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Analytical and numerical assessment of the accuracy of the approximated nuclear symmetry energy in the Hartree–Fock theory

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The nuclear symmetry energy is defined by the second derivative of the energy per nucleon with respect to the proton–neutron asymmetry, and is sometimes approximated by the energy difference between the neutron matter and the symmetric matter. The accuracy of this approximation is assessed analytically and numerically within the Hartree–Fock theory using effective interactions. By decomposing the nuclear-matter energy, the relative error of each term is expressed analytically; it is constant or is a single-variable function determined by the function type. The full errors are evaluated for several effective interactions, by inserting values for the parameters. Although the errors stay within 10% up to twice the normal density irrespective of the interactions, at higher densities the accuracy of the approximation significantly depends on the interactions.

Subject Index D10, D41

1. Introduction

The nuclear symmetry energy, which is defined as the second derivative of the energy per nucleon with respect to the proton–neutron asymmetry, is an important quantity for predicting the masses of neutron-rich nuclei and the structure of neutron stars. Calculation of the symmetry energy, however, is not always easy. Analytical approaches are impossible unless the energy of the system is given by a twice-differentiable function with respect to the asymmetry, while a numerical evaluation requires precise calculation of the energy of the system. For this reason, the symmetry energy is sometimes approximated as the energy difference between the pure neutron matter and the symmetric matter. Conversely, this approximation of the symmetry energy corresponds to a quadratic approximation of the neutron-matter energy with respect to the asymmetry, in its estimation from the symmetric-matter energy.

As Bethe originally pointed out [1], this quadratic approximation of energy is applicable to a small asymmetry but its validity is not obvious for systems with large asymmetry, like neutron matter. The accuracy of this approximation has been studied in several works. It was examined in the Brueckner–Hartree–Fock theory with the Paris potential by Bombaci and Lombardo [2], and with the AV18 plus three-nucleon force by Zuo et al. [3]. Chen et al. investigated this approximation at the saturation density with a modified Gogny interaction [4]. Drischler et al. also studied it in the chiral effective field theory [5]. It should be mentioned that Wellenhofer et al. pointed out that the approximation is not valid enough at certain temperatures [6].

We reinvestigate the accuracy of the approximation of the symmetry energy at zero temperature within the Hartree–Fock theory by using effective interactions which have been tested for the structure of finite nuclei. This enables analytical arguments, by which the origin of the errors can be examined term by term. Then, the errors can be numerically estimated by inserting values for the parameters. In practice we adopt six effective interactions: the Skyrme interactions SkM* [7] and SLy4 [8], the Gogny interactions D1S [9] and D1M [10], and the M3Y-type interactions M3Y-P6 and M3Y-P7 [11].

This paper is organized as follows. In Sect. 2, we decompose the nuclear-matter energy, the symmetry energy, and its error. From Sect. 3 to 5, we evaluate the relative errors of individual terms. Section 3 is devoted to the errors whose analytical forms are independent of the effective interaction. The errors specific to the Skyrme and finite-range (Gogny and M3Y-type) effective interactions are analytically investigated in Sects. 4 and 5, respectively. Numerical results for the errors for the symmetry energy are displayed and discussed in Sect. 6. Section 7 provides a summary of the paper.

2. Decomposition of energies and errors

In this section, we shall give expressions for the symmetry energy $a_t(\rho)$ and its approximation $\tilde{a}_t(\rho)$ by decomposing them into several terms. Correspondingly, the errors of the symmetry energy can be decomposed as well.

2.1. Effective Hamiltonian

In this paper, we consider homogeneous nuclear matter with spin degeneracy in the non-relativistic Hartree–Fock theory.

Because of the homogeneity, the single-particle wave function is written as a plane wave,

$$\varphi_{\boldsymbol{k}\sigma\tau}(\boldsymbol{r}) = \frac{1}{\sqrt{\Omega}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \chi_\sigma \chi_\tau, \tag{1}$$

where $\boldsymbol{k}$ denotes the momentum and Ω is the volume of the system, for which we shall take $\Omega \to \infty$ later. χ_σ (χ_τ) is the spin (isospin) wave function.

The effective Hamiltonian of the system consists of the kinetic energy and the effective interaction,

$$H = K + V\,; \quad K = \sum_i \frac{\boldsymbol{p}_i^2}{2M}, \quad V = \sum_{i<j} v_{ij}, \tag{2}$$

where i and j are nucleon indices. The effective interaction V is comprised of the central term, the LS term, and the tensor (TN) term. In homogeneous matter, the LS term and the TN term can be neglected. We therefore treat only the central term, which may contain the usual two-body interaction and a density-dependent interaction. For the latter we assume the contact form. We then have

$$\begin{aligned} v_{12} &= v_{12}^{(\mathrm{C})} + v_{12}^{(\mathrm{DD})}\,; \\ v_{12}^{(\mathrm{C})} &= \sum_n \left(t_n^{(\mathrm{SE})} P_{\mathrm{SE}} + t_n^{(\mathrm{TE})} P_{\mathrm{TE}} + t_n^{(\mathrm{SO})} P_{\mathrm{SO}} + t_n^{(\mathrm{TO})} P_{\mathrm{TO}}\right) f_n(r_{12}), \\ v_{12}^{(\mathrm{DD})} &= t_\rho^{(\mathrm{SE})} P_{\mathrm{SE}} \left[\rho(\boldsymbol{r}_1)\right]^\alpha \delta(\boldsymbol{r}_{12}) + t_\rho^{(\mathrm{TE})} P_{\mathrm{TE}} \left[\rho(\boldsymbol{r}_1)\right]^\beta \delta(\boldsymbol{r}_{12}), \end{aligned} \tag{3}$$

where $f_n(r_{12})$ is an appropriate function of $r_{12} = |\boldsymbol{r}_{12}|$, with $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$. In this paper, we consider the Skyrme, the Gogny, and the M3Y-type interactions. Correspondingly, $f_n(r_{12})$ is the δ-function (with and without momentum dependence), the Gauss function, or the Yukawa function.

The effective interactions have range parameters, and the index n distinguishes the range. All kinds of t in the above equation are strength parameters. $P_{\rm TE}$, $P_{\rm TO}$, $P_{\rm SE}$, and $P_{\rm SO}$ are projection operators to singlet–even, triplet–even, singlet–odd, and triplet–odd two-particle states, and they are related to the spin exchange operator P_σ and the isospin exchange operator P_τ as follows:

$$\begin{aligned} P_{\rm SE} &= \frac{1-P_\sigma}{2}\frac{1+P_\tau}{2}, \quad P_{\rm TE} = \frac{1+P_\sigma}{2}\frac{1-P_\tau}{2}, \\ P_{\rm SO} &= \frac{1-P_\sigma}{2}\frac{1-P_\tau}{2}, \quad P_{\rm TO} = \frac{1+P_\sigma}{2}\frac{1+P_\tau}{2}. \end{aligned} \tag{4}$$

We can rewrite $v_{12}^{\rm (C)}$ as follows,

$$\begin{aligned} v_{12}^{\rm (C)} &= \sum_n (t_n^{\rm (W)} + t_n^{\rm (B)} P_\sigma - t_n^{\rm (H)} P_\tau - t_n^{\rm (M)} P_\sigma P_\tau) f_n(r_{12})\,; \\ t_n^{\rm (W)} &= \left(t_n^{\rm (SE)} + t_n^{\rm (TE)} + t_n^{\rm (SO)} + t_n^{\rm (TO)}\right)/4, \\ t_n^{\rm (B)} &= \left(-t_n^{\rm (SE)} + t_n^{\rm (TE)} - t_n^{\rm (SO)} + t_n^{\rm (TO)}\right)/4, \\ t_n^{\rm (H)} &= \left(-t_n^{\rm (SE)} + t_n^{\rm (TE)} + t_n^{\rm (SO)} - t_n^{\rm (TO)}\right)/4, \\ t_n^{\rm (M)} &= \left(t_n^{\rm (SE)} + t_n^{\rm (TE)} - t_n^{\rm (SO)} - t_n^{\rm (TO)}\right)/4. \end{aligned} \tag{5}$$

2.2. Decomposition of energy per nucleon

With the wave function of Eq. (1) and the effective Hamiltonian of Eqs. (2) and (3), the energy per nucleon can be expressed in the following manner, as was derived in Ref. [12]. We consider a function $\mathcal{W}$ as

$$\mathcal{W}(k_\tau, k_{\tau'}) = \int_{k_1 \le k_\tau} d^3k_1 \int_{k_2 \le k_{\tau'}} d^3k_2 \tilde{f}(|\boldsymbol{k}_{12} - \boldsymbol{k}'_{12}|), \tag{6}$$

where $\tilde{f}(q)$ signifies the Fourier transform of $f(r)$: $\tilde{f}(q) = \int d^3r f(r) e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$. $\boldsymbol{k}_{12}$ and $\boldsymbol{k}'_{12}$ represent relative momenta ($\boldsymbol{k}_{12} = (\boldsymbol{k}_1 - \boldsymbol{k}_2)/2$) in the initial and the final states. Since $\boldsymbol{k}_{12} = \boldsymbol{k}'_{12}$ in the Hartree (i.e., direct) term and $\boldsymbol{k}_{12} = -\boldsymbol{k}'_{12}$ in the Fock (i.e., exchange) term, the contributions of these terms to the energy are represented by the function $\mathcal{W}$ as follows:

$$\begin{aligned} \mathcal{W}_n^{\rm H}(k_\tau, k_{\tau'}) &= \int_{k_1 \le k_\tau} d^3k_1 \int_{k_2 \le k_{\tau'}} d^3k_2 \tilde{f}_n(0) = \frac{16\pi^2}{9} k_\tau^3 k_{\tau'}^3 \tilde{f}_n(0) \quad \text{(Hartree term)}, \\ \mathcal{W}_n^{\rm F}(k_\tau, k_{\tau'}) &= \int_{k_1 \le k_\tau} d^3k_1 \int_{k_2 \le k_{\tau'}} d^3k_2 \tilde{f}_n(2k_{12}) \\ &= 8\pi^2 \Bigg[\int_0^{|k_{\tau'} - k_\tau|/2} dk_{12}\, \frac{16}{3} k_\tau^3 k_{12}^2 \tilde{f}(2k_{12}) + \int_{|k_{\tau'} - k_\tau|/2}^{(k_{\tau'} + k_\tau)/2} dk_{12} \Big\{ -\frac{1}{2}(k_{\tau'}^2 - k_\tau^2)^2 k_{12} \\ &\quad + \frac{8}{3}(k_\tau^3 + k_{\tau'}^3) k_{12}^2 - 4(k_\tau^2 + k_{\tau'}^2) k_{12}^3 + \frac{8}{3} k_{12}^5 \Big\} \tilde{f}_n(2k_{12}) \Bigg] \quad \text{(Fock term)}. \end{aligned} \tag{7}$$

We can express the total energy by using these functions. Under spin degeneracy, the total energy is represented by

$$\begin{aligned}
E = {} & \frac{\Omega}{10\pi^2 M}(k_{\rm p}^5 + k_{\rm n}^5) \\
& + \frac{\rho^\alpha \Omega}{36\pi^4}\left[t_\rho^{(\rm SE)}\rho^\alpha(k_{\rm p}^6 + k_{\rm n}^6) + (t_\rho^{(\rm SE)}\rho^\alpha + 3t_\rho^{(\rm TE)}\rho^\beta)k_{\rm p}^3 k_{\rm n}^3\right] \\
& + \frac{\Omega}{(2\pi)^6}\sum_n \Big[(2t_n^{(\rm W)} + t_n^{(\rm B)} - 2t_n^{(\rm H)} - t_n^{(\rm M)})\left\{\mathcal{W}_n^{\rm H}(k_{\rm p},k_{\rm p}) + \mathcal{W}_n^{\rm H}(k_{\rm n},k_{\rm n})\right\} \\
& \qquad + 2(2t_n^{(\rm W)} + t_n^{(\rm B)})\,\mathcal{W}_n^{\rm H}(k_{\rm p},k_{\rm n}) \\
& \qquad + (2t_n^{(\rm M)} + t_n^{(\rm H)} - 2t_n^{(\rm B)} - t_n^{(\rm W)})\left\{\mathcal{W}_n^{\rm F}(k_{\rm p},k_{\rm p}) + \mathcal{W}_n^{\rm F}(k_{\rm n},k_{\rm n})\right\} \\
& \qquad + 2(2t_n^{(\rm M)} + t_n^{(\rm H)})\,\mathcal{W}_n^{\rm F}(k_{\rm p},k_{\rm n})\Big],
\end{aligned} \tag{8}$$

where $k_{\rm p}$ ($k_{\rm n}$) denotes the Fermi momentum of protons (neutrons), and they are connected with the density $\rho_{\rm p}$ ($\rho_{\rm n}$) or with the total density ρ and the asymmetry η_t as

$$\begin{aligned}
k_{\rm p} = (3\pi^2\rho_{\rm p})^{1/3} = \left\{\frac{3\pi^2}{2}\rho(1-\eta_t)\right\}^{1/3}, \quad k_{\rm n} = (3\pi^2\rho_{\rm n})^{1/3} = \left\{\frac{3\pi^2}{2}\rho(1+\eta_t)\right\}^{1/3}; \\
\rho = \rho_{\rm p} + \rho_{\rm n}, \quad \eta_t = \frac{\rho_{\rm n} - \rho_{\rm p}}{\rho}.
\end{aligned} \tag{9}$$

The energy per nucleon $\mathcal{E} = E/A$ is acquired by dividing E by the nucleon number $A = \rho\Omega$. Here, we decompose $\mathcal{E}$ into the kinetic term ($\mathcal{E}_{\rm K}$), the density-dependent term ($\mathcal{E}_{\rm DD}$), the Hartree term between like nucleons ($\mathcal{E}_{\rm HO}$) and between unlike nucleons ($\mathcal{E}_{\rm HX}$), and the Fock term between like nucleons ($\mathcal{E}_{\rm FO}$) and between unlike nucleons ($\mathcal{E}_{\rm FX}$):

$$\begin{aligned}
\mathcal{E}(\rho,\eta_t) &= \mathcal{E}_{\rm K} + \mathcal{E}_{\rm DD} + \sum_n\left(\mathcal{E}_{{\rm HO}n} + \mathcal{E}_{{\rm HX}n} + \mathcal{E}_{{\rm FO}n} + \mathcal{E}_{{\rm FX}n}\right); \\
\mathcal{E}_{\rm K} &= \frac{1}{10\pi^2 M\rho}(k_{\rm p}^5 + k_{\rm n}^5), \\
\mathcal{E}_{\rm DD} &= \frac{1}{36\pi^4}\left[t_\rho^{(\rm SE)}\rho^{\alpha-1}(k_{\rm p}^6 + k_{\rm n}^6 + k_{\rm p}^3 k_{\rm n}^3) + 3t_\rho^{(\rm TE)}\rho^{\beta-1}k_{\rm p}^3 k_{\rm n}^3\right], \\
\mathcal{E}_{{\rm HO}n} &= \frac{1}{(2\pi)^6\rho}\cdot\left(2t_n^{(\rm W)} + t_n^{(\rm B)} - 2t_n^{(\rm H)} - t_n^{(\rm M)}\right)\left\{\mathcal{W}_n^{\rm H}(k_{\rm p},k_{\rm p}) + \mathcal{W}_n^{\rm H}(k_{\rm n},k_{\rm n})\right\}, \\
\mathcal{E}_{{\rm HX}n} &= \frac{1}{(2\pi)^6\rho}\cdot 2(2t_n^{(\rm W)} + t_n^{(\rm B)})\,\mathcal{W}_n^{\rm H}(k_{\rm p},k_{\rm n}), \\
\mathcal{E}_{{\rm FO}n} &= \frac{1}{(2\pi)^6\rho}\cdot\left(2t_n^{(\rm M)} + t_n^{(\rm H)} - 2t_n^{(\rm B)} - t_n^{(\rm W)}\right)\left\{\mathcal{W}_n^{\rm F}(k_{\rm p},k_{\rm p}) + \mathcal{W}_n^{\rm F}(k_{\rm n},k_{\rm n})\right\}, \\
\mathcal{E}_{{\rm FX}n} &= \frac{1}{(2\pi)^6\rho}\cdot 2(2t_n^{(\rm M)} + t_n^{(\rm H)})\,\mathcal{W}_n^{\rm F}(k_{\rm p},k_{\rm n}).
\end{aligned} \tag{10}$$

2.3. *Decomposition of symmetry energy*

The symmetry energy $a_t(\rho)$ is defined by the second-order derivative of $\mathcal{E}$ with respect to the asymmetry η_t,

$$a_t(\rho) := \frac{1}{2}\frac{\partial^2 \mathcal{E}}{\partial \eta_t^2}\bigg|_{\eta_t=0}. \tag{11}$$

Let us denote the νth-order partial derivative of a function $\mathcal{F}$ with respect to η_t by $\mathcal{F}^{(\nu)}$: $\mathcal{F}^{(\nu)} = (\partial^\nu/\partial\eta_t^\nu)\mathcal{F}$. Corresponding to the decomposition of $\mathcal{E}$, we also decompose the symmetry energy as follows:

$$\begin{aligned} a_t(\rho) &= \frac{1}{2}\mathcal{E}^{(2)}\bigg|_{\eta_t=0} = a_{\mathrm{K}} + a_{\mathrm{DD}} + \sum_n \left(a_{\mathrm{HO}n} + a_{\mathrm{HX}n} + a_{\mathrm{FO}n} + a_{\mathrm{FX}n}\right); \\ a_i &= \frac{1}{2}\mathcal{E}_i^{(2)}\bigg|_{\eta_t=0} \quad (i = \mathrm{K}, \mathrm{DD}, \mathrm{HO}n, \mathrm{HX}n, \mathrm{FO}n, \mathrm{FX}n). \end{aligned} \tag{12}$$

Formulas for calculating each a_i are given in Appendix B. Note that $\mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{p}}, k_{\mathrm{p}})^{(\nu)}\big|_{k_{\mathrm{p}}=k_{\mathrm{F}}} \neq \mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{p}}, k_{\mathrm{n}})^{(\nu)}\big|_{k_{\mathrm{p}}=k_{\mathrm{n}}=k_{\mathrm{F}}}$.

2.4. *Approximation of the symmetry energy and its error*

The symmetry energy $a_t(\rho)$ is often approximated by the difference in $\mathcal{E}$ between $\eta_t = 1$ and $\eta_t = 0$,

$$\begin{aligned} \tilde{a}_t(\rho) &:= \mathcal{E}(\rho, \eta_t = 1) - \mathcal{E}(\rho, \eta_t = 0) \\ &= a_t(\rho) + \sum_{\nu=2}^{\infty} \frac{\mathcal{E}(\rho, \eta_t)^{(2\nu)}}{(2\nu)!}\bigg|_{\eta_t=0}. \end{aligned} \tag{13}$$

This coincides with the quadratic approximation of $\mathcal{E}$ with respect to η_t,

$$\mathcal{E}(\rho, \eta_t) \approx \mathcal{E}(\rho, \eta_t = 0) + a_t(\rho)\,\eta_t^2. \tag{14}$$

Notice that $\mathcal{E}$ is an even function of η_t under charge symmetry.

We are now ready to consider the accuracy of the approximation of Eq. (13). As measures of the approximation, we will estimate the absolute error $\delta_t = \tilde{a}_t - a_t$ and the relative error $\Delta_t = (\tilde{a}_t - a_t)/a_t$. Corresponding to each term of Eq. (12) ($a_i(\rho)$; $i = \mathrm{K}, \mathrm{DD}, \mathrm{HO}n, \mathrm{HX}n, \mathrm{FO}n, \mathrm{FX}n$), we define its approximated value $\tilde{a}_i(\rho)$ by

$$\tilde{a}_i(\rho) := \mathcal{E}_i(\rho, \eta_t = 1) - \mathcal{E}_i(\rho, \eta_t = 0). \tag{15}$$

Then the relative error of each term $\Delta_i = (\tilde{a}_i - a_i)/a_i$ can be estimated analytically, as shown below. The full errors δ_t and Δ_t are expressed by using Δ_i,

$$\begin{aligned} \delta_t &= \tilde{a}_t - a_t = \sum_i \Delta_i\, a_i, \\ \Delta_t &= \frac{\tilde{a}_t - a_t}{a_t} = \sum_i \Delta_i \frac{a_i}{a_t}. \end{aligned} \tag{16}$$

Thus, through Δ_i, the error of the symmetry energy can be examined term by term. We shall estimate Δ_i in the following three sections.

3. Terms with constant relative errors

The relative errors (Δ_i) for the kinetic term, the density-dependent term, and the Hartree term of the central force are constant, or even vanish, independent of the density. In this section we discuss Δ_i for these terms.

3.1. Kinetic term

We first discuss the kinetic energy term in the exact and the approximated symmetry energy, $a_{\rm K}$ and $\tilde{a}_{\rm K}$. The 2νth-order coefficient of kinetic energy per nucleon ($1 \le \nu$) can be derived inductively:

$$\left.\frac{\mathcal{E}_{\rm K}^{(2\nu)}(\rho,\eta_t)}{(2\nu)!}\right|_{\eta_t=0} = \frac{k_{\rm F}^5}{5\pi^2 M\rho}\cdot\frac{1}{(2\nu)!}\prod_{i=0}^{2\nu-1}\frac{5-3i}{3}; \quad k_{\rm F}=\left(\frac{3\pi^2\rho}{2}\right)^{1/3}. \tag{17}$$

The relative error of the kinetic term in the symmetry energy turns out to be constant, independent of ρ:

$$\Delta_{\rm K} = \frac{\tilde{a}_{\rm K}-a_{\rm K}}{a_{\rm K}} = \sum_{\nu=2}^{\infty}\left.\frac{\mathcal{E}_{\rm K}^{(2\nu)}/(2\nu)!}{\mathcal{E}_{\rm K}^{(2)}/2}\right|_{\eta_t=0} = \sum_{\nu=2}^{\infty}\left[\prod_{i=1}^{\nu-1}\frac{(6i-5)(6i-2)}{(6i+3)(6i+6)}\right] = 0.057. \tag{18}$$

In Eq. (6) of Ref. [6], the result equivalent to Eq. (18) is presented as $(\tilde{a}_{\rm K}-a_{\rm K})/\tilde{a}_{\rm K} = \Delta_{\rm K}/(1+\Delta_{\rm K}) = 0.054$.

3.2. Density-dependent term

For the density-dependent contact term, we have

$$\tilde{a}_{\rm DD} = a_{\rm DD}, \quad \Delta_{\rm DD} = 0. \tag{19}$$

This is because $\mathcal{E}_{\rm DD}$ can be written up to the quadratic term with respect to η_t as

$$\mathcal{E}_{\rm DD}(\rho,\eta_t) = \frac{1}{16}\left[\, t_\rho^{\rm (SE)}\rho^{\alpha+1}(3+\eta_t^2) + 3t_\rho^{\rm (TE)}\rho^{\beta+1}(1-\eta_t^2)\,\right]. \tag{20}$$

3.3. Hartree term

The Hartree term of the central force in the energy per nucleon also depends quadratically on η_t. In practice, Eq. (7) yields

$$\begin{aligned}
\mathcal{W}_n^{\rm H}(k_{\rm p},k_{\rm p}) &= 4\pi^6\rho^2(1-\eta_t)^2\tilde{f}_n(0),\\
\mathcal{W}_n^{\rm H}(k_{\rm n},k_{\rm n}) &= 4\pi^6\rho^2(1+\eta_t)^2\tilde{f}_n(0),\\
\mathcal{W}_n^{\rm H}(k_{\rm p},k_{\rm n}) &= 4\pi^6\rho^2(1-\eta_t^2)\tilde{f}_n(0),
\end{aligned} \tag{21}$$

leading to $\Delta_{{\rm HO}n} = \Delta_{{\rm HX}n} = 0$.

4. Skyrme interaction

We next discuss the terms depending on the function types of the interaction, through which a full expression of the errors of the symmetry energy is obtained.

The Skyrme interaction has momentum-dependent terms, additional to Eq. (3), while some of the terms in Eq. (5) become equivalent to the exchange terms of others. The interaction is expressed, instead of Eq. (3), as

$$\begin{aligned} v_{12} &= t_0(1+x_0P_\sigma)\,\delta(\boldsymbol{r}_{12}) \\ &\quad + \frac{1}{2}t_1(1+x_1P_\sigma)\left[\boldsymbol{p}'^2_{12}\,\delta(\boldsymbol{r}_{12}) + \delta(\boldsymbol{r}_{12})\,\boldsymbol{p}^2_{12}\right] + t_2(1+x_2P_\sigma)\boldsymbol{p}'_{12}\cdot\delta(\boldsymbol{r}_{12})\,\boldsymbol{p}_{12} \\ &\quad + t_\rho^{(\mathrm{SE})}P_{\mathrm{SE}},[\rho(\boldsymbol{r}_1)]^\alpha\,\delta(\boldsymbol{r}_{12}) + t_\rho^{(\mathrm{TE})}P_{\mathrm{TE}}\,[\rho(\boldsymbol{r}_1)]^\beta\,\delta(\boldsymbol{r}_{12}). \end{aligned} \tag{22}$$

Here, $\boldsymbol{p}_{12} = (\nabla_1 - \nabla_2)/(2i)$, and $\boldsymbol{p}'_{12}$ is the Hermitian conjugate of $\boldsymbol{p}_{12}$ acting on the left. For the t_0 term that has $f(r_{12}) = \delta(\boldsymbol{r}_{12})$, we have

$$\mathcal{W}_0^{\mathrm{H}}(k_\tau,k_{\tau'}) = \mathcal{W}_0^{\mathrm{F}}(k_\tau,k_{\tau'}) = \frac{16\pi^2}{9}k_\tau^3k_{\tau'}^3. \tag{23}$$

For the t_1 and t_2 terms, $\frac{1}{2}(\boldsymbol{p}'^2_{12}\,\delta(\boldsymbol{r}_{12}) + \delta(\boldsymbol{r}_{12})\,\boldsymbol{p}^2_{12})$ and $\boldsymbol{p}'_{12}\cdot\delta(\boldsymbol{r}_{12})\,\boldsymbol{p}_{12}$ yield [12]

$$\begin{aligned} \mathcal{W}_1^{\mathrm{H}}(k_\tau,k_{\tau'}) = \mathcal{W}_1^{\mathrm{F}}(k_\tau,k_{\tau'}) &= \frac{4\pi^2}{15}k_\tau^3k_{\tau'}^3(k_\tau^2+k_{\tau'}^2) \quad (t_1\text{ term}), \\ \mathcal{W}_2^{\mathrm{H}}(k_\tau,k_{\tau'}) = -\mathcal{W}_2^{\mathrm{F}}(k_\tau,k_{\tau'}) &= \frac{4\pi^2}{15}k_\tau^3k_{\tau'}^3(k_\tau^2+k_{\tau'}^2) \quad (t_2\text{ term}), \end{aligned} \tag{24}$$

respectively.

Let us define

$$\begin{aligned} \mathcal{W}_{\mathrm{c}}^{\mathrm{o}} &= \frac{16\pi^2}{9}(k_{\mathrm{p}}^6+k_{\mathrm{n}}^6), \quad \mathcal{W}_{\mathrm{c}}^{\mathrm{x}} = \frac{16\pi^2}{9}k_{\mathrm{p}}^3k_{\mathrm{n}}^3, \\ \mathcal{W}_{\mathrm{p}}^{\mathrm{o}} &= \frac{8\pi^2}{15}(k_{\mathrm{p}}^8+k_{\mathrm{n}}^8), \quad \mathcal{W}_{\mathrm{p}}^{\mathrm{x}} = \frac{8\pi^2}{15}k_{\mathrm{p}}^3k_{\mathrm{n}}^3(k_{\mathrm{p}}^2+k_{\mathrm{n}}^2), \end{aligned} \tag{25}$$

where the superscript o (x) indicates the like- (unlike-) nucleon contribution and the subscript c (p) corresponds to the t_0 (t_1 and t_2) term. Then the energy per nucleon $\mathcal{E}$ is decomposed as follows:

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_{\mathrm{K}} + \mathcal{E}_{\mathrm{c}} + \mathcal{E}_{\mathrm{pO}} + \mathcal{E}_{\mathrm{pX}} + \mathcal{E}_{\mathrm{DD}}\,; \\ \mathcal{E}_{\mathrm{c}} &= \frac{1}{(2\pi)^6\rho}\left(t_{\mathrm{c}}^{\mathrm{o}}\,\mathcal{W}_{\mathrm{c}}^{\mathrm{o}} + t_{\mathrm{c}}^{\mathrm{x}}\,\mathcal{W}_{\mathrm{c}}^{\mathrm{x}}\right), \\ \mathcal{E}_{\mathrm{pO}} &= \frac{1}{(2\pi)^6\rho}\,t_{\mathrm{p}}^{\mathrm{o}}\,\mathcal{W}_{\mathrm{p}}^{\mathrm{o}}, \quad \mathcal{E}_{\mathrm{pX}} = \frac{1}{(2\pi)^6\rho}t_{\mathrm{p}}^{\mathrm{x}}\,\mathcal{W}_{\mathrm{p}}^{\mathrm{x}}, \end{aligned} \tag{26}$$

where

$$\begin{aligned} t_{\mathrm{c}}^{\mathrm{o}} &= t_0(1-x_0), \quad t_{\mathrm{c}}^{\mathrm{x}} = t_0(2+x_0), \\ t_{\mathrm{p}}^{\mathrm{o}} &= t_1(1-x_1) + 3t_2(1+x_2), \quad t_{\mathrm{p}}^{\mathrm{x}} = t_1(2+x_1) + t_2(2+x_2). \end{aligned} \tag{27}$$

Corresponding to the above decomposition, the symmetry energy and its error are decomposed into a_i and Δ_i ($i = \mathrm{K}, \mathrm{c}, \mathrm{pO}, \mathrm{pX}, \mathrm{DD}$). δ_t and Δ_t are expressed by

$$\begin{aligned}\delta_t &= \tilde{a}_t - a_t = \Delta_\mathrm{K} a_\mathrm{K} + \Delta_\mathrm{pO} a_\mathrm{pO} + \Delta_\mathrm{pX} a_\mathrm{pX},\\ \Delta_t &= \frac{\tilde{a}_t - a_t}{a_t} = \Delta_\mathrm{K}\frac{a_\mathrm{K}}{a_t} + \Delta_\mathrm{pO}\frac{a_\mathrm{pO}}{a_t} + \Delta_\mathrm{pX}\frac{a_\mathrm{pX}}{a_t},\end{aligned} \tag{28}$$

since we obviously have $\Delta_\mathrm{c} = \Delta_\mathrm{DD} = 0$.

We have shown, in the previous section, that $\Delta_\mathrm{K} = 0.057$. For the momentum-dependent terms, we consider the ratios $\tilde{a}_\mathrm{pO}/a_\mathrm{pO} = 1 + \Delta_\mathrm{pO}$ and $\tilde{a}_\mathrm{pX}/a_\mathrm{pX} = 1 + \Delta_\mathrm{pX}$, where $a_\mathrm{pO} = \frac{1}{2}\mathcal{E}^{(2)}_\mathrm{pO}\big|_{\eta_t=0}$, $\tilde{a}_\mathrm{pO} = \mathcal{E}_\mathrm{pO}\big|_{\eta_t=1} - \mathcal{E}_\mathrm{pO}\big|_{\eta_t=0}$, and likewise for a_pX, $\tilde{a}_\mathrm{pX}$. Because

$$\begin{aligned}\mathcal{W}^{\mathrm{o}(2\nu)}_\mathrm{p}(\rho,\eta_t)\Big|_{\eta_t=0} &= \frac{16\pi^2}{15}k_\mathrm{F}^8\prod_{i=1}^{2\nu}\frac{11-3i}{3},\\ \mathcal{W}^{\mathrm{x}(2\nu)}_\mathrm{p}(\rho,\eta_t)\Big|_{\eta_t=0} &= \frac{16\pi^2}{15}k_\mathrm{F}^8(3\nu-2)\prod_{i=1}^{2\nu}\frac{11-3i}{3},\end{aligned} \tag{29}$$

the ratios are

$$\begin{aligned}\frac{\tilde{a}_\mathrm{pO}}{a_\mathrm{pO}} &= \left.\frac{\sum_{\nu=1}^{\infty}\mathcal{E}^{(2\nu)}_\mathrm{pO}/(2\nu)!}{\mathcal{E}^{(2)}_\mathrm{pO}/2}\right|_{\eta_t=0} = \frac{9}{20}\sum_{\nu=1}^{\infty}\left[\frac{1}{3^{2\nu}(2\nu)!}\prod_{i=1}^{2\nu}(11-3i)\right] = 0.979,\\ \frac{\tilde{a}_\mathrm{pX}}{a_\mathrm{pX}} &= \left.\frac{\sum_{\nu=1}^{\infty}\mathcal{E}^{(2\nu)}_\mathrm{pX}/(2\nu)!}{\mathcal{E}^{(2)}_\mathrm{pX}/2}\right|_{\eta_t=0} = \frac{9}{20}\sum_{\nu=1}^{\infty}\left[\frac{3\nu-2}{3^{2\nu}(2\nu)!}\prod_{i=1}^{2\nu}(11-3i)\right] = 0.900.\end{aligned} \tag{30}$$

Therefore δ_t and Δ_t for the Skyrme interaction are calculated by the following formulas:

$$\begin{aligned}\delta_t &= \tilde{a}_t - a_t = 0.057 a_\mathrm{K} - 0.021 a_\mathrm{pO} - 0.100 a_\mathrm{pX},\\ \Delta_t &= \frac{\tilde{a}_t - a_t}{a_t} = 0.057\frac{a_\mathrm{K}}{a_t} - 0.021\frac{a_\mathrm{pO}}{a_t} - 0.100\frac{a_\mathrm{pX}}{a_t}.\end{aligned} \tag{31}$$

5. Finite-range interactions

In this section, the errors of each term in the symmetry energy are analytically investigated for finite-range effective interactions. The Gauss function $f_n(r_{12}) = e^{-(\mu_n r_{12})^2}$ and the Yukawa function $f_n(r_{12}) = e^{-\mu_n r_{12}}/\mu_n r_{12}$ are handled in practice; they are used in the central channels of the Gogny and M3Y-type interactions.

As we showed in Sect. 3, the density-dependent term and the Hartree term give no errors. Then, the errors in the approximated symmetry energy can be expressed as

$$\begin{aligned}\delta_t &= \tilde{a}_t - a_t = \Delta_\mathrm{K} a_\mathrm{K} + \sum_n \Delta_{\mathrm{FO}n} a_{\mathrm{FO}n} + \sum_n \Delta_{\mathrm{FX}n} a_{\mathrm{FX}n},\\ \Delta_t &= \frac{\tilde{a}_t - a_t}{a_t} = \Delta_\mathrm{K}\frac{a_\mathrm{K}}{a_t} + \sum_n \Delta_{\mathrm{FO}n}\frac{a_{\mathrm{FO}n}}{a_t} + \sum_n \Delta_{\mathrm{FX}n}\frac{a_{\mathrm{FX}n}}{a_t}.\end{aligned} \tag{32}$$

Recall that Δ_K is constant. Unlike the Skyrme interaction, the relative errors of the Fock terms in the symmetry energy depend on the density. However, for a given function form, each of $\Delta_{\mathrm{FO}n}$

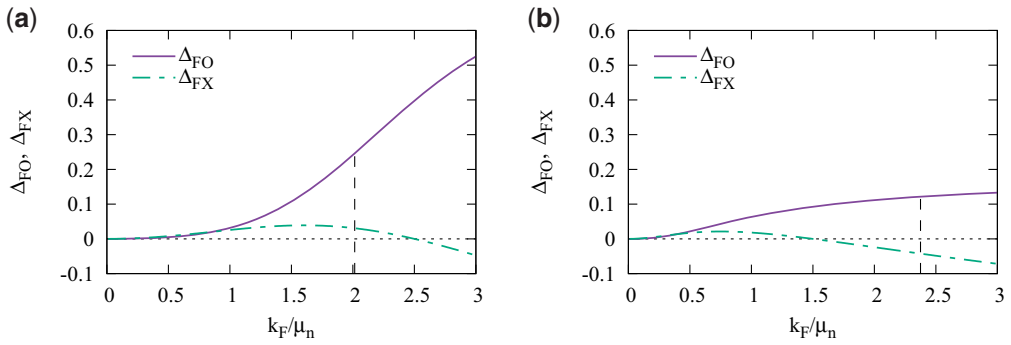

Fig. 1. k_F/μ_n dependence of Δ_{FOn} and Δ_{FXn} for (a) the Gauss interaction and (b) the Yukawa interaction. The dashed lines are explained in the text.

and Δ_{FXn} depends only on k_F/μ_n, where $1/\mu_n$ is the range parameter. Therefore, we calculate the relative errors Δ_{FOn} and Δ_{FXn} as functions of k_F/μ_n. We depict the results in Fig. 1 for the Gauss and the Yukawa functions, whose analytic expressions are given in Appendix C. It is emphasized that these results are determined only by the function type of the effective interaction, independent of the parameters. As k_F/μ_n increases, $|\Delta_{FOn}|$ and $|\Delta_{FXn}|$ tend to deviate from zero. The longest range of the nucleon–nucleon interaction should be given by the Compton wavelength of the pion (1.414 fm). Corresponding to the range $0 \le 1/\mu_n \le 1.414\,\text{fm}$ and the density $0 \le \rho \le 0.64\,\text{fm}^{-3}$ (i.e., $0 \le k_F \le 2.11\,\text{fm}^{-1}$), whose upper bound is about four times the normal density, Δ_{FOn} and Δ_{FXn} are displayed for $0 \le k_F/\mu_n \le 3$.

The dashed vertical line in Fig. 1(a) represents k_F/μ_n for $\rho = 0.32\,\text{fm}^{-3}$ and $1/\mu_n = 1.2\,\text{fm}$, which is the longer range in D1S. We find that $|\Delta_{FOn}|$ may reach about 0.25 at $\rho = 0.32\,\text{fm}^{-3}$ for the Gauss interaction, while $|\Delta_{FXn}|$ is kept within 0.05 in the full range in Fig. 1(a).

As in Fig. 1(a), the dashed line in Fig. 1(b) represents k_F/μ_n for $\rho = 0.32\,\text{fm}^{-3}$ and the longest range of the M3Y-type interaction $1.414\,\text{fm}^{-1}$. In contrast to the Gauss function, $|\Delta_{FOn}|$ and $|\Delta_{FXn}|$ stay within 0.1 in $0 \le \rho \le 0.32\,\text{fm}^{-3}$ for the Yukawa interaction. Even in the higher-density region up to $k_F/\mu_n = 3$, they are 0.13 at the most.

6. Numerical results for several interactions

By inputting the values of the parameters, Δ_t and δ_t are calculated at individual values of ρ. In Tables 1 to 3, δ_t at the saturation density ρ_0 is presented for the interactions SkM*, SLy4, D1S, D1M, M3Y-P6, and M3Y-P7. The values of the parameters in these interactions are given in Appendix A. The decomposed errors δ_i are also listed in Tables 1 to 3, unless they vanish.

In Fig. 2, we show the numerical results of Δ_t and δ_t as a function of ρ. Figure 2(a) reveals the interaction dependence of Δ_t. In $0 < \rho \le 0.32\,\text{fm}^{-3}$, we find that the relative error is $|\Delta_t| < 0.1$ for all of the effective interactions. To be more precise, it is within $|\Delta_t| < 0.05$ except for SkM*. In the higher-density region of $0.32 \le \rho < 0.64\,\text{fm}^{-3}$, $|\Delta_t| < 0.05$ holds for SLy4 and the M3Y-type interactions. In contrast, $|\Delta_t|$ exceeds 0.20 for SkM* and the Gogny interactions. This tendency is also seen for δ_t in Fig. 2(b). The $|\delta_t|$ of M3Y-P6, M3Y-P7, and SLy4 are significantly smaller than those of D1M, D1S, and SkM*.

Interestingly, although SkM* and SLy4 have the same function type, δ_t behaves quite differently between them. The origin of this difference may be accounted for on the basis of the decomposition in Sect. 4, via Eq. (31) to be more precise. In Fig. 3, $a_K(\rho)$, $a_{pO}(\rho)$, and $a_{pX}(\rho)$ for SkM* and SLy4 are

Table 1. $\delta_t(\rho_0)$ and $\delta_i(\rho_0)$ for SkM* and SLy4.

		SkM*	SLy4
ρ_0	(fm^{-3})	0.160	0.160
$a_t(\rho_0)$	(MeV)	30.03	32.00
$\delta_t(\rho_0)$	(MeV)	1.36	0.68
$\delta_{\rm K}(\rho_0)$	(MeV)	0.70	0.70
$\delta_{\rm pO}(\rho_0)$	(MeV)	−0.00	−0.33
$\delta_{\rm pX}(\rho_0)$	(MeV)	0.65	0.31

Table 2. $\delta_t(\rho_0)$ and $\delta_i(\rho_0)$ for D1S and D1M.

		D1S	D1M
ρ_0	(fm^{-3})	0.163	0.165
$a_t(\rho_0)$	(MeV)	31.12	28.55
$\delta_t(\rho_0)$	(MeV)	0.82	1.17
$\delta_{\rm K}(\rho_0)$	(MeV)	0.71	0.72
$\delta_{\rm FO1}(\rho_0)$	(MeV)	0.06	−0.71
$\delta_{\rm FO2}(\rho_0)$	(MeV)	−0.35	0.88
$\delta_{\rm FX1}(\rho_0)$	(MeV)	−0.68	−1.47
$\delta_{\rm FX2}(\rho_0)$	(MeV)	1.08	1.75

Table 3. $\delta_t(\rho_0)$ and $\delta_i(\rho_0)$ for M3Y-P6 and M3Y-P7.

		M3Y-P6	M3Y-P7
ρ_0	(fm^{-3})	0.163	0.163
$a_t(\rho_0)$	(MeV)	32.14	31.74
$\delta_t(\rho_0)$	(MeV)	1.10	1.20
$\delta_{\rm K}(\rho_0)$	(MeV)	0.71	0.71
$\delta_{\rm FO1}(\rho_0)$	(MeV)	−0.38	−0.43
$\delta_{\rm FO2}(\rho_0)$	(MeV)	0.15	0.22
$\delta_{\rm FO3}(\rho_0)$	(MeV)	−0.09	−0.09
$\delta_{\rm FX1}(\rho_0)$	(MeV)	0.00	0.12
$\delta_{\rm FX2}(\rho_0)$	(MeV)	0.78	0.74
$\delta_{\rm FX3}(\rho_0)$	(MeV)	−0.08	−0.08

depicted. As recognized from Eq. (31) and Fig. 3, $\Delta_{\rm pX}a_{\rm pX} = -0.1a_{\rm pX}$ contributes to δ_t positively. The contribution $(-a_{\rm pX})$ is larger for SkM* than for SLy4. Moreover, $\Delta_{\rm pO}a_{\rm pO} = -0.021a_{\rm pO}$ contributes negatively and it tends to cancel the positive contribution of $a_{\rm pX}$ for SLy4, while $a_{\rm pO}$ almost vanishes for SkM*.

Figure 1 accounts for the difference in δ_t between the Gogny and M3Y-type interactions. The large $|\Delta_{{\rm FX}n}|$ for the Gauss function at high $k_{\rm F}/\mu_n$ makes $|\delta_t|$ large. Note that Δ_t in D1S diverges at $\rho = 0.55\,{\rm fm}^{-3}$, as shown in Fig. 2(a), because a_t reaches 0.

7. Summary

We have investigated the accuracy of the approximation of the symmetry energy in nuclear matter within the Hartree–Fock theory. As measures of the accuracy of $\tilde{a}_t(\rho)$ (difference of the neutron-matter energy from the symmetric-matter energy) relative to $a_t(\rho)$ (second derivative of

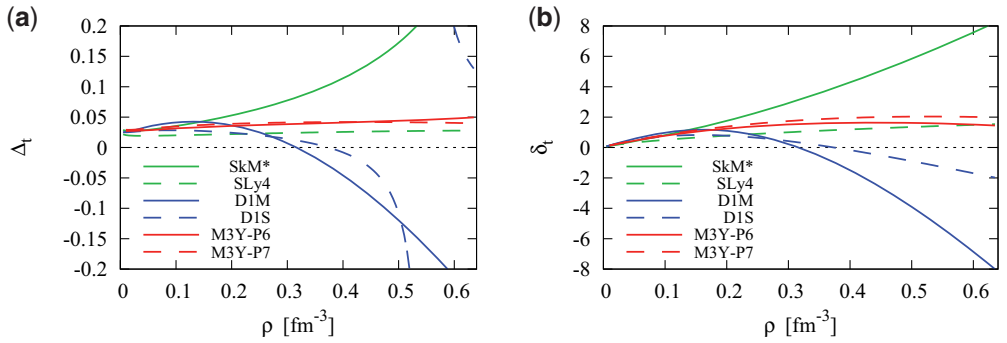

Fig. 2. Errors for the symmetry energy, (a) Δ_t and (b) δ_t (MeV), for several effective interactions.

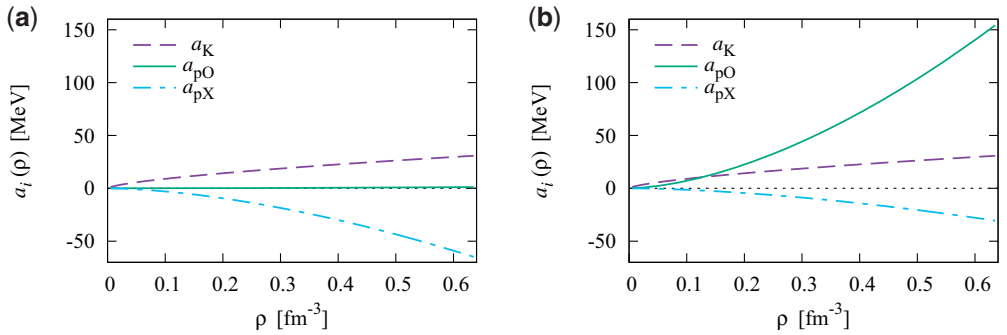

Fig. 3. Contribution of each term in the symmetry energy for (a) SkM* and (b) SLy4.

the energy with respect to the asymmetry η_t), we have estimated the absolute error δ_t and the relative error Δ_t. The errors are decomposed into several terms, associated with the decomposition of the nuclear-matter energy in Eq. (10). With this decomposition, δ_t can be expressed in terms of a_i and Δ_i, where i represents the individual components. We have derived analytical expressions for each of the Δ_i, which makes the origin of δ_t and Δ_t transparent. On this basis, δ_t and Δ_t are estimated for the six effective interactions by inserting values for the parameters. We have found $-2\,\mathrm{MeV} \lesssim \delta_t \lesssim 4\,\mathrm{MeV}$, which means $|\Delta_t| < 0.1$, up to twice the normal density for all of the effective interactions. At higher densities, they depend considerably on the effective interactions. The errors are kept small for the two M3Y-type interactions and SLy4, while the good accuracy of the approximation is lost for the two Gogny interactions and SkM*. We have explained this tendency from Δ_i, i.e., the relative errors of individual terms in the symmetry energy.

Recall that SLy4, D1M, M3Y-P6, and M3Y-P7 are the parameter sets that are fitted to the microscopic calculation of the equation of state of neutron matter. With the exception of D1M, this constraint might help to keep δ_t and Δ_t small even at high density. For the Skyrme interactions, all of Δ_i are constant, and δ_t can be written by only three terms, the kinetic term a_K, the momentum-dependent term between like nucleons a_pO, and that between unlike nucleons a_pX. Then, we found that the parameters of the momentum-dependent terms t_p^o and t_p^x produced different behavior of δ_t between SkM* and SLy4. For the finite-range interactions, δ_t can be expressed by the kinetic term a_K, the Fock terms between like nucleons $a_{\mathrm{FO}n}$ and between unlike nucleons $a_{\mathrm{FX}n}$. For given function types, the coefficients of the Fock terms $\Delta_{\mathrm{FO}n}$ and $\Delta_{\mathrm{FX}n}$ are single-variable functions of k_F/μ_n, which are independent of the parameters. As shown in Fig. 1, $\Delta_{\mathrm{FX}n}$ produces a significant difference

in δ_t and Δ_t between the Gauss and the Yukawa functions at high density. This suggests that the function type plays a certain role in the accuracy of the approximation.

Acknowledgements

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Appendix A. Parameters of interactions

The parameters of the effective interactions used in this paper, SkM* [7], SLy4 [8], D1S [9], D1M [10], M3Y-P6, and M3Y-P7 [11], are tabulated in Tables A1, A2, and A3.

Table A1. Parameter sets of SkM* and SLy4.

Parameters		SkM*	SLy4
t_0	(MeV fm^3)	−2645.0	−2488.91
t_1	(MeV fm^5)	410.0	483.13
t_2	(MeV fm^5)	−135.0	−549.40
x_0		0.090	0.778
x_1		0.000	0.328
x_2		0.000	−1.000
t_c^o	(MeV fm^3)	−382.291	−2406.95
t_c^x	(MeV fm^3)	−7064.94	−5528.05
t_p^o	(MeV fm^5)	5.000	324.663
t_p^x	(MeV fm^5)	550.00	575.327
$t_\rho^{(SE)}$	(MeV fm$^{3(1+\alpha)}$)	2599.17	−608.49
$t_\rho^{(TE)}$	(MeV fm$^{3(1+\beta)}$)	2599.17	5166.12
α		1/6	1/6
β		1/6	1/6

Table A2. Parameter sets of D1S and D1M.

Parameters		D1S	D1M
μ_1^{-1}	(fm)	0.7	0.5
μ_2^{-1}	(fm)	1.2	1.0
$t_1^{(W)}$	(MeV)	−1720.3	−12797.57
$t_1^{(B)}$	(MeV)	1300.0	14048.85
$t_1^{(H)}$	(MeV)	−1813.53	−15144.43
$t_1^{(M)}$	(MeV)	1397.6	11963.89
$t_2^{(W)}$	(MeV)	103.64	490.95
$t_2^{(B)}$	(MeV)	−163.48	−752.27
$t_2^{(H)}$	(MeV)	162.81	675.12
$t_2^{(M)}$	(MeV)	−223.93	−693.57
$t_\rho^{(SE)}$	(MeV fm$^{3(1+\alpha)}$)	0	0
$t_\rho^{(TE)}$	(MeV fm$^{3(1+\beta)}$)	2781.2	3124.44
α		—	—
β		1/3	1/3

Table A3. Parameter sets of M3Y-P6 and M3Y-P7.

Parameters		M3Y-P6	M3Y-P7
μ_1^{-1}	(fm)	0.25	0.25
μ_2^{-1}	(fm)	0.40	0.40
μ_3^{-1}	(fm)	1.414	1.414
$t_1^{(\mathrm{SE})}$	(MeV)	10766	10655
$t_1^{(\mathrm{TE})}$	(MeV)	8474	9592
$t_1^{(\mathrm{SO})}$	(MeV)	−728	11510
$t_1^{(\mathrm{TO})}$	(MeV)	12453	13507
$t_2^{(\mathrm{SE})}$	(MeV)	−3520	−3556
$t_2^{(\mathrm{TE})}$	(MeV)	−4594	−4594
$t_2^{(\mathrm{SO})}$	(MeV)	1386	1283
$t_2^{(\mathrm{TO})}$	(MeV)	−1588	−1812
$t_3^{(\mathrm{SE})}$	(MeV)	−10.463	−10.463
$t_3^{(\mathrm{TE})}$	(MeV)	−10.463	−10.463
$t_3^{(\mathrm{SO})}$	(MeV)	31.389	31.389
$t_3^{(\mathrm{TO})}$	(MeV)	3.488	3.488
$t_\rho^{(\mathrm{SE})}$	(MeV fm$^{3(1+\alpha)}$)	384	830
$t_\rho^{(\mathrm{TE})}$	(MeV fm$^{3(1+\beta)}$)	1930	1478
α		1	1
β		1/3	1/3

Appendix B. Formulas for symmetry energy

Here we present some formulas for a_i, which is defined in Eq. (12).

The second-order derivatives with respect to the asymmetry of the kinetic-energy term and the density-dependent term are

$$\begin{aligned}\mathcal{E}_{\mathrm{K}}^{(2)} &= \frac{\partial^2 \mathcal{E}_{\mathrm{K}}}{\partial \eta_t^2} = \frac{\pi^2 \rho}{4M}(k_{\mathrm{p}}^{-1} + k_{\mathrm{n}}^{-1}),\\ \mathcal{E}_{\mathrm{DD}}^{(2)} &= \frac{\partial^2 \mathcal{E}_{\mathrm{DD}}}{\partial \eta_t^2} = \frac{1}{8}\left(t_\rho^{(\mathrm{SE})}\rho^{\alpha+1} - 3t_\rho^{(\mathrm{TE})}\rho^{\beta+1}\right).\end{aligned} \tag{B.1}$$

The second-order derivatives of the $\mathcal{W}$ functions are:

$$\begin{aligned}&\mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{p}}, k_{\mathrm{p}})^{(2)} + \mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{n}}, k_{\mathrm{n}})^{(2)} = 16\pi^6\rho^2\tilde{f}_n(0),\\ &\mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{p}}, k_{\mathrm{n}})^{(2)} = -8\pi^6\rho^2\tilde{f}_n(0),\\ &\mathcal{W}_n^{\mathrm{F}}(k_{\mathrm{p}}, k_{\mathrm{p}})^{(2)} + \mathcal{W}_n^{\mathrm{F}}(k_{\mathrm{n}}, k_{\mathrm{n}})^{(2)} = 32\pi^6\rho^2\left[k_{\mathrm{p}}^{-4}\int_0^{k_{\mathrm{p}}} dk\, k^3\tilde{f}_n(2k) + k_{\mathrm{n}}^{-4}\int_0^{k_{\mathrm{n}}} dk\, k^3\tilde{f}_n(2k)\right],\\ &\mathcal{W}_n^{\mathrm{F}}(k_{\mathrm{p}}, k_{\mathrm{n}})^{(2)} = 4\pi^6\rho^2\int_{|k_{\mathrm{p}}-k_{\mathrm{n}}|/2}^{(k_{\mathrm{p}}+k_{\mathrm{n}})/2} dk\\ &\qquad\times\left[-\{(k_{\mathrm{p}}^{-4}+k_{\mathrm{n}}^{-4})(k_{\mathrm{p}}^2+k_{\mathrm{n}}^2) + 4k_{\mathrm{p}}^{-1}k_{\mathrm{n}}^{-1}\}k + 4(k_{\mathrm{p}}^{-4}+k_{\mathrm{n}}^{-4})k^3\right]\tilde{f}_n(2k).\end{aligned} \tag{B.2}$$

Each term of Eq. (12) is given by

$$
\begin{aligned}
a_{\mathrm{K}} &= \frac{\pi^2\rho}{4M}k_{\mathrm{F}}^{-1},\\
a_{\mathrm{DD}} &= \frac{\rho}{16}(t_\rho^{(\mathrm{SE})}\rho^{\alpha+1} - 3t_\rho^{(\mathrm{TE})}\rho^{\beta+1}),\\
a_{\mathrm{HO}n} &= \frac{1}{2(2\pi)^6}(2t_n^{(\mathrm{W})} + t_n^{(\mathrm{B})} - 2t_n^{(\mathrm{H})} - t_n^{(\mathrm{M})})\left\{\mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{p}},k_{\mathrm{p}})^{(2)} + \mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{n}},k_{\mathrm{n}})^{(2)}\right\}\Big|_{\eta_t=0}\\
&= \frac{\rho}{8}(2t_n^{(\mathrm{W})} + t_n^{(\mathrm{B})} - 2t_n^{(\mathrm{H})} - t_n^{(\mathrm{M})})\tilde{f}_n(0),\\
a_{\mathrm{HX}n} &= \frac{1}{(2\pi)^6}(2t_n^{(\mathrm{W})} + t_n^{(\mathrm{B})})\,\mathcal{W}_n^{\mathrm{H}}(k_{\mathrm{p}},k_{\mathrm{n}})^{(2)}\Big|_{\eta_t=0}\\
&= -\frac{\rho}{8}(2t_n^{(\mathrm{W})} + t_n^{(\mathrm{B})})\tilde{f}_n(0),\\
a_{\mathrm{FO}n} &= \frac{1}{2(2\pi)^6}(2t_n^{(\mathrm{M})} + t_n^{(\mathrm{H})} - 2t_n^{(\mathrm{B})} - t_n^{(\mathrm{W})})\left\{\mathcal{W}_n^{\mathrm{F}}(k_{\mathrm{p}},k_{\mathrm{p}})^{(2)} + \mathcal{W}_n^{\mathrm{F}}(k_{\mathrm{n}},k_{\mathrm{n}})^{(2)}\right\}\Big|_{\eta_t=0}\\
&= \frac{\rho}{2}(2t_n^{(\mathrm{M})} + t_n^{(\mathrm{H})} - 2t_n^{(\mathrm{B})} - t_n^{(\mathrm{W})})k_{\mathrm{F}}^{-4}\int_0^{k_{\mathrm{F}}} dk\,k^3\,\tilde{f}_n(2k)\\
a_{\mathrm{FX}n} &= \frac{1}{(2\pi)^6}(2t_n^{(\mathrm{M})} + t_n^{(\mathrm{H})})\,\mathcal{W}_n^{\mathrm{F}}(k_{\mathrm{p}},k_{\mathrm{n}})^{(2)}\Big|_{\eta_t=0}\\
&= \frac{\rho}{2}(2t_n^{(\mathrm{M})} + t_n^{(\mathrm{H})})k_{\mathrm{F}}^{-4}\int_0^{k_{\mathrm{F}}} dk\left(-k_{\mathrm{F}}^2k + k^3\right)\tilde{f}_n(2k).
\end{aligned}
\tag{B.3}
$$

Appendix C. Explicit expressions for $\Delta_{\mathrm{FO}n}$ and $\Delta_{\mathrm{FX}n}$

If the function form is specified, $\Delta_{\mathrm{FO}n}$ and $\Delta_{\mathrm{FX}n}$ can be expressed in more explicit forms. Here, we denote k_{F}/μ_n by x. Then $\Delta_{\mathrm{FO}n}$ and $\Delta_{\mathrm{FX}n}$ become functions only of x for a given function form.

For the Gauss function $f(r) = e^{-(\mu r)^2}$, whose Fourier transform is $\tilde{f}(2k) = (\sqrt{\pi}/\mu)^3\,e^{-(k/\mu)^2}$, we obtain

$$
\begin{aligned}
\Delta_{\mathrm{FO}n} &= \frac{3}{x^2}\cdot\frac{1}{1-e^{-x^2}-x^2e^{-x^2}}\Big[-1+3(1-2^{-1/3})x^2 + (2-x^2)\,e^{-x^2}\\
&\qquad - (1-2^{-1/3}x^2)\,e^{-2^{2/3}x^2} + \sqrt{\pi}x^3\{\mathrm{erf}(2^{1/3}x) - \mathrm{erf}(x)\}\Big] - 1,\\
\Delta_{\mathrm{FX}n} &= \frac{3}{x^2}\cdot\frac{2-3x^2-2e^{-x^2}+x^2e^{-x^2}+\sqrt{\pi}x^3\,\mathrm{erf}(x)}{-1+x^2+e^{-x^2}} - 1,
\end{aligned}
\tag{C.1}
$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-t^2}dt$.

For the Yukawa function $f(r) = e^{-\mu r}/\mu r$, whose Fourier transform is $\tilde{f}(2k) = 4\pi/\mu(\mu^2 + 4k^2)$, $\Delta_{\mathrm{FO}n}$ and $\Delta_{\mathrm{FX}n}$ are

$$\begin{aligned}\Delta_{\mathrm{FO}n} = \frac{3}{2x^2} \cdot \frac{1}{\left[4x^2 - \log(1+4x^2)\right]}\Big[&2(1-2^{-1/3})x^2 + 12(2^{1/3}-1)x^4 \\ &+ \frac{1}{4}\log\frac{1+4\cdot 2^{2/3}x^2}{(1+4x^2)^2} + 3\cdot 2^{2/3}x^2\log(1+4\cdot 2^{2/3}x^2) - 6x^2\log(1+4x^2) \\ &- 16x^3\{\arctan(2^{4/3}x) - \arctan(2x)\}\Big] - 1, \\ \Delta_{\mathrm{FX}n} = \frac{3}{4x^2} \cdot \frac{4x^2(1-6x^2) + 32x^3\arctan(2x) - (1+12x^2)\log(1+4x^2)}{4x^2 - (1+4x^2)\log(1+4x^2)} &- 1. \end{aligned} \tag{C.2}$$

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