The automated Matrix-Element reweighting and its applications at the LHC

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Abstract. Given a sample of experimental events and a set of theoretical models, the matrix element method (MEM) is a procedure to select the most plausible model that governs the production of these events. From a theoretical point of view, it is probably the most powerful multi-variate analysis technique since it maximally uses the information contained in the Feynman amplitudes. This technique is now widely known since it has been used for the precision top mass measurement at Tevatron, for example.

The MadWeight software is presented. MadWeight is a phase-space generator designed for the automated numerical estimation of matrix elements based on MadGraph amplitudes. With the modern computing resources, it allows the large-scale deployment of the MEM technique on high-statistics data and simulated samples.

Several applications of the method at LHC are discussed, including the measurement of the spin and parity of the recently discovered boson, signal-to-background discrimination, full differential spectrum estimation and other promising applications.

1. Introduction

The LHC has now delivered 5 and 20 fb^{-1} of data with 7 and 8 TeV in center of mass respectively. The first long shutdown, which will last until 2015, provides an opportunity to upgrade and develop new analyses. In this context, the Matrix Element Method (MEM) is a multivariate analysis (MVA) method in particle physics that takes into account all kinematic information of an experimental event. Using the final state 4-vectors, the MEM computes the posterior probability that a reconstructed event is produced under a given hypothesis. In comparison with other MVA techniques (NN, BDT), MEM is certainly the most theoretically motivated, since it fully exploits the process Feynman amplitude.

This method has been widely used and tested, especially for measurement in the top sector at the Tevatron and for Higgs searches/measurement at the LHC. An automation of this method has been implemented and dubbed *MadWeight* [1].

In the following, the theoretical context of the leading order MEM computation and MadWeight will be first presented before reviewing few applications (b-charge association, differential matrix element method, background rejection and hypothesis testing).

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2. The methods

The matrix element method

The probability to observe a reconstructed event x in the theoretical context α is given by:

$$P(x|\alpha) = \int dy P_\alpha(y) W(x,y) \quad (1)$$

Where we have defined:

α :	theoretical informations (signal hypothesis).
x :	experimental informations.
$P_\alpha(y)$:	The probability to have a partonic configuration y in the theoretical context α.
$W(x,y)$:	The transfer function represent the evolution from partonic configuration y to a reconstructed event x.

The previous expression of $P_\alpha(y)$ can be re-written in terms of squared Feynman amplitude $|\mathcal{M}_\alpha|^2(y)$ and the parton density functions ($f_1(q_1)$ et $f_2(q_2)$):

$$P(x|\alpha) = \frac{1}{\sigma_\alpha \epsilon_\alpha} \int d\phi(y) dq_1 dq_2 f_1(q_1) f_2(q_2) |\mathcal{M}_\alpha|^2(y) W(x,y) \quad . \quad (2)$$

Since $P(x|\alpha)$ is strictly a probability, the full set of statistical methods and tools on the market can be used.

As example, given a data sample of N events, one can define a likelihood build upon the MEM probability associated with these events.

$$\mathcal{L}(\alpha) = f(N) \prod_{i=1}^{N} P(x_i|\alpha) \quad (3)$$

where $f(N)$ is a renormalization factor.

ISR correction

When considering only leading order matrix element, two methods are usually used to take into account the ISR corrections. The most common approach is to apply a Lorentz transformation to boost the system back into a frame where the transverse momentum of the ISR is null. As illustration, let's consider a reconstructed event with two b-jets and two electrons (see Figure 1). Among others, two processes can lead to this final state: $Zb\bar{b}$ associated production and $t\bar{t}$.

Since the extra radiations are often soft and not fully reconstructed, it is more convenient to derive them from the reconstructed objects belonging to the LO process. If the process contains genuine missing transverse energy ($\not{E}_T$), such as $t\bar{t}$, the $\not{E}_T$ has to be included in the sum. Note that this procedure will not give the expected result in the presence of final state radiations.

- $P(x|Zb\bar{b} \to l^+l^-b\bar{b})$: $p_{ISR} = -(p_{b1} + p_{b2} + p_{e1} + p_{e2})$
- $P(x|t\bar{t} \to l^+l^-b\bar{b}\nu\bar{\nu})$: $p_{ISR} = -(p_{b1} + p_{b2} + p_{e1} + p_{e2}) - \not{E}_T$

The second method is to use LO calculation that contains an extra radiation (i.e. $P(x|Zb\bar{b}j)$), other extra-radiations are then taken into account using the first described method. This method provides a good modeling of the extra hard radiation from both initial and final state but can be time consuming.

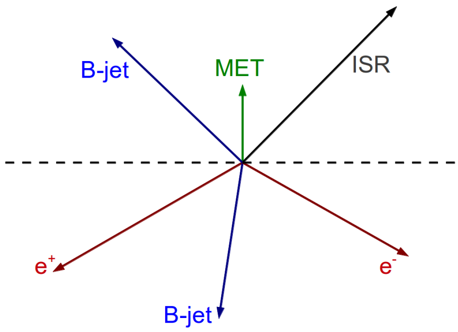

Figure 1. experimental event in which two b-jets and two opposite-sign electrons have been reconstructed.

MadWeight and automation of the MEM
The calculation of the probability entails an intricate numerical integration. A generic module dubbed `MadWeight` has been designed to perform this phase-space integration in an efficient way. The program is based on MadGraph/MadEvent [4] and can be used to test any process implemented in MadGraph. By default, the output probability is the average over all possible parton-jet assignements. MadWeight can also deal with any shape of transfer function provided by the user.

3. Applications at the LHC

3.1. b-Charge association

A method to measure the CP violation with semi-leptonically decayed $t\bar{t}$ events at the LHC has been provided in [2]. The CP violation asymmetries can be defined as mixing or as direct according to the following decay modes:

- **mixing**: $N(b \to \bar{b} \to \ell^+) - N(\bar{b} \to b \to \ell^-)$
- **direct**: $N(b \to \ell^-) - N(\bar{b} \to \ell^+)$

In order to extract the direct and mixing CP violation, one can find out which are b and $\bar{b}$ initially by using MadWeight. The method has been dubbed *b-charge association*. For each event, probabilities P_1 and P_2 corresponding to the two possible choices of the b-charge association can be computed. One can define $W = |(P_1 - P_2)/(P_1 + P_2)|$, which is a variable quantifying the quality of the association (figure 2, left). Selecting events with a high W value will provide a sample with correctly assigned events. For instance one can achieve $\leq 10\%$ mis-association rate with $\approx 70\%$ signal efficiency (figure 2, right). In addition, we observe on left of the figure 2 that this variable can also be used to reject the background.

3.2. Differential Matric Element Method (DMEM) and differential cross-section measurement

We can also use the MadWeight program to estimate the probability density function of arbitrary event kinematic variable. As example, the invariant mass of a $t\bar{t}$ event where both top decays leptonically.

$$\frac{\partial \mathcal{P}}{\partial m}|_{m=m_0} = \frac{1}{\sigma}\int d\phi(y)dq_1dq_2f_1(q_1)f_2(q_2)|\mathcal{M}_\alpha|^2(y)W(x,y)\times\delta(m-m_0) \qquad (4)$$

For each event, a $\frac{1}{\mathcal{P}}\frac{\partial \mathcal{P}}{\partial m_{tt}}$ curve is computed. Then the sum of the curves provides $\frac{\partial \sigma}{\partial m_{tt}}$. A prospect developed for the LHC was the search for new resonant productions of $t\bar{t}$ pairs decaying

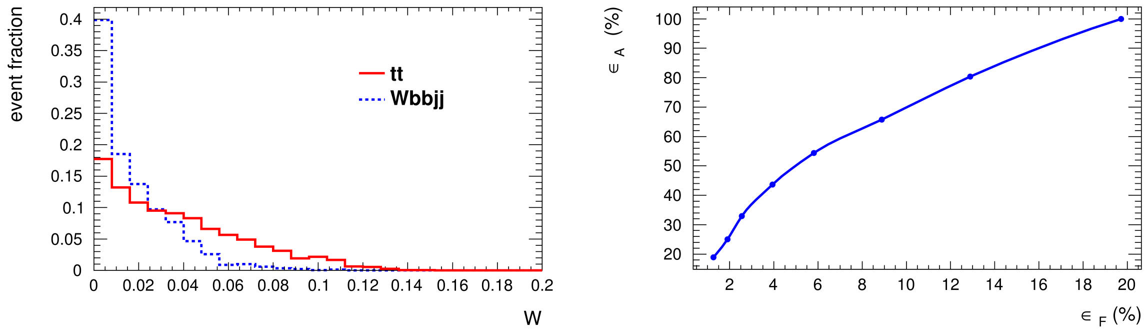

Figure 2. On the left: W variable distribution for $t\bar{t}$and $Wb\bar{b}jj$ events. On the right: Efficiency of the b-charge association as a function of the mis-association probability.

leptonically [9]. The injection of signal event, corresponding to a Z' (spin-1 particle) with a mass of 1 TeV, shows the good discrimination in the left of the figure 3. The possibility to measure the spin of the resonance using the angular variable $\cos(\theta^*_{tt})$ is illustrated in [9].

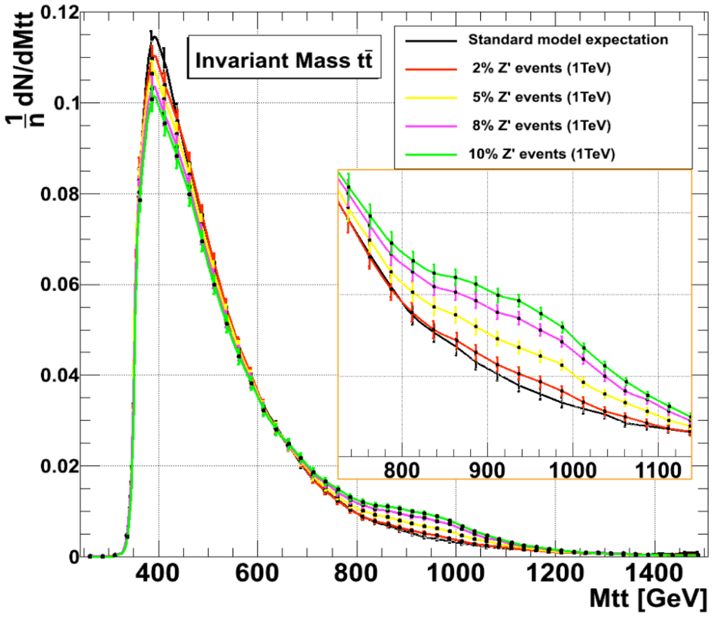

Figure 3. Reconstructed $t\bar{t}$-invariant mass for different ratios of SM and Z'.

3.3. Background discrimination and hypothesis testing

In order to illustrate the use of the MEM in case of background discrimination, let's look at the study of the Higgs boson decaying into four leptons. Benefiting from a very low background and a good mass resolution, the $4l$ final state provides the best spin/parity measurements. Several papers already treat this subject using MEM, see for example [3]. In this section, we will consider only the main background $pp \to ZZ$ and then neglect $pp \to Z + X$ and other backgrounds with a small contribution.

The events with the different spin and parity hypothesis and the background have been generated with MadGraph5 [4], using the "Higgs Characterization" model [5]. We then used Pythia6 [6] and Delphes3 [7] for the detector simulation.

Leptons are selected if the reconstructed transverse momentum is higher than 7 GeV/c and $|\eta|$ smaller than 2.4. An event passes the first selection if four leptons are reconstructed with at least one satisfying $p_T \geq 20$ and a second lepton with $p_T \geq 10$. The two Z-candidates, z_1 and z_2 are then reconstructed with $m_{z_1} \geq m_{z_2}$. A second set of selection is then applied, requiring:

$$50 \leq m_{z_1} \leq 120 \qquad 12 \leq m_{z_2} \leq 120 \tag{5}$$

Finally, a Higgs candidate is reconstructed by summing the four reconstructed lepton 4-vectors. A third selection is then applied, selecting events with a m_{4l} close to the observed excess:

$$110 \leq m_{4l} \leq 160 \qquad . \tag{6}$$

In order to increase the purity of the sample, a simple kinematic discriminant (KD) is defined as $P_{0^+}/(P_{0^+} + P_{ZZ})$. Typically, the background has a small value of KD, while the signal populates a region close to 1 (see Fig. 4, left).

The hypothesis testing is illustrated here by the characterization of the boson at 126 GeV. We will use an extended Likelihood defined as :

$$\mathcal{L}_k = e^{-n_s - n_b} \prod_i \Big(n_s \times P_s^k(x_i) + n_b \times P_b(x_i) \Big) \tag{7}$$

where k is represents the different spin-parity signal hypothesis for the new resonance (0^+, 0^-, 2^+, etc...). Here we will focus on $\mathcal{L}_{0^+}$ and $\mathcal{L}_{0^-}$. Assuming ≈ 25 fb^{-1}, and considering only the ZZ background. We expect a separation of $\approx 3\sigma$ while neglecting the systematic errors (see Fig. 4, right).

While similar to the CMS approach for the 4 leptons topology [10], this method can be easily extended to channels with neutrinos or jets. Recently searches as complex as $t\bar{t}h$ were treated with the same method [8].

4. Conclusion

The leading order formulation of the matrix element method has been presented as well as the MadWeight program, an automation of this method. Four types of analysis were discussed, namely the b-charge association, the search for new physics using differential matrix element, methods for background discrimination and hypothesis testing. We showed here that the MadWeight program has opened doors to an easy implementation of advanced application of methods. The MEM may therefore acquire a more important role in various type of analysis, ranging from simple four leptons final states to more complex $t\bar{t}h$ decays.

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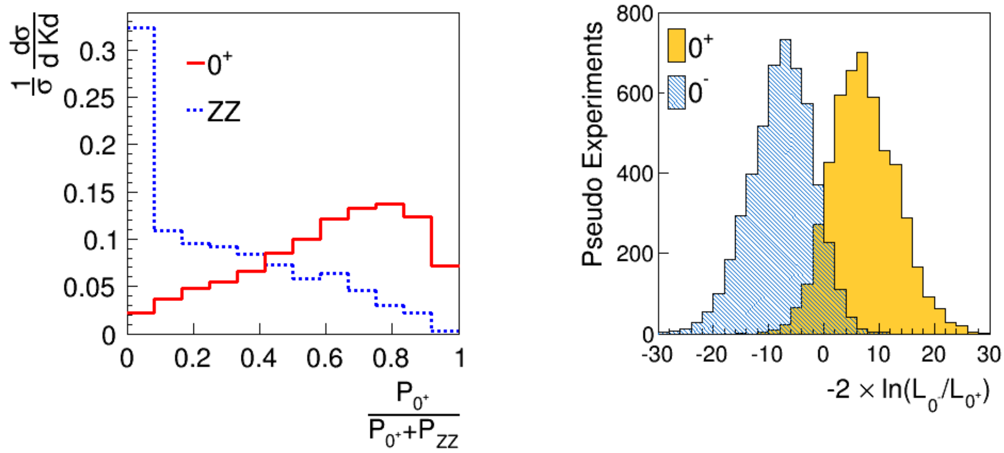

Figure 4. Left: Kinematic discriminant variable for Standard model Higgs boson (red solid line) and for the Background (dashed blue line). On the right: the dashed blue distribution correspond to extended likelihood value with pseudo-experiment considering the background and a odd parity signal The yellow distribution correspond to the background and the Standard model higgs signal.

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