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Review

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Quantum Machine Learning: Exploring the Role of Data Encoding Techniques, Challenges, and Future Directions

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Abstract: Quantum computing and machine learning (ML) have received significant developments which have set the stage for the next frontier of creative work and usefulness. This paper aims at reviewing various data-encoding techniques in Quantum Machine Learning (QML) while highlighting their significance in transforming classical data into quantum systems. We analyze basis, amplitude, angle, and other high-level encodings in depth to demonstrate how various strategies affect encoding improvements in quantum algorithms. However, they identify major problems with encoding in the framework of QML, including scalability, computational burden, and noise. Future directions for research outline these challenges, aiming to enhance the excellence of encoding techniques in the constantly evolving quantum technology setting. This review shall enable the researcher to gain an enhanced understanding of data encoding in QML, and it also suggests solutions to the current limitations in this area.

Keywords: quantum computing; machine learning (ML); data encoding; Quantum Machine Learning (QML)

MSC: 81P99

check for updates

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1. Introduction

Quantum computation [1–4] represents a groundbreaking means of harnessing nature's computational power. Widely regarded as the frontier of technology, it offers the potential for practical tasks to be executed through collaboration between parallel universes and subsequent result sharing [5–8]. However, amidst the anticipation surrounding quantum computing, demonstrating a "quantum advantage" remains a formidable challenge. This entails identifying tasks that a quantum computer can accomplish more efficiently or effectively than the best classical algorithms on conventional machines. While the realization of quantum advantage is yet to be proven, even the prospect of quantum computers outperforming classical ones in certain tasks within hours or minutes would signify a significant milestone.

Machine learning [9–11] refers to a versatile approach of teaching machines to recognize patterns within data so as to enable categories and group them, arrange them in clusters, or for some other purpose, without being programmed how to do so individually [12–15]. Hence, in this context, Quantum Machine Learning (QML), which has been proposed and explored in the literature [16–18], appears to be the most feasible solution and is in its infancy. While classical ML has expanded into a large field with many different uses, QML has the possibility of outcompeting traditional approaches given the use of quantum computation concepts.

The performance of QML algorithms highly depends on the used encoding schemes that lie at the heart of the classical data-to-quantum system transformation. It is possible to obtain relevant data sets compatible with further actions of quantum algorithms by encoding techniques that act as a transition of classical information to quantum states [19–22]. These approaches help integrate classical data into quantum structures, enhancing the speed and optimization of QML algorithms [23–26]. The specific properties of quantum computation make the utilization of QML focused on providing new opportunities in the handling, sorting, and analysis of data. In this review paper, the study of encoding techniques in QML addresses theoretical and practical problems and future developments. We examine the effectiveness of numerous encoder schemes to help us determine their compatibility in diverse contexts with considerations made on their computational time, precision, and complexity. This work defines current issues and discusses possible enhancements. We also highlight current developments and trends in QML encoding, with case studies that demonstrate their practical application. Concluding with recommendations for future research, this paper aims to advance the field of QML encoding. Table 1 shows a comparison of quantum encoding methods with existing papers.

Table 1. Comparison with existing papers: A = quantum computing basics, B = QML and its methods and discussions, C = discussions on quantum data encoding methods, D = research summary, E = challenges and future directions related to QML, F = challenges and future directions related to quantum encoding methods.

Year	Reference	A	B	C	D	E	F	Limitations
2018	Biamonte et al. [27]	✗	✓	✗	✗	✗	✗	The work's discussion of QML methodologies is limited, lacking basic quantum computing concepts, making it high-level for readers, and it restricts future scope to hardware challenges, ignoring other significant issues in QML.
2020	Abohashima et al. [28]	✗	✓	✗	✗	✓	✗	While aiming for a comprehensive survey, this paper's coverage of 30 publications is insufficient for recent QML advances, lacks detailed methodology, overemphasizes classification over other areas, and focuses too much on hardware limitations, neglecting algorithmic efficiency, error correction, scalability, and integration with classical systems.
2020	Zhang et al. [29]	✓	✓	✗	✗	✗	✗	The paper is not a comprehensive review as it focuses solely on quantum versions of specific supervised and unsupervised algorithms based on the quantum circuit model and insufficiently covers recent advancements by reviewing only a limited number of publications.
2022	Houssein et al. [30]	✓	✓	✗	✗	✓	✗	The work is similar to that by [28] and lacks novelty. Despite being published in 2022, it only discusses papers up to 2020, hindering its ability to present recent innovations. This limited scope and outdated coverage diminish the paper's relevance and contribution to current research.
2023	Tychola et al. [31]	✗	✓	✗	✗	✗	✗	The review focuses solely on SVM and QSVM, overlooking other quantum algorithms like QNN and QkNN, and lacks specific future research directions or practical steps for advancing QML in unsupervised learning and generative models.
2024	Pande et al. [32]	✗	✗	✓	✗	✗	✗	The review overlooks some novel data-encoding techniques and lacks detailed discussion on practical implementation challenges, including required quantum resources and computational complexity.
–	Our Study	✓	✓	✓	✓	✓	✓	–

✓: a characteristic is satisfied in our study; ✗: a characteristic is not satisfied in our study.

2. Contributions of This Study

The main contributions of this research are outlined below:

- Introduction to Quantum Computing Fundamentals: This paper commences with a concise yet comprehensive discussion of the rudimentary principles of quantum computing. This foundation equips readers with the essential knowledge required to comprehend the advanced topics discussed in subsequent sections.
- Exploration of QML: A comprehensive explanation of QML is presented that highlights historical background in addition to the existing state. As a distinct section, this

concept presents methodologies of data encoding in QML and can thus be considered valuable for readers.
- In-depth Analysis of Data Encoding Methods: This study offers an extensive analysis of the most common data encoding practices in QML at the time. Finally, an evaluation of the technique and its application in the current literature from the period of 2020 to 2024 is presented.
- Challenges and Future Directions: Finally, this paper discusses the most significant issues that exist in the encoding stage for QML at the present time. It also describes what future work may be worth pursuing that has the potential to further advance these techniques and, in turn, the overall field of quantum technology.

Organization of the Work

Following the introduction, this paper is organized as follows to ensure that the subsequent sections systematically and progressively address the main themes of the study. The first part, Section 3, familiarizes the reader with fundamental concepts such as Quantum Bits (Qubits), Superposition, and Entanglement, as well as the other important principles. Next, in Section 4, we describe pure QML, quantum-inspired ML, and classical–hybrid QML to provide the reader with the big picture of the area. Moreover, the discourse seamlessly transitions into a detailed discussion of various encoding strategies, highlighting their importance in the effective implementation of QML. In Section 5, a critical review and discussion from 2020 to 2024 follows, providing a contemporary analysis of the literature. The penultimate Section 6 addresses the pressing issues and potential advancements that could shape the future of QML. This paper culminates in Section 7, which synthesizes the insights garnered throughout the study. Table 2 shows abbreviations used in the paper.

Table 2. List of abbreviations used in the paper.

Abbreviation	Full Forms	Abbreviation	Full Forms
AEs	Autoencoders	AI	Artificial Intelligence
ANN	Artificial Neural Network	APM	Autonomous Perceptron Model
APS	American Physical Society	AUC	Area under the ROC Curve
CAN	Control Area Network	CFMs	Classical Feature Maps
CML	Classical Machine Learning	CNNs	Convolutional Neural Networks
CNOT	Controlled NOT	DDoS	Distributed Denial of Service
DEQSVC	Dimensionality Reduction and Encoding Technique for Quantum Support Vector Classifier	DL	Deep Learning
DQL	Deep Quantum Learning	DT	Digital Twin
DTQFL	Digital Twin-Assisted Quantum Federated Learning Algorithm	EF-QAE	Enhanced Feature Quantum Autoencoder
EO	Earth Observation	FL	Federated Learning
HHL	Harrow Hassidim Lloyd	HQCA	Hybrid Quantum Classical Architecture
HCQCs	Hybrid Classical–Quantum Classifiers	HC-QNN	Hybrid Classical–Quantum Neural Network
HQCNN	Hybrid Quantum–Classical Neural Network	IBM	International Business Machines
IoMT	Internet of Medical Things	IoT	Internet of Things
ML	Machine Learning	NIDS	Network Intrusion Detection System
NISQ	Noisy Intermediate Scale Quantum	NISQRC	Noisy Intermediate Scale Quantum Reservoir Computing
NNs	Neural Networks	PQCs	Parametrized Quantum Circuits

Table 2. *Cont.*

Abbreviation	Full Forms	Abbreviation	Full Forms
QA	Quantum Annealer	QC	Quantum Computing
QCC	Quality Control Circle	QC-CNN	Quantum Classical Convolutional Neural Network
QCL	Quantum Circuit Learning	QC-NNs	Quantum Convolutional Neural Networks
QENN	Quantum Embedding Neural Network	QFMs	Quantum Feature Maps
QGAN	Quantum Generative Adversarial Network	QGLMs	Quantum Generalized Linear Models
QGFormer	Quantum Gravitational Transformer	QKAR	Quantum Kernel Alignment based Regression
QKE-QSVR	Quantum Kernel Estimation-based Quantum Support Vector Regression	QKNN	Quantum k-Nearest Neighbour
QLDA	Quick Look Display Area	QML	Quantum Machine Learning
QNN	Quantum Neural Network	QNNN	Quantum Naive Neural Network
QPCA	Quantum Principal Component Analysis	QRAM	Quantum Random Access Memory
QSVR	Quantum Support Vector Regression	QSVM	Quantum Support Vector Machine
QuBits	Quantum Bits	RBF	Radial Basis Function
RS	Remote Sensing	SciML	Scientific Machine Learning
SVMs	Support Vector Machines	SVR	Support Vector Regression
UCI	University of California Intelligence	VQC	Variational Quantum Classifier
VQE	Variational Quantum Eigensolver	VQNN	Variational Quantum Neural Network
VQP	Variational Quantum Pulses	3D-QAE	3D-Quantum Autoencoder

3. Quantum Computing Fundamentals

Quantum computing represents a paradigm shift, utilizing the principles of quantum mechanics to process information in ways that classical computers cannot. Quantum mechanics introduces the concept of a quantum, the smallest discrete unit of any physical property. This same fundamental notion is applied in quantum computing to manipulate 'qubits'—quantum bits that can exist in multiple states simultaneously, rather than just two (as is the case with classical bits). Quantum computing relies on unique phenomena like superposition and entanglement to make computations not only more efficient but also faster. While both quantum computing and conventional computing use unique principles, quantum computing leverages non-classical properties of subatomic particles, allowing it to perform tasks beyond the capabilities of classical computation [33].

The fundamental principles of quantum computing that are essential for its implementation include the following.

3.1. Quantum Bits (Qubits)

A qubit is the fundamental unit of quantum information. It is the quantum analog of the classical binary bit and is represented using Dirac notation:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{1}$$

Unlike a classical bit, which can be either 0 or 1, a qubit can exist in a state of superposition, allowing it to be simultaneously both 0 and 1. This phenomenon is a consequence of quantum mechanics, which permits particles to exist in multiple states at once. The superposition state of a qubit can be mathematically expressed as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad \text{where } |\alpha|^2 + |\beta|^2 = 1. \tag{2}$$

The coefficients α and β are complex numbers, representing the probabilities of the qubit collapsing to the state $|0\rangle$ or $|1\rangle$ upon measurement.

Consider a two-qubit system expressed as follows:

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, \tag{3}$$

where $a, b, c, d \in \mathbb{C}$ and $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.

An example of a two-qubit system is:

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \tag{4}$$

where the subscripts 1 and 2 denote the order of the qubits.

The density matrix is another way to represent quantum states. For a pure state $|\psi\rangle$, the density matrix is:

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}. \tag{5}$$

Density matrices can also describe mixed states, which are a statistical ensemble of pure states $|\psi_i\rangle$ with probabilities p_i:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|, \tag{6}$$

where $\sum_i p_i = 1$.

3.2. Superposition

In quantum mechanics, a physical system can exist in multiple states simultaneously—a property known as superposition. This phenomenon arises when Schrödinger's equation is solved for a quantum system as wave functions are often expressed as combinations of different states [34–36]. These superpositions of possible states are precisely what quantum mechanics predicts: a quantum system exists in a "fuzzy" superposition of different states until a measurement reveals which specific state it is in. The key to enabling quantum computers to perform a vast array of calculations simultaneously lies in the principle of superposition. In quantum computing, each additional qubit exponentially increases computational power, unlike the linear increase seen in classical computing. For example, while two classical bits can represent 00, 01, 10, or 11, two qubits in superposition can represent all four combinations simultaneously. Born's rule [37,38] is used to calculate the probabilities of measuring specific outcomes from these superposed states.

3.3. Entanglement

Entanglement is quantum mechanics' most fundamental property and serves as the basis for both quantum computing and QML. The process describes two or more quantum particles, where the state of one particle instantly affects the other, regardless of the distance between them [39–42]. By entangling qubits, the quantum computer is able to process information more efficiently than a classical computer. With QNNs, entangled qubits represent input data in terms of quantum states and explain complex correlations between features. When a QNN processes a batch of photos, the entangled qubits can execute several calculations concurrently, leading to substantial accelerations and enhanced generalization across various image classes. Entanglement is a potent resource that not only expedites computation but also improves the precision of machine learning tasks, underscoring its essential role in the progression of quantum technologies [39,43].

3.4. Quantum Gates

Quantum gates are fundamental components in the quantum circuit model of computation, much like classical logic gates in conventional digital circuits. In contrast to classical logic gates, quantum gates are all reversible and are represented by unitary matrices. This implies that quantum gates always have the same number of inputs and outputs.

In reality, quantum gates are operators—unitary matrices that act upon a quantum state to transform it into a different quantum state. This transformation can be represented as:

$$U|\psi\rangle = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} a \\ b \end{bmatrix} \quad (8)$$

$$= |\phi\rangle \quad (9)$$

In quantum circuit models, qubits are manipulated by applying quantum gates sequentially. Quantum gates differ depending on the qubit system considered; there are different gates for single-qubit and multi-qubit systems. Table 3 briefly discusses the most important gates used for one-qubit, two-qubit, and three-qubit systems.

Table 3. Quantum gates.

Gate Type	Gate Name	Matrix	Working Principle
One-Qubit Gates	Identity Gate	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	It turns $\|0\rangle$ to $\|0\rangle$ and $\|1\rangle$ to $\|1\rangle$.
	Pauli-X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \sigma_x$	It converts $\|0\rangle$ to $\|1\rangle$ and $\|1\rangle$ to $\|0\rangle$. It is similar to the NOT gate.
	Pauli-Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \equiv \sigma_y$	It turns $\|0\rangle$ to $i\|1\rangle$ and $\|1\rangle$ to $-i\|0\rangle$.
	Pauli-Z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \sigma_z$	It turns $\|1\rangle$ to $-\|1\rangle$ and leaves the basis state $\|0\rangle$ unchanged. It is also called the phase-flip.
	Hadamard	$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	It converts $\|0\rangle$ to superposition state $\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}$ and $\|1\rangle$ to $\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}$.
	Phase shift	$S = \begin{pmatrix} 1 & 0 \\ 0 & \text{i} \end{pmatrix}$	It turns $\|1\rangle$ to $i\|1\rangle$ and $\|0\rangle$ remains unchanged.
	T gate	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	It turns $\|1\rangle$ to $e^{i\frac{\pi}{4}}\|1\rangle$ and $\|0\rangle$ remains unchanged.

Table 3. *Cont.*

Gate Type	Gate Name	Matrix	Working Principle
Two-Qubit Gates	Controlled I	$CI = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	It leaves $\|00\rangle$, $\|01\rangle$, $\|10\rangle$, $\|11\rangle$ unchanged.
	Controlled NOT	$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	It turns $\|10\rangle$ to $\|11\rangle$, $\|11\rangle$ to $\|10\rangle$ and leaves $\|00\rangle$, $\|01\rangle$ unchanged.
	SWAP Gate	$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	It turns $\|10\rangle$ to $\|01\rangle$, $\|01\rangle$ to $\|10\rangle$ and leaves $\|00\rangle$, $\|11\rangle$ unchanged.
	Controlled Z	$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	It turns $\|11\rangle$ to $-\|11\rangle$ and $\|00\rangle$, $\|01\rangle$, $\|10\rangle$ remain unchanged.
	Controlled S	$CS = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$	It turns $\|11\rangle$ to $i\|11\rangle$ and $\|00\rangle$, $\|01\rangle$, $\|10\rangle$ remain unchanged.
	Controlled T	$CT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	It turns $\|11\rangle$ to $e^{i\frac{\pi}{4}}\|11\rangle$ and $\|00\rangle$, $\|01\rangle$, $\|10\rangle$ remain unchanged.
Three-Qubit Gates	Toffoli Gate	$CCNOT = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	It turns $\|110\rangle$ to $\|111\rangle$, $\|111\rangle$ to $\|110\rangle$ and $\|000\rangle$, $\|001\rangle$, $\|010\rangle$, $\|011\rangle$, $\|100\rangle$, $\|101\rangle$ remain unchanged.
	Fredkin Gate	$CCSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	It turns $\|101\rangle$ to $\|110\rangle$, $\|110\rangle$ to $\|101\rangle$ and $\|000\rangle$, $\|001\rangle$, $\|010\rangle$, $\|011\rangle$, $\|100\rangle$, $\|111\rangle$ remain unchanged.

3.5. Measurement

The concept of measurement is pivotal in quantum mechanics and quantum computing. It is through measurement that quantum states are observed and their properties inferred. Despite ongoing debates regarding the interpretation of quantum mechanics, the practical application of measurement to physical systems remains of paramount importance.

A quantum measurement is characterized by a set of measurement operators, M_m, as stipulated by the third postulate of quantum mechanics. These operators act upon the state space of the system being measured, with the index m representing the possible outcomes of the experiment. The probability of obtaining a particular outcome m when the system is in state $|\phi\rangle$ is given by:

$$p(m) = \langle\phi|M_m^{\dagger}M_m|\phi\rangle, \tag{10}$$

and the post-measurement state of the system is:

$$\frac{M_m|\phi\rangle}{\sqrt{\langle\phi|M_m^{\dagger}M_m|\phi\rangle}}. \tag{11}$$

The measurement operators must satisfy the completeness relation:

$$\sum_m M_m^{\dagger}M_m = I, \tag{12}$$

which ensures that the sum of probabilities for all possible outcomes is equal to 1:

$$1 = \sum_m p(m) = \sum_m \langle\phi|M_m^{\dagger}M_m|\phi\rangle. \tag{13}$$

In the context of quantum computing, measurement typically refers to the observation of qubits in the computational basis, described by the operators $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$. For a system in state $|\phi\rangle = a|0\rangle + b|1\rangle$, the probabilities of measuring the qubit in state $|0\rangle$ or $|1\rangle$ are:

$$p(0) = \langle\phi|M_0^{\dagger}M_0|\phi\rangle = |a|^2, \tag{14}$$

and

$$p(1) = \langle\phi|M_1^{\dagger}M_1|\phi\rangle = |b|^2, \tag{15}$$

respectively. The corresponding post-measurement states are:

$$\frac{M_0|\phi\rangle}{|a|} = |0\rangle, \tag{16}$$

and

$$\frac{M_1|\phi\rangle}{|b|} = |1\rangle, \tag{17}$$

where the phase factors $\frac{a}{|a|}$ and $\frac{b}{|b|}$ can be disregarded as they do not affect the measurement outcome.

Since measurement allows us to extract classical information from quantum systems, we therefore view this first as a definition of measurement. At the end of most quantum algorithms, the computation's result is read out to be seen and used directly by classical interpretation.

3.6. The No-Cloning Theorem

The no-cloning theorem is one of the principles in quantum computing which should be taken into consideration by the experts. The no-cloning theorem asserts that it is impossible to replicate an exact duplicate of an arbitrary unknown quantum state [44]. This is significant because, unlike classical information, which can be freely duplicated, quantum information cannot. This is not merely a technical limitation, but it also necessitates the development of new techniques for information processing that significantly differ from classical methods. Rather, it is a result of fundamental quantum mechanical laws [45]. It has profound implications for the development of quantum algorithms and prompts a need for new techniques for information processing markedly different from the classical.

3.7. Decoherence in Quantum Systems

A phenomenon of decoherence represents a serious practical obstacle in the realization of quantum computing. A quantum state refers to a loss of quantum coherence, i.e., the probabilistic nature of quantum states leads to a mixture of states instead of a superposition of states [46]. Usually, this occurs either due to interactions between a quantum system and its environment, which causes its delicate phase relationships between quantum states to be disrupted, or from a nonlinear interaction between two quantum systems. Thus, the quantum system is maintained in a quantum state but transitions into a classical state, thereby losing exactly the properties that make quantum computing so special.

4. Quantum Machine Learning and the Role of Data Encoding

QML integrates quantum computing principles with ML algorithms, offering advantages like faster processing and the ability to handle vast datasets [16,47,48]. The potential of QML lies in its ability to solve complex problems that are challenging for classical computers.

The origins of QML can be traced back to Peter Shor and Lov Grover in the mid-1990s, whose algorithms demonstrated quantum advantages in factorization and database searching [49,50]. These foundational works laid the groundwork for using quantum mechanics in computational tasks, including machine learning. The development of quantum hardware, such as NISQ devices, has enabled the testing of QML algorithms, showcasing quantum speedups in specific tasks [51]. As quantum hardware improves, the potential applications of QML continue to expand.

QML approaches can be classified based on the dataset and algorithm employed [52], as shown in Figure 1. These include:

1. Classical–Classical (CC): Classical algorithms with classical datasets.
2. Quantum–Classical (QC): Classical algorithms processing quantum datasets.
3. Classical–Quantum (CQ): Classical datasets processed on quantum hardware.
4. Quantum–Quantum (QQ): Quantum algorithms with quantum datasets.

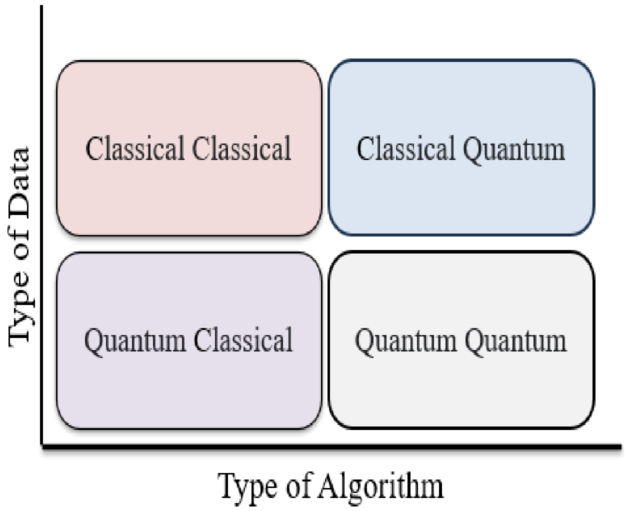

Figure 1. QML approaches for machine learning and quantum computing.

QML algorithms can also be categorized into three types: pure quantum, hybrid classical–quantum, and quantum-inspired.

4.1. Pure Quantum ML

Pure quantum ML algorithms operate entirely on quantum hardware, utilizing quantum mechanics properties such as superposition and entanglement to achieve potential speedups over classical methods. Examples include Quantum Support Vector Machines (QSVMs) and Quantum Neural Networks (QNNs), which leverage quantum properties to efficiently perform tasks such as matrix inversion and classification [16,53]. Despite their potential, these algorithms require advanced quantum hardware, which remains a challenge due to noise and error rates in current devices.

4.2. Quantum-Inspired ML

Quantum-inspired ML draws inspiration from quantum mechanics principles, applying them to classical algorithms. These approaches, such as quantum-inspired binary classifiers and neural networks, use classical hardware but achieve improved accuracy and efficiency by incorporating quantum mechanics concepts [54,55]. Although they do not achieve the same speedups as true quantum algorithms, quantum-inspired methods offer practical benefits with current technology.

4.3. Hybrid Classical-Quantum ML

Hybrid classical–quantum ML combines classical and quantum resources to optimize performance. Quantum computers are used for specific tasks, while classical components handle operations better suited to traditional hardware. Approaches such as Variational Quantum Classifiers (VQCs) and Quantum Circuit Learning (QCL) leverage this combination to optimize performance in tasks like classification and regression [56,57].

However, hybrid methods give us flexibility and use of near-term quantum devices, but balancing quantum and classical computation is complex.

It is important for us to understand the differences between classical and quantum ML because we see how QML could be useful. In classical ML, data are typically prepared in binary form, while QML demands encoding classical data to quantum states via qubits. This transformation allows one to execute quantum algorithms, which exploit quantum phenomena, such as superposition, to execute computation in parallel over a number of possibilities at once.

As depicted in Figure 2, the workflow of QML shares similarities with CML, including data preprocessing and model training. However, the introduction of quantum encoding, algorithms, and measurements adds unique complexities. The quantum model is trained once, and a system must then convert quantum measurements back into classical data for evaluation and decision-making. The heart of this process is data encoding, which, along with its implications for the mapping from classical data to quantum states, has direct consequences for the efficiency and performance of quantum algorithms.

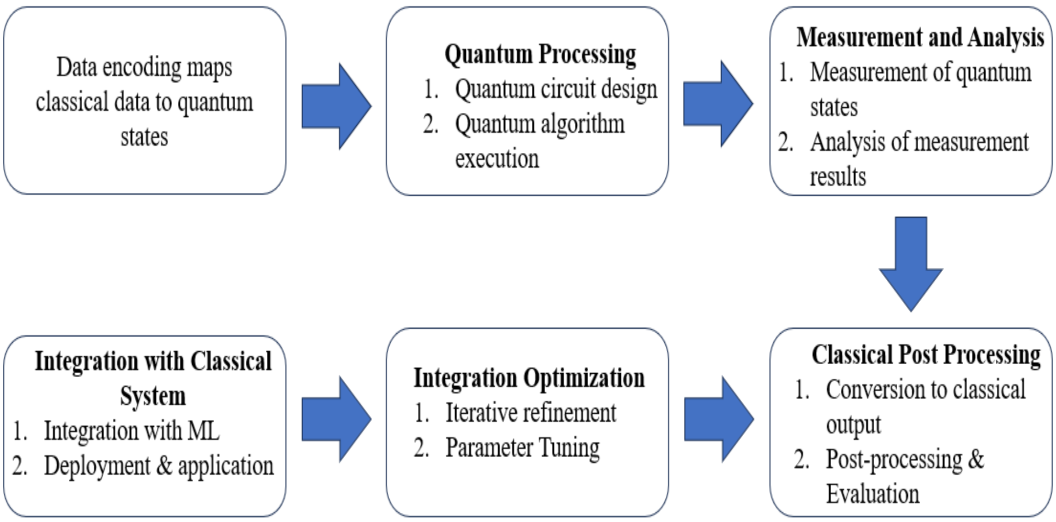

Figure 2. General working methodology of QML.

The data encoding process is a key differentiator from CML to QML. Classical information in QML is transformed to quantum states, which then perform better on quantum algorithms for processing the data. Different encoding techniques are good for different datasets or sometimes for different tasks. Below, we explore some of the most commonly used encoding methods in QML (Figure 3):

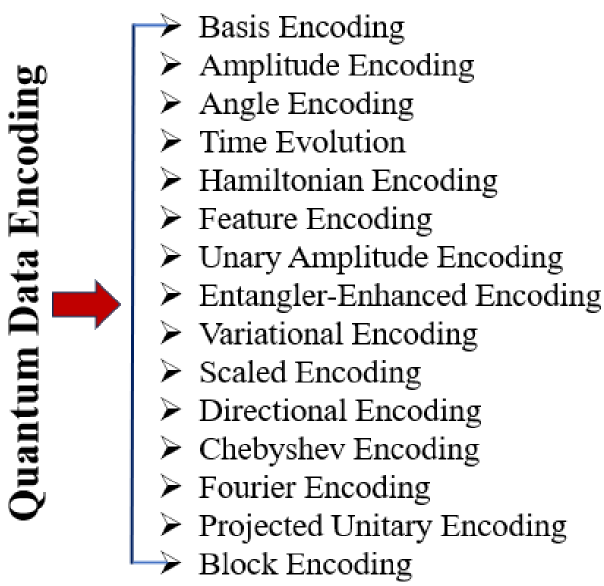

Figure 3. Types of data encoding methods in QML.

4.4. Basis Encoding

Basis encoding is a widely utilized method for translating classical data into quantum states, effectively bridging classical and quantum computational paradigms [58]. In this approach, an m-bit classical data string is encoded into an m-qubit quantum state. For example, the classical sequence 1001 is represented in the quantum domain as $|1001\rangle$.

The transformation of classical data X, consisting of vectors x^n, into quantum states can be mathematically formulated as:

$$|X\rangle = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} |x^n\rangle, \tag{18}$$

where each vector x^n is a binary string $x^n = (b_1, b_2, \ldots, b_M)$ indicating the presence (1) or absence (0) of M features. Binary thresholding is applied to convert grayscale images into binary vectors. Each binary vector x^n is then associated with a quantum state $|x^n\rangle$, and the entire dataset is represented in a superposition of these states.

Each individual data sample is depicted by:

$$|x^n\rangle = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} |b_i\rangle, \tag{19}$$

with b_i denoting the binary value that signifies the presence of a feature, and $|b_i\rangle$ representing the corresponding qubit in the quantum system.

4.5. Amplitude Encoding

Amplitude encoding is a technique that maps classical data to the amplitudes of a quantum state, establishing a fundamental connection between classical information vectors and quantum state vectors [33,59–61]. This method utilizes normalized classical vectors to encode information into the amplitudes of a quantum state [62,63]. The classical vector y is represented as:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \tag{20}$$

where y is a normalized vector within the complex vector space $\mathbf{C}^{2^n}$. The amplitude encoding process is described by the following quantum state:

$$|\psi_y\rangle = \sum_{i=1}^{2^n} y_i |i\rangle, \tag{21}$$

where $|\psi_y\rangle$ is the encoded quantum state residing in the Hilbert space $\mathcal{H}$, and the normalization condition $\sum_i |y_i|^2 = 1$ is satisfied.

This state-preparation strategy is pivotal for enhancing the efficiency of data encoding and facilitating the learning process in QML models [64,65].

4.6. Angle Encoding

Angle encoding, also referred to as qubit encoding, is a technique employed by various QML algorithms to encode input data features into the rotation angles of qubits [66–70]. For a feature vector $x = [x_1, x_2, \ldots, x_N]$ belonging to the space $\mathcal{X}^N$, angle encoding is mathematically represented as:

$$|x\rangle = \bigotimes_{i=1}^{N} (\cos(x_i)|0\rangle + \sin(x_i)|1\rangle), \tag{22}$$

where N features are encoded into an n-qubit system. The unitary operation for state preparation of a single qubit in qubit encoding is given by:

$$S_{x_j} = \bigotimes_{i=1}^{N} U_i, \tag{23}$$

with each U_i defined as:

$$U_i := \begin{pmatrix} \cos\left(x_j^{(i)}\right) & -\sin\left(x_j^{(i)}\right) \\ \sin\left(x_j^{(i)}\right) & \cos\left(x_j^{(i)}\right) \end{pmatrix}. \tag{24}$$

Dense qubit encoding, a variant of angle encoding, allows for the encoding of two features within a single qubit, thereby enhancing the density of the encoded information [71].

4.7. Time-Evolution Encoding

Time-evolution encoding, as described by the Schrödinger equation, is a pivotal concept in quantum mechanics that delineates the temporal progression of a quantum system. The equation is given by:

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle, \tag{25}$$

where $\hbar$ denotes Planck's constant, and H represents the Hamiltonian of the system, an observable that encapsulates the system's energy dynamics [16]. In the context of QML, time-evolution encoding is leveraged to correlate a scalar value $x \in \mathbb{R}$ with time t through the unitary evolution governed by the Hamiltonian H, as expressed in:

$$U(x) = e^{-ixH}. \tag{26}$$

The state of the system's evolution, $|\psi(x)\rangle = U(x)|\psi_0\rangle$, varies with x, as dictated by H. This encoding mechanism is particularly prevalent for embedding classical trainable parameters within a quantum circuit. The most commonly utilized Hamiltonians for this purpose are the Pauli rotation gates, defined by $H = \frac{1}{2}\sigma_a$, where a belongs to the set $\{x, y, z\}$. To encode a real-valued vector $x \in \mathbb{R}^N$, one can sequentially apply gates or evolutions of the form $U(x)$.

4.8. Hamiltonian Encoding

Hamiltonian encoding is a sophisticated technique that involves embedding matrices into the Hamiltonian of a quantum system's time evolution. This method is particularly instrumental in algorithms such as the HHL algorithm for matrix inversion [72]. The essence of this technique lies in associating a square matrix A with a Hamiltonian H, which is especially useful when A is not inherently Hermitian. In such cases, the Hamiltonian H_A is constructed as:

$$H_A = \begin{bmatrix} 0 & A \\ A^\dagger & 0 \end{bmatrix}, \tag{27}$$

allowing computations to be performed within two distinct subspaces of the Hilbert space. Hamiltonian encoding facilitates operations such as the multiplication of matrix A or its inverse A^{-1} with an amplitude-encoded vector, as well as the extraction and manipulation of the eigenvalues of A.

4.9. Feature Map Encoding

In QML, the encoding of inputs $x \in \mathcal{X}$ into a quantum system constitutes a mathematical mapping from the input space $\mathcal{X}$ to the state space of the quantum system. This mapping, when coupled with an inner product defined on the state space, is conceptualized as a feature map [56,73,74]. Feature maps are integral to the kernel method in machine learning, facilitating the transformation of data into high-dimensional Hilbert spaces where the computation of inner products between feature-encoded vectors is central to the analysis.

These inner products serve as a metric for quantifying the similarity between data points, which is essential for addressing new data in learning tasks. The construction of a data-encoding feature map—occasionally termed a "quantum feature map" [75,76]—can be approached in two predominant manners. The first method involves interpreting Dirac vectors as feature vectors, defining the feature map as:

$$\phi_1 : \mathcal{X} \rightarrow |\phi\rangle, \tag{28}$$

where the inner product is the conventional bra–ket inner product $\langle\phi|\phi|\phi|\phi\rangle$. Alternatively, a more natural choice is to employ density matrices to denote feature-encoding states, with the feature map delineated as:

$$\phi_2 : \mathcal{X} \rightarrow \rho(x), \tag{29}$$

and the inner product defined as $\mathrm{tr}\{\rho(x)\rho'(x)\}$. In instances where ρ represents pure states, the two representations are interconnected via $\rho = |\phi\rangle\langle\phi|$.

For this discussion, we will focus on feature maps of the type ϕ_1. It is noteworthy that the term "embedding", commonly used in natural language processing to describe the mapping of non-numerical data into a metric space, is analogous to the concept of a feature map in quantum states.

4.10. Unary Amplitude Encodings

Unary amplitude encoding offers a compact representation of unit vectors $y \in \mathbb{R}^n$ within the quantum framework [77]. The encoding is defined as:

$$|un(y)\rangle = \sum_{i\in[n]} y_i|e_i\rangle, \tag{30}$$

where e_i represents the computational basis state corresponding to the binary string $0^{i-1}10^{n-i}$. Unlike binary amplitude encodings that necessitate depth $O(n)$ or specialized hardware like quantum random access memory (QRAM), unary amplitude encodings can be efficiently prepared using parametrized circuits with a depth of $\log n$, rendering them particularly advantageous for QML applications. Unary data loaders, as described in [78], are specialized circuits designed to initialize quantum states for unary amplitude encodings.

4.11. Entangler-Enhanced Encoding

Entangler-enhanced encoding is a sophisticated technique that leverages the power of entanglement to enrich the representational capacity of quantum states [79]. The encoding circuit is characterized by:

$$V_{\Phi(y)} = \prod_{m=1}^{M} E_{ent}^{m} \mathcal{U}_{\phi(y)}, \tag{31}$$

where each layer m consists of a two-qubit entangling operation E_{ent}^{m}, typically implemented using CZ or CNOT gates, and a unitary operator $\mathcal{U}_{\phi(y)}$ that encodes the classical input data y. This approach, termed entangler-enhanced encoding, significantly boosts the expressive power of quantum circuits.

4.12. Variational Encoding

Variational encoding is a versatile method for embedding classical information into quantum states, as outlined in [80,81]. An N-qubit quantum state can be generally expressed as:

$$|\psi\rangle = \sum_{(p_1,p_2,\ldots,p_N)\in\{0,1\}^N} a_{p_1,\ldots,p_N} |p_1\rangle \otimes |p_2\rangle \otimes \cdots \otimes |p_N\rangle, \tag{32}$$

where $a_{p_1,\ldots,p_N} \in \mathbb{A}$ denotes the amplitude of each basis state, and $p_i \in \{0,1\}$. The probability of observing the post-measurement state $|p_1\rangle \otimes |p_2\rangle \otimes \cdots \otimes |p_N\rangle$ is given by the modulus squared of the amplitude $a_{p_1,\ldots,p_N}$, with the total probability summing to unity:

$$\sum_{(p_1,p_2,\ldots,p_N)\in\{0,1\}^N} |a_{p_1,\ldots,p_N}|^2 = 1. \tag{33}$$

The variational encoding scheme utilizes the classical input values, or their transformations, as rotation angles in quantum gates, thereby enabling the encoding of classical data into the amplitudes and phases of a quantum state.

4.13. Scaled Encoding

Scaled Encoding is an efficient quantum encoding scheme designed for environments with input values confined to specific ranges [82]. This method involves scaling each input to fall between 0 and 2π, followed by rotations along the $\mathbf{R}_x$and $\mathbf{R}_z$ axes by the corresponding radians. The simplicity of this approach, requiring merely two gates per qubit, has been validated experimentally for its effectiveness, as detailed in the Blackjack section of [82]. However, it is noteworthy that certain environments, such as CartPole, present challenges due to unbounded data ranges, rendering traditional scaling methods impractical.

4.14. Directional Encoding

To address the limitations of Scaled Encoding in environments with unbounded input values, Directional Encoding is employed as an alternative scheme. This method entails rotating each qubit along the $\mathbf{R}_x$ and $\mathbf{R}_z$ axes by either π or 0 radians. The decision for rotation is contingent upon a simple conditional: a rotation of π radians is applied if the data point is positive; otherwise, no rotation is performed. This binary approach also necessitates only two gates per qubit and has been demonstrated to be experimentally viable.

4.15. Chebyshev Encoding

Chebyshev polynomials serve as a robust basis for various computational tasks, including regression, function fitting, and integral evaluation [83]. Their application extends to spectral methods for differential equations, financial modeling, and the characterization of many-body systems. Within the realm of quantum scientific machine learning (SciML), Chebyshev polynomials have been utilized in derivative quantum circuits to solve differential equations [84,85], albeit without linear basis independence. Building upon a recently developed protocol, an orthonormal Chebyshev basis set can be generated through quantum feature maps, facilitating the construction of global Chebyshev models within the quantum latent space [86].

Chebyshev encoding is designed such that the amplitudes of a quantum state align with the x-dependent Chebyshev polynomials of the first kind, denoted as $T_k(x) = \cos(k \arccos x)$, where k is the polynomial degree. The encoded quantum state is given by:

$$|x\rangle = \frac{1}{\mathcal{N}_N(x)} \left(\frac{1}{2^{N/2}} T_0(x)|\phi\rangle + \frac{1}{2^{(N-1)/2}} \sum_{k=1}^{2^{N-1}} T_k(x)|k\rangle \right), \tag{34}$$

ensuring orthogonality of the states $\{|x\rangle\}$ over a Chebyshev grid comprising 2^N points. The normalization of this N-qubit state is achieved through:

$$\mathcal{N}_N(x) = \frac{1}{2^{(N-1)/2}} \left(\frac{1}{2} + \sum_{k=1}^{2^N-1} T_k^2(x) \right)^{1/2}, \tag{35}$$

noting that the normalization factor $\mathcal{N}_N(x)$ is generally x-dependent when evaluated off the Chebyshev grid. The aim is to utilize the Chebyshev polynomials as the basis functions themselves, without the scaling introduced by $\mathcal{N}_N(x)$.

4.16. Fourier Encoding

The Fourier basis, comprising functions $T_j^N := \exp\left(\frac{i2\pi jx}{2^N}\right)$, where N denotes the number of qubits, is a foundational element for constructing a variety of mathematical models. The phase feature map, which possesses an exponentially large capacity, is formed by this basis and was introduced in [84]. It consists of a layer of Hadamard gates applied to each qubit, succeeded by a layer of controlled phase gates $\hat{P}_N(x, j) = \text{diag}\{1, \exp(i2\pi x 2^{j-N})\}$, with j indicating the qubit index. When applied to the computational zero state, the phase feature map yields the state:

$$|x\rangle = \frac{1}{2^{N/2}} \sum_{j=0}^{2^N-1} T_j^N(x)|j\rangle, \tag{36}$$

resulting in an orthonormal set of Fourier states $\{|x_j\rangle\}_{j=0}^{2^N-1}$ evaluated at integer points.

4.17. Projected Unitary Encoding

Projected unitary encoding is a pivotal technique in quantum computing that facilitates the representation of a matrix within a higher-dimensional unitary operator. This method employs orthogonal projectors $\widetilde{\Pi}$ and Π, in conjunction with a unitary matrix U, to encode a target matrix A. The encoding is defined by the relation $A := \widetilde{\Pi} U \Pi$, as elucidated in the seminal work by Gilyén et al. [87].

The essence of this approach lies in the ability to embed a potentially non-unitary matrix A into a unitary framework, which is essential for quantum algorithms that require unitary operations. The projectors $\widetilde{\Pi}$ and Π serve to isolate a specific portion of the unitary matrix U that corresponds to the matrix A. When $\widetilde{\Pi}$ and Π are identical, the encoding is referred to as symmetric, ensuring that the same subspace is used for both the domain and the range of A in the unitary representation. In practical terms, projected unitary encoding allows for the quantum simulation of operations described by A by effectively 'projecting' them into the unitary operation U. This is particularly useful for implementing quantum algorithms that simulate the dynamics of physical systems, solve linear systems of equations, or perform other complex operations that can be described by a matrix A.

To illustrate, consider a simple example where A is a matrix representing a quantum gate, and U is a larger unitary operator within which A is encoded. The projectors $\widetilde{\Pi}$ and Π act as filters that, when applied to U, extract the functionality of A. This enables the use of A within quantum circuits and algorithms despite A itself not being unitary. The concept of projected unitary encoding is a cornerstone of quantum algorithm design, providing a versatile tool for the development of new quantum protocols and the enhancement of existing ones.

4.18. Block-Encoding

Block-encoding is a nuanced form of projected unitary encoding where the orthogonal projectors $\widetilde{\Pi}$ and Π are defined as $|0\rangle\langle 0|^{\otimes a} \otimes I$. In this scenario, the matrix A is effectively the top-left block of a larger unitary matrix U:

$$A = (\langle 0|^{\otimes a} \otimes I)U(|0\rangle^{\otimes a} \otimes I) \iff U = \begin{bmatrix} A & \cdot \\ \cdot & \cdot \end{bmatrix}. \tag{37}$$

Such a unitary U is referred to as an a-qubit block-encoding of A. If A operates on s qubits and we have $\alpha, \epsilon \in \mathbb{R}_+$ and $a \in \mathbb{N}$, then an $(s+a)$-qubit unitary U is an (α, a, ϵ)-block-encoding of A, provided that:

$$\|A - \alpha(\langle 0|^{\otimes a} \otimes I)U(|0\rangle^{\otimes a} \otimes I)\| \leq \epsilon. \tag{38}$$

Given that $\|U\| = 1$, it follows that $\|A\| \leq \alpha + \epsilon$. It is important to note that while the definition suggests a representation for square matrices of size $2^s \times 2^s$, this is not a limitation. For any matrix $A \in \mathbb{C}^{n \times m}$ with $n, m \leq 2^s$, an embedding matrix $A_e \in \mathbb{C}^{2^s \times 2^s}$ can be constructed such that A occupies the top-left block of A_e and all other elements are zero. This ensures that block encodings are indeed special cases of the projected unitary encodings previously defined.

5. Review and Discussion of Existing Studies (2020–2024)

Building upon the foundational discussions of various encoding techniques in the preceding section, this section aims to present a comprehensive review and discussion of the most recent studies within the domain of QML. An upward trend can be observed in the number of publications in various reputed databases such as IEEE Xplore, Springer, and Elsevier from 2015 to 2024 in the QML field, as depicted in Figure 4. These studies have pragmatically employed a multitude of encoding techniques across diverse datasets, yielding fruitful outcomes in a range of applications. While this section highlights the fact that research conducted before 2020 has already been reviewed extensively in the literature, there has been a notable absence of reviews encompassing the advancements made in the subsequent years. Since 2020, much more focus has been given to this field. Consequently, we endeavor to bridge this gap by examining the literature from 2020 to 2024, offering critical insights into each study, with a particular focus on the encoding techniques utilized.

More specifically, a study by Gouveia and Correia [88] introduced an innovative technique for network intrusion detection by employing unsupervised QML and quantum-assisted ML with Autoencode, utilizing both Basis and Amplitude Encoding on the KDD-NSL and NB15 datasets. Their simulation evaluations on IBM QX suggest that Quantum-Assisted Network Intrusion Detection Systems (NIDS) can achieve high accuracy levels, rivaling those of the best conventional SVMs, contingent upon the characteristics of the dataset. However, the study's reliance on simulation mode raises questions about the scalability and practical implementation of the technique in real-world quantum hardware. Another study by Cao et al. [68] explored the embedding of an ML cost function into a quantum circuit using qubit encoding on the canonical Iris flower dataset. Their method allows for the reception of a training dataset either in a superposed state or as a pre-prepared mixed state, and they detail a procedure for estimating the gradient of the cost function within the quantum state. Notably, their approach circumvents the need to encode the cost function directly in the quantum state, which is typically determined classically via repeated measurement outcomes. This strategy effectively insulates the cost function from direct quantum manipulation. A potential limitation of this study is the indirect estimation of the cost function's gradient, which may introduce complexities in gradient-based optimization algorithms commonly used in machine learning. Lloyd et al. [75] have proposed training the embedding segment of a quantum circuit as a strategic part of learning. In this work, they develop a method that maximizes the separation of data classes in a Hilbert space using feature map encoding over datasets, such as the two-dimensional moons, using a term that they dub as quantum metric learning. This technique simplifies the classification procedure by defining in advance the measurement that leads to the minimum loss in terms of the metric used, either when using the Helstrøm measurement when using the $\mathcal{L}^1$ distance or a simple overlap measurement when using the $\mathcal{L}^2$ distance. This strategy provides a promising analytic framework for QML given complex embeddings and measurements, but this requires quantum processors capable of doing so. Limitation of the study's potential applicability might also stem from the level of dependence on individual metrics and datasets used in the study, which will prompt further study to validate this technique's effectiveness on other quantum computing platforms and types of data.

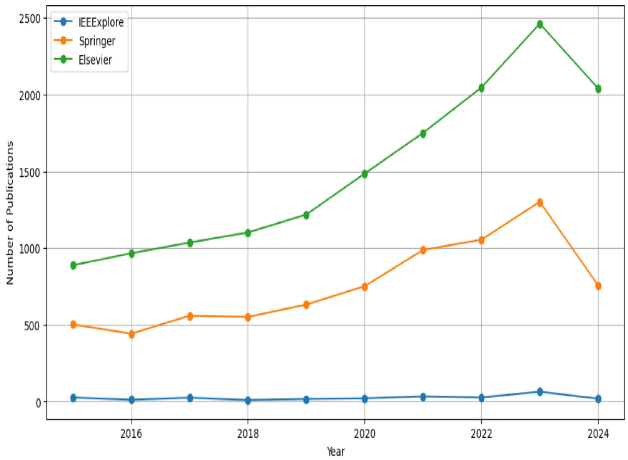

Figure 4. Number of publications in QML from 2015 to 2024 based on the keywords "Quantum Data Encoding Techniques".

In the other exploration, Kim et al. [89] employed feature map encoding in the cryptanalysis of the Caesar cipher by means of a QML framework. In contrast to classical neural networks, they made a departure from classical neural networks and used a parameterized quantum circuit as a learning model from KDD-NSL and NB15 datasets. Limitations in testing four-bit combinations were not possible because the simulations on IBM QX were perfect accuracy for two-bit plaintext and keys 0.84 for three-bit and four-bit combinations, but restrictions on a cloud environment could not take any more than that, so four-bit combinations had to be abandoned. It also found that the accuracy was not as good when going from simulating quantum processors to real ones, which suggests there is still work to be performed for practical application with good qubit efficiency and error mitigation. Payares et al. [90] demonstrated a trio of quantum models for DDoS threat detection based on angle encoding of an originally 'specialized' dataset. Their work compared the efficacy of QSVMs, hybrid quantum–classical NNs, and a dual-circuit ensemble model. The models demonstrated impressive performance, with the lowest accuracy being 96%. However, the study's focus on a specific threat type suggests a need for broader validation across various cybersecurity challenges to fully assess the versatility of the proposed quantum models. Stein et al. [91] presented GenQu, a hybrid framework that utilizes quantum states for classical data learning. Applied to the MNIST and circle datasets, GenQu showcased a reduction in qubit usage by up to 95.86% and a 33.33% increase in convergence speed compared to conventional methods. Despite these promising results, the study acknowledges the complexity of scaling such frameworks to larger, more complex datasets and the ongoing challenge of achieving consistent performance on actual quantum hardware versus simulations

In an innovative exploration of binary classification through QML, Maheshwari et al. [64] employed amplitude encoding within a VQC. This investigation spanned three datasets: an exclusive diabetes dataset, the UCI sonar dataset, and a synthetic dataset. The authors' methodology included a pre-processing strategy encompassing feature selection and quantum state preparation to mitigate the challenges posed by NISQ systems. The study underscored the significance of quantum state preparation, particularly amplitude encoding, in enhancing the QML models' data encoding and learning processes. The amplitude encoding-enhanced VQC demonstrated superior performance, occasionally surpassing the conventional VQC, achieving accuracies of 98.40%, 67.3%, and 74.50% on the synthetic, sonar, and diabetes datasets, respectively. However, the study's limitation lies in its reliance on amplitude encoding, which may not generalize across different quantum hardware or datasets. Dilip et al. [92] scrutinized the compression of classical data into quantum-efficient representations, utilizing amplitude encoding on the Fashion-MNIST dataset. Their approach enabled the adjustment of both the quantum circuit's qubit count and depth. This was achieved by drawing parallels between matrix-product states and quantum circuits. The researchers introduced a hardware-efficient quantum circuit design, tested on the Fashion-MNIST dataset, demonstrating that a quantum circuit-based classifier can rival existing tensor learning methods with a mere 11-qubit requirement. The limitation of this study is the potential scalability issue when dealing with larger datasets or more complex classification tasks. Islam et al. [58] presented a hybrid quantum–classical NN utilizing basis encoding to detect amplitude shift cyberattacks within an in-vehicle control area network (CAN) dataset. Their hybrid model achieved a commendable 94% attack detection accuracy, outperforming both the LSTM NN (87%) and the standalone quantum NN (62%). Despite the success, a limitation of this approach is the model's specificity to the CAN dataset, which may limit its applicability to other cybersecurity contexts or datasets.

Pushpak and Jain [76] delineated algorithms that leverage quantum feature maps to discern fraudulent activities in home insurance claims. The intricate nature of insurance fraud patterns necessitates robust detection mechanisms to preclude financial losses from spurious claims. Their work harnessed QSVM algorithms, complemented by feature engineering, selection, and parameter optimization, to identify dubious claims. A comparative analysis with traditional SVMs is also presented, highlighting the efficacy of QSVM in fraud detection within property insurance claims. Nonetheless, the study's limitation is the potential computational overhead associated with the quantum feature map, which may not be feasible for large-scale datasets. Hur et al. [93] evaluated fully parameterized QCNNs on MNIST and Fashion MNIST datasets for classifying classical data. Their QCNN model, inspired by conventional CNNs, operates with two-qubit interactions. The performance of various QCNN models, differentiated by their quantum circuit topologies, data encoding methods, pre-processing techniques, cost functions, and optimizers, was compared. Despite a limited number of parameters, QCNNs consistently achieved high classification accuracy, surpassing CNNs under the same training conditions. The QCNN approach, utilizing fully parameterized and shallow-depth quantum circuits, is deemed suitable for NISQ devices. However, the limitation of this study is the QCNN's dependence on specific circuit topologies, which may not be universally optimal across different quantum computing platforms. Nikoloska and Simeone [94] introduced a two-layer hybrid classical-quantum classifier. The first layer comprises quantum stochastic neurons using generalized linear models (QGLMs) with amplitude encoding, exploiting the exponential fan-in capability relative to the qubit count. The implementation of QGLMs is simplified through binary weights and activations. They proposed a stochastic variational optimization technique for the concurrent training of quantum and classical layers via stochastic gradient descent, overcoming the limitations of previous training methods restricted to exhaustive search or single-neuron bit-flip strategies. The effectiveness of this approach is validated through experiments with diverse activation functions in QGLM neurons. The study's limitation, however, is the binary nature of weights and activations, which may restrict the model's expressiveness and adaptability to complex datasets.

Qian et al. [95] conducted systematic numerical experiments using qubit encoding on the Quantum synthetic, Wine, and MNIST datasets. Their findings indicate that contemporary QNNs do not outperform conventional learning models. The study delivers two significant insights: first, QNNs have low effective model capacity, leading to poor generalization to real-world datasets; second, QNNs are trainable regardless of regularization strategies, unlike conventional models. These results suggest that the current functionality of current QNNs needs to be rethought and new protocols developed for maximizing the quantum advantages that are possible for solving real-world problems. A limitation of our work is that it relies on synthetic and small real-world datasets, which may not fully capture the issues encountered in real-world practice. Multi-amplitude encoding is proposed on MNIST and face datasets by Bar et al. [96] using a hybrid quantum–classical technique for image classification. On the quantum side, they use a VQC, encoding the data with multiple amplitude encoding while accounting for qubit constraints. The classical component of the VQC consists of image preprocessing, convolution, and optimization of single qubit rotation gates. The method was tested and evaluated for two datasets and compared two- and four-layer VQC accuracy rates. Compared to other similar studies, this hybrid strategy provided superior parameter efficiency and quantum cost. A limitation is that this is not ideal for large, more complex datasets that have a small number of anomalies. Digital twin-assisted quantum federated learning (DTQFL) was proposed by Qu et al. [97] using amplitude and angle encoding on the Breast Cancer Wisconsin and Fetal Health datasets. With this method, data from the IoMT devices are aggregated through a 5G mobile network to create digital twins (DTs) of patients, thereby reducing time for communication in federated learning. Synchronous training and updating of VQNNs without hampering real-world operations is enabled by DTQFL. For privacy, security, and training speed, final global parameters were also trained iteratively local personal VQNNs were trained personalized for each hospital. The results show that DTQFL can train effective VQNNs without collecting local data and achieve accuracy comparable to centralized methods. A limitation of this approach is the dependency on advanced network infrastructure, which may not be available in all healthcare settings.

Satpathy et al. [98] investigated the impact of a noisy quantum environment on two datasets (TWTDUS and SDWTT18) associated with IoT harsh environments using various QML methods. Their analysis employed analytical clustering approaches, with the variational UU method achieving a maximum accuracy of 98.10% on the TWTDUS dataset. On the SDWTT18 dataset, the $UU^{\dagger}$ method combined with k-means clustering attained an accuracy of 94.43%. These proposed quantum algorithms outperformed classical methods and demonstrated potential for predicting daily output power generation based on metrics used in the energy sector for decision-making. However, a limitation of this study is its dependency on specific datasets, which may not fully generalize to other IoT environments or datasets. Fan et al. [33] utilized amplitude encoding on five different Earth Observation (EO) datasets—Overhead-MNIST, So2Sat LCZ42, PatternNet, RSI-CB256, and NaSC-TG2. They described a hybrid quantum–classical convolutional neural network (QC-CNN) designed to extract key features from EO data for categorization. The use of amplitude encoding minimized the need for additional quantum bits. Complexity analysis revealed that the proposed model outperformed classical models in convolutional operations. The model's performance, evaluated using the TensorFlow Quantum platform, showed superior generalizability and accuracy on the EO benchmarks compared to classical counterparts. Nonetheless, a limitation of this approach is the potential challenge in scaling the model for larger datasets due to quantum hardware constraints. Tscharke et al. [99] presented a semisupervised anomaly detection method based on the reconstruction loss of an SVR with a quantum kernel, using amplitude and angle encoding on a toy dataset. This model was positioned as an alternative to variational quantum and quantum kernel one-class classifiers and was benchmarked against a quantum autoencoder, an SVR with a radial-basis-function (RBF) kernel, and a classical autoencoder. Extensive benchmarking on ten real-world anomaly detection datasets and one toy dataset demonstrated that the SVR model with a quantum kernel outperformed the SVR with an RBF kernel and all other models, achieving the highest mean AUC across all datasets. Additionally, the quantum SVR outperformed the quantum autoencoder on nine of the eleven datasets. A limitation of this study is the use of a toy dataset for method development, which may not entirely reflect real-world complexity.

Jiaxiang and Jiale [100] introduced the Quantum-Gravitational Transformer (QGFormer) using the Gravity Spy project dataset, presenting a novel quantum–classical hybrid transformer for detecting gravitational waves. The QGFormer architecture integrates a vision transformer encoder with a quantum classifier within an autoencoder framework. The encoder segments the input signal into small patches and maps them to higher dimensions, which are then fed into the quantum classifier for prediction. This parallelism with quantum computing enhances computational efficiency and scalability. Additionally, techniques such as super-density coding and quantum state compression help reduce noise. Experiments on the Gravity Spy dataset, sourced from the LIGO detector, demonstrate that QGFormer surpasses comparator models in both accuracy and F1-score, indicating significant potential for QML in this domain. However, the study's reliance on quantum hardware, which may not be widely accessible, poses a limitation for practical implementation. Hu et al. [101] introduced NISQRC, a machine learning algorithm for qubit-based quantum systems that analyzes temporal data over durations exceeding the coherence times of individual qubits. NISQRC balances input encoding, mid-circuit measurements, and reset to provide persistent temporal memory, capturing time-domain correlations in streaming data. This method addresses challenges such as finite coherence, information scrambling, and thermalization in monitored circuits. Using Volterra Series analysis from dynamical systems theory, the study determines the necessary measurement and reset conditions for maintaining a finite memory period in a monitored circuit. To validate their approach, the authors applied it to the channel equalization challenge, recovering a test signal of Nts symbols from a noisy and distorted channel. Their experiments with a seven-qubit quantum processor and numerical simulations showed that Nts is not limited by coherence time. However, the need for precise mid-circuit measurements, which may be challenging to achieve consistently with current quantum hardware, is a notable limitation. In this work, Ruan et al. [102] introduced VIOLET, a visual analytics tool to increase QNN explainability on the AMASS dataset via angle encoding. The development of VIOLET was informed by interviews with domain experts and a comprehensive literature review, resulting in three visualization views: It consists of the encoder view, which shows the classical input data-to-quantum states conversion; the ansatz view, depicting the temporal evolution of quantum states when trained; and the feature view, showcasing the features learned by a QNN after training. The visual representations, including satellite charts and augmented heatmaps, depict variational parameters and quantum circuit measurements, respectively. VIOLET's effectiveness was demonstrated through two case studies and in-depth interviews with 12 specialists, proving beneficial for QNN users and developers in understanding and exploring QNNs. Nonetheless, the complexity and learning curve associated with these visualization tools may limit their accessibility for users without a strong background in quantum computing.

Alomari et al. [103] introduced the DEQSVC strategy utilizing a quantum feature map on the DDoS Evaluation Dataset (CIC-DDoS2019) to enhance the accuracy of DDoS attack detection. The DEQSVC leverages fast dimensionality reduction techniques, a robust feature map method, and an efficient kernel estimation strategy to improve data encoding, learning, and detection accuracy. Performance evaluations using simulations on the Qiskit platform and an IBM quantum computer revealed that DEQSVC outperformed several benchmark algorithms commonly used in intrusion detection systems, achieving a detection accuracy of 99.49%. However, a limitation of this study is its reliance on simulations and specific datasets, which may not fully represent real-world network conditions. Rathi et al. [104] presented the first quantum autoencoder for 3D point clouds, employing amplitude encoding on the AMASS dataset. The 3D-QAE technique is entirely quantum, optimizing all data processing components for quantum hardware. The model is trained on 3D point clouds to generate compressed representations, addressing challenges such as 3D data normalization, parameter optimization, and the identification of an appropriate architecture. Experiments on simulated gate-based quantum hardware demonstrated that this method outperformed classical baselines, suggesting a promising new direction in 3D computer vision. Nevertheless, the study's dependence on simulated quantum hardware poses a limitation, as real-world quantum hardware may present additional challenges. Another study by Alomari et al. [105] proposed an HCQNN using angle encoding on a solar radiation space weather dataset to simulate space weather. The HCQNN achieved a 99.9% accuracy rate in identifying space weather events and issuing early warning signals to prevent adverse impacts on space-based systems. This approach has the potential to enhance space weather monitoring and improve the resilience of critical space-based technology, thereby reducing the economic and societal costs associated with space weather occurrences. However, the study's focus on a specific dataset and the use of angle encoding may limit the generalizability of the findings across different space weather phenomena.

Nguyen et al. [106], utilizing angle encoding on the MNIST and FashionMNIST datasets, introduced the concept of quantum scalable data. They presented a hybrid quantum–classical architecture designed to manage such data, encompassing scalable data definition, metadata, and operations within both classical and quantum components. Their QML experiments demonstrated the benefits of retaining more data layers, thereby improving classification performance. The ease of scaling implementation with quantum data, as opposed to classical data, was highlighted as a significant advantage. This study marks the first introduction and empirical validation of quantum scalable data. However, the reliance on specific datasets and experimental conditions may limit the generalizability of the results. Fan et al. [107] proposed two hybrid quantum–classical DL frameworks to efficiently extract features from multi-spectral images using quantum computing, followed by classification through classical computation. Their models were validated against the LCZ42 dataset using the TensorFlow Quantum platform, demonstrating improved feature extraction capabilities. Despite these promising results, the study's dependency on specific datasets and the hybrid nature of the model may pose challenges in fully leveraging the advantages of quantum computing. Zhou et al. [108], using a quantum feature map on the NASA airfoil self-noise dataset, proposed a QKE-QSVR model for regression tasks. The model encodes classical inputs as quantum feature vectors using a circuit with variable parameters. Their quantum kernel alignment-based regression (QKAR) technique optimizes the trainable quantum kernel, thereby enhancing the model's predictive accuracy for specific datasets. The optimized quantum kernel is then integrated into the classical support vector regression process to formulate the decision function and predict new data points. The efficacy of the proposed model was validated through three illustrative scenarios, with experimental results indicating superior predictive accuracy compared to classical SVM models. However, the complexity of integrating quantum kernels with classical algorithms and the reliance on specific datasets could limit the broader applicability of this approach.

Based on the above discussion, a summary of the studies categorized by encoding, dataset used, task performed, and result findings is presented in Table 4.

Table 4. Summary of studies categorized by encoding, dataset, task performed, and result findings.

References	Year	QML Model	Encoding	Dataset	Task	Result Findings
Gouveia and Correia [88]	2020	QSVM	Autoencode (Basis and Amplitude Encoding)	KDD-NSL and NB15 datasets	Classification	QASVM generated 76.75% accuracy, 82.3% recall, 86.15% precision, and 77.2% F-score.
Cao et al. [68]	2020	PQCs	Qubit Encoding	Canonical Iris flower dataset	Classification	–
Lloyd et al. [75]	2020	QGAN	Feature Map Encoding	2D moons dataset	Classification	–
Payares et al. [90]	2021	QSVM	Angle Encoding	DDoS Evaluation Dataset	Classification	QSVM generated 97.14% accuracy, 96.14% recall, 97.19% precision, and 96% F-score.
Stein et al. [91]	2021	GenQu	Quantum Encoding	MNIST and circle datasets	Classification	–
Kim et al. [89]	2021	QSVM	Feature Map Encoding	Caesar cipher dataset	Classification	–
Maheshwari et al. [64]	2021	Amplitude Encoding based VQC	Amplitude Encoding	Synthetic, Sonar, and diabetes datasets	Binary Classification	Results of proposed amplitude encoding based VQC on three datasets are 98.4%, 67.3%, and 74.4% accuracy, respectively.
Islam et al. [58]	2022	HQ-CNN	Basis Encoding	In-vehicle control area network (CAN) dataset	Classification	For both the training and testing datasets, the hybrid quantum-classical NN shows 98.7% and 93.9% accuracy, respectively.
Rohit Dilip et al. [92]	2022	QCC	Amplitude Encoding	Fashion-MNIST dataset	Classification	–
Hur et al. [93]	2022	QCNNs	Amplitude, Qubit, and Dense Encoding	MNIST and Fashion MNIST datasets	Classification	–
Pushpak and Jain [76]	2022	QSVM	Quantum Feature Map	–	Classification	QSVM generated 91.15%, 92.66%, and 92.67% accuracy with linear, circular, and full kernels, respectively.
Nikoloska and Simeone [94]	2022	HCQC	Amplitude Encoding	Prototypical image dataset	Classification	The proposed SVO scheme achieved higher classification accuracy for all response functions.
Qian et al. [95]	2022	QNNN, QENN, and QCNN	Qubit Encoding	Quantum synthetic, the wine, and MNIST datasets	Classification	–
Bar et al. [96]	2023	2 and 4-layer VQC	Multi-Amplitude Encoding	MNIST and face datasets	Image Classification	MNIST dataset achieved 79.26% accuracy in 2-layer VQC and 92.04% in 4-layer VQC. The face dataset achieved 71.05% accuracy in 2-layer VQC and 84.12% in 4-layer VQC.
Qu et al. [97]	2023	DTQFL	Amplitude and Angle Encodings	Breast Cancer Wisconsin and Fetal Health datasets	Fetal Health Classification	–
Satpathy et al. [98]	2023	$UU^{\dagger}$ classifier, Variational $UU^{\dagger}$ classifier, and QNN	–	TWTDUS and SDWTT18 Datasets	Classification	For the TWTDUS dataset, variational $UU^{\dagger}$ with analytical clustering methods achieved 98.10% accuracy. For the SDWTT18 dataset, the $UU^{\dagger}$ method with k-means clustering achieved 94.43% accuracy.

Table 4. *Cont.*

References	Year	QML Model	Encoding	Dataset	Task	Result Findings
Fan et al. [33]	2023	QC-CNN	Amplitude Encoding	Five different EO datasets (Overhead-MNIST, So2Sat LCZ42, PatternNet, RSI-CB256, and NaSC-TG2)	Image Classification	–
Jiaxiang and Jiale [100]	2023	QGFORMER	–	Gravity Spy project dataset	Glitch Classification	QGFormer generated 94.27% accuracy and 94.13% F-score.
Alomari et al. [105]	2023	HC-QNN	Angle Encoding	Solar radiation space weather dataset	Classification	Results of HCQNN on the IBM Quantum Computer: 95% accuracy, recall, precision, F-score, and 5% error rate.
Tscharke et al. [99]	2023	QSVR	Amplitude and Angle Encoding	Toy dataset	Regression	QSVR generated 100% AUC, 94% accuracy, 100% recall, 89% precision, and 94% F-score.
Hu et al. [101]	2023	NISQRC	Hamiltonian Encoding	Temporal Data	Logistic Regression	–
Ruan et al. [102]	2023	VIOLET	Angle Encoding	–	Classification	–
Alomari et al. [103]	2023	DEQSVC	Quantum Feature Map	DDoS Evaluation Dataset (CIC-DDoS2019)	Classification	DEQSVC generated 99.49% detection accuracy, 99% recall, 99% precision, and 99% F-score.
Rathi et al. [104]	2023	3D-QAE	Amplitude Encoding	AMASS dataset	Classification	–
Nguyen et al. [106]	2023	HQCA	Angle Encoding	MNIST and Fashion-MNIST datasets	Image Classification	The accuracy values of MNIST are over 95% while the accuracy values of FashionMNIST are below 90%.
Fan et al. [107]	2023	FQCNN and MQCNN	–	LCZ42 dataset	Classification	MQCNN model is superior to the FQCNN model (from 0.899 to 0.913).
Zhou et al. [108]	2024	QKE-QSVR	Quantum Feature Map	NASA airfoil self-noise dataset	Regression	–

Example Application: Quantum Machine Learning for Classification Tasks

In our classification task example, we employed a quantum support vector machine (QSVM) technique. The encoding procedure encompassed the subsequent steps:

- Data Encoding: Classical data were converted into quantum states by amplitude encoding. This method enables the simultaneous representation of many data points by encoding them into the amplitudes of a quantum state.
- Quantum Feature Map: We utilized a quantum feature map to embed the classical data into a higher-dimensional Hilbert space. This mapping improves the distinguishability of the data, facilitating the quantum algorithm's accurate classification.
- The Quantum Support Vector Machine (QSVM) was executed through a sequence of quantum gates that manipulate the stored information. The circuit's architecture is engineered to enhance classification according to the support vectors recognized during training.

Distinctive Features of the Quantum Algorithm

The quantum technique employed for this application possesses numerous distinctive characteristics:

- Exponential Speedup: In contrast to classical algorithms, QSVM can attain an exponential acceleration in specific situations, especially when managing extensive datasets.
- Quantum Interference: The method utilizes quantum interference to augment the likelihood of accurate classifications while diminishing the chances of erroneous ones, resulting in enhanced precision.
- Scalability: The inherent parallelism of quantum computing allows the algorithm to scale more effectively with increased data complexity, providing a robust solution for high-dimensional datasets.

6. Challenges and Future Directions

All the discussions in the previous sections of the paper highlight the critical role of data-encoding techniques in QML. While significant progress has been made, several challenges specific to data encoding remain, which must be addressed to fully exploit the potential of QML. Although our main emphasis is on data encoding, it is essential to recognize the substantial advancements in QML. Recent research in QML, especially utilizing hybrid classical–quantum models, presents exciting opportunities for enhancing encoding strategies. Hybrid methodologies utilize the advantages of classical systems for managing extensive datasets alongside the quantum benefits in particular computing functions, including state preparation and data classification. Integrating these improvements may enable the development of more efficient and scalable encoding methods that correspond with real-world applications, thus fostering growth in both domains synergistically. Disregarding these advancements will constrain the breadth and relevance of data encoding research. This section outlines these challenges and proposes future research directions.

6.1. Scalability of Quantum Computers for QML Applications

The scalability of quantum computers remains a formidable barrier to the widespread adoption of QML algorithms. Despite the rapid advancement in qubit counts, as detailed in Table 5, the current quantum computing infrastructure is insufficient for executing large-scale QML algorithms that require extensive data processing capabilities. The progression from the first 2-qubit system developed by IBM, the MIT Media Lab, and UC Berkeley in 1997 to the anticipated 5k-gate, 156-qubit 'Flamingo' system by IBM in 2025 underscores the significant strides made in quantum computing [109]. However, the practical implementation of QML algorithms is hindered by the limited qubit coherence times, error rates, and the quantum volume of existing systems.

The quantum volume, a metric that considers both the number of qubits and the depth of operations that can be performed before decoherence sets in, is a critical factor in determining the computational power of a quantum computer. As Herbster et al. [110] highlight, the small scale of quantum computers restricts their data processing capacity, leading to data loss when the number of features exceeds the available qubits. Furthermore, many algorithms for NISQ devices, as considered by Preskill et al. [111], suffer from inflexibility, requiring trade offs between the complexity of the algorithm and the quantum hardware's capabilities.

But to overcome these challenges, we desperately need progress in developing quantum error correction, qubit connectivity, and circuit optimization. In addition, error-tolerant algorithms capable of running on NISQ devices are also essential. According to Perdomo-Ortiz et al. [112], the future of QML rests on our capacity to operate quantum computers at scale not only in the number of qubits or quantum volume but also with regard to the size of the datasets, without compromising the integrity of the data or the efficiency of the algorithms.

Table 5. Progress in qubit counts of quantum computers from 2015 to 2024.

References	Year	Notable Progress in Quantum Computing	Number of Qubits
[109]	2019 I	IBM 27 qubits	27
	2020	IBM 65 qubits	65
	2021	IBM 127 qubits	127
	2022	IBM 433 qubits	433
	2023 S	IBM 133 qubits, IBM 1121 qubits	133, 1121
	2024 T	IBM 408 qubits, IBM 1386 qubits	408, 1386
[113]	2016	RIGETTI 3 qubits	3
	2016	IBM 5 qubits	5
	2017	IBM 50 qubits	50
	2018	INTEL 49 qubits	49
	2019	GOOGLE 72 qubits	72
	2019	RIGETTI 128 qubits	128

6.2. Advancements in Quantum Data Encoding

Integrating QML is a difficult task, given the intricacies of quantum data encoding. Furthermore, the conversion of classical information (e.g., images and large datasets)—a computationally expensive task—requires significant energy. A key part of this bottleneck is the exponential increase of the Hilbert space with another qubit and therefore efficient encoding schemes for representing data compactly but without losing information. More recently, LaRose and Coyle [71] have moved in this direction with the development of a robust binary quantum classifier that is resistant to noisy states. In this approach, they then select the most suitable encoding method to put input data into the quantum system. They showed to what extent accuracy can be enhanced by evaluating different encoding strategies on the same datasets.

Nevertheless, the optimal encoding methods continue to be sought after. Future work should investigate encoding schemes requiring the least quantum resources while maximizing the encoded information conveyed in the quantum states. Exploration of data representations in high dimensions, such as tensor network states and quantum feature maps, is included and can capture complex correlations in the data. Additionally, error-resilient encoding methods must be further developed that can withstand modern quantum hardware imperfections. These achievements are not only going to make the QML models more accurate but will also allow us to take advantage of QML models in many more real-world problems.

6.3. Innovations in Encoding-Based Quantum Algorithms

A critical frontier in the integration of quantum computing with machine learning is the evolution of encoding-based quantum algorithms. Efficiency in exploiting the computational power of quantum systems largely depends on efficient mapping of classical data onto quantum states, a task of great importance for realizing the power of quantum hardware. This approach is demonstrated with the very real seminal work by Maheshwari et al. [64] on implementing an amplitude encoding-based VQC for noisy quantum environments. This method optimizes the encoding scheme to improve the input-handling capability of the quantum system.

Specialized quantum subroutines coupled with quantum information theory are essential for addressing complex machine learning challenges and provide a performance boost to quantum algorithms. In combination, QNNs, DL, and quantum-inspired ML paradigms open up the opportunities to kickstart new advances in the field. The encoding-based QVCs hold deep potential for future research of classification algorithms that are suitably leveraged to achieve the highest performance with minimum circuit depth. These advances will enable the applying of classical machine learning to real quantum hardware and simulations, closing the gap between ideal theory and real-world engineering.

In addition, encoding strategies are also explored beyond amplitude encoding in basis, angle, and Hamiltonian encoding, among others. Different encoding schemes have their own advantages and disadvantages that need analysis in totality to understand their suitability for a given machine learning problem. With the advent of quantum technology, the encoding-based algorithms, which are not only theoretically sound but practical and viable on near-term quantum devices, will play an important role. It entails the development of robust quantum algorithms for few-qubit regimes utilizing sparse ensembles whose quantum resources scale efficiently as hardware improves.

6.4. Optimization of Quantum Encodings for Emerging Hardware

It is important for the practical deployment of QML algorithms that quantum computing hardware continue to advance. This poses an important engineering challenge for us, as we need to scale qubit counts significantly in order to generate and evaluate QML algorithms, as well as to process large datasets. An inherently qubit-intensive encoding technique included in basis encoding is exemplified. Fortunately, there is a pressing need to encode quantum problems in a fashion inherently congruent with the peculiar properties of emergent quantum architectures. For future research, more should be accomplished to encode data that are optimized for the landscape of the quantum hardware in order to minimize the quantum resource overhead. In this, we mean to minimize the number of qubits and quantum gates needed for encoding classical data into quantum states. It also makes use of hardware-specific features—such as limitations on connectivity and fidelity of the gates—to improve the efficiency and reliability of quantum operations.

Also, hardware-optimized encodings must take into account error rates and noise characteristics for quantum systems. If one can design encoding schemes for a quantum processor that minimizes its error profiles, this increased resistance and accuracy of QML could be achieved. To implement this approach, there is a need for deep understanding both of the theoretical aspects in quantum information and practical limitations of the quantum hardware while bridging the gap between quantum algorithm designs and hardware installations. As a purely technical pursuit, pursuing hardware-optimal encoding is simply a hardware engineering problem to solve. But in reality, it is a multidisciplinary effort to be achieved involving quantum algorithm theorists, hardware engineers, and machine learning practitioners. Ultimately the result of this research direction will dictate how well QML can scale to and be applicable to the real world.

6.5. Harnessing Entanglement for Feature Encoding in QML

In QML, entanglement-based feature encoding is a transformational approach where classical data are encoded into quantum states that are entangled corresponding to each data sample. Cao et al.'s [114] pioneering research in entangled encoding schemes for qutrits provides clear illustration of the scheme's potential. We find that the choice of encoding has a significant impact on classification accuracy and uncovers the subtle interplay between entanglement and computational performance.

The area of research aimed at developing feature maps or encoding systems that will generate entangled quantum states specifically suited for a specific set of attributes in input data is on the rise. Despite the powerful capability of quantum states to represent patterns and correlations in the data, we envision these feature maps to tap into the specific properties of entanglement to preserve this information in compact representations of quantum states.

Future research directions include:

- Exploring the Theoretical Foundations: Deepening our understanding of how entanglement can be harnessed to improve data representation in high-dimensional spaces.

- Designing Entanglement-Rich Feature Maps: Creating novel quantum feature maps that intrinsically generate highly entangled states, potentially leading to more powerful QML models.
- Investigating Entanglement Measures: Studying various measures of entanglement as a resource for QML to ascertain which aspects most significantly contribute to enhanced learning capabilities.
- Optimizing for Quantum Hardware: Tailoring entanglement-based encodings to align with the capabilities and limitations of current and future quantum hardware.
- **Benchmarking Against Classical Methods:** Comparing the performance of entanglement-based encodings with classical encoding methods to quantify the advantages offered by quantum approaches.

Incorporation of entanglement into feature encoding, while promising to increase QML algorithm efficacy beyond existing limits, could also be exploited as a vehicle exploring the richer interplay between quantum mechanics and machine learning. In this synergy, the focus is on unlocking the new paradigms of data processing and algorithm design taken from the analysis of quantum correlations.

6.6. Strategies to Overcome Barren Plateaus in Quantum Feature Maps

In the optimization landscape of QML, barren plateaus represent a key concept. These regions are where the gradient is going to be small, but small means that gradient-based optimization methods will not be able to find the global minimum. Usually, barren plateaus are probabilistic, but structural elements in variational circuits like large Hilbert spaces increase the probability of occurrence.

Future work should be directed at designing quantum feature maps, variational models, with less constrained structure in order to alleviate the impact of barren plateaus. In other words, this involves focusing the training process on a portion of the Hilbert space such that our gradients remain large enough for successful optimization. This necessitates an elegant control of the expressiveness of the quantum model and the fitness of its training landscape.

Exploring the following areas will be crucial for advancing this field:

- Constrained Variational Models: Developing variational quantum models that avoid large, unstructured Hilbert spaces prone to barren plateaus.
- Quantum Supremacy in Feature Maps: Investigating the potential of quantum supremacy to create feature maps that can efficiently solve industry-relevant problems.
- Data-Driven Learning: Utilizing data-driven techniques to optimize the form and parameters of quantum kernels, which is a nascent yet promising area of research.
- Performance Across Data Types: Assessing the performance of parameterized quantum circuits (PQCs) on diverse types of data, both classical and quantum, to establish benchmarks and identify best practices.

Beyond these, we find it critical to carry out a thorough exploration of barren plateaus across several domains like optimization and DL. This will yield important insights into the nature of these challenges to enable the development of robust QML quantum feature maps capable of traversing the daunting optimization landscapes intrinsic to QML.

7. Conclusions

The data encoding methods for QML are explored and show a high potential of improving the efficiency and effectiveness of machine learning systems by quantum computing fundamentals. As an encoding approach, this study highlights how encoding approaches can translate classical data into quantum states, thus enabling the operation of quantum algorithms for related datasets. While there have been many significant strides, scalability, optimization for emerging quantum hardware, and overcoming barren plateaus in quantum feature maps remain difficult challenges. The development of creative encoding-based quantum algorithms, as well as leveraging entanglement for feature encoding, can offer profound advances to the field.

Future work should aim at improving these encoding techniques and exploring further avenues for this promise to be fully achieved in practical use cases. Through these successes, we will continue to increase the material of quantum technology and provide this door to new pathways for applications in diverse domains.

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