{4}$$

In contrast to the absolute values of a_1, a_2 the ratio a_2/a_1 is not sensitive to the Mark III normalization of branching ratios, the D-lifetimes and the assumption of no inelasticity[F1]. The negative sign demonstrates the negative interference of two amplitudes in important D^+-decays. In Fig. 1 we represent the experimental branching ratios (with error bars corresponding to statistical errors) together with the theoretical values using $a_1 = 1.3$ and $a_2 = -0.55$. Isospin phases from final state interaction are only taken into account in $D \to \bar{K}\pi$, $D \to \bar{K}\rho$ and $D \to \bar{K}^*\pi$ transitions. The overall agreement is quite remarkable, considering the simplicity of the model and the fact that D-decays occur in a resonance region. To discuss some of the discrepancies between theory and experiment seen in Fig. 1 we note that the channels $\bar{K}^*\pi$ and $\bar{K}\rho$ can easily mix by final state interaction. Indeed, the results for the square of the isospin amplitudes improve if we sum over the two channels as one would expect if channel mixing takes place. The relatively large value of the $\bar{K}\omega$ amplitude could be due to a contribu-

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[F1] An early estimate of a_2/a_1 was obtained by Y. Koide in 1979[16].

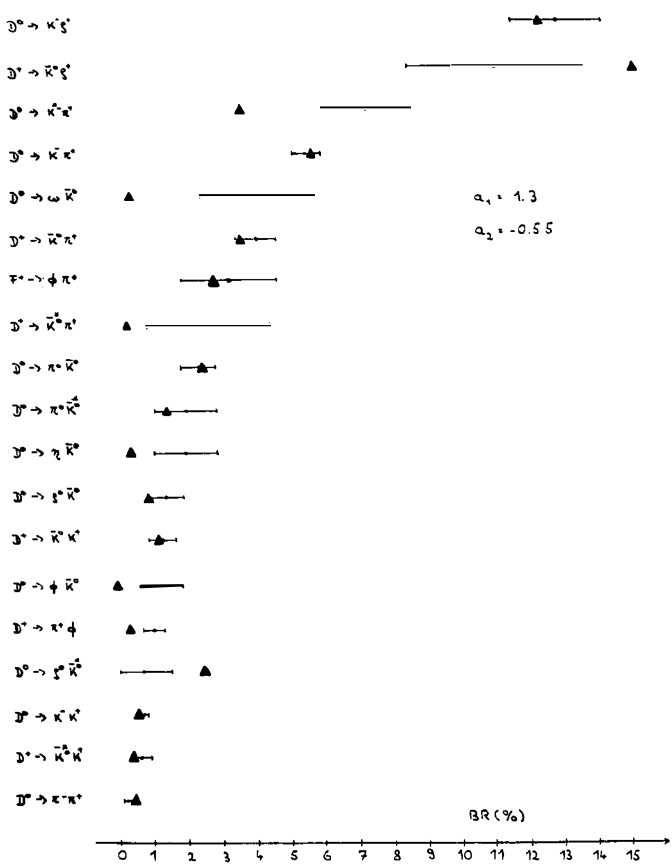

Fig. 1 Comparison between experimental and theoretical D and F branching ratios. Only statistical errors are indicated. The theoretical values denoted by ▲ are from factorized amplitudes with no annihilation contribution.

tion from the mixing of this weak channel (proportional to a_2) with the stronger I = 1/2 part of $\bar{K}^*\pi$ and $\bar{K}\rho$ channels by strong interaction. I = 1/2 channel mixing could also be responsable for the "annihilation process" $D \rightarrow \bar{K}\phi$ as suggested by Donoghue[17)] (section 4).

The success of our description of two-body decays encouraged us to compute many more two-body decays and summing them up[7,9]. This estimate can at best give an orientation since for transitions with little energy release, $D \to \bar{K}^* \omega$ for instance, the factorization approximation is very doubtful. Final state interaction effects, on the other hand, cancel to some extent in the sum because of the unitarity of the strong interaction S-matrix. It turns out that about 70 to 80 % of the total nonleptonic transition rates of D^o and D^+ can be accounted for by two-body decays. More important, the calculated ratio $\Gamma(D^o \to xy)/\Gamma(D^+ \to xy)$ for these nonleptonic two-body decays turned out to be ≈ 3. Thus, a sizeable part of the lifetime difference between D^o and D^+ could come from two-body decays due to the destructive interference ($a_2/a_1 < 0$) in several important D^+-decays[12,18]. Note that no annihilation contribution has been used for this estimate.

For the coefficients a_1 and a_2 one expects from factorization

$$a_1 = c_1 + \xi c_2, \quad a_2 = c_2 + \xi c_1, \quad c_1 \simeq 1.21, \quad c_2 \simeq -0.43 \tag{5}$$

where the charmed quark mass ($\simeq$ 1.5 GeV) is taken for the scale parameter μ. The parameter ξ expresses the colour mismatch for hadron formation: $\xi = 1$ corresponds to no colour suppression and $\xi = 1/N = 1/3$ to a colour mismatch expected from naive colour counting. $\xi = 0$ corresponds to maximal colour suppression. In this latter case only the quarks from the same charged or neutral current combine to form a hadron[4,7] in accord with the leading 1/N contribution[15]. As seen from the numbers in (4) and (5), $\xi = 1$ and $\xi \simeq 1/3$ are excluded while for $\xi \simeq 0$ agreement is reached.

4. Quark Annihilation. Weak decays can also proceed by the simultaneous annihilation of the two valence quarks in the decaying hadron and the subsequent creation of two other quarks which then hadronize. It is of great interest to know to what extent this process is important in two-body decays. On the level of hadrons the relevant graphs for Cabibbo allowed transitions are shown in Fig. 2. It should be noted that hadron diagrams in which the strong vertex proceeds the weak vertex are with regard to topology quark decay graphs and not quark annihilation graphs. More important, the corresponding contributions from heavy intermediate states are already contained in the factorization approach described in the previous section as a form factor effect.

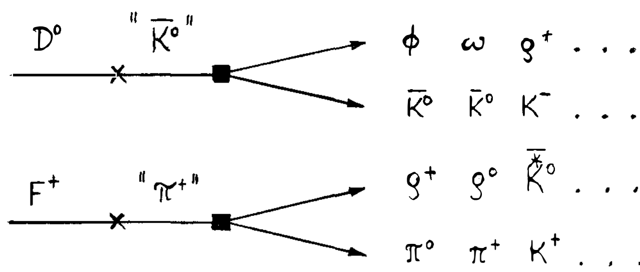

Fig. 2 Annihilation diagrams at the level of hadrons. The cross denotes the weak transition to virtual states with quantum numbers of $\bar{K}^o$ in D-decays and π^+ in F^+-decays, which then decay strongly.

It has been noted that the process $D^o \to \phi\bar{K}$ can only proceed via annihilation[19]. The experimentally observed branching ratio is $\approx 1\%$[20]. Because the D-decay channels in Fig. 2 are all related by flavour SU3, one can conclude from this number that annihilation can in general not be a dominant mechanism, at least not in $D^o \to K^-\rho^+$, $\bar{K}^{*-}\pi^+$ decays. Using the graph of Fig. 2 for $\phi\bar{K}^o$ with $\bar{K}^o$ as an intermediate state the fitted value for $\langle\bar{K}^o|H_w|D^o\rangle$ is rather big[21]. If a similar large number would hold for $\langle\pi^+|H_w|F^+\rangle$ the modes $F^+ \to \rho^o\pi^+$, $\rho^+\pi^o$ would dominate F-decays. This can hardly be the case. It seems more likely that the process $D^o \to \phi\bar{K}$ is caused by channel mixing[17] in a resonance scattering process. It can proceed via the large amplitudes $D \to (\bar{K}\rho)_{I=1/2}$, $(\bar{K}^*\pi)_{I=1/2}$ and $(\bar{K}^*\rho)_{I=1/2}$ from which the $\phi\bar{K}$ state can be reached by strong interaction[F2]. The corresponding diagram is shown in Fig. 3.

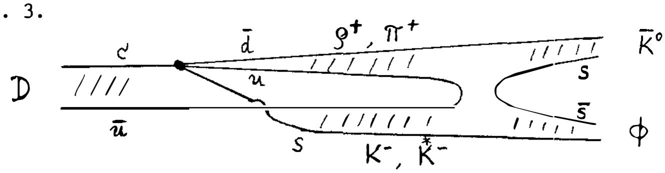

Fig. 3 The decay $D \to \bar{K}^o\phi$.
The weak process proceeds via "quark decay" followed by a strong interaction reaction with quark annihilation and creation.

[F2] This chain differs from the one suggested by Donoghue[17] $D^o \to \eta\bar{K}^{*o} \to \phi\bar{K}$, which is in our scheme not effective enough.

Since the time scale of weak interaction ($1/m_W$) and the time scale of strong interaction ($1/\Lambda_{QCD}$) differ drastically, the process of Fig. 3 has little to do with weak quark annihilation in its usual meaning. Only with regard to flavour flow topology it is still an annihilation process. The idea of channel mixing and little direct annihilation needs to be tested, of course. F-decays are very interesting in this respect: Compared to the bare amplitude from factorization, we expect an enhancement of $\bar{K}^{*o}K^+$ through its mixing with the $\eta\rho^+$, $\phi\rho^+$ channels by quark exchange scattering while the $\rho\pi$-decay mode should remain suppressed.

5. Nonleptonic B-Decays The method discussed in section 3 can easily be applied to nonleptonic B-decays. In table 2 a few important branching ratios are shown in terms of V_{ub}, V_{cb} and the two parameters a_1, a_2 relevant for B-decays. Table 2 was obtained using the factorization approximation (without annihilation) and with the help of overlap factors discussed earlier.

Table 2 Expected branching ratios from factorization for $\bar{B}^o$-decays taking $\tau_B = 1.2 \times 10^{-12}$ sec.

$\bar{B}^o \to$	$D^+\pi^-$	$0.5\ a_1^{\ 2}$	$\cdot\ \lvert V_{cb}/0.05\rvert^2$	%
	$D^{*+}\pi^-$	$0.4\ a_1^{\ 2}$	$\cdot\ \lvert V_{cb}/0.05\rvert^2$	%
	$D^{*+}\rho^-$	$1.2\ a_1^{\ 2}$	$\cdot\ \lvert V_{cb}/0.05\rvert^2$	%
	$D^{*+}F^{*-}$	$2.0\ a_1^{\ 2}$	$\cdot\ \lvert V_{cb}/0.05\rvert^2$	%
	$\bar{K}^o\ J/\psi$	$1.0\ a_2^{\ 2}$	$\cdot\ \lvert V_{cb}/0.05\rvert^2$	%
	$\bar{K}^{*o}\ J/\psi$	$4.4\ a_2^{\ 2}$	$\cdot\ \lvert V_{cb}/0.05\rvert^2$	%
	$\pi^+\pi^-$	$0.17\ a_1^{\ 2}$	$\cdot\ \lvert V_{ub}/0.05\rvert^2$	%
	$\pi^+\rho^-$	$0.4\ a_1^{\ 2}$	$\cdot\ \lvert V_{ub}/0.05\rvert^2$	%

The numbers in the last four entries of this table depend on the square of small overlap factors which are particular sensitive to the tails of the meson wave functions. Thus, the theoretical error can be a factor $\simeq 2$ in this case, but of course less for the relative rates of these decays.

At the scale of the B-meson one has $c_1 \simeq 1.1$ and $c_2 \simeq -0.24$. The value of a_1 is not very sensitive to the precise value of the colour suppression factor ξ. According to the table and taking $|V_{cb}| = 0.05$ we thus expect for the decay $\bar{B}^o \to D^{*+}\pi^-$ a branching ratio of ≈ 0.5 %. This value is lower than the number measured by CLEO[22], but in agreement with the smaller value mentioned by Argus[2]. The value of a_2 strongly depends on the amount of colour suppression. $\xi \approx 1/3$ would give practically zero. From our experience in D-decays we expect, however, $a_2 \approx -0.24$ to -0.3 compatible with $\xi \approx 0$. For this latter case the decay $\bar{B}^o \to \bar{K}^{*o}\ J/\psi$ is predicted to have a branching ratio of ≈ 0.3 % with the above mentioned error of about a factor 2. A first restriction on $|V_{ub}|$ from nonleptonic decays can be obtained from the limit[3] $BR(\bar{B}^o \to \pi^+\pi^-) < 0.02$ % giving $|V_{ub}| < 0.02$.

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