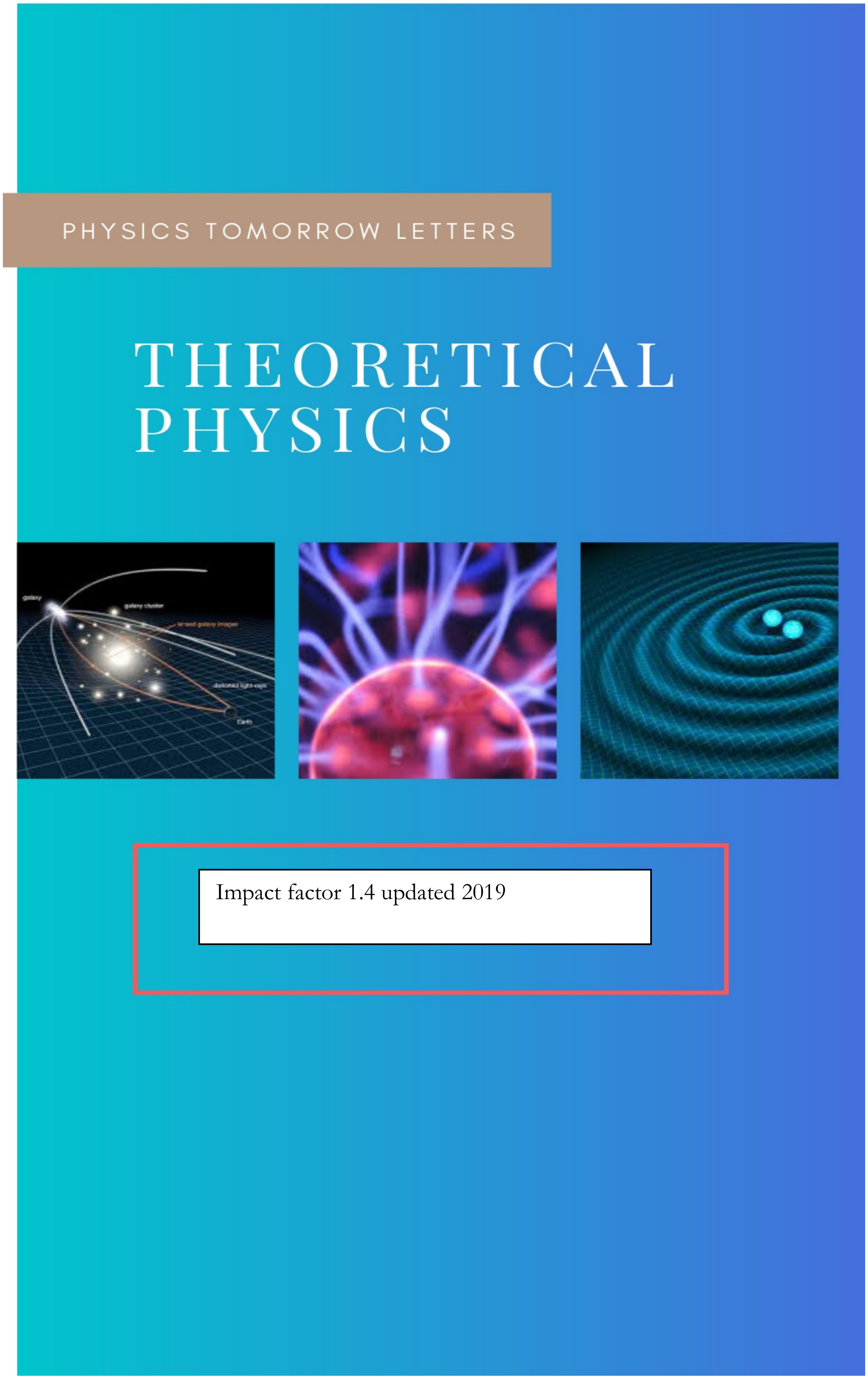
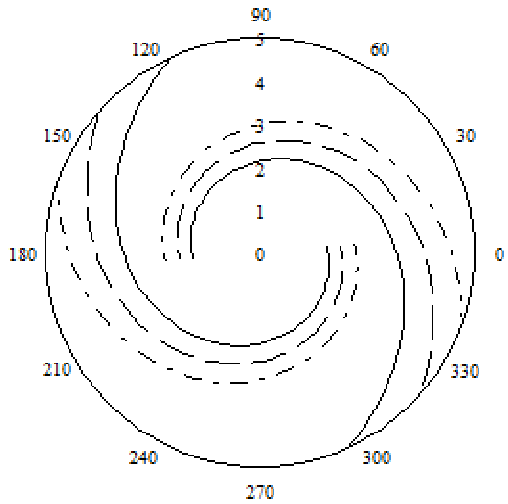
Theoretical Physics Letters (IF 1.4) : PTL Coresearch
2020 ° 06(06) ° 08-01
https://www.wikipt.org/tphysicsletters
DOI: 10.1490/ptl.dxdoi.com/08-01tpl-sci



Spiral Galaxy Model Free of Dark Matter

This article proposes a model of a spiral flat galaxy based on the general relativistic Einstein equations. An expression is found for the relativistic interval, consistent with observational data on spiral galaxies. An embedding of the two-dimensional plane of a galaxy with a curved metric into the Euclidean three-dimensional space is made, and the induced metric of the two-dimensional embedding surface is investigated. It is shown that an increase in the effective mass of the galaxy occurs due to relativistic effects and does not require the introduction of hidden or dark matter. It is shown that the nature of the rotation curve can be related to the quantum properties of baryonic matter on galactic scales.



Galactic sleeves

Keys: galaxy, dark matter, metric, Einstein equations, gravity, rotation curve

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06 June 20

1. Introduction

Recent decades have been marked by the appearance of a large number of astrophysical works devoted to the search for an explanation of the phenomenon known as “dark matter”. It was introduced due to contradictions between the ideas about the structure of galaxies and observational data. Initially, dark matter was supposed to explain the discrepancy between the apparent rotation speed of peripheral stars in spiral galaxies and the amount of observable matter in them, unable to provide the magnitude of this speed [1]. The presence of dark matter has not been confirmed by any observations, therefore its introduction speaks more about attempts to hide our ignorance of the real state of things. Nevertheless, this concept continues to be used to explain a wide range of astrophysical phenomena, such as the stability of the galactic disk [2], the formation of galactic structures [3], etc.

In this paper, we propose a relativistic model of a spiral galaxy, within the framework of which it is possible to carry out calculations to explain the “origin” of the additional mass without resorting to the concept of dark matter. The consideration is based on the general relativistic equations of A. Einstein.

Also, a method is proposed for calculating the rotational component of the velocity of galactic stars, which is consistent with observations and based on the calculation of the Newtonian gravitational potential acting on the star by solving the Newton-Schrödinger equations for the baryon component of matter. Specific estimates were made for the Milky Way galaxy (MW).

2. Model description

The structure of spiral galaxies has been thoroughly studied [4]. The spatial concentration of stars decreases with distance from the center of the galaxy from several million stars at one pc^3 in the center to several stars at one pc^3 at a distance of 1 Kpc and up to about 1 star at 8 pc^3 in the vicinity of the Sun [4]. This, as well as that the almost all mass of the Galaxy (up to 98%) falls on the stellar component, allows us to propose a model of the galaxy described below. Stars located on the periphery, such as the Sun in the MW galaxy, move under the influence of the gravitational field of the core and, to a much lesser extent, their immediate surroundings. Therefore, within the framework of the model, we assume that the entire mass of the galaxy is concentrated in its core and consider the motion of a single star in the field of this center, neglecting all other factors influencing it. Then the metric of curved space-time at a distance from the core has the Schwarzschild form [5]

$$ds^2 = \left(1 - \frac{r_g}{r'}\right)c^2 dt'^2 - \left(1 - \frac{r_g}{r'}\right)^{-1} dr'^2 - r'^2\left(d\theta'^2 + \sin^2\theta' d\varphi'^2\right) \tag{1}$$

r' – is a distance from the star to the center of the galaxy, θ', φ' – polar angles, t' – time, c – light speed, $r_g = 2KM/c^2$ – Schwarzschild radius of the galactic core [1], K – is the constant of gravity, $M = 9{,}55 \cdot 10^{41}$ kg is a MW mass. For the

[1] This Schwarzschild radius has nothing to do with the Schwarzschild radius of a real black hole, which according to modern concepts should be in the center of the galaxy.

MW $r_g \approx 1.4\cdot10^{15}$ $m \approx 0.46\cdot10^{-1}$ pc, which is much less than the size of the core $r_C \approx 10^3$ pc and the distance of the Sun to the center of the galaxy $r_S \approx 10^4$ pc [4].

The motion of stars at a distance from the center of the galaxy consists of rotation around the center and radial motion. In the vicinity of the Sun ($r' \approx 3.08\cdot10^{20}$ m), rotation linear speed V_φ is constant and equal to approximately 220 km / s [4].

Taking into account the flat character of the star's motion (a consequence of angular momentum conservation), expression (1) for the interval can be simplified by setting $\theta' = \pi/2$ and $d\theta' = 0$.

Let's move to the coordinate system, rotating with the star, making a coordinate transformation

$$d\varphi' = d\varphi + \frac{V}{r}dt', V \equiv V_\varphi \tag{2}$$

Performing elementary calculations, we obtain an expression for the interval

$$\begin{aligned} ds^2 &= e^\nu c^2 dt'^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 d\varphi^2 + 2ad\varphi dt', \\ e^\nu &= 1 - \frac{r_g}{r} - \frac{V^2}{c^2}, a = -Vr, r' = r \end{aligned} \tag{3}$$

To get rid of the cross term in (3), we perform the time transformation

$$dt = \eta\left(ad\varphi + e^\nu c^2 dt'\right) \tag{4}$$

Where the function $\eta(r)$ is found from the condition that (4) is a total differential. This leads to the expression of $\eta = e^\nu / c^2$. The final expression for the interval looks as follows

$$ds^2 = e^\nu c^2 dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2\left(1 + e^{-\nu} c^{-2} V^2\right) d\varphi^2 \tag{5}$$

The expression for the interval (5) differs from the Schwarzschild one [5] due to the term $\sim V^2/c^2$, which in our case is of the order $\sim 10^{-6}$ for $r >> r_g$. Note that the determinant of the 4-dimensional metric tensor obtained from (5) by adding the term $-r^2\sin^2\theta\cdot d\theta^2$ is equal to $-r^4\sin^2\theta$, i.e. coincides with the Schwarzschild one. Thus, the dynamics of a typical star, such as the Sun, in the periphery of the galaxy, practically coincides with the dynamics of the body in the vicinity of the Schwarzschild black hole, the parameters of which are determined by the parameters of the galaxy (see Appendix). This saves us from the detailed study of it, which can be found in any textbooks (see, for example [5]).

3. Investigation of the metric 5

Instead, we will study the metric (5). We embed the galactic plane with the spatial part of the metric (5) in a smooth flat three-dimensional manifold. The condition of smooth embedding defines a two-dimensional surface described by the equations $z = z(r), \varphi = \varphi(r)$ with the spatial metric obtained from the flat metric of 3-dimensional space

$$dL'^2 = dz^2 + dr^2 + r^2 d\varphi^2 \Rightarrow \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\varphi^2 \tag{6}$$

For this, we express the spatial part of the metric (5) in the form

$$dL^2 = \left[\left(1 - \frac{r_g}{r}\right)^{-1} + r^2 e^{-\nu} \frac{V^2}{c^2} \left(\frac{d\varphi}{dr}\right)^2\right] dr^2 + r^2 d\varphi^2 \tag{7}$$

From the embedding condition $dL = dL'$ we find the equation of the surface,

$$\left(\frac{dz}{dr}\right)^2 = \left(1 - \frac{r_g}{r}\right)^{-1} + r^2 e^{-\nu} \frac{V^2}{c^2} \left(\frac{d\varphi}{dr}\right)^2 - 1 \tag{8}$$

which at the distances $r >> r_g$ takes the form

$$\frac{dz}{dr} = \pm \alpha r \frac{d\varphi}{dr}, \alpha = \frac{V/c}{\sqrt{1 - (V/c)^2}} \tag{9}$$

It looks like a wave equation

$$\frac{\partial r}{\partial z} \pm \frac{1}{\alpha r} \frac{\partial r}{\partial \varphi} = 0 \tag{10}$$

Equation (10) is a special case of the equation for nonlinear hyperbolic waves ($z \to t$, $\varphi \to x$)[6]

$$\frac{\partial \rho}{\partial t} + c(\rho) \frac{\partial \rho}{\partial x} = 0 \tag{11}$$

in the plane t, x: t – is time, x- is a coordinate; $c(\rho)$ – is a given function of ρ, which is sufficiently studied. Sending for details to [6], we give here the main results.

The equation (11) is a condition of the constancy of $c(\rho)$ on x, t – plane along a curve Σ, defined by the equation

$$\frac{dx}{dt} = c(\rho) \tag{12}$$

i.e. represents a straight line with the slope $c(\rho)$. In the variables z, φ the equation (12) has the form of (11) with $c(\rho) = \pm 1/\alpha r, \rho = r$. For the problem with the initial conditions

$$r = f(\varphi), z = 0, -\infty < \varphi < \infty \tag{13}$$

the line Σ, which transverse an axis $z = 0$ in the point $\varphi = \xi$ preserves its slope $c(f(\xi))$ along its whole length. If we denote it as $F(\xi)$ then the equation of the line Σ has the form $\varphi = \xi + F(\xi)z$. Varying the parameter ξ we receive the family of lines Σ. It is easy to show that in this way the solution to the original equation (10) is obtained [6]. Equation (10) is an example of a hyperbolic wave equation, which means that some disturbance propagates along the straight lines (in the general case, curves) Σ, which are called the characteristics of the equation.

Equation (10) demonstrates the peculiarity inherent in all nonlinear hyperbolic equations related to that the propagation velocity of the perturbation depends on the wave amplitude. This leads to the intersection of the characteristics Σ corresponding to different values of the parameter ξ. The first (in order of increasing z) intersection occurs at the point $z_B = -[F'(\xi_B)]^{-1}$ and $|F'(\xi)|$ takes its maximum [6]. This phenomenon is called wave rollover.

A lot of attention was paid to the construction of solutions of equation (11) after the intersection of characteristics. We briefly list the main results [6, 7].

1. The solution of equation (11) at the intersection point becomes discontinuous and a shock wave appears on its front. Such a scenario is accepted in many astrophysical works devoted to modeling the behavior of spiral galaxies having a jumper.

2. Taking into account the terms of equation (8), omitted in the derivation of equation (10), can restore the continuity of the solution and allows us to talk about the structure of the shock wave [6]. If we interpret the discarded terms as the interaction of simple waves described by equation (10) or (11), then we can use the concept of characteristics that, due to the interaction, do not intersect each other, but behave on the plane z, φ (10) or x, t (11) in a more complex way, forming a shock wave front [7].

3. Integration of the equation (8) leads to the expression

$$z - z_0 = \int dr \sqrt{\left(1 - \frac{r_g}{r}\right)^{-1} + r^2 e^{-\nu} \frac{V^2}{c^2} \left(\frac{d\varphi}{dr}\right)^2 - 1} \tag{14}$$

where in the integrand $d\varphi/dr$ should be expressed as a function of r from the equations of motion (Appendix). This solution applies to single-phase ones. Two-phase solutions can be constructed, as was done in [8], taking into account the interaction of two harmonics - the main and secondary. This is interesting for describing complex dynamic processes, which, as is commonly believed [4], underlying the formation of some galaxies.

Let's define the shape of a two-dimensional surface in three-dimensional flat space, on which a trajectory lies whose projection onto the plane $z = 0$ we investigate. For $r >> r_g$ we can write the general solution of equation (10) in an implicit form

$$r = R(z \mp \alpha r \varphi) \tag{15}$$

where R – is a function defined from the boundary condition. From (15) it follows that the solution is a nonlinear traveling wave, the front of which is determined by the constant phase condition $\Psi_\pm = z \pm \alpha r \varphi$ which gives the equation of the needed surface. In Figure 1 the behavior of $r(\varphi)$ is shown for different values z for $\Psi_- = 0, \pi$. It follows that the projection of a 3-dimensional trajectory onto the $z = 0$ plane has the shape of the sleeves of the galaxy, and the region of small r where the wave front rollover occurs corresponds to a jumper (not shown).

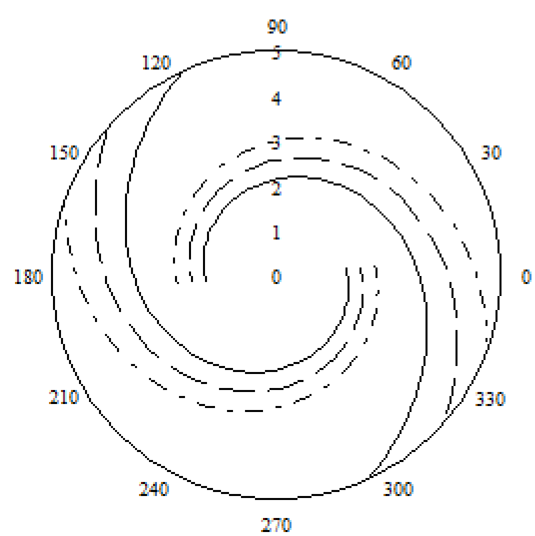

Figure 1 | The lines of constant phase $\Psi\_=0,\pi$ for different z: $z=1$ – solid lines, $z=1.2$ – dashed lines, $z=1.4$ – dash-dotted lines. In the region of small r, where the rollover takes place, this consideration is not applicable.

4. The origin of the hidden mass

Let's show that this model allows us to give the concept of hidden mass physical meaning, allowing not to associate it with hypothetical dark matter. To do this, recall the connection between the g_{00} component of the metric tensor and the Newtonian gravitational potential $\Phi = -KM/r$ [5] (where M as before denotes the true mass of the galaxy)

$$g_{00} = 1 + \frac{2\Phi}{c^2} = 1 - \frac{r_g}{r}$$
$$r_g = \frac{2KM}{c^2} \tag{16}$$

In the metric (5) $g_{00} = e^{\nu}$. We introduce the effective Schwarzschild radius r_g^{ef} with the help of formula $r_g^{ef}=2KM^{ef}/c^2$, where M^{ef} - is an effective mass of a galaxy. Then

$$e^{\nu} = 1 - \frac{r_g}{r} - \frac{V^2}{c^2} = 1 - \frac{r_g^{ef}}{r} \tag{17}$$

From here we find the connection between M^{ef} and M

$$M^{ef} = M\left[1 + \frac{r}{r_g}\frac{V^2}{c^2}\right] \tag{18}$$

For $r >> r_g$. According to estimations, for the Milky Way galaxy at a distance of the Sun from the galactic center , the additional mass is about 30% in the observed mass M^{ef}. Since this mass does not reveal itself except by gravity, there is every reason to call it hidden. As follows from the above, there is no reason to associate it with any additional type of matter.

5. The rotation curve derivation

As is clear from the publications, all the results obtained are related to the peculiarity of the rotation curve $V(r)$, which indicates the deviation of the law of gravitational interaction at galactic scales from Newtonian one. Indeed, if, using the equations of Newtonian dynamics, we write down the condition for the equality of the centrifugal force and the force which attracts the star to the center of galaxy, then for the velocity V we get the expression

$$V = \sqrt{-U}, U = -K\frac{M}{r} \tag{19}$$

M – galaxy mass[2]. It follows that at large r, the velocity $V \sim r^{½}$, which contradicts the experiment. Various theories have been proposed to eliminate this contradiction. Since their review is not the aim of this work, we only mention the so-called modified Newtonian dynamics (MOND) [9], which in its name clearly expresses their common feature. Since

[2] Given that almost the entire mass of the galaxy is concentrated in a region smaller than the distance r of the star from the center of the galaxy

the modification of theories, generally speaking, requires more serious motivation than the noted discrepancy, there is reason to doubt the fruitfulness of this direction.

A different approach is proposed below, which does not require a radical restructuring of existing theories, although it may seem more drastic. It is about taking into account the quantum properties of matter at the galactic scales. Although, at first glance, this approach seems unreasonable because of the belief that atomic and subatomic phenomena are a typical arena for the manifestation of the quantum properties of matter, nevertheless, it can be substantiated, which is done below. In addition to the above advantages over known approaches, he has one more thing - he allows us to limit ourselves to the well-known sources of baryonic matter in reasoning without involving hypothetical dark matter.

The is the following reason for referring to the quantum description of baryonic matter at galactic distances. It is known that a nonrelativistic quantum unit of length (m – neutron mass, $\hbar = h/2\pi$, h – Planck constant) is about 1.15 *Kpc*, which is close to the distance at which the behavior of the rotation speed of spiral galaxies begins to deviate from that predicted by Newton's theory: $1 \div 5$ *Kpc* [4][3] .

To calculate $V(r)$, we write the dimensionless Newton-Schrödinger equation[4] for a typical baryon - a neutron

$$\eta_{\rho\rho} + 2[\nu - U]\eta = 0$$

$$\rho^2 U_{\rho\rho} + 2\rho U = 4\pi|\eta|^2$$

$$\eta = \frac{\hbar}{\sqrt{Km^3}}\chi, U = \frac{\hbar^2}{\sqrt{K^3 m^4}}\Phi, \nu = \frac{\hbar^2}{K^2 m^5}\omega, \rho = \frac{Km^3}{\hbar^2} r \tag{20}$$

$$\chi = \begin{pmatrix}\chi_1 \\ \chi_2\end{pmatrix}, |\eta|^2 = |\eta_1|^2 + |\eta_2|^2$$

where the corresponding units are indicated; r – is a distance, ω – is an energy minus rest energy, Φ - is a gravitational potential and χ – is radial two-component neutron wave function, which components $\chi_{1,2}$ correspond to different projections of the spin [11]. Using (20), we find the relationship between the velocity $V(r)$ (in *m/s*) and the dimensionless potential per baryon U

$$V = \frac{Km^2}{\hbar}\sqrt{-U} \approx 1{,}77 \cdot 10^{-30}\sqrt{-U} \tag{21}$$

The total gravitational potential is NU, where $N \approx 1{,}19 \cdot 10^{68}$ – the number of baryons in the galaxy MW. Taking this into account the velocity V (in *km/s*) is equal

$$V \approx 19{,}3\sqrt{-U} \tag{22}$$

This relation allows us to determine the boundary conditions under which equations (20) should be solved. The results of calculating $V(r)$ under various boundary conditions are shown in Figure 2.

[3] As R. Feynman noted [10], this distance is an analog of the Bohr radius for a neutron

[4] More precisely, its radial part.

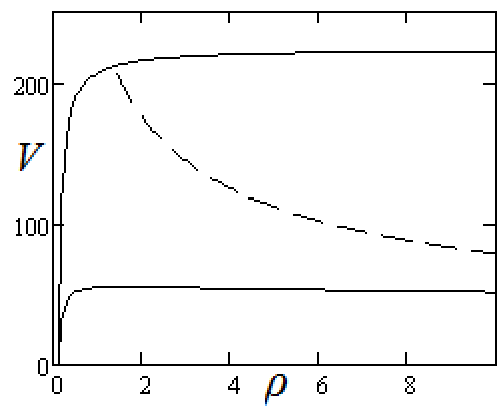

Figure 2 | Solid lines - the rotation speed V versus dimensionless distance ρ determined from the solution of equations (20) for various boundary conditions at the left turning point $U(a) = v$, $a = 0.15$. Upper case $U' = -9\cdot10^2$, down case $U' = -7\cdot10^1$. The rest boundary conditions are: $\eta_1 = \eta_2 = 0.01$, $\eta_1' = \eta_2' = 1$, $U = 0$. The dashed line shows the rotation speed in the classical Newtonian approximation

6. Discussions

The information in this section has two goals: firstly, to show that the proposed model does not contradict known facts, and secondly, to give an explanation for the observed phenomena without resorting to a radical restructuring of the established physical theory, on which the whole picture of the structure of the universe at the galactic level is based. This primarily refers to attempts to harmonize theory and experiment by hasty minor corrections of existing theories (MOND, dark matter).

Section 3 shows that this model is consistent with both observations of the structure of spiral galaxies and accepted theories that explain this structure. The results presented in many respects overlap with the known ones using the concepts of wave theory, adding some new ones related to multiphase solutions that describe complex dynamic modes responsible for the formation of the structure of galaxies.

Let us dwell on the results related to the implementation of the second goal.

We turn to formula (18), which determines the relationship between the true and effective mass of the galaxy. It follows that more distance the effective mass of the galaxy grows. This effect does occur but is interpreted as the inclusion of new regions filled with dark matter. This conclusion is made based on measuring the mass/luminosity ratio, which, with the transition from the center of the galactic disk to its periphery, increases from 2 [12] to 20 [13]. Applying formula (18) for r ~ 30 *Kpc* and r ~ 1 *Kpc* (galactic core size), we obtain an increase in the effective mass of the galaxy by 30 times, in contrast to 10 according to of radio astronomical observations.

The results presented are not related to any modification of the Newtonian theory of gravitation. Strictly speaking, the use of quantum theory also entails certain refinements being introduced into the classical Newtonian picture of gravitational phenomena, but this cannot be called a modification, especially for one specific reason.

Other effects, using dark matter for their explanation, can also find an interpretation in the framework of this model. For example, the effect of gravitational lensing can be described by analogy with the known effect of the deflection of a light beam in a gravitational field based on the formulas given in the Appendix.

The purpose of the present work wasn't to explain all galactic phenomena without invoking the concept of dark matter, which has already gained widespread dissemination. However, the latter circumstance cannot be an argument in its favor.

7. Conclusion

A relativistic model of a planar spiral galaxy containing only baryonic matter is proposed. It is based on the assumption that the space-time metric on the periphery of the galaxy is approximately Schwarzschild-like, distorted taking into account the constancy of the speed of rotation of the periphery of the galaxy with a distance to its center. It is shown that this effect can be associated with the quantum properties of baryonic matter without resorting to a modification of Newtonian dynamics. The obtained metric is embedded in the metric of the Euclidean three-dimensional manifold and the properties of the resulting induced metric are investigated. The possibility of constructing multiphase wave solutions for the induced metric associated with complex dynamic behavior responsible for the formation of galactic structures is shown. It is shown that this model leads to an apparent increase in the mass of the galaxy in comparison with its real mass. This increase is due to relativistic effects and does not require the introduction of dark matter.

Appendix

Relativistic equations of motion have the form [5]

$$\frac{d^2 x^i}{ds^2} + \Gamma^i_{kl} \frac{dx^l}{ds} \frac{dx^k}{ds} = 0$$

$$x^0 = ct, x^1 = r, x^2 = \theta, x^3 = \varphi$$

For the flat motion ($\theta = \pi/2$), which corresponds to the interval (5), calculations taking into account non-zero Christoffel symbols Γ^i_{kl} lead to equations of the form (s is the proper time, $i,k,l = 0,1,3$)

$$\frac{dx^0}{ds} = \frac{1}{g_{00}}$$

$$\frac{dx^1}{ds} = \frac{1}{\sqrt{-g_{11}}} \left(A + \frac{1}{g_{00}} + \frac{\rho^2}{g_{33}} \right)^{1/2}$$

$$\frac{dx^3}{ds} = \frac{\rho}{g_{33}}$$

where $\rho = L/mc$ - orbital perihelion, L – angular momentum of a star with mass m, and the constant A is related to the energy of the star, and in the case, $V_\varphi = 0$ is equal to $-(mc^2/E)^2$ (E – energy). The components of the metric tensor are equal

$$g_{00} = 1 - \frac{r_g}{r} - \frac{V_\varphi^2}{c^2}$$

$$g_{11} = -\left(1 - \frac{r_g}{r}\right)^{-1}$$

$$g_{33} = -\frac{r^2}{1 - \left(1 - \frac{r_g}{r}\right)^{-1} \left(\frac{V_\varphi}{c}\right)^2}$$

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