NUCLEAR BETA DECAY AND WEAK INTERACTIONS

D. Tadić

Institute "Ruđer Bošković", Zagreb, Yugoslavia

1. INTRODUCTION

The purpose of this note is to outline some problems of weak interactions of elementary particles, trying to show how they are connected with nuclear β-decay. It does not pretend to be complete or exhaustive, it merely tries to establish relations and a general framework for various topics included in the Symposium.

The understanding of many problems of weak interactions has been obtained by the mutual guidance and influence of two fields seemingly wide apart, weak decays of elementary particles and nuclear β decay. Nuclear β decay, being an extremely complicated process going on in an intricate interplay of weak, strong and electromagnetic interactions of a many-body system, is certainly more difficult to interpret theoretically than the decays of a single elementary particle. Nevertheless, some vital information, such as the nucleon weak coupling constant, is for the moment accessible only through nuclear decay measurements.

The general plan of this lecture is to outline the present status of the universal V-A theory of weak interactions, stressing especially the details relevant to nuclear problems. A brief outline of the amazing historical development leading to this simple and generally successful theory can be found in Marshak's paper[1].

2. UNIVERSAL V-A THEORY

This theory attempts and succeeds, at least qualitatively and to a growing extent quantitatively, in describing all weak processes by a single simple interaction Hamiltonian[2]

$$H_I = \frac{G}{\sqrt{2}} (J_\lambda + \ell_\lambda) (J_\lambda + \ell_\lambda)^\dagger , \qquad (1)$$

which is the product of currents containing hadrons and leptons, J_λ and ℓ_λ , respectively.

As it is well known, experiments support the V-A transformation properties of the current, as predicted on the basis of various symmetry arguments[2-5]. The leptonic current is symmetric in muon and electron contributions, each paired with a different neutrino ν_e and ν_μ

$$\ell_\lambda = \bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_{\nu_e} + \bar{\psi}_\mu \gamma_\lambda (1 + \gamma_5) \psi_{\nu_\mu} \quad . \qquad (2)$$

The structure of the leptonic current has been rather well established by studying the muon decay where the measurement of the asymmetry parameter[6] strongly supports the V-A form. The muon-electron universality (i.e. symmetry) can be ascertained by comparison of the $\to e\nu_e$ and $\to \mu\nu_\mu$ decay rates of pions, kaons, Λ, and Σ particles. The agreement between experimental measurements and theoretical predictions is always within the experimental error[1].[*] To complete the picture, one would like to have some direct evidence of the self-current interaction as

$$e^+ + e^- \to \nu_e + \bar{\nu}_e ,$$

which is claimed to play an important and necessary role as a star cooling mechanism[7,8]. The existence of the two neutrinos is confirmed by observing that the neutrino produced in the decay

$$\pi^+ \to \mu^+ + \nu_\mu$$

can induce

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*) As an example[1,6]

		Theory	Experiment
$\dfrac{\Gamma(\pi^+ \to e^+ + \nu_e)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)}$	=	$1.23\cdot10^{-4}$	$(1.24\pm0.03)\cdot10^{-4}$
$\dfrac{\Gamma(K^+ \to e^+ + \nu_e)}{\Gamma(K^+ \to \mu^+ + \nu_\mu}$	=	$2.47\cdot10^{-5}$	$(3 \pm 1.89)\cdot10^{-5}$

$$\nu_\mu + Z \to (Z+1) + \mu^- ,$$

where Z symbolizes some nuclei, but never[6)]

$$\nu_\mu + Z \not\to (Z+1) + e^- .$$

In order to account for semileptonic decays of nucleons and strange particles, a special structure of the hadronic current being a member of an SU(3) octet of currents was proposed[9) *)]

$$J_\lambda(x) = \cos\theta\,(J^1_\lambda + iJ^2_\lambda) + \sin\theta\,(J^4_\lambda + iJ^5_\lambda) \quad . \qquad (3)$$

The angle $\theta \approx 0.26$ was introduced in order to explain why strangeness changing semileptonic decay rates are systematically lower by a factor of about 20 compared to those of strangeness conserving decays. There is a vast amount of facts concerning semileptonic transitions that can be explained by this assumption, notably the $|\Delta I| = 1/2$ and the $\Delta S = \Delta Q$ rules, which seem to be almost valid[1,7,8,9)**)]. Furthermore, the interaction Hamiltonian (1) together with recent ideas concerning the current

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*) Indices 1,2,4,5 denote transformation properties under SU(3). 1+i2 corresponds to $\tau_1 + i\tau_2$, i.e. the first combination transforms as π^+ particle. The second combination transforms as K^+. Under the Lorentz group they behave as V-A, typical terms being $\bar{p}\,\gamma_\mu(1+\gamma_5)n$ or $\bar{p}\,\gamma_\mu\,(1+\gamma_5)\Lambda$ etc.

**) The recent measurements of $K^0 \to \pi + \ell + \nu$ decays are in agreement, although small violation is possible[10)].

algebra[11] and the soft pion approximation[12] has been successfully used to establish relations among various nonleptonic weak decays[1,7].

The investigation of the space-time transformation properties of hadronic currents meets with many experimental and theoretical difficulties. The measurements of elementary particle decays and of a free neutron decay are difficult. On the other hand it is also rather hard to extract some information from nuclear β decay. Various angular correlation measurements seem to support strongly the V-A form for J_λ (refs. 1,6). It should also be noted that the ratios quoted in the footnote on p. 27 follow only if the contributing hadronic current transforms as V-A. A recent analysis[13] of $\Sigma^{\pm} \to$ $\to \Lambda^0 + e^{\pm} + \nu$ also seems to be in agreement with V-A.

According to the interaction (1) the coupling constants in semileptonic decays of hadrons should be the same as the one measured in the muon decay. The only difference can come from the scale factors introduced by strong and electromagnetic renormalization effects and corrections. As all of these effects have to be accounted for, the task is not a very simple one. Strong interaction effects can be estimated by the conserved vector current (CVC) and by the partially conserved axial vector current (PCAC) hypotheses. Additional details connected with these hypotheses will be discussed in sections 4 and 5, respectively. Definite information about the magnitude of the weak coupling constant can be obtained at present only from nuclear β decay, while some strong interaction renormalization effects are estimated by combining these results from nuclear β decay with the free neutron decay data[*)]. To complete the material already presented in some contributions to this Symposium, in section 6 we shall briefly mention some additional corrections needed in analysing nuclear β decay. The results concerning the coupling constants seem to support the total universality and symmetry within a few percent, as formulated in (1).

Experimental evidence seems to point against the existence of neutral leptonic currents, which would lead to the processes such as $K_2^0 \to \mu^+ + \mu^-$, $K_2^0 \to e^+ + e^-$ etc.[15)]. There is no direct experimental evidence about neutral hadronic currents. A possible experimental test will be discussed in section 7.

To complete the picture, one should mention the possibility that there is a vector boson W which mediates weak interactions through a coupling of the form

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*) For additional information and a review of the present situation see contributions to this Symposium by G. Alaga, A. Nielsen and G. Källen.

$$H_I = (J_\lambda + \ell_\lambda) W_\lambda + h.c. \tag{4}$$

Experimental attempts to discover such a particle have only ascertained that the boson mass m_W is larger than 2 BeV (ref. 6). In this connection it is interesting to mention a recent calculation of the nonleptonic K decay in the intermediate boson model using Cabbibo's currents which reproduced the observed K_1^0 decay rate with $m_W \approx 8$ BeV (ref. 14). The calculation is, however, based on certain ad hoc assumptions and approximations.

3. SOME REMARKS ON ELECTROMAGNETIC AND STRONG INTERACTION CORRECTIONS IN NUCLEAR β DECAY

Had a weak interaction Hamiltonian been independently established from some other sources, then the study of nuclear β decay could serve as a tool for investigating nucleons and nuclear structure and various interaction effects. Not being so, we are in fact forced to do it in both ways. We estimate non-weak effects on the basis of a reasonable hypothesis and then justify this hypothesis by the fact that in combination with the weak interaction Hamiltonian it does reasonably well describe the general experimental situation.

Part of the influence of electromagnetic interactions can be extracted and described by an effective external field produced by charged atomic nuclei*), while the rest has to be calculated using field theory or Feynman diagram techniques.[16)]

We would like to stress that the first part of the effect represents the collective contribution of all charged particles in nuclei, while the second part is calculated as a γ quantum exchange process between an electron and a particular nucleon which has undergone β decay, making the theory from the outset dependent on the usual hypothesis about the structure of nuclei. The first part of the effect is accounted for by the calculation of the electron Coulomb wave function for the point or finite size[18)], or even for deformed nuclei[19)]. Electron wave functions get further modified by the screening effect of atomic electrons[20)]. As they actually appear in the transition probability as operators inside the nuclear matrix elements**), an additional approximation is required to make the calculations tractable. Some recent discussions of these problems show the importance of a detailed

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*) In the papers by B. Stech and L. Schülke[17)] one can find a systematic and pedagogical report of the approximations involved from the point of view of quantum field theory. Their results, although in the momentum space, are essentially equivalent with the standard ones.

**) As an example see formula (15), section 4.

knowledge and/or assumptions concerning nuclear structure[21,22,23], mostly in the cases where the leading nuclear matrix element is hindered.

Naturally, the estimation of strong interaction corrections is even more dependent on the usual way of visualizing the nucleus as a conglomerate of particles, retaining almost all of their individual properties. Keeping this in mind, one can say that strong internucleon interactions manifest themselves in, broadly speaking, three different ways:

a) An individual nucleon interacts with virtual fields of all other elementary particles, (as an example visualize the nucleon surrounded by its mesonic "cloud") which leads to coupling constant renormalization effects and induced terms in the effective beta decay interaction. Additional detailes concerning these effects are going to be presented in the next sections.

b) A nucleon interacts with other nucleons, therefore one has to use nuclear wave functions in calculating hadronic matrix elements[24].

c) There could be special mesonic exchange effects, which generate two-body operators. In principle, they could generate many-body operators. One can argue, from analogy with nuclear forces, that the two-body contribution is a dominant one. For example, in the nonrelativistic limit one could have the following contribution to the axial vector term[25]:

$$H_\beta(\text{exchange}) = G_A \sum_{i<j} [g_I(r_{ij}) \cdot (\vec{\sigma}_i \times \vec{\sigma}_j) + g_{II}(r_{ij}) \cdot$$

$$\cdot \frac{\vec{r}_{ij}(\vec{r}_{ij} \cdot (\vec{\sigma}_i \times \vec{\sigma}_j)}{r_{ij}^2} \cdot (\vec{\tau}_i \times \vec{\tau}_j) + \text{other terms} \qquad (5)$$

and the summation goes over the nucleons. $g_{I,II}$ are some arbitrary unknown functions, estimable either on the basis of some crude field theoretical calculation or treated as free parameters. Several attempts to estimate a possible contribution of (5) give an order of magnitude of about 10%, but the results are not yet conclusive[25]. Somewhat related is also the treatment of the whole nuclei as an elementary particle, which could automatically include exchange corrections[26].

4. CONSERVED VECTOR CURRENT

The conserved vector current hypothesis[27] has met with considerable success. Its predictions about the absence of strong renormalization effects and the weak magnetism term[28] are connected to the identification of CVC with the isospin current I_+ or I_-, just as the electromagnetic current can be identified with I_3. The corresponding form factors can then be related

$$\langle p | J_\lambda^V | n \rangle = \bar{u}_p(q')(G_V(k^2)\gamma_\lambda + F_V(k^2)\sigma_{\lambda\nu} k_\nu) u_n(q)$$

$$k = (q - q') \qquad (6)$$

$$G_V(q^2) = G F_E(q^2) \qquad (7)$$

$$F_V(q^2) = G \frac{\mu_p - \mu_n}{2m_N} F_M(q^2) \qquad (8)$$

These formulas are strongly supported by the experimental verification of weak magnetism effects, due to (8), in the study of the decay of the triplet ^{12}B, $^{12}C^*$ and ^{12}N (ref. 29). There is also good agreement between the theoretical and experimental values for the lifetime of the decay $\pi^+ \to \pi^0 + e^+ + \nu_e$ (ref. 6) which, when electromagnetic corrections are neglected[16], has a contribution from J_λ^V only. The relevant number for the ratio $\frac{\Gamma(\pi^+ \to \pi^0 + e^+ + \nu_e)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)}$ is $(1.00 \pm 0.02) \cdot 10^{-8}$ as predicted and $(1.14 \pm 0.09) \cdot 10^{-8}$ as found experimentally. In view of this one can also understand the near equality of weak coupling constants from μ meson decay, G_μ, and from the decay of ^{14}O, G_β. Leaving a detailed comparison to other contributors, we would like to note that Cabibbo's theory[9] predicts[*]

[*] Electromagnetic corrections are neglected.

$$G_\beta = G_\mu \cos\theta \,. \qquad (9)$$

The exact value of θ is still unknown*), which together with uncertainties in the estimation of electromagnetic corrections, and recently measured variations in ft values[29] leaves this problem open to further research.

From the point of vies of SU(3) theory the vector current, transforming as the F type coupling, is divergenceless only in the limit of exact symmetry. (When, for example, all baryon masses and all meson masses are equal.) Even without electromagnetic interactions, the intermediate strong symmetry breaking effects could cause modifications. There are arguments that the renormalization happens only to the second order in the symmetry breaking interactions[33]. As the isospin symmetry, which is a subgroup of SU(3), seems to be broken mainly by electromagnetic interactions, one does not need to worry about any such effects in nuclear β decay.

*) For example: Freeman et al.[29] have noticed that θ spreads as $0.17 \leq \theta \leq 0.21$ when various ft values are fitted, N. Brene et al.[30] obtain $\theta = 0.268$, while on the basis of K_{ℓ_3} data[31] $\theta = 0.22$. For a very critical report see ref. 32 where also the measurement of the branching ratio for the neutron two-body weak decay is suggested.

The CVC hypothesis has also been used to obtain the relation among nuclear matrix elements in nuclear β decay[34,35]. The continuing equation in the presence of an external electromagnetic fields A_μ reads[35,36)

$$\partial_\mu \; J^V_{\mu\pm} = \pm \, ie \, J^V_{\mu\pm} \; A_\mu \quad . \tag{10}$$

Here, J^V_μ is different from J^V_μ $(A_\mu = 0)$, because it contains the derivative terms. It does represent the actual current contributing to nuclear β decay. Using (10) and the standard formalism[17,26] the following relation for nuclear matrix elements is derived

$$(\Phi_f | k_\mu \, \delta^3(\vec{R}) \sum_k \, [\gamma_4 \gamma_\mu \tau^\pm]_k \; e^{-i\vec{k}\vec{r}_k} \, | \, \Phi_i) = 0 \; . \tag{11}$$

Φ_i and Φ_f correspond to the initial and final nuclei, respectively. $\vec{R}$ is the centre of mass coordinate, and the summation goes over all individual contributing nucleons. $k_\mu = [\vec{k}, \; i(W_0 - V_C)]$ is the Coulomb field modified momentum transfer. V_C, which takes care of the Coulomb field influence, can be averaged over the nucleus using the method of Ahrens and Feenberg[33,34,37], or it can be kept inside the nuclear matrix element in the cases where the radial variation is important[22,38]. Recently J. Fujita has proposed a possible test for the Ahrens-Feenberg approximation based on the investigation of suitable β and γ transitions[39].

A simplified form of relation (11), with the summation over the individual particles omitted

$$< k_0 \, e^{-i\vec{k}\vec{r}} > + < \vec{\alpha} \, \vec{k} \, e^{-i\vec{k}\vec{r}} > = 0 \qquad (12)$$

can be written after a straightforward angular momentum algebra as

$$(-i)^J < (W_0 - V_C) \, Y_J^{M_J} \, j_J(kr) > =$$

$$= - \sum_{\ell} C^{J\,0}_{1\,0\,\ell\,0} \left[\frac{2\ \ +1}{2J\ +1} \right]^{1/2} (-i)^{\ell+1} \, k < j_\ell(kr) \, V^M_{J\ell} >$$

$$V^M_{J\ell} = \sum_{\mu, m} C^{JM}_{1\mu\,\ell m} \, \alpha_1^\mu \, Y_\ell^m$$

$j_\ell(kr)$ is a spherical Bessel function.

For $\ell = 0$, $J = 1$ relation (13) goes into the well-known formula for the $1^- \to 0^+$ transition[34,35]. In the nonrelativistic limit, the substitution $\alpha_1^\mu \to - \frac{p_1^\mu}{M}$ is suggested by the Foldy-Wouthuysen transformation.

According to some opinions[40] the difference between CVC theories and non-CVC theories where the exchange mesonic effects are present, seem to be too small to make use of this formula for the experimental test of CVC. However, if CVC is ascertained otherwise, it certainly can be used to establish the connection among nuclear matrix elements, thus limiting the number of parameters in the analysis of the experimental data[22]. In this connection one can also think of an alternative formula, which is derived as follows. The effective fourth vector component interaction Lagrangian for β decay is of the form

$$L_{nt}(J_4^V) = \int d^3k < e^{-i\vec{k}\vec{r}} > \int d^3r' \, L_V(r') \, e^{i\vec{k}\vec{r}'} \qquad (14)$$

$$L_V(r') = \bar{\psi}_e(r') \, \gamma_4 \, \psi_\nu(r') \quad ,$$

where $\psi_e(r')$ and $\psi_\nu(r')$ are electron and neutrino wave functions, respectively.The former contain all influences of the Coulomb field. Using (12) to replace the nuclear matrix element and integrating over the momentum k, one obtains

$$(W-V_C) < L_V(r) > = - < i(\vec{\alpha}\,\vec{\nabla}) \, L_V(r) > . \qquad (15)$$

Thus in the whole interaction Lagrangian for vector transitions one would have one operator $\vec{\alpha}$ only instead of 1 and $\vec{\alpha}$. The actual lepton operators had been connected before any multipole expansion has been made (instead of connecting only some terms in the multipole expansion) which can probably be of advantage in some nuclear situations. The weak point of the above replacement formula is that V_C, generally some function of r, cannot be left inside the matrix element and some stimation of its average value has to be done.

5. PARTIALLY CONSERVED AXIAL VECTOR CURRENT

The PCAC hypothesis[41)] seems to have an impressive support from the Goldberger-Treiman relation[42)] and recent successes of Adler and Weisberger in calculating the axial vector coupling constant from the pion-nucleon scattering data[43)]. It also seems to work satisfactorily in connection with soft pion methods[12)], giving connections among elementary particle decay rates[1)]. The basic PCAC relation is

$$\partial_\mu J^i_{\mu 5} = C \;\; \phi^i_\pi \qquad\qquad C = \frac{m^2_\pi}{f_\pi(-m^2_\pi)} \;, \tag{16}$$

where $J_{\mu 5}$ is the hadronic axial vector current, i marks the isospin component, ϕ^i_π the pion field, and f_π is proportional to the amplitude for the decay $\pi \to \ell + \nu$. Combining (16) and equal time current-current commutation relations[11,12)] Adler and Weissberger[43)] obtained the relation

$$1 - \left(\frac{G_V}{G_A}\right)^2 = \left(\frac{2m_N}{g_{\pi N}}\right)^2 \frac{1}{2\pi} \int \frac{d\nu}{\nu} \left(\sigma^+(\nu) - \sigma^-(\nu)\right) , \tag{17}$$

where $g_{\pi N}$ is the pion nucleon coupling constant, and $\sigma^\pm$ is the total $\pi^\pm p$ cross section for a pion of zero mass. The most recent calculation of the r.h.s. (ref. 44) gave the result

$$\left| \frac{G_A}{G_V} \right| = 1.16 \quad ,$$

which could be compared with the most recent measurement described in Nielsen's contribution. Although the amount of renormalization is in the right direction, there is some discrepancy. Some corrections, rather hard to estimate, must be done when using real scattering cross sections in (17) instead of those with $m_\pi = 0$. It should be noted that the Adler–Weisberger relation determines only the absolute magnitude and not the sign of the G_A/G_V ratio.*) Further confirmation of PCAC seems to follow from the PCAC consistency condition requirement[43,46] which seems to be satisfied experimentally. Roughly speaking, it follows from the fact that because of (16) the pion nucleon scattering amplitude should be of the form

$$\left(\psi^{out}_{p'_N q'_\pi} , \phi_\pi \; \psi^{in}_{p_N} \right) \sim q_\mu \left(\psi^{out}_{p'_N q'_\pi} , J_{\mu 5} \, \psi^{in}_{p_N} \right) \qquad (18)$$

In the limit $q \to 0$ only the terms with singularity survive. The contribution from these pole terms can be located thus leading to the requirement

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*) Recent group theoretical arguments, based on the partial chiral symmetry introduced by Weisberg has led to estimate $\frac{g_A}{g_V} = - \frac{5}{3\sqrt{2}}$, see J. Schwinger, Phys. Rev. Lett. 18 (1967) 923.

$$\frac{g_{\pi N}^2}{m_N} = \frac{2}{\pi} \int \frac{d\nu}{\nu} \ \mathrm{Im}\, A^{(+)} (\nu, 0, 0) \ . \qquad (19)$$

$A^{(+)}$ is the imaginary part of the forward scattering amplitude, again for zero mass pions. This requirement seems to be satisfied within a 10% accuracy. Actually, the whole amplitude $T^{(+)} = A^{(+)} + \omega B^{(+)}$ should vanish as a consequence of PCAC, which is already well satisfied with its physical threshold value in the scattering length approximation[45,46]. So speaking extremely pessimistically, the fulfilment of the consistency condition could be accidental.

In the case of $\Delta S \neq 0$ transitions, caused by this part of the unitary octet current which transforms as $K^{\pm}$, one can also formulate a similar PCAC condition, the K meson playing the role of a pion. Although this has been used in applications (see for example ref. 14)), the consistency condition analogous to (19) fails in the scattering length approximation[45] and gives a large negative number. More elaborate calculations either disagree[46] or are inconclusive[45]. The Adler-Weisberger type of the integral has also been estimated several times[46], being important in the test for Cabibbo's theory[9], but the results are again inconclusive. The situation is much more involved due to the bigger mass of the kaon and the existence of the unphysical contributions to the integrals, all necessitating corrections and estimations whose validity is always doubtful[46].

A further important feature of the PCAC hypothesis is the prediction of the induced pseudoscalar term[42], giving in nuclear β decay or in capture the contribution with the effective coupling constant

$$g_P = \frac{2M\ g_A}{m_\pi^2 + m_\ell^2}\ m_\ell \qquad g_A = \frac{G_A}{G_V} , \tag{20}$$

m_ℓ being the mass of the electron or the muon, respectively. The numerical estimation shows that while the term is important in muon capture where $g_P \approx (7-8)\ g_A$, it should be completely negligible in β decay where $g_P \approx 0.04\ g_A$. The analysis of the $0^- \to 0^+$ β transitions and allowed transitions shows that the results are fairly insensitive to the much greater strength of the effective pseudoscalar coupling constant,[47,48] the effective limit being

$$\left| \frac{g_P}{g_A} \right| < 90 .$$

Recent measurements performed by the Heidelberg group[49] do not seem to alter significantly these limits. Some additional information can be inferred from the absolute measurement of the β-longitudinal polarization in the ^{32}P decay[49]. In the energy range 200-500 keV it is almost constant, while the existence of the large g_P would lead to an energy variation[50,51]. Until recently the results based on muon capture gave either large limits

$$5 < \frac{g_P}{g_A} < 28 \qquad \text{for example in ref. 52}$$

or even indicated that there must be a contribution from the induced tensor[53], the discussion of which will be given in the next section. In muon capture the theoretical analysis is much more involved, as one has to estimate somehow some rather complicated nuclear matrix elements in order to obtain information on coupling constants.

Some very recent estimations seem again to speak in favour of the general agreement between theory and experiment[54], without any need for the induced tensor contribution. For illustration, using Migdal´s theory to estimate nuclear matrix elements and adjusting the parameters of the residual interaction, M. Rho[54] could fit all data with

$$6 \lesssim \frac{g_P}{g_A} \lesssim 10$$

and without any G parity irregular terms.

All conclusions about g_P depend on the treatment of the pseudoscalar matrix element, for which the nonrelativistic approximation must be introduced, creating certain amgibuties[47,48]. It is worth asking whether anything about nuclear matrix elements could be learned from PCAC itself. When the Coulomb field is switched on, Adler[36] gave a general argument that (16) ought to be replaced by

$$(\partial_\mu - i\, e\, A_\mu)\, J'_{\mu 5} = C'\phi_\pi \quad , \qquad (21)$$

where C' can have only second order electromagnetic corrections. If these corrections behave in a similar way as radiative corrections to nuclear β decay[16] then it is very reasonable to suppose that $C' \approx C$ within a few percent. As the left hand side of (16), according to some models[41,55] contains derivatives, one actually has to make the replacement

$$J_{\mu 5}\,(\partial_\mu) \rightarrow J'_{\mu 5}\,(\partial_\mu - i\, e\, A_\mu) \quad .$$

It does not have any practical consequences because $J'_{\mu 5}$ is the actual operator appearing in nuclear problems.

If one were able to estimate $\langle f|\phi_\pi|i\rangle$ for complex nuclei, then the relations among the axial vector and maybe even pseudoscalar matrix elements would be established. This follows from the fact that on the basis of the general invariance arguments, $\langle f|\phi_\pi|i\rangle$ must contain a term proportional to the pseudoscalar matrix element $\langle f|P|i\rangle$. The actual value of this term is going to be inferred only from future pion nucleon scattering or pion capture experiments. The first, most crude approximation would be to locate the nucleons in the nuclei as completely independent. Then (21) can be approximated as

$$k_\mu < f \,|\, -J_{\mu 5} |\, i > = \frac{2m_N \, m_\pi^2 \, G_A}{m_\pi^2 + k^2} < f | P | i > \qquad (22)$$

$$k_\mu = (\vec{k}, \, i(W_o - V_C))$$

and converted into[55)]

$$< \beta \gamma_5 L_5(r) > = \frac{1}{2m_N} [< i\vec{\sigma}\nabla L_5(r) > + <(W_o - V_C) \, \gamma_5 L_5(r) >] \, . \qquad (23)$$

As (23) terms ought to be an expression which would hold for any Fermi gas of independent nucleons, irrespectively of PCAC, one is left with doubts about the validity of the approximation made in deriving (23).

In connection with PCAC one should also mention the works of Kim and Primakoff[26)], who treat nuclei as composite particles describing all processes by general form factors[*)]. The relations among the form factors are then established by the use of PCAC, An example is

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*) Actually the matrix elements of currents are described by form factors[17)] and Coulomb corrections are treated separately. Otherwise, the decay would be a general parity nonconserving two by two body process involving a much larger number of invariant amplitudes.

$$\left| \frac{G_A(N_i \to N_f)}{G_A\ (n - p)} \right| = \left| \frac{G_{\pi\ N_i N_f}}{G_{\pi\ np}} \right| , \qquad (24)$$

where the upper quantities refer to atomic nuclei and the lower ones to nucleons. On the left side one has the axial vector coupling constant and on the right side the pion coupling constant. $G_{\pi\ N_i N_f}$ had to be estimated in their calculations. In the same spirit they were able to derive the analogue of the Adler-Weisberger sum rule (17) involving integrals over the total cross sections for a pion nucleus reaction. The result was shown to depend on the contribution from mesonic exchange currents. Kim and Ram[56)] made an attempt to connect the current-current commutator[11)] with the total muon capture rate in He^3. Although the calculation met with considerable numerical success one should keep in mind that they assumed the equality of all form factors involved, the validity of the closure approximation, and finally they approximated the bilinear product of nuclear matrix elements by an equal time commutator.

6. INDUCED TENSOR

Historically it all started with the persistent claim of the Bloomington group[57)] that all theoretical spectrum shape factors ought to be multiplied by $1+b/W$ ($b \approx 0.2$ and W is the electron energy) in order to fit measurements. It is extremely hard to account for such a factor in the standard theory, because all imaginable induced terms can only produce factors of the type $1 \pm b/W \pm aW$ (ref. 50), the sign varying with the charge of the emitted lepton contrary to the experimental expectation. In my knowledge, Kuchowicz[58)] was the first to suggest to try the induced tensor term, which served to work in the righ direction. His proposal was done using the plane wave approximation for electrons. Fortunately for the theory, there are also many measurements not requiring the horrid $1 + b/W$ factor.

The interest in the induced tensor term has been revived recently by the already mentioned muon capture discrepancies and by the great experimental difference among ft values of two mirror transitions ^{12}B and ^{12}N (ref. 59)[*)]. According to a very detailed analysis[60)] even when all possible corrections and charge dependent effects are taken into account there is still a 3% discrepancy with experiment (There are, however, some more optimistic estimates[61)].) The difference in ft values can be accounted for[59,60)] by the introduction of the in-

*) There is a claim[60)] that some results concerning the $\Sigma - \Lambda$ leptonic decay are easily accounted for by the introduction of wrong G parity[61)] currents.

duced tensor term of the same order of magnitude as the one needed in muon capture[53]. The Heidelberg group measurements of $0^- \to 0^+$ transitions[62] could also be reproduced by the induced tensor of the same order of magnitude, if nuclear structure effects are neglected[63]. Most recently it seems that muon capture data indicate against the existence of the induced tensor[54]. A detailed analysis[64] of some unique forbidden transitions[65] suggests that if there is any induced tensor at all, the sign of its form factors must be opposite to that used in refs. 60,63,53) thus making any induced tensor explanation even less probable.

7. NEUTRAL HADRONIC WEAK CURRENTS AND PARITY NON-CONSERVING NUCLEAR FORCES

Recent experiments seem to give some evidence that the effect of P nonconserving forces among nucleons is present[66], but its intensity if still somewhat doubtful. The interesting part of the weak interaction Hamiltonian density is

$$H_W = J_\mu^{\dagger Va} J_\mu^{Ab} + \text{c.c.} \qquad (25)$$

Here, V, A mean the vectors and the axial vector respectively, while a and b refer to the isospin and other quantum numbers necessary for identification. The study of the general invariance properties[67,68] shows that the exchange of the low-

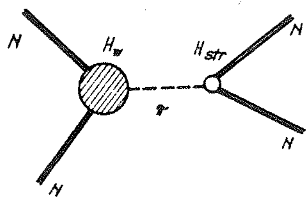

Fig. 1

est mass particles, π and ρ meson, contributes to the isospin exchange $\Delta\vec{I} = 1$ and $\Delta\vec{I}=0$ respectively. The π meson exchange can be diagramatically pictured as in Fig. 1. The first vertex can be estimated using SU(3) symmetry from the nonleptonic decays of hyperons[x)], while the strong vertex is proportional to the well-known pion nucleon coupling constant. The ρ meson exchange dominates the form factors of the vector current in (25), and it can be pictured by the diagram of Fig. 2. The existing estimate concludes that the main contribution comes from the weak magnetism term.

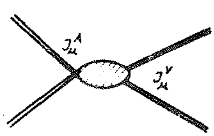

Fig. 2

It seems that both contributions, by π and ρ exchange could be of the same order

x) For example, $\Sigma \to N + \pi$.

of magnitude.

By studying suitable γ transitions, with or without isospin exchange involved, in principle one could discriminate between the two contributions[*)]. That, combined with the SU(3) symmetry assumption and Cabibbo's model, could allow us to conclude something about the neutral weak hadronic currents. The product of charged currents gives the contributions $\Delta\vec{I} = 0,2$ multiplied by cos θ (θ is Cabibbo's angle) and the contribution $\Delta\vec{I} = 1$ multiplied by sin θ (ref. 68). If there are no neutral currents, which do give the $\Delta\vec{I} = 1$ contribution multiplied by cos θ, the $\Delta\vec{I} = 1$ effects should be weaker than the $\Delta\vec{I} = 0$ ones[68)] by the factor $tg\, \theta \approx 15$. The $\Delta\vec{I} = 1$ rule for the nonleptonic decays of hadrons should be due to some dynamic effects, a conclusion which seems to have already been indirectly supported by soft pion calculations.

The existence of currents with wrong G parity or those producing the induced tensor term in nuclear β decay would introduce the $\Delta\vec{I} = 1$ contribution times cos θ even with charged currents only[70)]. The estimation of the magnitude of this contribution is not yet available.

H. Barclay-Adams has remarked recently[71)] that the initial or final state interaction among nucleons scattered by H_W as estimated in the nuclear matter model can seriously reduce the magnitude of

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*) ^{10}B was suggested[68)].

the effective force. Unfortunately, he has treated only the ρ meson exchange contribution, so the important information as to how this would affect the comparison between $\Delta\vec{I} = 0$ and $\Delta\vec{I} = 1$ is missing[*].

8. CP NONINVARIANCE

Although several years have passed since the discovery of CP noninvariance, the situation is still not clear[72]. The $K_L^o \to 2\pi$ experiment[73] has been supported by the discovery of CP violating effects in the semileptonic decays of K^o mesons[74]. Apart from the decays of neutral kaons which constitute the system particularly sensitive to the CP violation, the definite positive evidence has been missing in other decays. Small contributions of the terms such as $\vec{J}_i \cdot (\vec{p}_i \times \vec{p}_f)$ have been searched for in the β decay of the neutron[75], ^{19}Ne (ref. 76) and charged kaon[77]. ($\vec{J}$ is the angular momentum of one of the spinor particles, and p_i are their linear momenta). A general conclusion, when final state electromagnetic interactions are taken into account, is that the effect could be consistent with

[*] The ρ and π exchange contributions have been calculated[71] for nonmesonic hypernuclear decays, where the responsible interaction is very similar. The respective ratio is changed there by about 2 times. The change of the same magnitude would not affect the suggested test.

the CP conservation. In the case of nuclear decay, the final state electromagnetic interaction would give the term of the same form, its intensity proportional to the weak magnetism term and of too small a magnitude to be detected by the present experimental accuracy. In the case of non-leptonic decay $\Lambda^0 \to p + \pi^-$ the presence of the term $\vec{p}_p \cdot (\vec{J}_\Lambda \times \vec{J}_p)$ is expected to come both from the final state pn interaction and from the CP violation. The observed effects[78] again could be in agreement with the CP conservation. Similarly the situation is not clear in connection with the existence of the electric dipole momentum (EDM) of the neutron, which can exist only if both P and T are violated[79].

If the electromagnetic interaction[80] is responsible for the violation of T and C, then the expected EDM would be of the order of magnitude $\mu_E \approx e\, C_V\, m_N \approx 10^{-19} - 10^{-12}$ ecm (e the electron charge, C_V the weak coupling constant, m_N the neutron mass). If the weak interaction itself violates T, then $\mu_E \approx e\, C_V\, m_N \sin\theta_T$ $10^{-21} \sim 10^{-22}$ ecm, where $\theta_T = (180.2 \pm 5.5)^0$ from neutron and ^{19}N measurements. EDM can also be derived from other theories explaining the CP violation, the results varying from 10^{-21} to 10^{-22} e cm (ref. 81). The only two existing measurements give the same absolute magnitude with opposite signs:

$\mu_E = (-2 \pm 3)\cdot 10^{-22}$ e cm (ref. 82) and
$\mu_E = (+2.4 \pm 3.9)\cdot 10^{-22}$ (ref. 83)[x)] leaving the very question of existence open.

The existing experimental uncertainties concerning the CP violation are reflected in the great profusion of theoretical models, attributing it to weak interactions, strong interactions[84)], semistrong interactions[85)], electromagnetic interactions[80)], superweak interactions[86)] or other effects[87)]. The difficulties are increased by the fact that in most cases the experimental consequences of the CP violation cannot be predicted without involved, model-dependent, and not very reliable calculations.

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