Light propagation through cosmic structures using cosmological simulations

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We discuss numerical investigations of the effects of cosmological inhomogeneities on light propagation. A new algorithm directly integrating the geodesics equations has been developed and has been run on very large scale cosmological simulations up to a redshift of $z = 30$. Comparisons have been made for several dark energy models and for several observers, revealing percent-level discrepancies between the angular diameter distance interpreted in a friedmannian background, and the angular diameter distance computed from geodesics integration. Here, we focus on the methodology that has been deployed to obtain quantitative results in cosmological simulations.

Keywords: Inhomogeneous cosmology; large scale structures; light propagation; simulation.

1. Introduction: Inhomogeneous cosmologies and numerical simulations

Historically, cosmology has been developed on the top of an idea of symmetry, taking the form of the cosmological principle, and making the equations of general relativity tractable. However, large scale redshift surveys such as the 2dF Galaxy Redshift Survey (see Ref. 1) and the Sloan Digital Sky Survey (see Ref. 2) have revealed a cosmic web structure with galaxies and galaxy clusters distributed in filaments, walls and superclusters, thus breaking the symmetry locally. These local inhomogeneities of matter induce a local metric slightly different from the global averaged metric. This has directly observable consequences, and weak lensing (see Ref. 3) is one of them. Other consequences include dynamic effects, as stated by the backreaction conjecture (see Ref. 4). The whole question about these dynamic effects is not whether they exist or not. Taking an averaged metric first and evolving it does not lead to the same result as evolving an inhomogeneous Universe and taking its averaged metric afterwards. The two procedures are not mathematically equivalent. Instead of that, the whole question is whether the differences between the two procedures are significant, or if they can be completely neglected. Answering this question has implications that go far beyond the sole study of the dynamical backreaction conjecture. Even if the dynamic effects are negligible, quantifying the kinematic effects of inhomogeneities on the propagation of light is of primary importance for the interpretation of cosmological data and the accurate estimation of cosmological parameters.

However, one of the main problem the above-mentioned quantification is facing is that large scale structure formation is a highly non-linear process. Consequently, even if pure analytical calculations allow to explore the effects discussed here, observations and numerical computations will eventually be required to bring a final answer to the problem of backreaction. Observationally, measuring the redshift drift (see Ref. 5) would allow to directly probe the expansion rate of the Universe in a model-independent manner. Numerically, fully general relativistic simulations of structure formation would provide a unique way of understanding, testing and characterizing both the kinematic and dynamic aspects of background-free inhomogeneous cosmologies. But making such fully relativistic cosmological simulations possible will require to solve theoretical, algorithmic and computational challenges.

Present day large scale cosmological simulations are mostly based on a splitting between a homogeneous, averaged, metric background, and an inhomogeneous foreground of matter (see Ref. 6). In this approach, the background evolution deduced from the value of cosmological parameters can be pre-computed separately from the structure formation. Then, starting from a matter distribution statistically compatible with CMB data, the collapse of matter into larger and larger structures can be followed using cosmological N-body codes that are solving the Vlasov-Poisson equations in an expanding background. By using a background expansion based on a cosmological constant or alternative models of dark energy, and by simulating the purely gravitational dynamics of a set of N dark matter particles, with or without additional baryonic physics, these simulations are able to reproduce the large scale properties of the Universe. But while they allow to study the formation of structures, they also allow to study how light propagates through these structures, thus offering opportunities to analyze the kinematic effects of inhomogeneities in the weak-field approximation without requiring fully relativistic cosmological simulations. In the following, we focus on the design and implementation of such analyses through our experience of raytracing within the weak-field limit in the Full Universe Run simulation set (see Ref. 7 and 8).

2. Redshifts, cosmological distances and raytracing

Being able to associate the redshift of a source to a cosmological distance, plays a central role in observational cosmology. In a traditional Hubble diagram, the redshift is directly measured from the light coming from the source, while the luminosity distance can be calculated from both the measured apparent magnitude and the inferred absolute magnitude of that same object. It establishes a $z\left(D_L\right)$ relation, or equivalently a $z\left(D_A\right)$ relation, with the angular diameter distance D_A being related to the luminosity distance through the Etherington's reciprocity relation, independently from any underlying cosmology. As in a FLRW metric, the redshift can be directly linked to the expansion factor, and as different set of cosmological parameters lead to different $D_A\left(z\right)$, the cosmological parameters can be reversely inferred from an observed $D_A\left(z\right)$ relation. However, this last approach suppose that the observational data are interpreted within a perfectly homogeneous framework. And even if the Universe is not strictly homogeneous in practice, most cosmological data are still interpreted assuming that strict homogeneity which allows to directly link the redshift, the scale factor, the cosmological distances and the cosmological parameters in a very convenient manner.

But reality is more complex: the light that is used to probe the Universe not only contains information from sources, but also retains information on its trajectory between the place where the photons have been produced and where they have been analyzed. Or, in other words, light is affected by the inhomogeneities it encounters on its paths, creating effects such as gravitational lensing and the integrated Sachs-Wolfe effect (ISW, see Ref. 9). But while these effects can be used themselves as cosmological probes, they also break the simple laws linking redshift, the scale factor and cosmological distances in a pure FLRW metric, making the interpretation of observational data far more difficult. This is where the idea of simulating light propagation in cosmological simulations comes into play, in order to understand and to quantify the effects affecting these laws.

Simulations of the propagation of photons on cosmological distances have already been done in the past, for example to study the effects of weak lensing (see Ref. 10). However these simulations were generally focused on a particular effect, and were using approximations to simplify the study of that particular effect. But as simulating the formation of large scale structures already requires an accurate computation of their gravitational field, a more generic approach can be used to compute null geodesics with the same level of accuracy as the trajectory of dark matter particles, directly using the value of the gravitational potential. This is the approach we developed and implemented in order to analyze the Full Universe Run.

3. Building light-cones from the Full Universe Run

The Full Universe Run, a subset of the Dark Energy Universe Simulation project, is a series of three simulations that were run on the entire Curie supercomputer, in 2012 (see Ref. 7). Each of these simulation followed the gravitational collapse of 8192^3 dark matter particles in a volume of $(21\ \mathrm{Gpc/h})^3$ throughout the entire history of the Universe and up to $z = 0$. The simulations are based on three different dark energy models: one is based on a Ratra-Peebles quintessence model (RPCDM) (see Ref. 11), one is based on the concordance model of cosmology (ΛCDM), and the last one is based on a phantom dark energy with an equation of state $w < -1$ (wCDM). Regarding the growth of structures, RPCDM tends to form less structures than ΛCDM, while wCDM leads to the formation of more massive structures. Moreover, they are all based on the same random phase to facilitate comparisons. Throughout the entire project, more than 1 petabyte of data were extracted. In top of usual snapshots and halo catalogs, we built, for the first time, accurate light-cones with both the matter and the gravitational field distribution, from $z = 30$ to $z = 0$. Three full sky light-cones for three virtual observers at $z = 0$ were extracted for each simulation, plus two extra ones for observers at different redshifts, making a total of 15 light-cones allowing unique comparisons between different histories of cosmic expansion.

The procedure followed to construct these cones is the following. Because the structure formation computation was already taking the entire machine memory, it was impossible to propagate hundreds of billions of photons during the run. Furthermore, to limit as much as possible numerical approximations while computing null geodesics, it was excluded to do the raytracing only in a pure post-processing phase from the few time steps that would have been saved. Instead, we opted for an hybrid approach which consisted in first extracting all the relevant data during the run for few predefined virtual observers, and then to propagate light using these preselected data sets. To do that, we had to assume that photon deviations would remain smaller than a given value so that a redshift z could be associated to a range of comoving distances $[\chi_{min}, \chi_{max}]$. Then, for every coarse time step of a simulation starting at t_{old} and ending at t_{new}, we can compute the value of the scale factor at these times, and therefore two redshifts, and two comoving distances ranges: $[\chi_{old_{min}}, \chi_{old_{max}}]$ and $[\chi_{new_{min}}, \chi_{new_{max}}]$. By extracting all the data in a spherical shell $[\chi_{new_{min}}, \chi_{old_{max}}]$ around a predefined observer at each time step, we end up with all the data required to compute null geodesics. In practice, it means that throughout a simulation, we are extracting concentric shells, starting at long distances and coming closer and closer to the observer while the structures are forming. These shells are overlapping each other to take into account light deflection. At the end of each run, shells can be merged, leading to a full sky light cone for each virtual observer, containing both the matter distribution and the gravitational field distribution in the Adaptive Mesh Refinement (AMR) grid used by the simulation code. Additionally to light cones, we saved the matter distribution and the gravitational potential in a subvolume corresponding to $1/512^{\text{th}}$ of the total box size, at each time step, in order to be able to relax and test the hypothesis of small deviations.

4. Computing null geodesics using backward raytracing

Using the above-mentioned light-cones, we can then propagate photons using backward raytracing, starting from the observer and going to higher and higher redshifts, up to $z = 30$. As we have a direct access to the gravitational potential Φ, we can directly integrate the geodesics equation:

$$\frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}\lambda^2} = -\Gamma^\alpha_{\beta\gamma} \frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} \frac{\mathrm{d}x^\gamma}{\mathrm{d}\lambda} \tag{1}$$

without other approximation than using a weak-field metric, compatible with our cosmological simulations:

$$\mathrm{d}s^2 = -c^2 \left(1 + 2\frac{\Phi}{c^2}\right) \mathrm{d}t^2 + a\left(t\right)^2 \left(1 - 2\frac{\Phi}{c^2}\right) \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) \tag{2}$$

Moreover, as Φ has been sampled at greater resolutions around large scale structures using an Adaptive Mesh Refinement strategy, we can use the exact same refinement grid to maximize the accuracy of the integrator around these structures. This has been easily achieved by setting the integration step at a fixed fraction of the current cell, regardless of its level of refinement, so that for a same distance, the integrator was producing more steps in a refined region than in a coarse one.

Even if this procedure seems conceptually simple, it represents a computational challenge in itself: sampling the whole light cone requires hundreds of thousands of photon trajectories in an AMR grid of hundreds of billion cells, using thousands of processors. To overcome this challenge, we developed a new library, MAGRATHEA, and a new code based on it, using new tree algorithms and advanced template metaprogramming techniques in C++11, to achieve both genericity and performance. The code was run on 8192 cores of the Curie supercomputer.

Our goal was to compute the relation $D_A(z)$ for a weakly perturbed metric corresponding to the large structures seen by virtual observers. However, the angular diameter distance cannot be computed from isolated, independent photon trajectories. Instead, we used a ray bundle approach, launching 8 photons at a time from the observer, distributed on a circle at $z = 0$. Depending on Φ and its derivatives, the shape of the circle is modified, allowing to study lensing effects, but also allowing to compute the angular diameter distance from the size of the circle x and its apparent angle θ on the sky at $z = 0$. In the same time, the perturbed redshift is computed at each point of a trajectory using its generic form:

$$1 + z = \frac{\nu_{\mathrm{S}}}{\nu_{\mathrm{O}}} = \frac{\left(g_{\mu\nu} k^{\mu} k^{\nu}\right)_{\mathrm{S}}}{\left(g_{\mu\nu} k^{\mu} k^{\nu}\right)_{\mathrm{O}}} \tag{3}$$

Launching hundreds of thousands of photons across the whole light-cone, we end up with an accurate sampling of $D_A(z)$ for a non strictly homogeneous metric, for several observers, and for several dark energy models.

5. Results and conclusion

We outlined the procedure we used to reconstruct the relation between the angular diameter distance and the redshift in a non-homogeneous context, building unique full sky light cones from $z = 0$ to $z = 30$ for several dark energy models, and deploying a very generic approach to integrate null geodesics while minimizing usual approximations. Our simulations led to a comparison of $D_A(z)$ for several observers, in ΛCDM, in a model less structured than ΛCDM, and in a model more structured than ΛCDM.

While the relative difference on the redshift between the homogeneous and the perturbed cases along a photon trajectory are of the order of 10^{-4} in average, the relative standard deviation between the homogeneous and inhomogeneous $D_A(z)$ are of the order of few percent: around 1% at $z = 1$ for RPCDM, around 2% for ΛCDM and around 3% for wCDM. Therefore, if measured, the power spectrum of the angular diameter distance can become itself a cosmological probe to put constraints on dark energy models.

But these few percent differences have more fundamental implications for observations: a less structured model is interpreted as more structured when inhomogeneities are not taken into account. As it is directly related to the distance-redshift relation, it can have a significant impact on the estimation of cosmological parameters, particularly when observations are aiming for a 1% level accuracy. As we used a very generic approach to integrate geodesics, this work will be extended and refined to get both a better understanding and a better estimation of the kinematic effects of inhomogeneities on observational quantities.

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