27 The S_3 flavour symmetry: quarks, leptons and Higgs sectors.

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Abstract We present a brief overview of the minimal S_3 extension of the Standard Model, in which the concept of flavour is extended to the Higgs sector by introducing in the theory three Higgs fields which are $SU(2)$ doublets. In both the quark and lepton sectors the mass matrices are reparametrized in terms of their eigenvalues, thus allowing to express the mixing angles in terms of mass ratios. In the leptonic sector, the $S_3 \times Z_2$ symmetry implies a non-vanishing θ_{13}, which is different from zero and in very good agreement with the latest experimental data.

27.1 The S_3 symmetry

The success of the Standard Model (SM) in describing the fundamental particles and their interactions has been confirmed with the recent observation of a state compatible with a (SM-like) Higgs boson at the LHC [1, 2]. Despite this success, the SM leaves many open questions and has too many free parameters whose value can only be determined by the experiment. Among these open questions is the whole subject of flavour physics, namely the origin of the masses and mixings of quarks and leptons.

It is possible to ask if the data on quark and lepton masses suggests already a flavour symmetry at the Fermi scale. The fact that the third generation is heavier than the first two already proposes a path to follow: if instead of looking at the masses alone we look at the mass ratios obtained by dividing the masses of each type of quarks and leptons by the heaviest of each sector, then a clear pattern emerges. The first two generations belong to a doublet representation and the third generation to a singlet one. The simplest flavour symmetry with doublet and singlet irreducible representations is the permutational flavour symmetry S_3 [3–5].

In the Standard Model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak and electromagnetic interactions. Prior to electroweak symmetry breaking, the Lagrangian is chiral and invariant under the action of the group of permutations acting on the flavour indices of the matter fields. Since the Standard Model has only one Higgs $SU(2)_L$ doublet, which can only be an S_3 singlet, it is necessary to break the S_3 symmetry in order to give different masses to all quarks and leptons. Hence, in order to impose S_3 as a fundamental symmetry, unbroken at the Fermi scale, we are led to extend the Higgs sector of the theory [6–12]. The quark, lepton and Higgs fields are $Q^T = (u_L, d_L)$, u_R, d_R, $L^T = (\nu_L, e_L)$, e_R, ν_R, and H_S, H_1 and H_2, in an obvious notation. All these fields have three species, and we assume that each one forms a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$ of the

S_3 group. The doublets carry capital indices I and J, which run from 1 to 2 and the singlets are denoted by $Q_3,\ u_{3R},\ d_{3R},\ L_3,\ e_{3R},\ \nu_{3R}$ and H_S. Note that the subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions for the lepton sector of this model are given by $\mathcal{L}_Y = \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}$, where

$$\begin{aligned}\mathcal{L}_{Y_E} \;=\; & -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} - Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\ & - Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{ h.c.},\end{aligned} \tag{27.1}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} \;=\; & -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} - Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} \\ & - Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{ h.c.},\end{aligned} \tag{27.2}$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{27.3}$$

We also add to the Lagrangian the Majorana mass terms for the right-handed neutrinos, $\mathcal{L}_M = -\nu_R^T C \mathbf{M}_{\nu_R} \nu_R$, where C is the charge conjugation matrix and $\mathbf{M}_{\nu_R} = \text{diag}\,\{M_1, M_2, M_3\}$ is the mass matrix for the right-handed neutrinos. The Lagrangian for the quark sector of this model has a similar form [7].

Due to the presence of the three Higgs fields, the Higgs potential is more complicated than that of the Standard Model [7, 13–15]. The S_3 Higgs potential was first analyzed by Pakvasa and Sugawara [3] who found that in addition to the S_3 symmetry, it has an accidental permutational symmetry S'_2: $H_1 \leftrightarrow H_2$, which is not a subgroup of the flavour group S_3. With these assumptions,the Yukawa interactions, eqs. (27.1)-(27.2) yield mass matrices, for all fermions in the theory, of the general form [6]

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \tag{27.4}$$

In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavour symmetry S_3.

27.2 Quarks

Quark models using the S_3 permutational symmetry as the flavour symmetry have been already explored, see for instance [3–6, 15–18]. Some of these studies have been done through numerical approaches, and all of them show very good agreement with the experimental data. Recently we have been able to reparametrize the quark mass matrices in terms of their eigenvalues, which allowed us to express the mixing angles in terms of quark mass ratios, an independent phase parameter, and a free parameter, when we do not consider the accidental S'_2 symmetry [3], for more details see [19].

An interesting result from this analysis of the S_3 quark model is the fact that, after electroweak symmetry breaking, the generic mass matrix that comes from the S_3-invariant Yukawa interactions is equivalent, when demanding hermiticity, to the four-zero Fritzsch-like texture and, when hermiticity is not demanded, to the Nearest Neighbour Interaction mass matrix form (NNI), both of which have been found not only to have a viable phenomenology, but also to allow a unified treatment of both quarks and leptons [20, 21]. The mass matrices are,

$$\mathbf{M}^f_{Hermitian} = \begin{pmatrix} 0 & |\mu_2^f|\sin\theta\cos\theta(3-\tan^2\theta) & 0 \\ |\mu_2^f|\sin\theta\cos\theta(3-\tan^2\theta) & -2|\mu_2^f|\cos^2\theta(1-3\tan^2\theta) & \mu_8^f\sec\theta \\ 0 & \mu_8^{f*}\sec\theta & |\mu_3^f|-\Delta_f \end{pmatrix} \quad (27.5)$$

$$\mathbf{M}^f_{NNI} = \begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f & 0 \\ +\frac{2}{\sqrt{3}}\mu_2^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_8^f & \mu_3^f-\Delta_f \end{pmatrix}, \quad (27.6)$$

where $\Delta_f << 1$, $\tan\theta = w_1/w_2$, $\mu_2^f \equiv Y_3^f w_2$, $\mu_3^f \equiv 2Y_1^f v_S$, $\mu_7^f \equiv \sqrt{2}Y_5^f w_2$, $\mu_8^f \equiv \sqrt{2}Y_6^f w_1$, where Y_i^f are the complex Yukawa couplings, and v_S, w_1 and w_2 are the vacuum expectation values of the three Higgs $SU(2)_L$ doublets, H_S and $H_D = (H_1, H_2)^T$, singlet and doublet of S_3, respectively [19].

27.3 Lepton masses and mixings

A further reduction of the number of parameters in the leptonic sector may be achieved by means of an Abelian Z_2 symmetry [6], which forbids the following Yukawa couplings $Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$. Therefore, the corresponding entries in the mass matrices vanish. The resulting expression for $\mathbf{M}_e$, reparametrized in terms of its eigenvalues and written to order $\left(m_\mu m_e/m_\tau^2\right)^2$ and $x^4 = \left(m_e/m_\mu\right)^4$, is

$$\mathbf{M}_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}. \quad (27.7)$$

This approximation is numerically exact up to order 10^{-9} in units of the τ mass. Notice that this matrix has no free parameters other than the Dirac phase δ_e [6, 22].

The mass matrix of the left-handed Majorana neutrinos, $\mathbf{M}_{\nu_L}$, is generated by the type I seesaw mechanism. The mass matrix $\mathbf{M}_{\nu_L}$ takes the form [22, 23]:

$$\mathbf{M}_{\nu_\mathbf{L}} = \begin{pmatrix} \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\lambda\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\lambda\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\left(\mu_4^\nu\right)^2}{\overline{M}} + \frac{\left(\mu_3^\nu\right)^2}{M_3} \end{pmatrix}, \text{ with } \quad \begin{array}{l} \lambda = \frac{1}{2}\left(\frac{M_2-M_1}{M_1+M_2}\right), \\ \overline{M} = 2\frac{M_1M_2}{M_2+M_1}. \end{array} \qquad (27.8)$$

where M_i ($i = 1, 2, 3$) are the masses of right-handed neutrinos, whereas the μ_2, μ_3 and μ_4 are complex parameters that come from the mass matrix of the Dirac neutrinos, which is obtained from the $S_3 \otimes Z_2$ flavour symmetry [22, 23].

The identity of the leptons is encoded in the mass matrices $\mathbf{M}_e$ and $\mathbf{M}_{\nu_\mathbf{L}}$, for charged leptons and left-handed neutrinos, respectively. Nevertheless, these matrices are basis dependent, since given any pair $\mathbf{M}_e$, $\mathbf{M}_{\nu_\mathbf{L}}$ one can obtain other pairs of matrices through a unitary rotation, without affecting the physics. On the other hand, the phase factors may be factored out of $\mathbf{M}_{\nu_L}$ if $\phi_2 = \arg\left\{\mu_2^\nu\right\}$ and $\phi_3 = \arg\left\{\mu_3^\nu\right\}$ satisfy the relation $\phi_2 = \phi_3$ [23]. Hence, the mass matrix of the left-handed Majorana neutrinos can be written as:

$$\mathbf{M}_{\nu_L} = \mathbf{Q}\mathcal{U}_{\frac{\pi}{4}}\left(\mu_0\mathbf{I}_{3\times3} + \widehat{\mathbf{M}}\right)\mathcal{U}^\dagger_{\frac{\pi}{4}}\mathbf{Q}, \qquad (27.9)$$

where $\mu_0 = 2|\mu_2^\nu|^2|\overline{M}|^{-1}(1-|\lambda|)$, $\mathbf{Q} = e^{i\phi_2}\text{diag}\left\{1, 1, e^{i\delta_\nu}\right\}$ with $\delta_\nu = \arg\left\{\mu_4^\nu\right\} - \arg\left\{\mu_2^\nu\right\}$,

$$\mathcal{U}_{\frac{\pi}{4}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}, \quad \text{and} \quad \widehat{\mathbf{M}} = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & 2d \end{pmatrix}, \qquad (27.10)$$

with $A = \sqrt{2}|\mu_2^\nu||\mu_4^\nu|(1-|\lambda|)|\overline{M}|^{-1}$, $B = 2|\mu_4^\nu|^2|\overline{M}|^{-1} + |\mu_3^\nu|^2M_3^{-1} - 2|\mu_2^\nu|^2|\overline{M}|^{-1}(1-|\lambda|)$, $C = \sqrt{2}|\mu_2^\nu||\mu_4^\nu||\overline{M}|^{-1}(1+|\lambda|)$ and $d = 2|\lambda||\mu_2^\nu|^2|\overline{M}|^{-1}$. The diagonalization of $\mathbf{M}_{\nu_L}$ is reduced to the diagonalization of matrix $\widehat{\mathbf{M}}$, which is a matrix with two texture zeroes of class I [20]. For this we use all the information we already have about the diagonalization of a matrix with two texture zeroes [4, 5, 20, 21]. Thus, the matrix $\mathbf{M}_{\nu_L}$ is diagonalized by a unitary matrix $\mathbf{U}_\nu = \mathbf{Q}^\nu\mathcal{U}_{\frac{\pi}{4}}\mathbf{O}_\nu^{N[I]}$ where the orthogonal matrix $\mathbf{O}_\nu^{N[I]}$ reparametrized in terms of the neutrino masses is given by [23]:

$$\begin{pmatrix} \sqrt{\frac{[-1](m_{\nu_3}-\mu_0)(m_{\nu_2}-\mu_0)f_1}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{\left(m_{\nu_{3[1]}}-\mu_0\right)\left(\mu_0-m_{\nu_{1[3]}}\right)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}} & -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})(m_{\nu_2}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}} \\ \sqrt{\frac{[-1]2d(\mu_0-m_{\nu_1})f_1}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{2d(m_{\nu_2}-\mu_0)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}} & \sqrt{\frac{[-1]2d(m_{\nu_3}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}} \\ -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})f_2^{N[I]}f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{(m_{\nu_2}-\mu_0)f_1f_3^{N[I]}}{\mathcal{D}_2^{N[I]}}} & -\sqrt{\frac{(m_{\nu_3}-\mu_0)f_1f_2^{N[I]}}{\mathcal{D}_3^{N[I]}}} \end{pmatrix}, \qquad (27.11)$$

where $f_1 = (2d+\mu_0-m_{\nu_1})$, $f_2^{N[I]} = [-1](2d+\mu_0-m_{\nu_2})$, $f_3^{N[I]} = [-1](m_{\nu_3}-\mu_0-2d)$, $\mathcal{D}_1^{N[I]} = 2d(m_{\nu_2}-m_{\nu_1})\left(m_{\nu_{3[1]}}-m_{\nu_{1[3]}}\right)$, $\mathcal{D}_2^{N[I]} = 2d(m_{\nu_2}-m_{\nu_1})\left(m_{\nu_{3[2]}}-m_{\nu_{2[3]}}\right)$ and $\mathcal{D}_3^{N[I]} = 2d\left(m_{\nu_{3[1]}}-m_{\nu_{1[3]}}\right)\left(m_{\nu_{3[2]}}-m_{\nu_{2[3]}}\right)$. The values allowed for the parameters μ_0 and $2d+\mu_0$ are in the following ranges: $m_{\nu_{2[1]}} > \mu_0 > m_{\nu_{1[3]}}$ and $m_{\nu_{3[2]}} > 2d+\mu_0 > m_{\nu_{2[1]}}$. The superscripts N and I denote the normal and inverted hierarchies, respectively.

The neutrino mixing matrix

The neutrino mixing matrix $\mathbf{V}_{_{PMNS}}$ is the product $\mathbf{U}_{eL}^{\dagger}\mathbf{U}_{\nu}\mathbf{K}$, where $\mathbf{K}$ is the diagonal matrix of the Majorana phase factors, defined by $\mathbf{K} = diag(1, e^{i\alpha}, e^{i\beta})$ [24]. The theoretical expression for the lepton mixing matrix, $\mathbf{V}^{th}_{_{PMNS}}$ is:

$$\begin{pmatrix} \frac{\tilde{m}_e}{\tilde{m}_\mu}O_{11}^{N[I]} - O_{21}^{N[I]}e^{i\delta_l} & \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{12}^{N[I]} - O_{22}^{N[I]}e^{i\delta_l}\right)e^{i\alpha} & \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{13}^{N[I]} - O_{23}^{N[I]}e^{i\delta_l}\right)e^{i\beta} \\ -O_{11}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu}O_{21}^{N[I]}e^{i\delta_l} & \left(-O_{12}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu}O_{22}^{N[I]}e^{i\delta_l}\right)e^{i\alpha} & \left(-O_{13}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu}O_{23}^{N[I]}e^{i\delta_l}\right)e^{i\beta} \\ O_{31}^{N[I]} & O_{32}^{N[I]}e^{i\alpha} & O_{33}^{N[I]}e^{i\beta} \end{pmatrix} \tag{27.12}$$

where the elements $O^{N[I]}$ are given in eq. (27.11) and $\delta_l = \delta_\nu - \delta_e$.

The reactor mixing angle

In the case of an inverted neutrino mass hierarchy ($m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$), the theoretical expression for the neutrino reactor mixing angle as function of the ratios of the lepton masses and, in a preliminary analysis, the numerical values are:

$$\sin^2\theta_{13}^{l} \approx \frac{(\mu_0 + 2d - m_{\nu_3})(\mu_0 - m_{\nu_3})}{(m_{\nu_1} - m_{\nu_3})(m_{\nu_2} - m_{\nu_3})}, \quad \sin^2\theta_{13}^{l} \approx 0.029 \longrightarrow \theta_{13}^{l} \approx 9.8^\circ, \tag{27.13}$$

with the following values for the neutrino masses $m_{\nu_2} = 0.056$ eV, $m_{\nu_1} = 0.053$ eV and $m_{\nu_3} = 0.048$ eV, and the parameter values $\delta_l = \pi/2$, $\mu_0 = 0.049$ eV and $d = 8 \times 10^{-5}$ eV, we get θ_{13} in very good agreement with experimental data [25, 26].

27.4 The Higgs potential

As already mentioned, the Higgs potential of this model is more complicated than the one of the SM, because of the extended Higgs sector. The correct computation of the potential is important for a meaningful and significative comparison of theory and experiment. Thus, any conclusions on the Higgs phenomenology depend strongly on the form of the Higgs potential. Work on the phenomenology implied by the Higgs potential has been done before [3, 14, 16–18, 27–34]. Nevertheless, it has not been made clear how the S_3 symmetry should be used among the different and independent terms of the Higgs potential expression, such that it has the highest degree of flavour symmetry. In this sense, we have explored the possibility that the different combinations of $SU(2)_L$ indices contractions belonging to the same S_3-invariant term are assigned to the same self coupling parameter. A preliminary analysis following this proposal can be found in ref. [35].

27.5 Conclusions

The permutational group S_3 is well motivated by the data on quarks and lepton masses as an underlying flavour symmtery. To be able to impose S_3 as a fundamental, unbroken symmetry, the Higgs sector of the theory has to be extended with two extra Higgs $SU(2)$ doublets. Both in the quark and lepton sectors it is possible to reparametrize the mass matrices in terms of their eigenvalues, thus allowing to express the mixing angles in terms of mass ratios. The resulting form of the quark and lepton mixing matrices allows for a unified treatment of quarks and leptons under the flavour symmetry. In the quark sector it is possible to express the mixing angles in terms of the quark mass ratios, an independent phase parameter, and a free parameter. In the leptonic sector, the flavour symmetry, together with the seesaw mechanism, imply a non-vanishing reactor mixing angle θ_{13}, and from a preliminary numerical analysis, gives results for all mixing angles in very good agreement with the most recent experimental data. Finally, from symmetry arguments regarding the gauge and flavour symmetries, it is possible to find the most general form of the Higgs potential with the highest degree of flavour symmetry.

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