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Dynamics of test particles around Hayward Black Holes

Nozima Juraeva[1] and Malika Xudoyberdiyeva[2]

[1]*Ulugh Beg Astronomical Institute, Astronomy St. 33, Tashkent 100052, Uzbekistan*

[2]*National University of Uzbekistan, Tashkent 100174, Uzbekistan*

Abstract: In this article, we study the Hayward black hole surrounded by quintessence. The Hayward metric is a spherically symmetric charged regular black hole, a modification of the Reisnner–Nordström black holes of Einstein equations coupled to nonlinear electrodynamics. The critical mass values and the normalization factor are obtained. We also describe the horizons and the extremal condition of the Hayward black hole surrounded by quintessence. Setting the quintessence state parameter in the particular case of $\omega_q = -2/3$, we study the effective potential. We discuss its essential geometrical properties such as scalar invariants and the size of innermost stable circular orbits. The dynamics of test particles around the Hayward black hole is also studied.

Keywords: Black holes, Nonlinear Electrodynamics, Quintessential Field, Kiselev, Hayward black hole.

I. INTRODUCTION

A spacetime singularity is the breakdown of the geometry or other physical structure related to the properties of the spacetime and gravity. Usually the breakdown of the geometry means thed edge or end of the spacetime. The general relativity proposed by the Albert Einstein not only predicts the singularity but it also claims that the existence of the singularity is unavoidable. Most of the singularity in general relativity appears when one considers the solution of black holes which contains the curvature singularity hidden by the event horizon. The understanding the nature of the physical singularities meets physical and philosophical problems [1]. On the other hand there are several attempts to resolve the singularity problems within general relativity or using alternative theories of gravity. Particularly, the quantum gravity theory which is not yet constructed is also believed to resolve the fundamental problems of the general relativity [2] [3][4]. Following the idea of the limiting curvature Hayward [5] proposed a static spherically symmetric black hole that near the origin behaves like a de Sitter spacetime, its curvature invariants being everywhere finite and satisfying the weak energy condition. Variations have been studied as the rotating Hayward [6] and Hayward charged [7]. Also, diverse investigations have been focused on the properties of the Hayward black hole. For example, in [8], the interior of the regular Hayward black hole was explored with The Painlev'e-Gullstrand coordinates and in [9] [10] the quasinormal modes were studied.

Current resolution of the observations and experiments are compatible with the general relativity. Particularly, the observation of the first ever image of the black hole shadow of the elliptical galaxy M87 by Event Horizon Telescope and gravitational wave detection from black hole-black hole merger allowed to test the general relativity and left open window for other models of gravity. In order to resolve the singularity problem of the general relativity Bardeen has proposed to couple the latter with the nonlinear electrodynamics [2]. The solution of the field equation of the general rlativity coupled to nonlinear electrodynamics describes the black hole without curvature singularity or so-called regular black hole. Other modifications of the nonlinear electrodynamics ay lead to other types of the regular black holes: e.g. Hayward [5], Ayon-Beato and Garcia regular black holes. Note, that the rotating analogue of these static regular black holes have been obtained in [11][12].

Basing on the ideas used by Kiselev, we study the Hayward black hole surrounded by quintessence. By setting for the quintessence state parameter at the special case of $\omega = -\frac{2}{3}$, using the metric of the black hole surrounded by quintessence and the definition of the effective potential, we analyzed in detail the null geodesics for different energies. We also described the horizons of the Hayward black hole surrounded by quintessence as well as the shadow of the black hole. For the space-time and geometrized unit system $G = c = 1$ (However, for an astrophysical application we have written the speed of light explicitly in our expressions). Latin indices run from 1 to 3 and Greek ones from 0 to 3.

II. HAYWARD BLACK HOLES IN QUINTESSENTIAL MEDIUM

In this section we study a Hayward black hole surrounded by quintessential matter obtained by Kiselev [13]. The line element of Hayward black hole is given by,

$$ds^2 = -fdt^2 + f^{-1}dr^2 + d\Omega^2 \tag{1}$$

where

$$f(r) = 1 - \frac{2Mr^2}{r^3 + 2M\epsilon^2}\,. \tag{2}$$

Kiselev [13] proposed a new solution of the metric for static and spherically symmetric space-time considering that the energy-momentum tensor for quintessence should satisfy;

$$T_{\phi\phi} = T_{\theta\theta} = -\frac{1}{2}(3\omega+1)T_{rr} = \frac{1}{2}(3\omega+1)T_{tt} \tag{3}$$

where ω is taken to be a constant and the dominant energy condition requires $T_{tt} \geq 0$ and $\mid 3\omega + 1 \mid \leq 2$. Following these ideas, the expression of the metric function of such a black hole surrounded by quintessence is obtained by adding the therm $-c/r^{3\omega+1}$. Thus, we can write.

$$f_\omega(r) = 1 - \frac{2Mr^2}{r^3 + 2M\epsilon^2} - \frac{c}{r^{3\omega+1}}\,, \tag{4}$$

where c is a normalization factor and ω has the range $-1 < \omega < -1/3$. For the Schwarzschild black hole surrounded by quintessence $\epsilon = 0$. We present new static spherically-symmetric exact solutions of Einstein equations with the quintessential matter surrounding a black hole charged or not as well as for the case without the black hole [14] condition of additivity and linearity in the energy-momentum tensor is introduced, which allows one to get correct limits to the known solutions for the electromagnetic static field implying the relativistic relation between the energy density and pressure, as well as for the extraordinary case of cosmological constant, i.e. de Sitter space [15]

A. Scalar Invariants

The curvature invariants are the quantities which help to understand the properties of spacetime of a geometrical object like a black hole. Well known scalar invariants include the Ricci scalar, the square of the Ricci tensor and the Kretschmann scalar (the square of the Riemann curvature tensor). In this section, we calculate and analyse these quantities by observing their behaviors graphically.

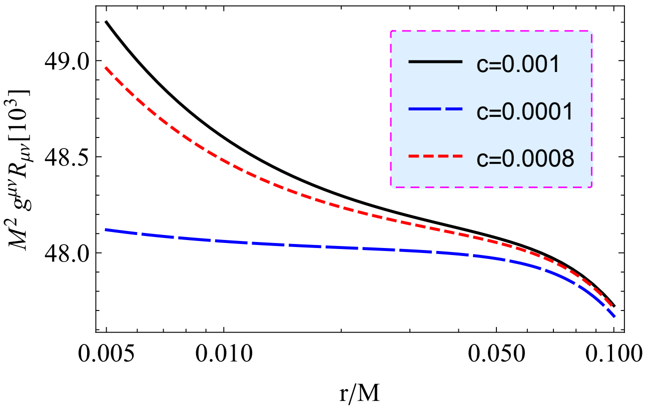

FIG. 1. Dimensionless Ricci scalar as a function of dimensionless radial coordinate (r/M) for different values of the parameters ω, ϵ and normalization factor , c .

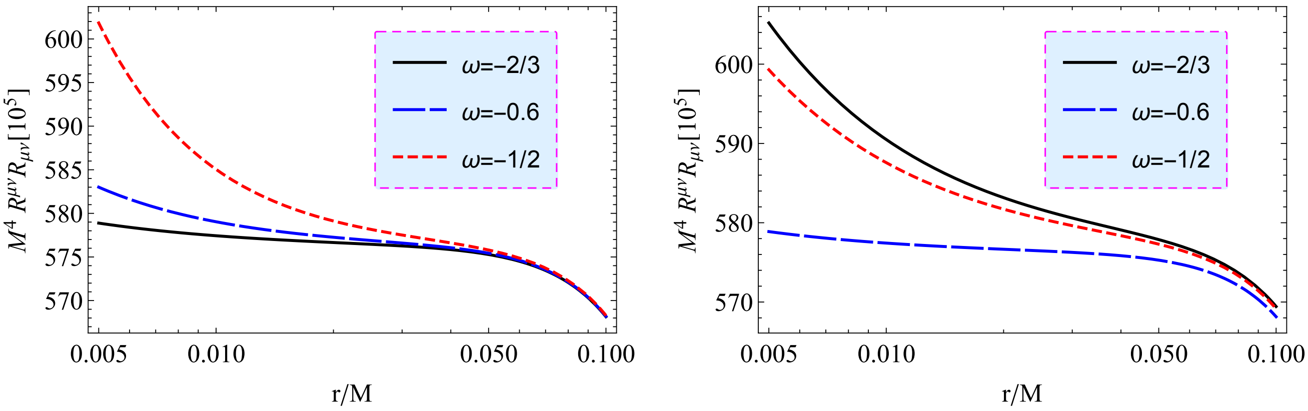

FIG. 2. Dimensionless Ricci tensor as a function of dimensionless radial coordinate (r/M) for different values of the parameters ω, ϵ and normalization factor , c .

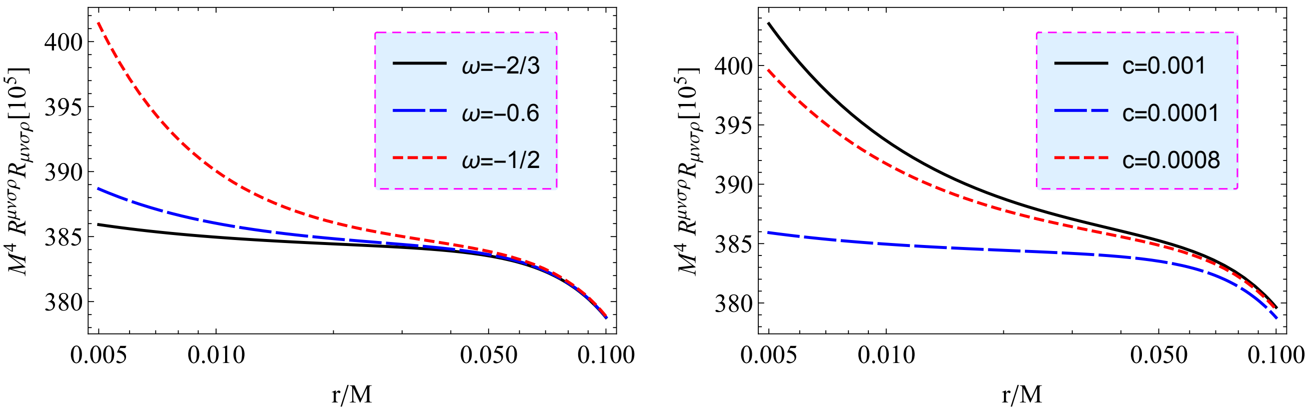

FIG. 3. Dimensionless Kretschmann scalar as a function of dimensionless radial coordinate (r/M) for different values of the parameters ω, ϵ and normalization factor , c .

III. DYNAMICS OF TEST PARTICLES AROUND HAYWARD BLACK HOLE

For the massive neutral particles motion is governed by timelike geodesics of the spacetime and the equations of motion can be derived using

$$\dot{r}^2 = \mathcal{E}^2 + g_{tt}\left(1+\frac{\mathcal{K}}{r^2}\right) , \tag{5}$$

$$\dot{\theta} = \frac{1}{g_{\theta\theta}^2}\left(\mathcal{K}-\frac{l^2}{\sin^2\theta}\right) , \tag{6}$$

$$\dot{\phi} = \frac{l}{g_{\phi\phi}} , \tag{7}$$

$$\dot{t} = -\frac{\mathcal{E}}{g_{tt}} , \tag{8}$$

where $\mathcal{K}$ is the Carter constant corresponding to the total angular momentum.

Restricting the motion of the particle to the plane with $\theta = \text{const}$ and $\dot{\theta} = 0$ (that is justified by the conservation of the angular momentum), the Carter constant takes the form $\mathcal{K} = l^2/\sin^2\theta$. Then the equation of the radial motion can be expressed in the form

$$\dot{r}^2 = \mathcal{E}^2 - V_{\text{eff}} , \tag{9}$$

where the effective potential of the motion of neutral particles reads

$$V_{\rm eff} = f_\omega(r)\left(1+\frac{L^2}{r^2}\right) \tag{10}$$

$$V_{\rm eff} = \left(1+\frac{L^2}{r^2}\right)\left(1-\frac{2Mr^2}{r^3+2M\epsilon^2}-\frac{c}{r^{3\omega+1}}\right) \tag{11}$$

Now we consider the conditions for the circular motion corresponding to zero radial velocity $\dot{r}=0$) and acceleration $\ddot{r}=0$. Then one may obtain the radial profiles of the specific angular momentum and specific energy for circular orbits at the equatorial plane ($\theta=\pi/2$) in the following form maximums of potential then exist unstable null geodesics with a radius of r_C. Then we can consider three different scenarios depending on the values of E for the motion

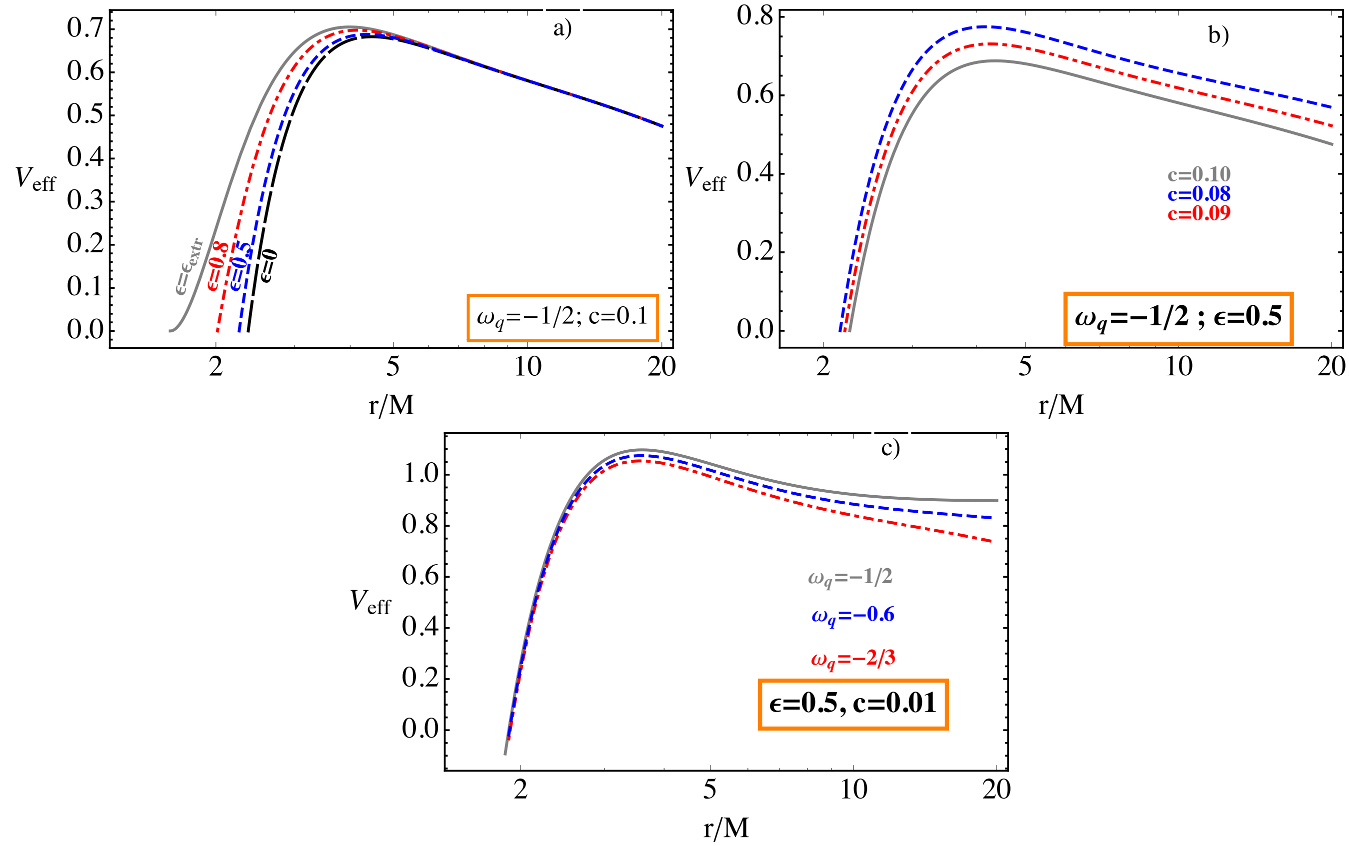

FIG. 4. Dependence of V_{eff} on the radial coordinate r/M for the different values of the parameter ω_q, ϵ and normalization factor , c.

A. Innermost stable circular orbits (ISCO)

We use the following standard conditions to study of stable circular orbits in the equatorial plane

$$V_{\rm eff}=\mathcal{E}\,, \qquad V'_{\rm eff}=0\,, \qquad V''_{\rm eff}\geq 0\,. \tag{12}$$

In figure(5) dependence of ISCO of a neutral particle moving around HKBH on the parameters ϵ, c and ω is studied graphically. It is observed that as the value of c increases (decreases) the minimum value of radius of ISCO also increases (decreases). Also, for smaller (larger) value of ω the value of minimum radius for ISCO is larger (smaller).

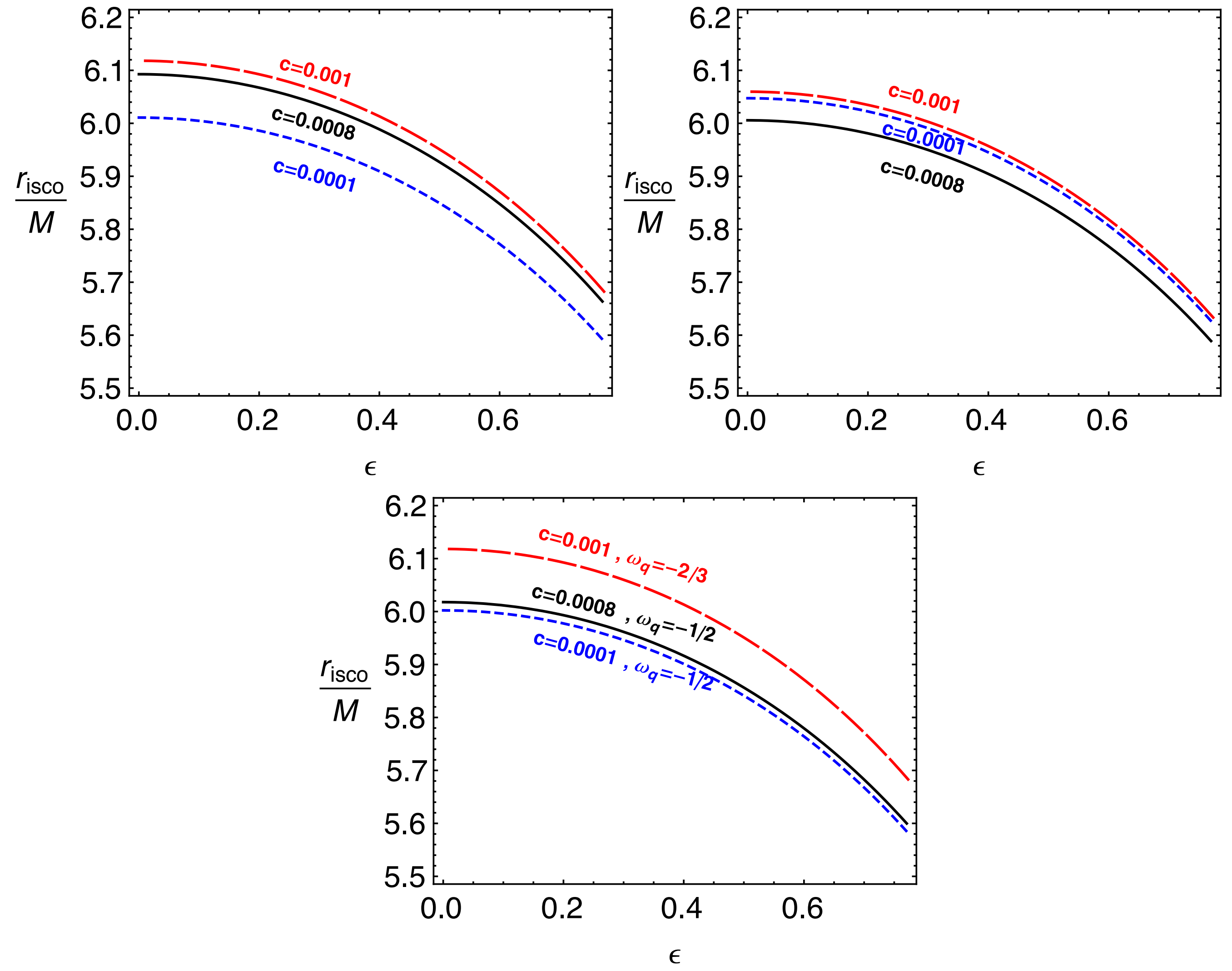

FIG. 5. The dependence of ISCO radius for the different values of the parameter ω, ϵ and normalization factor , c .

IV. CONCLUSIONS

In this research article we have studied the dynamics of neutral particle moving around Hayward-Kiselev BH (HKBH). This research work is devoted to focus on the effects of the normalization parameter c of the Hayward BH and equation of state parameter ω on the dynamics of particle. It is observed that all these scalars are defined at the centre of BH so there is no curvature singularity and as values of c increases the scalar invarients at the origin of the BH also increase in values.

A study is made of the horizons of the Hayward black hole surrounded by quintessence. Depending on a critical value of the normalization factor, the Hayward black hole surrounded by quintessence has one, two, or three horizons.

By analyzing the effective potential of test particles (photon), we have investigated the null geodesics and the kinds of orbits of the Hayward black hole surrounded by quintessence corresponding to different energy levels. The movement of the photons can be located within the quintessence horizon, but if it passes within the horizon $r = r_{out}$, the photons fall into the black hole, then quintessence horizon is an apparent horizon. Finally is possible to mention that the shadow area increases when the factor c increases.

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