Eigensolution of the various potentials and its application in different fields

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Summary

The exact or approximate solutions of the Schrodinger, Dirac, and Klein–Gordon equations can be solved using either analytical or numerical techniques. The importance of these equations in nuclear and particle physics, statistical physics, solid-state physics, quantum field theory, and molecular physics has piqued the interest of researchers in finding solutions for a variety of potentials in both relativistic and non-relativistic regions. To solve the various potentials, a variety of strategies are available. The plinth of the study we are acquainted with depends on 4 pillars namely Equations, Potentials, Methodology, and applications in areas of physics or chemistry. These methods are the Nikiforov-Uvarov (NU) method [1], Parametric Nikiforov-Uvarov (pNU) method [2], Supersymmetric quantum mechanics (SUSYQM) [3], Proper Quantization Rule method (PQR) [4], Asymptotic Iterations Methods (AIM) [5]. In our study, we propose six types of different potentials such as (1) Screened cosine Kratzer potential $V(r) = -2D_e\left(\frac{a}{r}-\frac{b}{2r^2}\right)e^{-\alpha r}\cosh\delta\alpha r$ [6] (2) Modified Yukawa-Kratzer potential $V(r) = \frac{a_1 e^{-2\alpha r}}{r^2} - \frac{a_2 e^{-\alpha r}}{r} + a + D_e - \left(\frac{A_1}{r}-\frac{A_2}{r^2}\right)$ [7](3)Linear plus modified Yukawa Potential $V(r) = A_1 r + \frac{A_2 e^{-\alpha r}}{r} - \frac{A_3 e^{-2\alpha r}}{r^2} + A_4$ [8] (4) Hulthen-screened cosine Kratzer potential $V(r) = -\frac{V_0 e^{-\alpha r}}{1-e^{-\alpha r}} - 2D_e\left(\frac{a}{r}-\frac{b}{2r^2}\right)e^{-\delta\alpha r}\cosh\delta\lambda\alpha r$ [9] (5) Extended Hulthen Yukawa with Inverse Square and Columb term plus Ring Shape potential $V(r,\theta) = \frac{(A+kC)e^{-2kr}}{(1-e^{-kr})^2} - \frac{Be^{-kr}}{(1-e^{-kr})r} + \frac{Dk\ e^{-kr}}{1-e^{-kr}} + \frac{J}{r} + \frac{G}{r^2} + F + \frac{\left(a\cos^2\theta+b\cos\theta+c\right)}{r^2\sin^2\theta}$ [10] (6) Attractive Radial Potential plus class of Yukawa potential $V(r) = \frac{\left(ae^{-2\alpha r}+be^{-\alpha r}+c\right)}{(1-e^{-\alpha r})^2} - \frac{A}{r} + \frac{Be^{-\alpha r}}{r} - \frac{Ge^{-2\alpha r}}{r^2}$.[11] We were able to solve the Schrodinger equation and the D-dimensional Klein-Gordon equation for the SCKP, HSCKP, EHYIC-RSP, MYKP, ARP-CYP, and LIMYP. We calculate the bound state energy eigenvalues and corresponding normalized wave functions using the extended N-U method, PQR method, SUSYQM methods, and AIM method. By adjusting the potential parameters of the SCKP, HSCKP, and MYKP, we deduced potentials and their energy eigenvalues such as screen Kratzer potential, the standard Kratzer potential, generalized Yukawa potential, screened Coulomb potential or Yukawa potential and Coulomb potential, inversely quadraticYukawa potential, Hellmann potential, generalized cosine Yukawa potential, and Hulthén potential. Obtained the energy spectrum for SCKP and MYKP under the effect of the magnetic field and Aharanov–Bohm flux field via the pNU method we also derive a series expansion method for MYKP and findings are consistent with the pNU method. We obtained various properties of the SCKP such as thermodynamical properties (partition function, vibrational mean energy, vibrational mean free energy, vibrational specific heat capacity, and vibrational entropy. We obtained rotational–vibrational energy for a few heterogeneous and homogeneous diatomic molecules in three dimensions. The numerical results obtained for LiH, HCl, NO, and I_2, O_2 diatomic molecules are in very good agreement with the results previously obtained by others. For Linear plus modified Yukawa poten-

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tial we obtain Heavy-Heavy and Heavy-Light flavor mesons for the K-G equation via the NU method. LIMYP has successfully calculated the mass spectra of Heavy-Heavy and Heavy-Light flavor mesons. We have successfully calculated the mass spectra of all heavy-light mesons (HLMs) combination sets utilizing the combined potential framework, where the potential was used to derive the energy eigenvalue using the linear plus modified Yukawa potential. The mass spectra data has a high degree of similarity compared to experimental data that is currently known, as well as the minimum percentage error compared to other theoretical study data. This study has demonstrated the significance of non-relativistic correction in a proposed model for accurate spectroscopic parameter prediction for the $c\bar{s}$, $c\bar{q}$, $b\bar{s}$, $b\bar{q}$ mesons. To quantify this HLM precisely in the future, more experimental work will be required. A forthcoming experimental facility $\bar{\text{P}}$ANDA and other experimental facilities like BABAR, Belle, and LHCb will be in a special position for that. The potential also utilized in this study can also be useful in nuclear particle physics (decay properties for heavy-heavy and heavy-light mesons), atomic and molecular physics, hot and dense QCD media, etc.

Results and Discussion

We present numerical results and graphical presentations of the SCKP. Graphical representations of the effective potential, energy spectra, and thermodynamical properties with respect to various parameters. Using the different methods, we obtain the eigenspectrum and momentum for time-independent and time-dependent Hulth'en-screened cosine Kratzer potentials. Also, we proposed and solved the extended Hulthen-Yukawa with inverse square and Coulombic term plus ring shape potential (EHYICRSP). The Schrödinger equation has been solved in two dimensions for the modified Yukawa-Kratzer potential (MYKP) under the influence of the magnetic field and the Aharanov-Bohm flux field (external fields). We proposed attractive radial potential plus class of Yukawa potential and obtained the rotational vibrational partition function from the energy eigenvalues using the asymptotic iteration method . The bound state solution of the K-G equation has been successfully used to determine the mass spectra of $c\bar{c}$, $b\bar{b}$, $b\bar{c}$ and $c\bar{s}$, $c\bar{q}$, $b\bar{s}$, $b\bar{q}$ for states ranging from $1S$, $2S$, $1P$, $2P$, $3S$, $4S$, $1D$, $2D$, and $1F$ by using energy eigen value which includes linear plus modified Yukawa potential, which finds good co-relation of mass spectra with the experimental data as well as some recent research.

References

[1] Kaushal Purohit, A. K. Rai, R. H. Parmar, AIP Conference Proceedings 2220,(1):120004 (2019)

[2] R. H. Parmar, Kaushal Purohit, A. K. Rai, AIP Conference Proceedings 2220,(1):140071(2019)

[3] Kaushal Purohit, A. K. Rai, R. H. Parmar, Proceedings of the DAE Symp. on Nucl. Phys. **64** 661(2019)

[4] Kaushal Purohit, A. K. Rai, R. H. Parmar, XXIV DAE-BRNS Symposium on high Energy Physics. (2020)

[5] Kaushal Purohit, A. K. Rai, R. H. Parmar, Proceedings of the DAE Symp. on Nucl. Phys. **66** 533 (2021).

[6] Kaushal Purohit, R. H. Parmar, A. K. Rai, Eur. Phys. J. P. **135**, 3 (2020).

[7] Kaushal Purohit, R. H. Parmar, A. K. Rai, Annals of Physics **424**, 12:168335 (2020).

[8] Kaushal Purohit, R. H. Parmar, A. K. Rai, Journal of Molecular Modeling **27** 358 (2021)

[9] Kaushal Purohit, Pooja Jakhad, A. K. Rai, Physica Scripta **97** 044002 (2022).

[10] Kaushal Purohit, R. H. Parmar, A. K. Rai, Journal of Mathematical Chemistry 60,9 (2022).

[11] Kaushal Purohit, R. H. Parmar, A. K. Rai, Indian J Phys. **doi :/doi.org/10.1007/s12648-023-02852-3**