Fermi National Accelerator Laboratory

FERMILAB-Conf-81/78-THY
November 1981

MODELS FOR HADRONS

CHRIS QUIGG

Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, Illinois 60510 USA

and

Laboratoire de Physique Théorique de l'Ecole Normale Supérieure+
24, rue Lhomond, 75231 Paris-Cédex 05, France

Lectures given at l'Ecole d'Eté de Physique Théorique, Les Houches
"Gauge Theories," August 3 - September 11, 1981

*Permanent address. Fermilab is operated by Universities Research Association, Inc., under contract with the United States Department of Energy.

+Laboratoire propre du Centre National de la Recherche Scientifique, associé à l'Ecole Normale Supérieure et à l'Université de Paris-Sud.

CONTENTS

1 Introduction :

In the course of the past fifteen years, high-energy physicists have developed a strong collective conviction that the smallest subunits of matter are the quarks and leptons. "Smallest" is of course to be interpreted as the smallest we have yet observed. Truly elementary particles must be indivisible and structureless. What is known so far about the quarks and leptons is that they are structureless down to a scale of a few times 10^{-16} cm. Nothing more complex than quarks and leptons qualifies as elementary. Nothing smaller is yet indicated by experiment. Thus, the possibility that quarks and leptons themselves have constituents is entertained largely by tradition, in hopes of bringing order to the burgeoning spectrum of apparently fundamental fermions.

We have also achieved a certain understanding of the weak and electromagnetic interactions of quarks and leptons. The Weinberg-Salam theory is calculable, incorporates all observational systematics, and agrees with experiment insofar as it has been tested. The theory as developed until now has some remaining arbitrariness embodied in the weak mixing angle θ_W, the mass of the Higgs boson, and fermion masses. It is also incompletely motivated, in that the left-handedness of the charged currents is not explained. Progress may come in the form of a deeper understanding of the mechanism of spontaneous symmetry breaking.

If, however, we are willing to regard the fermion spectrum as given, then the electroweak interactions of quarks and leptons may be considered as understood, at least at an engineering level. This means that apart from rare, lepton-number violating processes (and of course

gravity) the lepton sector is completely disposed of.

What about the strongly-interacting particles represented by the quarks ? We have a gauge theory, Quantum Chromodynamics, which purports to describe the strong interactions among quarks. It incorporates—by construction—all observational systematics, it has no experimental embarrassments, and it is calculable in perturbation theory under restrictive circumstances. This is promising ! But is it enough ?

If you know the elementary particles and their interactions, and you call yourself a physicist, you ought to be able to calculate the conscequences — or at least you should feel guilty if you can't ! The desire to share my guilt, which is amplified by the fact that ordinary laboratory experience concerns hadrons rather than quarks, will then be a motive force for these lectures. Our specific aspirations should include these :

- to compute the properties of hadrons, explain the absence of unseen species, and predict the existence new varieties of hadrons ;
- to explain why quarks and the quanta of the color forces, gluons, are not observed ;
- to derive the interactions among hadrons as a collective effect of the interactions among constituents.

These lectures will be concerned principally with the first task, understanding the hadron spectrum. The second and third will be discussed only in passing. My approach will be to review the basis of the quark hypothesis (Gell-Mann, 1964 ; Zweig, 1964 abc), and deduce the consequences of this picture. Because I do not know how to solve QCD in general, it will be necessary to proceed by iteration, using a progression of models motivated by simplicity, or QCD, or both.

The detailed plan or these lectures is to be found in the Table of Contents, but a short summary may be useful at this point. Following a brief review of basic concepts, the elementary quark model will be introduced as a classification tool. The quark model will then be extended to the consideration of static properties of hadrons. The ensuing description of magnetic moments, charge radii, and such is often called the naïve quark model. What is meant is that detailed dynamical assumptions play little or no rôle in the predictions of the model. Many consequences of the quark model are shared with symmetry schemes. I have not been fastidious in pointing out these overlaps, because I regard the quark description as the more fundamental. The relative success of the predictions argues that the constituent (quark) picture be taken seriously, and that dynamical explanations for its validity be sought. These considerations lead us to the idea of QCD, and to the construction of that theory. With QCD then serving as inspiration, we shall consider a variety of idealized descriptions of hadrons and of the force between quarks, including strings, bags, nonrelativistic potential models, and the 1/N expansion. As applications of these ideas we shall examine, in addition to the conventional hadrons, novel configurations such as glueballs and baryonium. A few words will follow on future prospects, and on the possibilities of unconfined quarks.

Within the conceptual framework to which this Les Houches session has been devoted, it is easy to identify three Great Questions - in addition to the overlying issue of whether the entire structure is defective. The first question concerns the spectroscopy of fundamental fermions : how many are there, and why do they have the properties they do? The second question has to do with the spectroscopy of gauge bosons or equivalently, with the identification of the gauge groups involved.

A corollary to these basic questions, which apply at the level of field theories of the quarks and leptons, is more in the nature of an applied science problem. This third Great Question, which may in some ways be the most difficult to answer, concerns the problem of hadron structure : to understand why hadrons have the form they do, and why hadrons interact as they do.

These lectures are intended to be relatively self-contained, and to begin at a rather elementary level. Nevertheless there are some prerequisites, including a general knowledge of particle physics as conferred by any of the standard textbooks, such as Frauenfelder and Henley (1974) , Frazer (1966), Gasiorowicz (1966), Perkins (1972), and Perl (1974). In addition to the specific references cited within, the reader will find much of interest in the following quark model reviews and monographs : Close (1979), Dalitz (1965, 1977), Feld (1969), Feynman (1972, 1974), Greenberg (1978), Hendry and Lichtenberg (1978), Kokkedee (1969), Lipkin (1973a), Rosner (1981a). It will also be useful to have at least a cultural appreciation of the logic of gauge theories, which may be gained from my St. Croix lectures (Quigg, 1981), or from the lectures at this school by Wess (1981).

2. Quarks for SU(2) :

In order to recall some basic concepts and to simplify arithmetic, let us open our exploration of constituent models by considering only the nonstrange hadrons. The generalization to SU(3) can be accomplished largely by transcription. That will be done in Section 3, where the use of SU(2) subgroups will be found convenient. Here we shall introduce terminology and notation in the somewhat simplified context of flavor SU(2), or isospin. After brief reviews of the idea and the calculus of isospin symmetry, we shall construct the nonstrange baryons out of the fundamental representation of SU(2). This will lead us to consider the spin × flavor group SU(4). We shall next make use of the baryon wavefunctions to investigate some elementary static properties. Techniques developed there will be applied repeatedly throughout the course of these lectures. The section concludes with a short discussion of the nonstrange mesons.

2. 1. The idea of Isospin :

The concept of isospin arose in nuclear physics during the 1930s. Heisenberg (1932) first suggested a relation between the neutron and proton because of the near equality of their masses (Particle Data Group, 1980) :

$$M(n) = 939.5731 \pm 0.0027 \quad MeV/c^2,$$

$$M(p) = 938.2796 \pm 0.0027 \quad MeV/c^2, \qquad (2.1)$$

which is to say $\Delta M / M \approx 1.4 \times 10^{-3}$. It was subsequently noticed by

Breit, Condon, and Present (1936), among others, that the neutron-proton and proton-proton forces in nuclei are exceedingly similar. This is made strikingly apparent by modern data on nuclear levels.

The binding energy of the ^3H ground state, a (pnn) composite, is 8.482 MeV, whereas that of the ^3He ground state, a (ppn) composite, is 7.718 MeV (Fiarman and Hanna, 1975). These are equal, up to the Coulomb repulsion in ^3He which we may estimate using the mean charge radius measured in electron scattering to be of order $\alpha/(1.88\ fm) \approx 0.77$ MeV.

The level structures of mirror nuclei further exhibit the charge-independence of nuclear forces. The energy levels of ^7Li (3p + 4n) and ^7Be (4p + 3n) are shown in Fig. 1, and those of ^{11}B (5p + 6n) and ^{11}C (6p + 5n) are compared in Fig. 2. In both cases the correspondence between energy levels of the mirror nuclei is precise.

Following these early observations, the use of isospin as a good quantum number of the nuclear interaction was discussed by Cassen and Condon (1936) and by Wigner (1937), who codified the idea of isospin symmetry as invariance under SU(2), the group of 2×2 unitary matrices with determinant =+1. Elaboration of the concept occupied more than a decade of interplay between theory and experiment. The evolution may be traced in any of the standard textbooks on nuclear physics, such as Blatt and Weisskopf (1952) or Bohr and Mottelson (1969).

We regard the proton and neutron as an isospin doublet of nucleons

$$|p\rangle = |I=1/2, I_3=1/2\rangle = \begin{pmatrix}1\\0\end{pmatrix},$$

$$|n\rangle = |I=1/2, I_3=-1/2\rangle = \begin{pmatrix}0\\1\end{pmatrix}. \qquad (2.2)$$

Evidence for isospin invariance in nuclei can also be found in the energy spectra, in the form of "isobaric analog states." Thus in the

A = 7 and 11 systems depicted in Figs. 1 and 2, we may note that the ^{7}He and ^{7}B ground states are identifiable as $I = {}^3/_2$ partners of the ^{7}Li (11.24 MeV) and ^{7}Be (11.01) levels, while the ^{11}Be and ^{11}N spectra have analog levels in ^{11}B and ^{11}C. However, it is more revealing to consider two-nucleon states systematically.

From the basis states (2.2) we may construct isospin triplet states

$$|1,1\rangle = |p_1\rangle|p_2\rangle,$$
$$|1,0\rangle = (|p_1\rangle|n_2\rangle + |n_1\rangle|p_2\rangle)/\sqrt{2},$$
$$|1,-1\rangle = |n_1\rangle|n_2\rangle, \tag{2.3}$$

and an isospin singlet state

$$|0,0\rangle = (|p_1\rangle|n_2\rangle - |n_1\rangle|p_2\rangle)/\sqrt{2}. \tag{2.4}$$

Among two-nucleon states, only the isoscalar deuteron is bound. Thus, to look for evidence of isobaric analog levels we must consider two nucleons outside a core which is hoped to be inert. The textbook example is the A = 14 system of ^{12}C plus two nucleons. Neglecting the core, we classify the isobars as

$$^{14}O = {}^{12}C + (pp), \quad I_3 = 1;$$
$$^{14}N = {}^{12}C + (pn), \quad I_3 = 0;$$
$$^{14}C = {}^{12}C + (nn), \quad I_3 = -1. \tag{2.5}$$

The energy levels are shown in Fig. 3 In addition to the I = 1 levels common to all three isobars, ^{14}N has a large number of additional levels, which may be identified as isoscalar. The isospin assignments in ^{14}N are confirmed in ^{14}N (α,α) ^{14}N* reactions, among others.

Data such as these, and measurements of nucleon-nucleon scattering provide abundant evidence that isospin is useful both as a classification symmetry, and as a dynamical symmetry of the nuclear interaction. The extension to elementary particle physics, motivated in the first instance by degeneracies in particle masses, is a central element in the description of hadrons. In combination with charge-conjugation invariance, isospin yields the useful discrete symmetry of G-parity. The relevant formalism is explained in the textbooks cited in section 1, and in the monograph by Sakurai (1964). Problems 1-3 provide some practice in the application of these elementary ideas.

2.2. The Isospin Calculus and the Baryon Wavefunctions :

We have just recalled how the similarity of the proton and neutron motivated the invention of the first flavor symmetry, isospin. The two nucleons are nearly degenerate in mass, experience the same nuclear forces, both have $J^P = {}^1/_2{}^+$, and differ only in electric charge. Isospin of course proved to be a "good" symmetry of the strong interaction among hadrons.

With the proliferation of "elementary" particles in the early 1960s, it was natural to try to extend the idea of isospin symmetry and seek familial relations among the then known pseudoscalar mesons, vector mesons, and the baryons of $J^P = {}^1/_2{}^+$ or ${}^3/_2{}^+$. Great effort went into finding the correct generalization, which was by no means obvious. Some of the steps toward SU(3) symmetry may be traced in the retrospective by Zweig (1980).

Before proceeding to flavor SU(3), let us review the calculus of isospin and introduce a notation that will match the conventional notation for larger groups. This will serve as preparation for the SU(N) calculations to follow. Accessible introductions to the group theory are given by Carruthers (1966), Close (1979), Hamermesh (1963), Lichtenberg (1978), and Lipkin (1966).

The lowest nontrivial isospin is $I = 1/2$. Two isospinors can be combined in two ways, leading to total isospin $I = 1$ (symmetric) or $I = 0$ (antisymmetric). It is convenient to label representations by the multiplicity of states they contain. Thus we write

Isospin	Dimension
$I = 0$	$(\mathbf{1})$
$I = 1/2$	$(\mathbf{2})$
$I = 1$	$(\mathbf{3})$
⋮	⋮
I	$(\mathbf{2I+1})$

With this notation we can reexpress the coupling of two isospinors as

$$(\mathbf{2}) \otimes (\mathbf{2}) = (\mathbf{1}) \oplus (\mathbf{3}), \tag{2.6}$$

which emphasizes the fact that the number of states which can be formed is the product of the numbers of states in the representations being combined. For a general representation $(\mathbf{n})$, it is easy to see that

$$(\mathbf{2}) \otimes (\mathbf{n}) = (\mathbf{n+1}) \oplus (\mathbf{n-1}). \tag{2.7}$$

We also note that

$$(\mathbf{2})^{k} \supset (\mathbf{k+1}) \oplus (\mathbf{k-1}) \oplus \ldots \oplus (\mathbf{2/1}). \tag{2.8}$$

Any isospin state $|I, I_3\rangle$ can be constructed out of products of $|1/2, \pm 1/2\rangle$. In other words, products of $(\mathbf{2})$ can lead to an arbitrary representation $(\mathbf{n})$. We may therefore imagine constructing the observed isospin multiplets out of fundamental objects which lie in a $(\mathbf{2})$, as shown in Fig. 4. The constituents are labeled u and d, for isospin up and down. This idea, like the alternative view of "nuclear democracy" (Chew and Frautschi, 1961; Chew 1965), in which all hadrons are regarded as composites of each other, finds its modern roots in the work of Fermi and Yang (1949). The lowest-lying nonstrange baryons are the nucleons (2.2) and the quartet of nucleon resonances

$$\begin{aligned} \Delta^{++} &= |3/2, 3/2\rangle, \\ \Delta^{+} &= |3/2, 1/2\rangle, \\ \Delta^{0} &= |3/2, -1/2\rangle, \\ \Delta^{-} &= |3/2, -3/2\rangle, \end{aligned} \tag{2.9}$$

depicted in Fig. 5.

To build the quartet we require (at least) the product

$$(\mathbf{2}) \otimes (\mathbf{2}) \otimes (\mathbf{2}) = (\mathbf{2}) \oplus (\mathbf{2}) \oplus (\mathbf{4}) \tag{2.10}$$

of three fundamental objects. In the interest of simplicity, we choose the minimal configuration of three constituents. Here we face a choice : should we invent new fundamental constituents, or should we use the nucleons as our elementary states (in terms of which $\Delta^{++} = pp\bar{n}$, for example) ? We reject the nucleon alternative on two grounds. First, nucleons are manifestly composite and have a finite geometrical size. Second, when generalized to include strange particles the nucleon scheme becomes the Sakata (1956) model, which generates the wrong multiplet structure. We are thus led to quarks.

The nucleon resonances may now be constructed as

$$\begin{aligned} \Delta^{++} &= uuu, \\ \Delta^{+} &= u\{ud\}, \\ \Delta^{\circ} &= d\{ud\}, \\ \Delta^{-} &= ddd, \end{aligned} \tag{2.11}$$

where the notation $\{a_1, a_2, \ldots a_n\}$ has been introduced for the totally symmetrized product of n objects, which in this case denotes the I =1 configuration of (ud + du)/ $\sqrt{2}$. If there are no further flavor distinctions among quarks, it is obvious that both up and down quarks have baryon number $B = {}^1/_3$ and that the electric charge assignments must be $e_u = {}^{2e}/_3$, $e_d = {}^{-e}/_3$ in agreement with the Gell-Mann (1953) - Nakano and Nishijima (1955) formula for displaced charge multiplets.

To construct explicit wave functions for the baryons, we first transcribe eqns. (2.3) and (2.4) to list the two-body states of definite isospin in the quark basis. These are

$$\begin{aligned} |1,1\rangle &= uu, \\ |1,0\rangle &= (ud+du)/\sqrt{2}, \\ |1,-1\rangle &= dd, \end{aligned} \tag{2.12}$$

and

$$|0,0\rangle = (ud-du)/\sqrt{2}. \tag{2.13}$$

These two-body states all have definite permutation symmetries. The isovector states are symmetric under interchange of the constituents, whereas the isoscalar state is antisymmetric.

States with isospin = ${}^3/_2$ can be obtained only by combining the third quark with an isovector pair, with the result

$$\begin{aligned} |{}^3/_2, {}^3/_2\rangle &= uu\underline{u}, \\ |{}^3/_2, {}^1/_2\rangle &= [uu\underline{d} + (ud+du)\underline{u}]/\sqrt{3}, \\ |{}^3/_2, -{}^1/_2\rangle &= [(ud+du)\underline{d} + dd\underline{u}]/\sqrt{3}, \\ |{}^3/_2, -{}^3/_2\rangle &= dd\underline{d}. \end{aligned} \tag{2.14}$$

The last quark added has been underlined, to emphasize the structure of the wavefunctions. For I = ${}^3/_2$, the wavefunctions are symmetric under the interchange of any pair of quarks, as expected.

There are two distinct paths to isospinor final states. First, the third quark can be combined with an isoscalar pair to give

$$\varphi^{p}_{M,A} \equiv |{}^1\!/_2, {}^1\!/_2\rangle_0 = (ud-du)\underline{u}/\sqrt{2},$$

$$\varphi^{n}_{M,A} \equiv |{}^1\!/_2, -{}^1\!/_2\rangle_0 = (ud-du)\underline{d}/\sqrt{2}, \tag{2.15}$$

which are antisymmetric under interchange of quarks 1 and 2. Alternatively the final quark may be added to an isovector pair ; this yields

$$\varphi^{p}_{M,S} \equiv |{}^1\!/_2, {}^1\!/_2\rangle_1 = -[(ud+du)\underline{u} - 2uu\underline{d}]/\sqrt{6},$$

$$\varphi^{n}_{M,S} \equiv |{}^1\!/_2, -{}^1\!/_2\rangle_1 = [(ud+du)\underline{d} - 2dd\underline{u}]/\sqrt{6}, \tag{2.16}$$

which are symmetric under interchange of particles 1 and 2.

The isospinor wavefunctions (2.15) and (2.16) are said to be of mixed symmetry because permutations involving quark 3 mix the states with definite symmetry properties under (12). For example, interchange of 1 and 3 yields

$$(13)\,\varphi^{p}_{M,A} = (\varphi^{p}_{M,A} - \sqrt{3}\,\varphi^{p}_{M,S})/2. \tag{2.17}$$

Further elaboration of the meaning of mixed symmetry for these states is to be found in problem 4. Notice that the construction just undertaken has led to an explicit realization of all the three-quark states anticipated in (2.10), namely a quartet and two doublets.

To give a complete description of the baryons it is necessary to specify the spin wavefunctions as well. The spin wavefunctions can be read off from the isospin wavefunctions (2.14) - (2.16), with the replacements

$$\text{flavor} \left.\begin{matrix} u \\ d \end{matrix}\right\} \longrightarrow \left\{\begin{matrix} \uparrow \\ \downarrow \end{matrix}\right. \quad \text{spin}. \tag{2.18}$$

The spin $-{}^3\!/_2$ wavefunctions are then

$$|s={}^3\!/_2,\ s_z={}^3\!/_2\rangle = \uparrow\uparrow\uparrow,$$

$$|s={}^3\!/_2,\ s_z={}^1\!/_2\rangle = [\uparrow\uparrow\downarrow + (\uparrow\downarrow+\downarrow\uparrow)\uparrow]/\sqrt{3},$$

$$|s={}^3\!/_2,\ s_z=-{}^1\!/_2\rangle = [(\uparrow\downarrow+\downarrow\uparrow)\downarrow + \downarrow\downarrow\uparrow]/\sqrt{3},$$

$$|s={}^3\!/_2,\ s_z=-{}^3\!/_2\rangle = \downarrow\downarrow\downarrow, \tag{2.19}$$

while for spin $-{}^1\!/_2$ the possibilities are

$$\chi^{+}_{M,A} = (\uparrow\downarrow-\downarrow\uparrow)\uparrow/\sqrt{2},$$

$$\chi^{-}_{M,A} = (\uparrow\downarrow-\downarrow\uparrow)\downarrow/\sqrt{2}, \tag{2.20}$$

and

$$\chi^{+}_{M,S} = -[(\uparrow\downarrow+\downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow]/\sqrt{6},$$

$$\chi^{-}_{M,S} = [(\uparrow\downarrow+\downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow]/\sqrt{6}. \tag{2.21}$$

Consequently we may write the wavefunctions for Δ^{++} in definite spin states as

$$|\Delta^{++}; {}^3\!/_2\rangle = u_\uparrow u_\uparrow u_\uparrow,$$

$$|\Delta^{++}; {}^1\!/_2\rangle = (u_\uparrow u_\uparrow u_\downarrow + u_\uparrow u_\downarrow u_\uparrow + u_\downarrow u_\uparrow u_\uparrow)/\sqrt{3}, \tag{2.22}$$

etc.

These wavefunctions, and indeed all those for the Δ-resonances, are symmetric in spin × flavor (isospin). Because Δ (1232) is the $J^P = \frac{3}{2}^+$ object with the lowest mass, it is reasonable to suppose that all the quarks are in relative s-waves. This assumption yields the correct parity for the Δ states at the price of fermion wavefunctions that apparently are in conflict with the generalized Pauli principle because the wavefunctions are symmetric in space × spin × flavor. This is a matter to which we shall have to return if we find that the model otherwise makes theoretical and experimental sense.

What has been accomplished so far ? We have found that the nucleons and spin-$^3/_2$ nucleon resonances can be generated from three-body configurations of spin-$^1/_2$ fundamental particles. We may further notice that all known nonstrange baryons have isospin = $^1/_2$ or $^3/_2$, and that the model in which baryons are constructed of three isospinors limits them to precisely these values.

Now let us construct the nucleon wavefunctions explicitly. The result of problem 5 will show that the nucleons and Δ resonances are plausibly members of the same flavor × spin supermultiplet, a 20 - dimensional representation of SU(4) (Young tableau □□□) which is decomposed under SU(2) ⊗ SU(2) as $(\underset{\sim}{2I+1}, \underset{\sim}{2s+1}) = (\underset{\sim}{4}, \underset{\sim}{4}) \oplus (\underset{\sim}{2}, \underset{\sim}{2})$. Thus the nucleon wavefunctions, like those for the Δ resonances, must be symmetric in flavor × spin. The fully symmetrized, normalized wavefunction for a proton with spin up is given in terms of the flavor and spin wavefunctions (2.15, 16) and (2.20, 21) by

$$|p\uparrow\rangle = (\varphi^p_{M,A}\,\chi^+_{M,A} + \varphi^p_{M,S}\,\chi^+_{M,S})/\sqrt{2}\,. \qquad (2.23)$$

Thus the proton and neutron wavefunctions have the explicit forms

$$\begin{aligned}|p\uparrow\rangle = (1/\sqrt{18})(&2u_\uparrow d_\downarrow u_\uparrow - u_\downarrow d_\uparrow u_\uparrow - u_\uparrow d_\uparrow u_\downarrow \\ &- d_\uparrow u_\downarrow u_\uparrow + 2d_\downarrow u_\uparrow u_\uparrow - d_\uparrow u_\uparrow u_\downarrow \\ &- u_\uparrow u_\downarrow d_\uparrow - u_\downarrow u_\uparrow d_\uparrow + 2u_\uparrow u_\uparrow d_\downarrow),\end{aligned} \qquad (2.24)$$

and

$$\begin{aligned}|n\uparrow\rangle = -(1/\sqrt{18})(&2d_\uparrow u_\downarrow d_\uparrow - d_\downarrow u_\uparrow d_\uparrow - d_\uparrow u_\uparrow d_\downarrow \\ &- u_\uparrow d_\downarrow d_\downarrow + 2u_\downarrow d_\uparrow d_\uparrow - u_\uparrow d_\uparrow d_\downarrow \\ &- d_\uparrow d_\downarrow u_\uparrow - d_\downarrow d_\uparrow u_\uparrow + 2d_\uparrow d_\uparrow u_\downarrow).\end{aligned} \qquad (2.25)$$

Several remarks about the spin structure of these wavefunctions are in order. Note first that in the proton wavefunction the pair of up quarks is always in a symmetric (I=1) flavor state. Therefore, because of the total symmetry of the wavefunction, the up quarks must always be in a symmetric spin state : $|s=1, s_z=1\rangle$ for the terms with coefficient 2, and $|s=1, s_z=0\rangle$ for the terms with coefficient (-)1. For pairs of quarks with total spin s, the expectation value of $\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2$ is easily evaluated as

$$\begin{aligned}\langle\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2\rangle &= \langle 4\underset{\sim}{s}_1\cdot\underset{\sim}{s}_2\rangle = 2\langle\underset{\sim}{s}^2 - \underset{\sim}{s}_1^2 - \underset{\sim}{s}_2^2\rangle \\ &= 2[s(s+1) - 2\times 3/4] \\ &= \begin{cases}1, & s=1;\\ -3, & s=0.\end{cases}\end{aligned} \qquad (2.26)$$

Thus, in the proton wavefunction (2.24) we have at once that

$$\langle \sigma_u \cdot \sigma_u \rangle = 1, \tag{2.27}$$

while in the neutron wavefunction (2.25),

$$\langle \sigma_d \cdot \sigma_d \rangle = 1. \tag{2.28}$$

For three quarks with total spin s, a similar trick applies :

$$\begin{aligned}\langle \sum_{i<j} \sigma_i \cdot \sigma_j \rangle &= 4 \langle \sum_{i<j} s_i \cdot s_j \rangle \\ &= 4\langle (s_1 \cdot s_2 + s_2 \cdot s_3 + s_1 \cdot s_3) \rangle \\ &= 2\langle (s^2 - s_1^2 - s_2^2 - s_3^2) \rangle \\ &= 2[s(s+1) - 3 \times 3/4] \\ &= \begin{cases} 3, & s = 3/2; \\ -3, & s = 1/2. \end{cases}\end{aligned} \tag{2.29}$$

In a proton, then,

$$\begin{aligned}\langle p\uparrow | \sum_{i<j} \sigma_i \cdot \sigma_j | p\uparrow \rangle &= 2\langle p\uparrow | \sigma_u \cdot \sigma_d | p\uparrow \rangle + \langle p\uparrow | \sigma_u \cdot \sigma_u | p\uparrow \rangle \\ &= -3\end{aligned} \tag{2.30}$$

implies, in combination with (2.27), that

$$\langle \sigma_u \cdot \sigma_d \rangle_p = -2. \tag{2.31}$$

Similar arithmetic shows that

$$\langle \sigma_u \cdot \sigma_d \rangle_n = -2, \tag{2.32}$$

as well. We shall find numerous applications for these and related results.

2.3. Baryons : Electromagnetic properties :

As a first application and test of the wavefunctions we have constructed, let us consider some elementary electromagnetic properties of the baryons. The total charge carried by a hadron is

$$Q(h) = \langle h | Q | h \rangle, \tag{2.33}$$

where the charge operator Q is the sum of the charge operators for the constituents. For baryons containing three quarks, this is simply

$$Q = Q_{(1)} + Q_{(2)} + Q_{(3)}.$$

Because of the total symmetry of the wavefunction, the total charge of a baryon can be expressed as

$$\begin{aligned}Q(B) = \langle B | Q | B \rangle &= \sum_{i=1}^{3} \langle B | Q_{(i)} | B \rangle \\ &= 3\langle B | Q_{(3)} | B \rangle,\end{aligned} \tag{2.34}$$

for example. One immediately verifies that $Q(p) = +1$, $Q(n) = 0$, etc.

Although this success is a trivial one, having been assured by construction, the distribution of charge within a hadron presents a sterner challenge. As we shall see in greater detail in § 3.4.5., a convenient parameter of the charge distribution is the mean-squared charge radius, defined as

$$\langle r_{EM}^2 \rangle = \int d^3r \; \rho(r)\, r^2, \tag{2.35}$$

where $\rho(r)$ is the charge density. In a nonrelativistic constituent picture, $\langle r_{EM}^2 \rangle$ is therefore given by

$$\langle r_{EM}^2 \rangle = \left\langle \sum_{i=1}^{3} e_i (\underset{\sim}{r}_i - \underset{\sim}{R})^2 \right\rangle , \tag{2.36}$$

where $\underset{\sim}{r}_i$ is the coordinate of the i-th quark and $\underset{\sim}{R}$ gives the position of the baryon center-of-mass which for equal-mass quarks is

$$\underset{\sim}{R} = (\underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3)/3 . \tag{2.37}$$

In the present approximation the baryon wavefunctions are completely symmetric, so eq.(2.36) is equivalent to

$$\langle r_{EM}^2 \rangle = \langle (\underset{\sim}{r}_3 - \underset{\sim}{R})^2 \rangle \sum_{i=1}^{3} e_i . \tag{2.38}$$

Because $\sum e_i = 0$ for the neutron, our model implies that

$$\langle r_{EM}^2 \rangle_n = 0. \tag{2.39}$$

or equivalently that the neutron is uniformly neutral. This is not in fact the case (Hofstadter, 1963), and an understanding of the neutron's charge distribution will have to await a less ingenuous model of the nucleon. (See § 5.3.).

Let us next analyze the implications of the quark model wavefunctions for electromagnetic mass differences. Within a multiplet one may identify three sorts of contributions to electromagnetic mass differences (Dolgov, et al., 1965 ; Thirring, 1966, Gerasimov, 1966) :

- a difference between the masses of the up- and down-quarks ;
- pairwise Coulomb interactions among quarks ;
- pairwise hyperfine or magnetic moment interactions among quarks.

A plausible expression for particle masses within a multiplet is then

$$M = M_0 + N_d(m_d - m_u) + \left\langle \frac{\alpha}{r} \right\rangle \sum_{i<j} e_i e_j - \frac{8\pi}{3} |\Psi_{ij}(0)|^2 \left\langle \sum_{i<j} \mu_i \mu_j \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \right\rangle \tag{2.40}$$

In this equation, N_d is the number of down-quarks in the hadron, e_i and μ_i are the electric charge and magnetic moment of the i-th quark, and $\Psi_{ij}(0)$ is the wavefunction at zero separation between the i-th and j-th quark. The symmetry of the wave function has been exploited in writing the Coulomb term. The Fermi (1930) hyperfine interaction will be derived for general values of the orbital angular momentum in Problem 16 .

A dynamical theory of the interactions among quarks would permit the calculation of quantities such as $\langle r^{-1} \rangle$ and $|\Psi(0)|^2$ from first principles. In the temporary absence of such a theory, it is necessary to parametrize the unknown dynamical quantities in eq. (2.40). The baryon masses then take the form

$$M = M_0 + N_d(m_d - m_u) + \sum_{i<j} e_i e_j \, \delta M_c + \left\langle \sum_{i<j} e_i e_j \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \right\rangle \delta M_m , \tag{2.41}$$

where it has been assumed provisionally that μ_i = constant × e_i. This would be the case for Dirac particles, if $m_u \approx m_d$. The quantities M_0, δM_C, and δM_m evidently may vary from one multiplet to another. Those for the $\Delta(N^*)$ multiplet will be denoted by an asterisk. Contributions to the EM mass differences are gathered in Table 1. In constructing Table 1, use has been made of the spin averages (2.26-28) and (2.31, 32). The resulting baryon masses are

$$M(p) = M_0 + (m_d - m_u) \qquad + \tfrac{4}{3}\delta M_m, \tag{2.42}$$

$$M(n) = M_0 + 2(m_d - m_u) - \tfrac{1}{3}\delta M_c + \delta M_m, \tag{2.43}$$

$$M(\Delta^{++}) = M_0^* \qquad + \tfrac{4}{3}\delta M_c^* + \tfrac{4}{3}\delta M_m^*, \tag{2.44}$$

$$M(\Delta^+) = M_0^* + (m_d - m_u) \tag{2.45}$$

$$M(\Delta^0) = M_0^* + 2(m_d - m_u) - \tfrac{1}{3}\delta M_c^* - \tfrac{1}{3}\delta M_m^*, \tag{2.46}$$

$$M(\Delta^-) = M_0^* + 3(m_d - m_u) + \tfrac{1}{3}\delta M_c^* + \tfrac{1}{3}\delta M_m^*. \tag{2.47}$$

In § 5.3 , where the strong hyperfine splitting is discussed, we shall attempt to relate $M_0^* - M_0$ to the magnetic moment term δM_m. For the moment we make the plausible approximations

$$\delta M_c = \delta M_c^* \tag{2.48}$$

and

$$\delta M_m = \delta M_m^*, \tag{2.49}$$

and note the following simple relations :

$$M(n) - M(p) = (m_d - m_u) - \tfrac{1}{3}\delta M_c - \tfrac{1}{3}\delta M_m$$
$$= (1.29343 \pm 0.00004)\ \mathrm{MeV}/c^2\ ; \tag{2.50}$$

$$M(\Delta^-) - M(\Delta^{++}) = 3(m_d - m_u) - \delta M_c - \delta M_m$$
$$= 3[M(n) - M(p)]\ ; \tag{2.51}$$

$$M(\Delta^0) - M(\Delta^+) = (m_d - m_u) - \tfrac{1}{3}\delta M_c - \tfrac{1}{3}\delta M_m$$
$$= M(n) - M(p). \tag{2.52}$$

Regrettably, these predictions have not been tested experimentally, in large part because of the breadth ($\Gamma \approx 115$ MeV) of the Δ. Upon making the generalization to flavor SU(3), however, we will immediately deduce similar mass differences for the strange particles, for which experimental comparisons are possible.

Of more immediate experimental interest are the magnetic moments of baryons. In terms of quark constituents, the magnetic moment of a hadron of spin s is defined to be

$$\mu_h = \sum_i \langle h; s_z = s | \mu_i \sigma_z^{(i)} | h; s_z = s \rangle . \tag{2.53}$$

If quarks are assumed to be Dirac particles, with

$$\mu_i = e_i \hbar / 2 m_i c, \tag{2.54}$$

then

$$\mu_d = -\tfrac{1}{2}\mu_u \equiv -\tfrac{1}{3}\mu. \tag{2.55}$$

Again using the symmetry of the baryon wavefunctions, we may rewrite (2.53) as

$$\mu_h = 3\mu \langle h | e_i \sigma_z^{(i)} | h \rangle \quad \text{not summed}, \tag{2.56}$$

which leads to

$$\mu_p = \mu, \tag{2.57}$$

and

$$\mu_n = -\frac{2}{3}\mu, \tag{2.58}$$

from which

$$\mu_n / \mu_p = -2/3. \tag{2.59}$$

Experimentally (Particle Data Group, 1980), the nucleon magnetic moments are Known to impressive accuracy*:

$$\mu_p = (2.7928456 \pm 0.000\,0011) \text{ n.m.}, \tag{2.60}$$

$$\mu_n = (-1.91304184 \pm 0.000\,000\,88) \text{ n.m.}, \tag{2.61}$$

whence

$$\mu_n/\mu_p = -0.68497945 \pm 0.000\,000\,58. \tag{2.62}$$

The agreement with the simple quark model prediction is quite satisfying, even if theory and experiment do differ by $\pi \times 10^4$ standard deviations !

[A general description of nucleon magnetic moment measurements is given by Ramsey (1956), especially chapters VI and VII. The neutron moment is reported by Greene, et al. (1979). Measurement of the fundamental properties of the neutron can be expected to advance greatly with the collection of ultracold neutrons, described by Golub et al. (1979).]

---

* The standard unit for baryon magnetic moments is the nuclear magneton (n.m.) $\equiv e\hbar / 2m_p c \approx 3.15 \times 10^{-12}$ eV/gauss.

If μ_u and μ_d are treated as independent parameters, the nucleon magnetic moments become

$$\mu_p = (4\mu_u - \mu_d)/3, \tag{2.63}$$

$$\mu_n = (4\mu_d - \mu_u)/3, \tag{2.64}$$

from which one may extract

$$\mu_u = 1.85166812 \text{ n.m.} \tag{2.65}$$

and

$$\mu_d = -0.97186433 \text{ n.m.} \tag{2.66}$$

The ratio of quark magnetic moments

$$\mu_d / \mu_u = -0.5248 \tag{2.67}$$

differs only slightly from the symmetry limit of eq. (2.55), but not in the direction expected if the down-quark is more massive than the up-quark , as suggested by the neutron-proton mass difference. Indeed, if the quarks have Dirac moments (2.54), then the effective quark masses may be determined from the quark moments (2.60, 61). The results are

$$m_u \approx 338 \text{ MeV}/c^2, \tag{2.68}$$

$$m_d \approx 322 \text{ MeV}/c^2. \tag{2.69}$$

What seems to me noteworthy is not the precise values, but the fact that the masses are so reasonable—approximately one-third of a proton mass.

It is also straightforward, as Problem 6 will demonstrate, to compute the magnetic moments of the Δ resonances. The lifetimes of these resonances are too short to permit magnetic moment measurements by the standard precession techniques. The Δ^{++} moment has, however, recently been determined indirectly in measurements of the bremsstrahlung reaction $\pi^+ p \to \pi^+ p\gamma$ by Nefkens, et al. (1978). Their determination,

$$\mu_{\Delta^{++}} = (5.7 \pm 1)\ \text{n.m.} \tag{2.70}$$

is in reasonable agreement with the quark model prediction

$$\mu_{\Delta^{++}} = 3\mu_u \approx 2\mu_p \approx 5.6\ \text{n.m.} \tag{2.71}$$

Before leaving the subject of magnetic moments, let us mention the transition magnetic moment that characterizes the M1 decay $\Delta^+ \to p\gamma$. It is straightforward to calculate that

$$\begin{aligned} \mu^* &\equiv \langle \Delta^+; 1/2 | \sum_{i=1}^{3} \mu_i \sigma_z^{(i)} | p; 1/2 \rangle \\ &= \frac{2\sqrt{2}}{3}(\mu_u - \mu_d) \approx \sqrt{2}\,\mu_u \approx \sqrt{\frac{8}{9}}\,\mu_p . \end{aligned} \tag{2.72}$$

This is in fair agreement with experiment (Dalitz and Sutherland, 1966 ; Gilman and Karliner, 1974), but for higher-lying resonances it is essential to incorporate recoil corrections. A vast literature exists on the subject of resonance photoproduction. This may be traced from chapter 7 of the book by Close (1971), or from the lecture notes by Rosner (1981a).

2. 4. Mesons :

The introduction of isospin quarks has been motivated by an analysis of the baryon spectrum. Let us now understand the implications of the model for mesons. The simplest meson configuration is a quark-antiquark pair $(q\bar{q})$. The quark basis can be written as $\binom{u}{d}$, which tranforms as a doublet under SU(2). In this basis the G-parity operator may be represented as

$$G = \mathbb{C}\, e^{i\pi\tau_2/2} = \mathbb{C}\, i\tau_2 = \mathbb{C}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2.73}$$

where $\mathbb{C}$ is the charge conjugation operator. The antiquark doublet

$$G\begin{pmatrix} u \\ d \end{pmatrix} = \mathbb{C}\begin{pmatrix} d \\ -u \end{pmatrix} = \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix} \tag{2.74}$$

also transforms as an isospin doublet. The minus sign conforms to the usual SU(2) phase convention (Carruthers, 1966, p.5 ; Close, 1979, p.26).

In the language of flavor × spin SU(4), the $(q\bar{q})$ mesons lie in the

$$\underset{\sim}{4} \otimes \underset{\sim}{4}^* = \underset{\sim}{1} \oplus \underset{\sim}{15} \tag{2.75}$$

representations. It is easy to verify that these SU(4) representations decompose under SU(2) ⊗ SU(2) as

$$\underset{\sim}{1} = (\underset{\sim}{1}, \underset{\sim}{1}) \tag{2.76}$$

$$\underset{\sim}{15} = (\underset{\sim}{3}, \underset{\sim}{1}) \oplus (\underset{\sim}{1}, \underset{\sim}{3}) \oplus (\underset{\sim}{3}, \underset{\sim}{3}) \tag{2.77}$$

in the $(2I + 1, 2s + 1)$ notation we have employed before. The elements of the $\underset{\sim}{15}$ correspond to the π, ω, and ρ respectively, and we may label the SU(4) singlet as "η" for the moment. With the results of problem 2 in hand, we readily conclude that for s-wave configurations the quantum numbers $I^G J^P$ of the $(q\bar{q})$ bound states justify these assignments.

Explicit wavefunctions for the mesons are formed by combining flavor and spin wavefunctions as before. The wavefunctions presented here are manifestly eigenstates of G-parity, but for many applications that is an unnecessary frill :

$$|``\eta"\rangle = \frac{(u\bar{u}+d\bar{d})+(\bar{u}u+\bar{d}d)}{2}\cdot\frac{(\uparrow\downarrow-\downarrow\uparrow)}{\sqrt{2}}, \tag{2.78}$$

$$|\pi^+\rangle = \left(\frac{u\bar{d}+\bar{d}u}{\sqrt{2}}\right)\cdot\left(\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}}\right)$$
$$= (u_\uparrow\bar{d}_\downarrow - u_\downarrow\bar{d}_\uparrow + \bar{d}_\uparrow u_\downarrow - \bar{d}_\downarrow u_\uparrow)/2, \tag{2.79}$$

$$|\pi^0\rangle = \left(\frac{(-u\bar{u}+d\bar{d})+(-\bar{u}u+\bar{d}d)}{2}\right)\frac{(\uparrow\downarrow-\downarrow\uparrow)}{\sqrt{2}}, \tag{2.80}$$

$$|\pi^-\rangle = -\left(\frac{d\bar{u}+\bar{u}d}{\sqrt{2}}\right)\left(\frac{\uparrow\downarrow-\downarrow\uparrow}{\sqrt{2}}\right), \tag{2.81}$$

$$|\omega\rangle = \left(\frac{(u\bar{u}+d\bar{d})-(\bar{u}u+\bar{d}d)}{2}\right)\times \text{spin}, \tag{2.82}$$

$$|\rho^+\rangle = \left(\frac{u\bar{d}-\bar{d}u}{\sqrt{2}}\right)\times \text{spin}, \tag{2.83}$$

$$|\rho^0\rangle = \left(\frac{(-u\bar{u}+d\bar{d})-(-\bar{u}u+\bar{d}d)}{2}\right)\times \text{spin}, \tag{2.84}$$

$$|\rho^-\rangle = \left(\frac{-d\bar{u}+\bar{u}d}{\sqrt{2}}\right)\times \text{spin}. \tag{2.85}$$

Ths neutral states are also seen to be eigenstates of charge conjugation, as required.

The $(q\bar{q})$construction has satisfactorily reproduced the spectrum of what may be called the ground state nonstrange mesons. As for the

baryons, it is interesting to explore whether the quark model provides more than a classification symmetry. Again we begin by considering electromagnetic mass differences.

In analogy with eqns.(2.40, 41) we write the meson masses within a multiplet as

$$M = M_0 + N_d(m_d - m_u) + \langle e_q e_{\bar{q}}\rangle \delta M_C + \langle e_q e_{\bar{q}}\, \underset{\sim}{\sigma}_q\cdot\underset{\sim}{\sigma}_{\bar{q}}\rangle \delta M_m \tag{2.86}$$

Contributions to the meson masses are summarized in Table 2. The resulting meson masses are

$$M(``\eta") = M_0 + (m_d - m_u) - \tfrac{5}{18}\delta M_C + \tfrac{5}{6}\delta M_m = M(\pi^0), \tag{2.87}$$

$$M(\pi^+) = M_0 + (m_d - m_u) + \tfrac{2}{9}\delta M_C - \tfrac{2}{3}\delta M_m = M(\pi^-), \tag{2.88}$$

$$M(\omega) = M_0' + (m_d - m_u) - \tfrac{5}{18}\delta M_C' - \tfrac{5}{18}\delta M_m' = M(\rho^0), \tag{2.89}$$

$$M(\rho^+) = M_0' + (m_d - m_u) + \tfrac{2}{9}\delta M_C' + \tfrac{2}{9}\delta M_m' = M(\rho^-). \tag{2.90}$$

The equalities of the $\pi^{\pm}$ masses and $\rho^{\pm}$ masses are ensured by the construction of particle-antiparticle wavefunctions. The equalities of the ρ^o and ω^o masses (which is desirable), and of the π^o and "η" masses (which is not) follow from the common quark compositions of these particles. We may hope, with some justification, that it is incorrect to identify "η" with the physical $\eta(549)$. Only one nonstrange meson electromagnetic mass difference is precisely known (Particle Data Group, 1980) ; it is recorded here for future reference :

$$M(\pi^+) - M(\pi^o) = (\delta M_c - 3\delta M_m)/2$$
$$= (4.6043 \pm 0.0037)\ \mathrm{MeV}/c^2. \tag{2.91}$$

It is worth noting that, as indicated in Table 2, the electromagnetic Hamiltonian mediates transitions between ρ^o and ω^o. It was observed long ago by Glashow (1961) that because ρ and ω are nearly degenerate the $\rho\omega$ mixing could give rise to substantial interference effects in the $\pi^+\pi^-$ mass spectrum. Such effects were observed in quasi-two body final states (G. Goldhaber, 1970) and were shown to provide useful information on the ρ and ω production mechanisms (A.S. Goldhaber, et al., 1969). Neglecting the ρ-ω mass difference, one may write the branching ratio for the isospin-violating decay $\omega \to \pi^+\pi^-$ as

$$\frac{\Gamma(\omega\to\pi^+\pi^-)}{\Gamma(\omega\to \mathrm{all})} \approx \frac{4\,|\langle\rho^o|M|\omega\rangle|^2}{\Gamma_\rho\,\Gamma_\omega}. \tag{2.92}$$

Approximating the matrix element for mixing as

$$\langle\rho^o|M|\omega\rangle \approx (M(\pi^+) - M(\pi^o))/3, \tag{2.93}$$

which entails neglecting the hyperfine interaction and $m_d - m_u$ and suppressing the distinction between vector meson and pseudoscalar

meson wavefunctions, we estimate

$$\langle\rho^o|M|\omega\rangle \approx 1.5\ \mathrm{MeV}. \tag{2.94}$$

This implies a branching ratio on the order of a percent, in rough agreement with the current experimental average. We shall see below how to improve upon these gross approximations. It is of some interest that the decay $\omega \to \pi^+\pi^-$ has now been observed directly, without benefit of interference with $\rho^o \to \pi^+\pi^-$, in the final state $\psi \to \pi^+\pi^-$ + anything (Gidal, et al., 1981), in a data sample corresponding to about 1.3 million produced ψs. In much of this sample, G-parity conservation ensures the incoherence of semifinal states containing ρ^o and ω. This is but a single illustration of how the enormous data samples becoming available in electron-positron annihilations may contribute to the study of light-meson spectroscopy.

The static magnetic moments of the mesons are not of particular interest, because they are zero or are unmeasurable or both. Nevertheless, let us verify that the quark model predictions do no violence to common sense. The meson magnetic moments may be evaluated as

$$\mu_h = \langle h; s_z = s|(\mu_q\sigma_z^{(q)} + \mu_{\bar q}\sigma_z^{(\bar q)})|h; s_z = s\rangle. \tag{2.95}$$

The magnetic moments of the spinless mesons and of the neutral mesons vanish. The charged ρ-meson moments are

$$\mu_{\rho^+} = (\mu_u - \mu_d) = -\mu_{\rho^-}. \tag{2.96}$$

If quark magnetic moments have the same value in mesons as in baryons, we expect

$$\mu_{\rho^+} \approx \mu_p. \tag{2.97}$$

The possibility of relating observables for mesons and for baryons distinguishes the explicit quark model from the related symmetry schemes.

Of considerably more experimental interest are the transition magnetic moments which mediate radiative decays of vector mesons. For transitions between s-wave states, the magnetic dipole transition rate is

$$\Gamma(i\to f+\gamma)=\frac{\omega^3}{3\pi}\left|\langle f|\mu_q\sigma^{(q)}+\mu_{\bar q}\sigma^{(\bar q)}|i\rangle\right|^2, \tag{2.98}$$

where ω is the energy of the emitted photon. The decay rate is independent of the polarization of the initial state, so we may choose the polarization to make the calculation as short as possible. For a pseudoscalar final state it is apt to choose $s_z = 0$ in the initial state, so that only σ_z contributes.

The matrix elements of interest are

$$\begin{aligned}\langle\pi|M1|\rho\rangle &= \mu_u+\mu_d\\ &= 0.88\ \text{n.m.},\end{aligned} \tag{2.99}$$

and

$$\begin{aligned}\langle\pi^0|M1|\omega\rangle &= \mu_u-\mu_d\\ &= 2.82\ \text{n.m.},\end{aligned} \tag{2.100}$$

where the numerical values derive from (2.65, 66), determined by the nucleon magnetic moments. Neglecting the $\rho\omega$ mass difference, we expect

$$r=\frac{\Gamma(\omega\to\pi^0\gamma)}{\Gamma(\rho\to\pi\gamma)}=\frac{(\mu_u-\mu_d)^2}{(\mu_u+\mu_d)^2}=10.3, \tag{2.101}$$

or approximately 9:1 in the SU(3) limit. Current experimental values,

$$\Gamma(\omega\to\pi^0\gamma)=\begin{cases}(889\pm50)\ \text{keV}, & \text{Particle Data Group (1980)}\\ (789\pm92)\ \text{keV}, & \text{Ohshima (1980)}\end{cases} \tag{2.102}$$

and

$$\Gamma(\rho\to\pi\gamma)=(67\pm7)\ \text{keV},\ \text{Berg, et al. (1980ab)}, \tag{2.103}$$

imply a ratio

$$r=\begin{cases}13.3\pm2.1\\ 11.7\pm2.6\end{cases} \tag{2.104}$$

that is consistent with the theoretical expectation. A more critical discussion, with attention to the absolute rates, will be given in §3.4.4.

With respect to experimental technique, note that whereas $\Gamma(\omega\to\pi^0\gamma)\ /\ \Gamma(\omega\to\text{all})\approx 8\%$, the ρ branching ratio for radiative decay is approximatively 4×10^{-4}. It is therefore relatively straightforward to measure the rate for radiative decay of ω by measuring the total width and branching ratio, although the measurement is still uncertain at the 10% level. This procedure would be unthinkable for the ρ. What must be done instead (Jensen, 1980, 1981) is to measure the cross section for the Primakoff (1951) effect-the excitation of the pion in the Coulomb field of a heavy nucleus (Halprin et al., 1966).

3. SU(3) and Light-Quark Spectroscopy :

We now broaden our interest to include all of the "light" mesons and baryons by incorporating strange particles. The observed multiplet structure is that of the low-dimensionality representations of the flavor group SU(3) : the 3 x 3 unitary matrices with determinant = + 1 (Gell-Mann, 1961, 1962; Ne'eman, 1961 ; Gell-Mann and Ne'eman, 1964). Thorough discussions of SU(3) and its applications are to be found in the books by Carruthers (1966), Gasiorowicz (1966), Gourdin (1967), Lichtenberg (1978), and Lipkin (1966). Although the pure symmetry aspects of SU(3) and its SU(2) subgroups will be discussed to a limited extent in this section, the main emphasis will be upon the SU(3) quark model as conceived by Gell-Mann (1964) and Zweig (1964 abc, 1980). The applications will principally be those of the preceding section, with two important additions : a first look at formulas for strong-interaction mass differences, and a discussion of meson electromagnetic form factors.

3 . 1. The SU(3) Quark Model :

Up, down, and strange quarks are assigned to the fundamental $[\underset{\sim}{3}]$ representation as shown in Fig. 6. The properties of the quarks are summarized in Table 3. It is instructive to decompose the triplet under $SU(2)_{isospin} \otimes$ strangeness as

$$[\underset{\sim}{3}] = (\underset{\sim}{2})_0 \oplus (\underset{\sim}{1})_{-1} , \qquad (3.1)$$

where square brackets denote SU(3) representations, parentheses denote SU(2) representations, and the subscripts label the strangeness S.

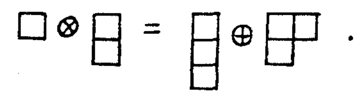
Using the isospin calculus reviewed in §2, we may now build up mesons and baryons as $(q\bar{q})$ and (qqq) composites, respectively. Because the wavefunction factors into flavor $\times$ spin, the spin analysis of §2 can be transplanted intact.

3.. 1.1. Mesons.

As before, mesons are composed of a quark and antiquark, so they lie in the

$$[\underset{\sim}{3}] \otimes [\underset{\sim}{3}^*] = [\underset{\sim}{1}] \oplus [\underset{\sim}{8}] \qquad (3.2)$$

dimensional representations of SU(3). In the notation of Young tableaux, the arithmetic is simply



(3.3)

Expanding (3.2) by inserting (3.1) and its conjugate we find

$$[\underset{\sim}{3}] \otimes [\underset{\sim}{3}^*] = \{(\underset{\sim}{2})_0 \oplus (\underset{\sim}{1})_{-1}\} \otimes \{(\underset{\sim}{2})_0 \oplus (\underset{\sim}{1})_1\}$$

$$= (\underset{\sim}{1})_0 \oplus (\underset{\sim}{3})_0 \oplus (\underset{\sim}{2})_1 \oplus (\underset{\sim}{2})_{-1} \oplus (\underset{\sim}{1})_0 . \qquad (3.4)$$

The first two terms are precisely the nonstrange mesons discussed in §2. In the vector meson nonet they correspond to ω and ρ^+, ρ^0, ρ^-. The doublet with strangeness S = +1 corresponds to the $K^{*+}(u\bar{s})$ and K^{*0} $(d\bar{s})$. Their charge conjugates lie in the S=-1 doublet. The remaining singlet arises in the product of $(\underset{\sim}{1})_1 \otimes (\underset{\sim}{1})_{-1}$. It is to be identified with the $\varphi(s\bar{s})$. The weight diagram for the vector nonet is presented in Fig. 7.

It is often useful to represent the flavor content of the mesons in matrix form. For the vector states, the flavor matrix is

$$\mathcal{V} = \begin{array}{c} \\ \bar{u} \\ \bar{d} \\ \bar{s} \end{array} \begin{array}{c} \begin{array}{ccc} u & d & s \end{array} \\ \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^- & K^{*-} \\ \rho^+ & \frac{\omega-\rho^0}{\sqrt{2}} & \bar{K}^{*0} \\ K^{*+} & K^{*0} & \varphi \end{pmatrix} \end{array} . \tag{3.5}$$

The SU(3) single state, in which all flavors receive equal weights, is then

$$\underset{\sim}{[1]} = (1/\sqrt{3})\,\mathrm{tr}\,\mathcal{V} = \frac{\varphi+\sqrt{2}\,\omega}{\sqrt{3}} \equiv V_1 . \tag{3.6}$$

The orthogonal isoscalar combination, which corresponds to the SU(3) generator

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{3.7}$$

is

$$V_8 = \frac{\omega - \sqrt{2}\,\varphi}{\sqrt{3}} . \tag{3.8}$$

The physical states ω and φ thus are mixtures of the SU(3) singlet and octet states. The idea of singlet-octet mixing and the flavor assignments of ω and φ will be reviewed in § 3..3.2.

Ths decomposition of SU(3) into $SU(2)_{\text{isospin}} \oplus$ (strangeness or hypercharge) was convenient but not obligatory. For other purposes it may be preferable to single out other additive quantum numbers, such as eleetric charge, and other SU(2) subgroups. The remaining

possibilities are illustrated in Fig. 8. The virtues of the SU(2) subgroups have been emphasized by Levinson, et al. (1962) and by Lipkin (1966).

3 . 1. 2. Baryons :

The baryons are three-quark states, which lie in the

$$\underset{\sim}{[3]} \otimes \underset{\sim}{[3]} \otimes \underset{\sim}{[3]} = \underset{\sim}{[1]} \oplus \underset{\sim}{[8]} \oplus \underset{\sim}{[8]} \oplus \underset{\sim}{[10]} \tag{3.9}$$

representations of SU(3). Again in terms of Young tableau, the product (3.9) is computed as

$$\square \otimes \square = \begin{array}{c}\square\\[-4pt]\square\end{array} \oplus \square\!\square \tag{3.10}$$

i.e. $\underset{\sim}{[3]} \otimes \underset{\sim}{[3]} = \underset{\sim}{[3^*]} \oplus \underset{\sim}{[6]}$, and thus that

$$\square \otimes \square \otimes \square = \begin{array}{c}\square\\[-4pt]\square\\[-4pt]\square\end{array} \oplus \begin{array}{l}\square\!\square\\[-4pt]\square\end{array} \oplus \begin{array}{l}\square\!\square\\[-4pt]\square\end{array} \oplus \square\!\square\!\square . \tag{3.11}$$

The SU(3) representations that occur in the product have the following decompositions with respect to isospin and strangeness :

$$\underset{\sim}{[10]} = (\underset{\sim}{4})_0 \oplus (\underset{\sim}{3})_{-1} \oplus (\underset{\sim}{2})_{-2} \oplus (\underset{\sim}{1})_{-3} , \tag{3.12}$$

$$\underset{\sim}{[8]} = (\underset{\sim}{2})_0 \oplus (\underset{\sim}{3})_{-1} \oplus (\underset{\sim}{1})_{-1} \oplus (\underset{\sim}{2})_{-2} , \tag{3.13}$$

$$\underset{\sim}{[1]} = (\underset{\sim}{1})_{-1} . \tag{3.14}$$

For the s-wave ground state, symmetric spin × flavor wavefunctions can be constructed for a spin-$^3/_2$ decimet and for one spin-$^1/_2$ octet. These are to be identified with the familiar $J^P = {^3/_2}^+$ and ${^1/_2}^+$ baryons, as indicated in Figs. 9 and 10.

3.1.3. Explicit Wavefunctions :

For explicit construction of the hadron wavefunctions, it is efficient to make use of the SU(2) subgroups of SU(3) : I-spin, U-spin, and V-spin. The action of the flavor-changing SU(2) operators upon the quarks and antiquarks is summarized in Fig. 11. The minus signs which appear in the antiquark figure reflect the fact that the proper SU(2) doublets of antiquarks are

$$\begin{pmatrix}\bar{d}\\ -\bar{u}\end{pmatrix}_I, \quad \begin{pmatrix}\bar{s}\\ -\bar{d}\end{pmatrix}_U, \quad \begin{pmatrix}\bar{s}\\ -\bar{u}\end{pmatrix}_V, \tag{3.15}$$

so that, for example,

$$I_+|\bar{u}\rangle = -|\bar{d}\rangle. \tag{3.16}$$

As an illustration of how wavefunctions may be constructed, let us build up the vector meson flavor wavefunctions already summarized in the matrix (3.5). Apart from the detailed identification of the isoscalars with physical particles, the same procedure applies for the pseudoscalars mesons as well. We begin with any convenient known wavefunction, for example

$$|\rho^+\rangle = u\bar{d}, \tag{3.17}$$

and compute as follows :

$$I_-|\rho^+\rangle = d\bar{d} - u\bar{u} = \sqrt{2}\,|\rho^0\rangle; \tag{3.18}$$

$$I_-|\rho^0\rangle = \sqrt{2}\,d\bar{u} = \sqrt{2}\,|\rho^-\rangle; \tag{3.19}$$

$$U_+|\rho^+\rangle = -u\bar{s} = -|K^{*+}\rangle; \tag{3.20}$$

$$I_-|K^{*+}\rangle = d\bar{s} = |K^{*0}\rangle. \tag{3.21}$$

The action of

$$U_-|K^{*0}\rangle = s\bar{s} - d\bar{d} = \sqrt{2}\,|U=1, U_3=0\rangle \tag{3.22}$$

yields a mixture of ρ^0 and V_8 (compare eq.(3.8)) which may be recognized as

$$|U=1, U_3=0\rangle = -\frac{|\rho^0\rangle + \sqrt{3}\,|V_8\rangle}{2}. \tag{3.23}$$

The octet is completed by the operations

$$V_-|\rho^+\rangle = s\bar{d} = |\bar{K}^{*0}\rangle, \tag{3.24}$$

$$-I_-|\bar{K}^{*0}\rangle = s\bar{u} = |K^{*-}\rangle. \tag{3.25}$$

An analogous procedure can be devised for any SU(3) multiplet. The slightly more onerous task of constructing wavefunctions for the baryon octet is posed as Problem 7.

With meson wavefunctions now in hand, we may return to the simple description of masses begun in §2. Contributions to the masses of mesons containing strange quarks are listed in Table 4, which may be viewed as an extension of Table 2. There N_s denotes the number of strange quarks in a hadron, and it has been assumed that $\mu_s = \mu_d$, the SU(3)-symmetric relation. Symmetry breaking is easily incorporated. In analogy with (2.82)-(2.85) we may write (compare the early work by Zel'dovich and Sakharov, 1966; Sakharov, 1980)

$$M(K^+) = M_0 + (m_s - m_u) + \tfrac{2}{9}\,\delta M_C - \tfrac{2}{3}\,\delta M_m, \tag{3.26}$$

$$M(K^0) = M_0 + (m_d - m_u) + (m_s - m_u) - \tfrac{1}{9}\,\delta M_C + \tfrac{1}{3}\,\delta M_m, \tag{3.27}$$

for the pseudoscalars and

$$M(K^{*+}) = M_0' + (m_s - m_u) + \frac{2}{9}\delta M_C + \frac{2}{9}\delta M_m, \quad (3.28)$$

$$M(K^{*0}) = M_0' + (m_d - m_u) + (m_s - m_u) - \frac{1}{9}\delta M_C - \frac{1}{9}\delta M_m, \quad (3.29)$$

$$M(\varphi) = M_0' + 2(m_s - m_u) - \frac{1}{9}\delta M_C - \frac{1}{9}\delta M_m \quad (3.30)$$

for the vectors.

Among the pseudoscalar mesons, let us form the electromagnetic mass difference

$$M(K^+) - M(K^0) = \frac{\delta M_C - 3\delta M_m}{3} - (m_d - m_u)$$

$$= -(4.01 \pm 0.13)\ \mathrm{MeV}/c^2. \quad (3.31)$$

The first term is expected to be positive. Indeed from (2.86) we may estimate it as

$$\frac{\delta M_C - 3\delta M_m}{3} = \frac{2}{3}\left(M(\pi^+) - M(\pi^0)\right) = 3.07\ \mathrm{MeV}/c^2, \quad (3.32)$$

which implies that the down quark is more massive than the up quark :

$$m_d - m_u \approx 7\ \mathrm{MeV}/c^2. \quad (3.33)$$

The sign of the quark mass difference is compatible with the observation that M(n) > M(p).

For the vector mesons, we may also write

$$M(\rho^+) - M(\rho^0) = \frac{\delta M_C + \delta M_m}{2}, \quad (3.34)$$

$$M(K^{*+}) = \frac{\delta M_C + \delta M_m}{3} - (m_d - m_u) \quad (3.35)$$

$$= -(6.7 \pm 1.3)\ \mathrm{MeV}/c^2,$$

and

$$\langle \rho | M | \omega \rangle = M(\rho^+) - M(\rho^0) - M(K^{*+}) + M(K^{*0})$$

$$= (m_d - m_u) + \frac{\delta M_C + \delta M_m}{6}. \quad (3.36)$$

Notice that the $\rho^\pm - \rho^0$ mass difference, like the $\pi^\pm - \pi^0$ mass difference has an explicit electromagnetic origin. In contrast, the $K^{*+} - K^{*0}$ mass difference and the off-diagonal $\rho\omega$ coupling are sensitive to the u-d quark mass difference, which may be identified with the tadpole term of Coleman and Glashow (1964).

Although mass differences have received only a superficial treatment in these introductory sections, they are of more than passing interest. More attention should be devoted to the experimental determination of mass differences for the ρ, K^*, and other meson families. The new tool of quarkonium decay makes possible measurements which are free from the biases of earlier experiments. We shall take up the question of quark masses again in § 5.3 . Until then, a survey of traditional methods for calculating electromagnetic mass difference has been given by Zee (1972).

As a preview of our discussion of strong-interaction mass formulas, let us observe that eqns. (2.85) and (3.28-30) imply that

$$M(\varphi) - M(K^{*0}) = M(K^{*+}) - M(\rho^+) = m_s - m_d. \quad (3.37)$$

The tabulated masses, which do not distinguish between different charges, yield

$$128\ \mathrm{MeV}/c^2 \approx 116\ \mathrm{MeV}/c^2, \quad (3.38)$$

which is not bad for a begining.

3. 2. Some Applications of U-spin :

The utility of the SU(2) subgroups of SU(3) for constructing wavefunctions has been demonstrated in § . 3 . 1.3. and in Problem 7. The SU(2) subgroups also yield, in straightforward fashion, extremely useful dynamical predictions. Because all the particles of a specified charge within an SU(3) multiplet lie in a single U-spin family, U-spin symmetry has many significant conscequences for electromagnetic interactions.

The electromagnetic current is of the form

$$j^\mu = e\left[\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\left(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s\right)\right] . \tag{3.39}$$

The up quark is a U-spin singlet. So too is the combination $\bar{d}d + \bar{s}s$. Thus the photon transforms as a U-spin singlet. Consequently the electromagnetic contribution to a hadron mass must be the same for all mambers of a U-spin multiplet. This is obviously satisfied by the quark-model predictions given above for mesons.

For baryons, let us write the massses of the $J^P = {}^1/_2{}^+$ octet as

$$M(p) = M_N + \delta m(p), \tag{3.40}$$

$$M(n) = M_N + \delta m(n), \tag{3.41}$$

etc., where M_N has a nonelectromagnetic origin and δm is purely electromagnetic in character. U-spin symmetry immediately implies that

$$\delta m(p) = \delta m(\Sigma^+), \tag{3.42}$$

$$\delta m(n) = \delta m(\Xi^0), \tag{3.43}$$

and

$$\delta m(\Sigma^-) = \delta m(\Xi^-). \tag{3.44}$$

Therefore, the hadron mass differences

$$M(p) - M(n) = \delta m(p) - \delta m(n) = -1.29 \text{ MeV}/c^2, \tag{3.45}$$

$$M(\Sigma^+) - M(\Sigma^-) = \delta m(p) - \delta m(\Xi^-) = -(7.98 \pm 0.08)\text{ MeV}/c^2, \tag{3.46}$$

$$M(\Xi^-) - M(\Xi^0) = \delta m(\Xi^-) - \delta m(n) = (6.39 \pm 0.62)\text{ MeV}/c^2, \tag{3.47}$$

are simply related :

$$M(\Sigma^+) - M(\Sigma^-) + M(\Xi^-) - M(\Xi^0) = \delta m(p) - \delta m(n)$$

$$= M(p) - M(n). \tag{3.48}$$

This is the Coleman-Glashow (1961,1964) relation, which is satisfied within experimental errors, the left-hand side yielding $-(1.59 \pm 0.63)\text{MeV}/c^2$.

U-spin symmetry makes similarly strong and simple predictions for baryon magnetic moments (Coleman and Glashow, 1961 ; Okubo, 1962a). One has immediately that

$$\mu_p = \mu_{\Sigma^+} , \tag{3.49}$$

$$\mu_n = \mu_{\Sigma^0_u} = \mu_{\Xi^0} , \tag{3.50}$$

$$\mu_{\Xi^-} = \mu_{\Sigma^-} , \tag{3.51}$$

where the neutral member of the U-spin triplet has been denoted as

$$|\Sigma_u^\circ\rangle = \frac{|\Sigma^\circ\rangle + \sqrt{3}\,|\Lambda^\circ\rangle}{2} . \tag{3.52}$$

The U-spin singlet in the octet is therefore the orthogonal combination

$$|\Lambda_u^\circ\rangle = \frac{\sqrt{3}\,|\Sigma^\circ\rangle - |\Lambda^\circ\rangle}{2} . \tag{3.53}$$

Using the inverse relations

$$|\Lambda^\circ\rangle = \frac{\sqrt{3}\,|\Sigma_u^\circ\rangle - |\Lambda_u^\circ\rangle}{2} \tag{3.54}$$

and

$$|\Sigma^\circ\rangle = \frac{|\Sigma_u^\circ\rangle + \sqrt{3}\,|\Lambda_u^\circ\rangle}{2} , \tag{3.55}$$

it is straightforward to compute that

$$\mu_\Lambda = \langle\Lambda|\mu|\Lambda\rangle = \tfrac{3}{4}\mu_{\Sigma_u^\circ} + \tfrac{1}{4}\mu_{\Lambda_u^\circ} , \tag{3.56}$$

$$\mu_{\Sigma^\circ} = \langle\Sigma^\circ|\mu|\Sigma^\circ\rangle = \tfrac{3}{4}\mu_{\Lambda_u^\circ} + \tfrac{1}{4}\mu_{\Sigma_u^\circ} , \tag{3.57}$$

$$\mu_{\Sigma\Lambda} = \langle\Sigma^\circ|\mu|\Lambda\rangle = \frac{\sqrt{3}}{4}\left(\mu_{\Sigma_u^\circ} - \mu_{\Lambda_u^\circ}\right)$$

$$= \frac{\sqrt{3}}{2}\left(\mu_\Lambda - \mu_{\Sigma^\circ}\right) . \tag{3.58}$$

Using the fact that the photon is a combination of isoscalar and isovector, we may define the otherwise unmeasurable Σ^0 moment as (Marshak, et al., 1957).

$$\mu_{\Sigma^\circ} = \tfrac{1}{2}\left(\mu_{\Sigma^+} + \mu_{\Sigma^-}\right). \tag{3.59}$$

The comparison with data (sources are listed in Table 6 below) is thus (in n.m.) :

$$2.79 \overset{?}{=} 2.33 \pm 0.13 , \tag{3.49'}$$

$$-1.91 \overset{?}{=} -1.15 \pm 0.07 \overset{?}{=} -1.25 \pm 0.01 , \tag{3.50'}$$

$$-0.75 \pm 0.06 \overset{?}{=} -1.41 \pm 0.25 , \tag{3.51'}$$

$$-1.82^{+0.25}_{-0.18} \overset{?}{=} -0.93 \pm 0.12 . \tag{3.58'}$$

While there is qualitative agreement, the U-spin argument fails to account for the measured moments in a quantitative fashion. The explicit quark model analysis presented in §3.4 will reveal that much, but not all, of the discrepancy is resolved by relaxing the assumption that $\mu_s = \mu_d$.

In contrast to the situation for mesons, enough information exists for baryon mass differences that it is possible to extract and test the reasonableness of all the parameters in the simple quark description of masses. Because particle-antiparticle restrictions do not apply within the batyon octet, there are eight independent masses, to be compared with five for the mesons. It is this added richness that makes for a more incisive confrontation of the model with experiment.

Still approximating $\mu_s = \mu_d$ in the hyperfine term we may write, in the notation of eqs. (2.41-43)

$$M(p) = M_0 + (m_d - m_u) + \tfrac{4}{3}\,\delta M_m , \tag{2.42}$$

$$M(n) = M_0 + 2(m_d - m_u) - \tfrac{1}{3}\,\delta M_C + \delta M_m , \tag{2.43}$$

$$M(\Sigma^+) = M_0 + (m_s - m_u) + \tfrac{4}{3}\,\delta M_m , \tag{3.60}$$

$$M(\Sigma^\circ) = M_0 + (m_d - m_u) + (m_s - m_u) - \tfrac{1}{3}\,\delta M_C\,, \tag{3.61}$$

$$M(\Sigma^-) = M_0 + 2(m_d - m_u) + (m_s - m_u) + \tfrac{1}{3}\,\delta M_C - \tfrac{1}{3}\,\delta M_m\,, \tag{3.62}$$

$$M(\Lambda) = M_0 + (m_d - m_u) + (m_s - m_u) - \tfrac{1}{3}\delta M_C + \tfrac{2}{3}\,\delta M_m\,, \tag{3.63}$$

$$M(\Xi^\circ) = M_0 + 2(m_s - m_u) - \tfrac{1}{3}\,\delta M_C + \delta M_m\,, \tag{3.64}$$

$$M(\Xi^-) = M_0 + (m_d - m_u) + 2(m_s - m_u) + \tfrac{1}{3}\,\delta M_C - \tfrac{1}{3}\,\delta M_m\,, \tag{3.65}$$

$$M(\Sigma_u^\circ) = M_0 + (m_d - m_u) + (m_s - m_u) - \tfrac{1}{3}\delta M_C + \delta M_m\,, \tag{3.66}$$

$$M(\Lambda_u^\circ) = M_0 + (m_d - m_u) + (m_s - m_u) - \tfrac{1}{3}\,\delta M_C - \tfrac{1}{3}\delta M_m\,, \tag{3.67}$$

$$\langle \Lambda^\circ | M | \Sigma^\circ \rangle = (1/\sqrt{3})\,\delta M_m\,. \tag{3.68}$$

It is evident at once that the U-spin relations (3.42-4) are respected ; the electromagnetic terms are equal in (2.42) and (3.60), in (2.43), (3.64), and (3.66), and in (3.62) and (3.65).

A modicum of arithmetic serves to isolate the parameters

$$m_d - m_u = M(n) - M(p) + \tfrac{1}{3}\left[M(\Sigma^+) + M(\Sigma^-) - 2M(\Sigma^\circ)\right]$$

$$= (1.89 \pm 0.04)\ \mathrm{MeV}/c^2\,, \tag{3.69}$$

$$\delta M_C = M(p) - M(n) + M(\Sigma^-) - M(\Sigma^\circ)$$

$$= (3.59 \pm 0.06)\ \mathrm{MeV}/c^2\,, \tag{3.70}$$

$$\delta M_m = M(n) - M(p) + M(\Sigma^+) - M(\Sigma^\circ)$$

$$= (-1.81 \pm 0.10)\ \mathrm{MeV}/c^2\,, \tag{3.71}$$

$$m_s - m_u = \tfrac{1}{2}\left[M(\Xi^\circ) - M(n)\right] + (m_d - m_u)$$

$$= (189.55 \pm 0.3)\ \mathrm{MeV}/c^2. \tag{3.72}$$

The quark mass differences (3.69) and (3.72) are of the same order of magnitude as the values (3.33) and (3.38) deduced from the mesons, but are not identical with those values. The defining equations (2.40, 41) show that the Coulomb term is to be identified as

$$\delta M_C \equiv \left\langle \frac{\alpha}{r} \right\rangle, \tag{3.73}$$

from which the effective radius is found to be

$$\left\langle \frac{1}{r} \right\rangle = 1/(0.4\ \mathrm{fm}), \tag{3.74}$$

a reasonable value. Similarly, the hyperfine term is

$$\delta M_m = -\,\frac{2\pi\alpha\,|\Psi(0)|^2}{3m^2}\,, \tag{3.75}$$

which should be, and is in (3.71), a negative energy. If m is chosen as a representative quark mass, say as one-third of

$$M_0 = 938.81\ \mathrm{MeV}/c^2, \tag{3.76}$$

bearing in mind our continued approximation of $\mu_s = \mu_d$, one may determine an effective value for

$$|\Psi(0)|^2 = 0.0116\ \mathrm{GeV}^3$$

$$= (0.87\ \mathrm{fm})^{-3}\,, \tag{3.77}$$

which is again a sensible value. Such an estimate for the square of the wavefunction at zero interquark separation is a necessary ingredient in estimates of the proton lifetime (reviewed by Langacker, 1981), and of the electric dipole moment of the neutron (Ellis, et al., 1981).

The parameters given in eqns. (3.69-72) and (3.76) lead to the baryon masses shown in Table 5. The mass differences within isospin multiplets are reproduced extremely well, and this success is nontrivial. Three parameters with reasonable values reproduce four mass differences. There is, however, a fly in the ointment. The $\Lambda - \Sigma^0$ mass difference, here attributed to the hyperfine interaction, is grossly underestimated. Indeed, if the hyperfine term had been estimated from the $\Lambda - \Sigma$ splitting, the result would have been

$$``\delta M_m" = \frac{3}{2}[M(\Lambda) - M(\Sigma^0)] \simeq -115 \text{ MeV}/c^2, \quad (3.78)$$

which is decidely not the scale of an electromagnetic mass shift ! We shall see in § 5.3 that the N-Δ level splitting and the $\Lambda - \Sigma$ mass difference have a common plausible interpretation as effects of the color hyperfine interaction.

As a closing remark on U-spin invariance, let us note the selection rules

$$Y_1^{*-} \not\rightarrow \Sigma^- \gamma \quad (3.79)$$

and

$$\Xi^{*-} \not\rightarrow \Xi^- \gamma \quad (3.80)$$

and the prediction that the matrix elements for the transitions $\Delta^+ \rightarrow p\gamma$ and $Y_1^{*+} \rightarrow \Sigma^+\gamma$ should be equal. The high-energy hyperon beams at Fermilab and at the CERN SPS bring tests of these predictions within reach. For further discussion see Kane (1972), Lipkin (1973b) and Quigg and Rosner (1976).

3.3. Strong-Interaction Mass Formulas :

3.3.1. The Gell-Mann - Okubo Mass Formula :

We have just seen that counting strange quarks does not give a perfect description of masses in the baryon octet. For the other multiplets of immediate interest, however, the situation is somewhat more satisfying, as our experience with vector mesons indicated in (3.37, 38). In the baryon decimet, for example, strange-quark counting leads to an equal-spacing rule

$$M(\Delta) - M(Y_1^*) = M(Y_1^*) - M(\Xi^*) = M(\Xi^*) - M(\Omega^-), \quad (3.81)$$

which agrees reasonably well with the measured charge-averaged intervals of 152, 149, and 139 MeV/c^2.

The hypothesis that masses can be determined by counting strange quarks can be intepreted to mean that the departures from exact SU(3) symmetry are governed by the hypercharge operator Y or λ_8. Because

$$Y = U_3 + \frac{1}{2} Q, \quad (3.82)$$

and because all members of a U-spin multiplet have the same charge, masses may be expressed as a sum of a U-spin scalar contribution, plus a contribution proportional to U_3, as

$$M = A + BU_3. \quad (3.83)$$

This constitutes an equal-spacing rule within U-spin multiplets. In the case of the vector meson octet, relation (3.83) implies that

$$M(K^{*0}) - M(U_8) = M(U_8) - M(\bar{K}^{*0}) \tag{3.84}$$

which, because of the equality of K^{*0} and $\bar{K}^{*0}$ masses, requires that

$$M(U_8) = M(K^{*0}). \tag{3.85}$$

Here I have introduced the notation

$$|U_8\rangle = |U=1, U_3=0\rangle \tag{3.86}$$

for the state defined in (3.23), which according to (3.8) is

$$|U_8\rangle = \frac{|\rho^0\rangle + |\omega^0\rangle - \sqrt{2}\,|\varphi\rangle}{2}. \tag{3.87}$$

Thus the implication of (3.85) is that

$$892\ MeV/c^2 = M(K^*) = \frac{M(\rho^0) + M(\omega) + 2M(\varphi)}{4} = 898\ MeV/c^2, \tag{3.88}$$

which is well satisfied.

Application of the equal-spacing rule (3.83) to the baryon octet yields the relation

$$M(n) - M(\Sigma^0_u) = M(\Sigma^0_u) - M(\Xi^0), \tag{3.89}$$

where $|\Sigma^0_u\rangle$ is defined in (3.52). When rewritten in the form

$$\frac{M(n) + M(\Xi^0)}{2} = \frac{M(\Sigma^0) + 3M(\Lambda^0)}{4}, \tag{3.90}$$

this connection is known as the Gell-Mann (1961, 1962) - Okubo (1962ab) mass formula. The measured masses give

$$1127.24\ MeV/c^2 \overset{?}{=} 1134.82\ MeV/c^2, \tag{3.91}$$

in excellent agreement. Note, however, that the baryon masses given by the simple quark model in Table 5 yield 1128.14 MeV/c^2, for the right-hand side, which is also splendid agreement. Reproducing $M(\Sigma^0_u)$

is thus not the same as successfully describing the Λ-Σ hyperfine splitting.

Applied to the meson octet, the Gell-Mann-Okubo formula makes a prediction for the mass of the isoscalar member. For the moment, let us assume η(548.8) to be the eighth member of the octet. Then the Gell-Mann-Okubo formula predicts

$$M(\eta) = \frac{4M(K^0) - M(\pi^0)}{3} = 618.6\ MeV/c^2, \tag{3.92}$$

which is not a spectacular success. Among many purported explanations for this failure is the suggestion that the Gell-Mann-Okubo formula should be applied to mass-squared, rather than mass, whereupon

$$M^2(\eta) = \frac{4M^2(K^0) - M^2(\pi^0)}{3} = (569.4\ MeV/c^2). \tag{3.93}$$

This proposal has provoked much learned debate and occasional puckish commentary. (See footnote 21 of Cahn and Einhorn, 1971). Until we have a true theory of hadron structure, I am willing to concede that ambiguity exists.

3.3.2. Singlet-Octet Mixing:

In our initial discussion of vector meson masses in terms of quarks, we skimmed over the identification of singlet and octet isoscalars. The Gell-Mann-Okubo formula for the vector octet reads

$$M(V_8) = \frac{4M(K^*) - M(\rho)}{3} = 930.4 \text{ MeV}/c^2, \tag{3.94}$$

which describes neither $\omega(782.4)$ nor $\varphi(1019.6)$. It is then natural to conclude that neither physical state is a pure octet member, and that ω-φ (or singlet-octet) mixing must be considered. In the quark basis, the singlet state (3.6) and the octet state (3.8) correspond to

$$|V_1\rangle = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}} \tag{3.95}$$

and

$$|V_8\rangle = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}, \tag{3.96}$$

both of which are orthogonal to

$$|\rho^0\rangle = \frac{d\bar{d} - u\bar{u}}{\sqrt{2}}. \tag{3.97}$$

The isoscalar mass matrix can be written in the singlet-octet basis as

$$\mathcal{M} = \begin{pmatrix} M_1 & \Delta \\ \Delta & M_8 \end{pmatrix}, \tag{3.98}$$

which has eigenvalues

$$\begin{pmatrix} M(\varphi) \\ M(\omega) \end{pmatrix} = \frac{M_1 + M_8}{2} \pm \sqrt{\frac{(M_1 - M_8)^2}{4} + \Delta^2}. \tag{3.99}$$

The value of M_8 has already been determined in (3.94). The eigenvalue expression yields

$$M_1 = M(\varphi) + M(\omega) - M_8 = 871.6 \text{ MeV}/c^2. \tag{3.100}$$

Therefore the mixing parameter Δ is given by

$$\Delta^2 = \frac{[M(\varphi) - M(\omega)]^2 - (M_1 - M_8)^2}{4} = (114.9 \text{ MeV}/c^2)^2. \tag{3.101}$$

It is convenient to parametrize the singlet-octet mixing trigonometrically , as

$$|\varphi\rangle = |V_1\rangle \sin\theta - |V_8\rangle \cos\theta, \tag{3.102a}$$

$$|\omega\rangle = |V_1\rangle \cos\theta + |V_8\rangle \sin\theta. \tag{3.102b}$$

The mixing angle can then be determined from the eigenvalue conditions

$$\mathcal{M}|\varphi\rangle = M(\varphi)|\varphi\rangle, \tag{3.103a}$$

$$\mathcal{M}|\omega\rangle = M(\omega)|\omega\rangle, \tag{3.103b}$$

which can be expanded as

$$M(\varphi) = M_1 - \Delta \operatorname{ctn}\theta = M_8 - \Delta \tan\theta, \tag{3.104a}$$

$$M(\omega) = M_1 + \Delta \tan\theta = M_8 + \Delta \operatorname{ctn}\theta. \tag{3.104b}$$

The solution of these equations is

$$\tan^2\theta = \frac{M(\omega) - M_1}{M(\omega) - M_8} = \frac{M_8 - M(\varphi)}{M_1 - M(\varphi)} = (0.776)^2, \tag{3.105}$$

which is not far from the "ideal mixing" value

$$\tan\theta_{ideal} = 1/\sqrt{2}. \tag{3.106}$$

The implied angles are $\theta = 37.8°$, corresponding to (3.105), and $\theta_{ideal} = 35.3°$, corresponding to (3.106). For ideal mixing, the physical particle wavefunctions are those given in (3.5) :

$$|\omega\rangle = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \tag{3.107a}$$

and

$$|\varphi\rangle = s\bar{s}. \tag{3.107b}$$

The quark composition of these states provides immediate insight into the pattern of masses. The ω^0 and ρ^0 contain the same light quarks in equal weights, and are nearly degenerate in mass. The hidden strangeness state φ is more massive because it contains the heavier strange quark, as was made quantitative in (3.37).

The $\eta'(957.57)$ suggests itself as a ninth pseudoscalar meson to be considered within a nonet containing $\eta(548.8)$. The singlet-octet mixing analysis can be transcribed directly from the vector meson case, with the definitions

$$|\eta'\rangle = |\eta_1\rangle \sin\theta - |\eta_8\rangle \cos\theta, \tag{3.108a}$$

$$|\eta\rangle = |\eta_1\rangle \cos\theta + |\eta_8\rangle \sin\theta. \tag{3.108b}$$

For the linear mass formula we have, upon reinterpreting (3.92) as M_8,

$$M_1 = 887.8 \ \mathrm{MeV}/c^2, \tag{3.109}$$

$$\Delta^2 = (153.8 \ \mathrm{MeV}/c^2)^2, \tag{3.110}$$

$$\theta = 65.6^\circ \tag{3.111}$$

and the convenient approximate forms

$$|\eta'\rangle = \sqrt{\frac{1}{8}}\,(u\bar{u}+d\bar{d}) + \sqrt{\frac{3}{4}}\, s\bar{s}, \tag{3.112a}$$

$$|\eta\rangle = \sqrt{\frac{3}{8}}\,(u\bar{u}+d\bar{d}) - \frac{1}{2}\, s\bar{s} \tag{3.112b}$$

An analysis using quadratic mass relations, with (3.93) interpreted as M_8^2, yields

$$M_1^2 = (945 \ \mathrm{MeV}/c^2)^2, \tag{3.113}$$

$$\Delta^2 = (343 \ \mathrm{MeV}/c^2) \tag{3.114}$$

$$\theta = 78.85^\circ \tag{3.115}$$

and the approximate flavor wavefunctions

$$|\eta'\rangle = \frac{u\bar{u}+d\bar{d}}{2} + \frac{s\bar{s}}{\sqrt{2}} \tag{3.116a}$$

$$|\eta\rangle = \frac{u\bar{u}+d\bar{d}}{2} - \frac{s\bar{s}}{\sqrt{2}} \tag{3.116b}$$

In neither case is a particularly transparent interpretation of hadron masses in terms of quark masses possible. For the linear mass formula the implied mass difference $m_s - m_u$ is more than twice as large as the values encountered before in (3.37, 38) and (3.72). Quark counting with a quadratic mass formula would lead to the expectation of degenerate η and η' states. This may be taken as a hint that there is something yet to be understood about the pseudoscalar mesons. Whether the clue lies in the special role of the almost-massless pion, or in a deeper understanding of the η-η' mixing mechanism, or both is a question to which we shall have to return.

3.4. Electromagnetic Properties Redux :

In this section we shall discuss the electromagnetic properties of hadrons in considerable quantitative detail. Such detail is justified by the impressive quality of recent measurements of baryon magnetic moments, transition moments in mesons, and meson charge radii. The first two subjects have been reviewed by Rosner (1980b, 1981a). What emerges from the comparison of the quark model and experiment is a level of agreement which is at least to my eye miraculous, but at the same time imperfect. The challenge to deeper theoretical approaches is to explain the successes while repairing the failures. To a limited extent this will be achieved in subsequent sections.

3.4.1. Baryon Magnetic Moments :

Problems 8 and 9 provide the opportunity to compute the baryon magnetic moments using the quark-model wavefunctions derived in Problem 7. The results are given in Table 6, together with the experimental measurements. Three sets of quark model predictions are shown. The first, designated "exact SU(3)" , is based on the assumption that the quark magnetic moments are proportional only to the quark charges, so that

$$\mu_u = -2\mu_d = -2\mu_s = 2\mu_p/3. \tag{3.117}$$

This simple description reproduces the trends of the data, insofar as signs and relative sizes are concerned, but it is not adequate quantitatively.

To attempt to improve the degree of agreement, we may break SU(3) symmetry by allowing μ_s to differ from μ_d. It is then natural to fix μ_s by fitting to the well-measured Λ^0 magnetic moment as

$$\mu_s = \mu_\Lambda = -0.614 \text{ n.m.} \tag{3.118}$$

This is smaller in magnitude than $\mu_d = -0.931$ n.m., consistent with the evidence from the spectrum that $m_s > m_d$. The assumption (3.118) makes a noticable improvement in the predictions, but leaves us short of a perfect description. Finally if we indulge in the fine-tuning of μ_u and μ_d discussed in § 2.3, eqns. (2.65, 66) the overall situation is not markedly changed.

Although the general pattern of baryon moments is extremely well reproduced, there are noticeable quantitative failures. Particularly bad is the combination

$$3(\mu_{\Xi^-} - \mu_{\Xi^0}) = \mu_u - \mu_d = \frac{3}{5}(\mu_p - \mu_n), \tag{3.119}$$

for which the measurements yield

$$(1.50 \pm 0.18) \text{ n.m.} \neq 2.82 \text{ n.m.} \tag{3.120}$$

However, if the effective moments of the up and down quarks are altered in strange hadrons, it is not in a systematic way, at least at the level of present measurements. The quark-model relation

$$\frac{3}{4}(\mu_{\Sigma^+} - \mu_{\Sigma^-}) = \mu_u - \mu_d = \frac{3}{5}(\mu_p - \mu_n) \tag{3.121}$$

is well satisfied by experiment :

$$(2.81 \pm 0.21) \text{ n.m.} = 2.82 \text{ n.m.} \tag{3.122}$$

Forthcoming measurements of the Σ moments will provide a more incisive test of (3.121).

We have already found in § 2.3, eqns. (2.63, 64) that if the quarks are regarded as Dirac particles, the inferred masses of the up and down quarks are reasonable—about one-third of a proton mass. From (3.118) and (2.66) we conclude that

$$m_s \approx 1.58\, m_d \approx 510\ \mathrm{MeV}/c^2. \tag{3.123}$$

This is both a sensible value on its own (half the φ mass) and implies that

$$m_s - m_u \approx 172\ \mathrm{MeV}/c^2, \tag{3.124}$$

consistent with the determination (3.72) from the baryon spectrum of approximately 190 MeV/c^2.

The availability of precise data on the hyperon moments and the suggestion that the elementary quark model is close to, but not exactly, the truth has stimulated many attempts at improving the model. The suggestions include relativistic corrections, configuration mixing, various dilution effects, and anomalous quarks moments. Many are interesting, but none is yet compelling. A representative sampling may be gleaned from the mini-review by Rosner (1980b), the papers by Bohm and Teese (1981), Cohen and Lipkin (1980), Dothan (1981), Geffen and Wilson (1980), Isgur and Karl (1980), Lichtenberg (1981), Lipkin (1981a), and Teese (1981), and references therein.

The now-standard method for measuring hyperon magnetic moments by exploiting the polarization of inclusively-produced hyperons is described in Schachinger, et al. (1978), and in the Les Houches seminar by Cox (1981). These spin-rotation measurements rely upon the self-analyzing weak decays of hyperons, which are the subject of problems 10 and 11.

3 . 4. 2. When Fermions are Confined :

The foregoing analysis has demonstrated that if quarks are regarded as Dirac particles with masses that seem reasonable for

constituents of hadrons a systematic understanding of the baryon magnetic moments emerges. On the one hand, this state of affairs provides motivation to elaborate or "improve" the simple quark model. On the other, we are impelled to ask whether it is indeed reasonable that the simple picture should work so well.

Because quarks appear (Jones, 1977 ; Lyons, 1981) to be securely bound within hadrons, it is important to ask whether a bound fermion behaves as a Dirac particle. That there is room for discussion is shown by an elementary example due to Lipkin and Tavkhelidze (1965). The Dirac equation for a particle of mass m is an electromagnetic field may be written in the form

$$\gamma^\mu(p_\mu - eA_\mu)\Psi = m\Psi. \tag{3.125}$$

It is obvious from common experience that the magnetic moment of an electron does not depend upon the strength of the magnetic field in which it is measured. In the same way, if the particle is subjected to an additional four-vector interaction V_μ, the ensuing Dirac equation,

$$\gamma^\mu(p_\mu - eA_\mu + V_\mu)\Psi = m\Psi \tag{3.126}$$

is still that of a particle with mass m, but with a redefined momentum. In the specific case of a static potential

$$V_0 = \mathcal{V}$$
$$\underset{\sim}{V} = 0 \tag{3.127}$$

the change amounts merely to a shift in the energy scale,

$$E \rightarrow E^* = E + \mathcal{V}. \tag{3.128}$$

Consider instead the case of a fermion interacting with a four-scalar potential $\mathcal{V}$. The Dirac equation is then

$$\left[\gamma^\mu(p_\mu - eA_\mu) + \mathcal{V}\right]\Psi = m\Psi\,, \tag{3.129}$$

which has the look

$$\gamma^\mu(p_\mu - eA_\mu)\Psi = (m - \mathcal{V})\Psi \tag{3.130}$$

and the effect of a Dirac equation for a particle with mass $m^* = m - \mathcal{V}$, which is to say that the fermion has acquired an apparent anomalous moment. Thus it is proper to be concerned about the effect of confinement upon the apparent magnetic moment of a quark.

More insight into the effective properties of a quark within a hadron may be gained by considering a free fermion confined within a rigid sphere. This is a textbook problem in relativistic quantum mechanics (cf. Akhiezer and Berestetskii, 1965) and has been applied in one guise (Bogoliubov, 1967) or another (Chodos, Jaffe, Johnson, and Thorn, 1974 ; DeGrand, Jaffe, Johnson, and Kiskis, 1975) to the quark model by many authors. (A brief summary of the problem is also to be found in chapter 18 of Close, 1979, but watch out for misprints!)

Suppose that within a rigid static sphere of radius R the fermion behaves as a free particle, and satisfies

$$\not p\,\Psi = m\Psi. \tag{3.131}$$

The confinement hypothesis requires that no probability current flow across the boundary of the sphere, which is characterized covariantly by the outward normal n_μ. The boundary condition is thus

$$n^\mu \bar\Psi \gamma_\mu \Psi = 0\,\Big|_{r=R}\,, \tag{3.132}$$

or simply $\bar\Psi \not n \Psi = 0$ at $r = R$. Ths lowest mode solution of the Dirac

equation is

$$\Psi(\mathbf{r},t) = \eta(x)\begin{pmatrix} \left(\frac{\omega+m}{\omega}\right)^{1/2} i\, j_0(xr/R)\,\chi \\ -\left(\frac{\omega-m}{\omega}\right)^{1/2} j_1(xr/R)\,\boldsymbol\sigma\cdot\hat{\mathbf{r}}\,\chi \end{pmatrix} e^{-i\omega t} \tag{3.133}$$

where ω is the particle energy, x/R its momentum, χ is a two-component spinor, the j_u are spherical Bessel functions, and $\eta(x)$ is a normalization factor.

The boundary condition (3.132) will be satisfied if at $r = R$

$$\not n\,\Psi = i\Psi \tag{3.134}$$

for then

$$\bar\Psi \not n = -i\bar\Psi \tag{3.135}$$

and thus

$$\bar\Psi \not n \Psi = i\bar\Psi\Psi = -i\bar\Psi\Psi = 0. \tag{3.136}$$

Since the outward normal is

$$n_\mu = (0, \hat{\mathbf{r}})\,, \tag{3.137}$$

the boundary conditinn (3.134) is

$$i\boldsymbol\gamma\cdot\hat{\mathbf{r}}\,\Psi = \Psi\,\Big|_{r=R} \tag{3.138}$$

or explicitly,

$$\begin{pmatrix} 0 & -i\boldsymbol\sigma\cdot\hat{\mathbf{r}} \\ i\boldsymbol\sigma\cdot\hat{\mathbf{r}} & 0 \end{pmatrix}\begin{pmatrix} i\sqrt{\omega+m}\; j_0(x) \\ -\sqrt{\omega-m}\;\boldsymbol\sigma\cdot\hat{\mathbf{r}}\, j_1(x) \end{pmatrix} = \begin{pmatrix} i\sqrt{\omega+m}\; j_0(x) \\ -\sqrt{\omega-m}\;\boldsymbol\sigma\cdot\hat{\mathbf{r}}\, j_1(x) \end{pmatrix}, \tag{3.139}$$

which implies, because $\underset{\sim}{\sigma}\cdot\underset{\sim}{a}\;\underset{\sim}{\sigma}\cdot\underset{\sim}{b} = \underset{\sim}{a}\cdot\underset{\sim}{b} + i\underset{\sim}{\sigma}\cdot\underset{\sim}{a}\times\underset{\sim}{b}$, that

$$j_1(x) = \sqrt{\frac{(\omega+m)}{(\omega-m)}}\; j_0(x). \qquad (3.140)$$

Enforcing the boundary condition (3.139) thus leads to the eigenvalue condition

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{1/2}}. \qquad (3.141)$$

In passing from (3.140) to (3.141) we have used the explicit forms of the spherical Bessel functions

$$j_0(x) = \sin x / x, \qquad (3.142a)$$

$$j_1(x) = (\sin x - x\cos x)/x^2, \qquad (3.142b)$$

and the connection

$$\omega = \left(m^2 + x^2/R^2\right)^{1/2}. \qquad (3.143)$$

The first eigenvalue of (3.141), corresponding to the ground-state momentum (in units of $^1/_R$) of the confined fermion, is plotted in Fig. 12(a). It ranges between the values

$$x(mR=0) = 2.043, \qquad (3.144a)$$

$$x(mR\to\infty) = \pi. \qquad (3.144b)$$

For any value of mR, the ratio of the free-particle mass to the confined -particle energy depends only upon mR. This is shown in Fig.12(b). In the extreme-relativistic limit $mR \to 0$, all the energy is a consequence of the confinement, and

$$\omega = 2.043/R, \quad mR = 0. \qquad (3.145)$$

In the nonrelativistic, or free particle limit $mR \to \infty$, the confined-particle energy approaches the free-particle mass :

$$\lim_{mR\to\infty} \omega = m. \qquad (3.146)$$

It is of interest to study the behavior of the confined-particle energy in two special cases. In Fig. 13 we see the energy of a massless particle confined within a sphere of radius R. Note some typical values : a massless particle confined with a radius of 1 fm acquires an energy, or effective mass, of 400 MeV/c^2. A system confined to a fixed radius of $^4/3$ fm is portrayed in Fig. 14. The two limits described by (3.145) and (3.146) are readily apparent.

What implications does confinement have for the magnetic moment ? With the Dirac wavefunction (3.133) for the ground state in hand, it is straightforward to compute the magnetic moment from the definition

$$\underset{\sim}{\mu} \equiv \frac{1}{2}\int_{|\underset{\sim}{r}|<R} d^3\underset{\sim}{r}\; \underset{\sim}{r}\times\underset{\sim}{J} = \frac{e}{2}\int_{|\underset{\sim}{r}|<R} d^3\underset{\sim}{r}\; \underset{\sim}{r}\times\bar{\Psi}\underset{\sim}{\gamma}\Psi. \qquad (3.147)$$

I note in passing an elegant discussion of the nature of intrinsic dipole moments (Jackson, 1977a), from which everyone can learn something. The result (Chodos, Jaffe, Johnson, and Thorn, 1974 for the massless case ; Allen, 1975 ; Golowich, 1975 ; De Grand, et al., 1975) is

$$\mu = \frac{e}{2m}\left\{\frac{mR}{3}\cdot\frac{4\omega R + 2mR - 3}{2(\omega R)^2 - 2\omega R + mR}\right\} \qquad (3.148)$$

which can lead to a large reduction of the magnetic moment, compared with that of the free particle. In particular, a confined massless fermion acquires a finite magnetic moment.

This result has been cited by the developers of the MIT Bag (e.g. De Grand, et al., 1975) as an example of the unreliability of the nonrelativistic quark model. While I do not disagree, I would emphasize a different interpretation. We may recast (3.148) as

$$\mu = \frac{e}{2\omega}\left\{\frac{\omega R}{3}\cdot\frac{4\omega R + 2mR - 3}{2(\omega R)^2 - 2\omega R + mR}\right\}, \qquad (3.149)$$

recognizing that the energy ω plays the role of an effective mass for the confined fermion. Comparing the magnetic moment of the confined fermion with the Dirac moment of a free fermion with mass ω, we find (see the plot of the ratio in Fig. 15) that the two differ by less than 20 %, even for the extreme case of a confined massless fermion. I take this very stylized calculation to indicate that it is not nonsensical for confined quarks to display Dirac moments characteristic of their constituent masses. Quark model phenomenology is therefore likely to make sense, although it may well be the case that a quark manifests slightly different moments in different hadrons, and that the model succeeds for complicated reasons. DeGrand (1980) is in grudging agreement with this view. This issue recurs for models of composite quarks and fermions. See a recent comment by Bander, et al., (1981).

3. 4.3. Axial Charges :

The semileptonic decays of hadrons provide information about both the weak current and the properties of the hadrons themselves. For exhaustive reviews, see Willis and Thompson (1968), and Chounet, et al. (1972). In the limit of zero momentum transfer, the matrix element for the baryon semileptonic decay $B \rightarrow B'\ell\nu$ is of the form

$$\bar{u}(B')\gamma_\mu(g_V + g_A\gamma_5)u(B). \qquad (3.150)$$

The vector couplings are determined by the Cabibbo (1963) hypothesis and by the assumption that the vector charges are generators of SU(3). They are protected from symmetry breaking effects (Ademollo and Gatto, 1964 ; Bouchiat and Meyer, 1964) and thus yield little information about baryon structure. The axial charges may have a greater sensitivity to hadron physics.

In the nonrelativistic quark model (e.g. Kokkedee, 1967), the ratio g_A/g_V is given by

$$g_A/g_V = \frac{\langle B'|I_+\sigma_z|B\rangle}{\langle B'|I_+|B\rangle} \qquad (3.151)$$

for $\Delta S = 0$ decays, and by

$$g_A/g_V = \frac{\langle B'|V_+\sigma_z|B\rangle}{\langle B'|V_+|B\rangle} \qquad (3.152)$$

for $\Delta S = 1$ decays. The predictions for various semileptonic decays are gathered in Table 7. The quark model predictions correspond to those of the SU(3) algebra, also given in Table 7, with the specific choice of symmetric and antisymmetric octet couplings $D \equiv 1$, $F/D = {}^2/_3$. Measurements of individual decay rates determine only linear combinations of $g_A{}^2$ and $g_V{}^2$, so can only constrain g_A within broad limits. All existing measurements are compatible with the Cabibbo parametrization, with

$$F/D = 0.54 \pm 0.02 \qquad (3.153)$$

(Shrock and Wang, 1978), not terribly far from the quark model value.

For the cases in which g_A/g_V or $|g_A/g_V|$ has been determined directly by correlation or polarization measurements, the pattern of the data is systematically that of the quark model. However, the quark model overestimates the axial charges by approximately one-third.

(Note that the sign convention adopted by Particle Data Group, 1980, is opposite to mine). Existing measurements are shown in Table 7. Here again, experiments using high-energy hyperon beams hold considerable promise for dramatically extending our knowledge.

The effects of confinement, which were treated schematically in the preceding section, are significant for quark matrix elements $\langle q|\sigma_z|q\rangle$ as well because the lower components of the Dirac spinor may may have the "wrong" spin projection. Confinement tends to increase the effective mass of a fermion, and thus to increase the importance of the lower components. The static spherical cavity was studied by Golowich (1975) who found for the nucleon ground state

$$g_A/g_V = \frac{5}{3}\left\{\frac{2(\omega R)^2+4mR\omega R-3mR}{6(\omega R)^2-6\omega R+3mR}\right\}, \tag{3.154}$$

which is plotted in Fig. 16 as a function of the dimensionless parameter mR. The values range from $g_A/g_V = 1.09$ for massless quarks to the free-fermion value of $5/3$ as $mR \to \infty$. The extension to other systems was made by Donoghue , et al. (1975) and by DeGrand et al. (1975). See also the treatment by Le Yaouanc, et al. (1977).

3.4.4. M1 Transitions in Mesons :

In § 2 . 4 we discussed the interpretation of radiative (M1) decays of vector mesons as single-quark spinflip transitions, and compared the relative rates for $(\rho,\omega) \to \pi\gamma$. In the course of extending the quark model predictions to the full vector meson nonet, we may also consider the expectations of the quark model for absolute decay rates. The treatment given in § 2.. 4 was complete so far as the flavor and spin aspects of the simple quark model are concerned,

but did not consider the degree of overlap between initial and final spatial wavefunctions. In the static limit, which corresponds to initial and final hadrons of equal mass, this overlap is likely to be complete. However, the vector and pseudoscalar mesons are decidedly different in mass, so recoil effects and possible differences in the radial wavefunctions due to the distortion of the strong hyperfine splitting can be expected to make the overlap incomplete. A true dynamical theory of hadrons should permit the computation of the overlap integral. Lacking that, we shall merely parametrize the anticipated effect by writing

$$\langle P|M1|V\rangle = \langle P|M1|V\rangle_{\text{flavor-spin}} \cdot \vartheta , \tag{3.155}$$

where P and V are generic labels for pseudoscalar and vector mesons, M1 is the transition operator of eq. (2.98), and ϑ will be called the overlap factor. It will be considered encouraging if, with the values of quark magnetic moments determined from the baryon magnetic moments, the overlap factor is less than unity, but of order unity. See also Rosner (1980b, 1981a), and O'Donnell (1981).

The quark model predictions and the comparison with experiment are given in Table 8, which requires a good deal of explanation and comment. After tabulating the energy ω of the emitted photon and the measured decay rate Γ, I have chosen to characterize the experimental matrix-element-squared by the dimensionless quantity

$$\frac{3\pi\,\Gamma(V\to P\gamma)}{\omega^3\,\mu_N}, \tag{3.156}$$

where the nuclear magneton μ_N is defined for a Dirac proton, so that

$$\mu_N^2 = \pi\alpha/m_p^2 . \tag{3.157}$$

This is free of the trivial kinematic dependence, and can readily be compared with the square of the flavor-spin matrix element, $3\pi\Gamma/\omega^3\mu_N^2\,\mathcal{O}$, evaluated in the quark model. Comparison of the experimental and theoretical numbers leads to a determination of the overlap factor $\mathcal{O}$. In Table 8, the quark model prediction has been evaluated for two sets of assumptions about the quark magnetic moments : the SU(3)-symmetric case

$$\mu_u = -2\mu_d = -2\mu_s = 2\mu_p/3, \tag{3.117}$$

and the broken-symmetry case with the strange quark moment given by

$$\mu_s = \mu_\Lambda . \tag{3.118}$$

This parallels our discussion of baryon magnetic moments.

The isoscalar pseudoscalar mesons were left in some disarray in § .3 .3.2. As a result the flavor wavefunctions of η and η' are not well specified. I have therefore presented calculations for two cases, labelled Q (for quadratic mass formula)and L (for linear mass formula) in Table 8. The approximate flavor wavefunctions (3.116) and (3.112) have been used in the calculations.

A reading of Table 8 shows that the quark model is rather successful in predicting the ratios of M1 transition rates, and that the reduced strange-quark moment of (3.118) improves the agreement between theory and experiment. The quark-model-forbidden transition $\varphi \to \pi^0\gamma$ is enormously suppressed. The predicted absolute rates are also sensible ; an overlap factor of approximately $^1/_2$ brings predictions and observations into reasonable agreement. This is shown in graphic form in Fig. 17, which compares the overlap factors deduced from individual decay rates, for the broken sym-

metry case. An average value of

$$\mathcal{O} = 0.56 \tag{3.158}$$

accomodates most of the measurements. Ohshima's (1980) reanalysis of the $\omega \to \pi^0\gamma$ rate, quoted above in eqn. (2.102), leads to a reduced overlap factor for that decay of

$$\mathcal{O}(\omega \to \pi^0\gamma)_{\text{Ohshima}} = 0.67 \pm 0.07, \tag{3.159}$$

which is more in line with the other inferred values. This underscores the already obvious remark that there is much room for improved experiments. The advantages of high-energy beams, already apparent from recent data, are stressed in the Les Houches seminar by T. Jensen (1981) ; see also Jensen (1980). An application of quark model techniques to M1 transitions among charmed mesons is posed as Problem 12.

3 . 4.5. Electromagnetic Form Factors :

Hadron form factors constitute an important piece of evidence that hadrons have a finite size and are composite in nature. A brief but lucid introduction to the subject appears in chapter 19 of the book by Perl (1974), and a thorough review with special emphasis on the vector dominance interpretation has been given by Gourdin (1974).

For spinless hadrons, the cross section for electron-hadron elastic scattering can be written as

$$\left.\frac{d\sigma}{dq^2}\right)_{\text{elastic}} = \left.\frac{d\sigma}{dq^2}\right)_{\text{point}} \times \left|F_h(q^2)\right|^2 , \tag{3.160}$$

where the form factor $F_h(q^2)$ is the Fourier transform of the charge distribution within the hadron. For a spherically symmetric charge distribution,

$$F_h(q^2) = \int d^3\underset{\sim}{r}\; e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}\, \rho(r)$$
$$= \int_0^\infty r^2 dr \int_{-1}^{1} dz \int_0^{2\pi} d\varphi\; e^{iqrz}\, \rho(r)$$
$$= 4\pi \int_0^\infty r^2 dr\; \rho(r)\, \sin(qr)/qr\,, \tag{3.161}$$

so that

$$F_h(0) = \int d^3\underset{\sim}{r}\; \rho(r) = Q_h\,. \tag{3.162}$$

At small values of the momentum transfer q^2, it is appropriate to approximate the integral by Taylor-expanding $\sin(qr)$:

$$F_h(q^2) \approx 4\pi \int_0^\infty r^2 dr\, \rho(r) \left[1 - \frac{q^2 r^2}{3!} + \frac{q^4 r^4}{5!} - \cdots \right]$$
$$\approx Q_h - \frac{q^2}{3!} \langle r_{EM}^2 \rangle_h\,. \tag{3.163}$$

The mean-squared charge radius of the hadron has been defined in a natural way as

$$\langle r_{EM}^2 \rangle_h = \int d^3\underset{\sim}{r}\; \rho(r)\, r^2. \tag{3.164}$$

[Sometimes a factor of Q_h is divided out of F_h and $\langle r_{EM}^2\rangle_h$. This is an unappealing convention for neutral particles.]

It is frequently assumed (e.g. Chou and Yang, 1968) that the charge distribution within a hadron is representative of the matter distribution. This is a simple, but not unavoidable assumption. It appears (Amaldi, et al., 1976) not to be grossly misleading.

The definitions (3.163, 164) show that a measurement of the slope of the form factor as a function of q^2 yields a determination of the charge radius as

$$\langle r_{EM}^2 \rangle_h = -6 \left. \frac{\partial F_h(q^2)}{\partial q^2} \right|_{q^2=0} \tag{3.165}$$

Experiments to measure the electromagnetic form factors are of three basic types.

The form factors of stable targets, which is to say nucleons and nuclei, are measured directly by the scattering of high energy electron beams. The classic experiments have been reviewed by Hofstadter (1963). For spin -$^1/_2$ particles, there are of course two independent form factors, which may be conveniently defined as electric and magnetic. Useful summaries are given by Weber (1967) and Rutherglen (1969). We have already failed in § 2..3. to explain the charge distribution within the neutron, and again promise enlightenment only in § 5.3 .

To determine the form factors of unstable particles it has until recently been necessary to rely upon indirect means, analogous to the Goebel (1958)- Chew and Low (1959) extrapolation method. To study the interactions of hadrons with the pion, it is useful to regard the nucleon as surrounded by a pion cloud, and to treat the virtual pions in the cloud as target particles. This is illustrated in Fig. 18(a) for the case of $\pi\pi$ scattering, which is studied in the reaction

$$\pi N \rightarrow \pi \pi N'\,. \tag{3.166}$$

The momentum transfer $t \equiv (p_N - p_{N'})^2$ between initial and final nucleons gives a measure of the mass of the virtual pion. The proposal of Goebel, Chew, and Low is to measure the properties of $\pi\pi$ scattering as a function of the virtual pion mass and to determine by extrapolating to the pion pole the properties of physical $\pi\pi$ scattering. Despite numerous technical complications, this suggestion has been enormously fruitful. It played an important part in the development of the peripheral exchange picture (e.g. Jackson, 1966 ; Fox and Quigg, 1973) and created an industry devoted to the study of meson-meson scattering (e.g. Williams and Hagopian, 1973 ; Estabrooks, 1977). In similar fashion, one may study single pion electroproduction

$$eN \rightarrow e\pi N' , \tag{3.167}$$

for which the pion pole contribution is depicted in Fig. 18(b). Typical of attempts to measure the pion form factor using this technique is the work of Mistretta, et al. (1969).

Until the advent of high-energy pion beams, electroproduction was the only method available for the measurement of the pion form factor in the spacelike region. Why high energies make a difference can be seen from an elementary kinematical argument. In pion scattering from a stationary (i.e. atomic) electron,

$$s \approx 2 m_e p_\pi \tag{3.168}$$

is kept small by the smallness of the electron mass. The maximum momentum transfer is characterized by

$$q^2_{max} \approx s . \tag{3.169}$$

If the charge radius is characterized by a typical hadronic dimension, perhaps on the order of 1 GeV, demanding that $q^2_{max} \cdot \langle r^2_{EM} \rangle \sim 0.1$ imposes the requirement that $p_\pi \sim 100$ GeV/c. With the availability of such beams, the direct study of reactions such as

$$\pi e \rightarrow \pi e \tag{3.170}$$

has become possible. The experimental results are summarized in Table 9.

Two interpretations of these data are instructive. First, let us consider the consequences of SU(3) symmetry and vector meson dominance. We assume that the photon couples to hadrons through the ideally mixed vector mesons ρ^0, ω, and φ, and that the photon transforms as a U-spin singlet, so that

$$|\gamma\rangle \sim \frac{3|\rho^0\rangle + |\omega\rangle - \sqrt{2}\,|\varphi\rangle}{\sqrt{12}} \tag{3.171}$$

Thus the pion form factor will be controlled only by the ρ^0, by G-parity, but the kaon will be influenced by ρ^0, ω, and φ. An elementary SU(3) calculation gives

$$F_{\pi^+}(q^2) = m_\rho^2 / (q^2 + m_\rho^2) , \tag{3.172}$$

$$F_{K^+}(q^2) = \frac{1}{2} \cdot \frac{m_\rho^2}{q^2 + m_\rho^2} + \frac{1}{6} \cdot \frac{m_\omega^2}{q^2 + m_\omega^2} + \frac{1}{3} \cdot \frac{m_\varphi^2}{q^2 + m_\varphi^2} , \tag{3.173}$$

$$F_{K^0}(q^2) = -\frac{1}{2} \, \frac{m_\rho^2}{q^2 + m_\rho^2} + \frac{1}{6} \cdot \frac{m_\omega^2}{q^2 + m_\omega^2} + \frac{1}{3} \cdot \frac{m_\varphi^2}{q^2 + m_\varphi^2} . \tag{3.174}$$

The definition (3.165) of the mean-squared charge radius then yields

$$\langle r_{EM}^2 \rangle_{\pi^+} = 6 / m_\rho^2 = 0.388 \text{ fm}^2 , \tag{3.175}$$

$$\langle r_{EM}^2\rangle_{K^+} = 3/m_\rho^2 + 1/m_\omega^2 + 2/m_\varphi^2 = 0.332\ \text{fm}^2, \tag{3.176}$$

$$\langle r_{EM}^2\rangle_{K^0} = -3/m_\rho^2 + 1/m_\omega^2 + 2/m_\varphi^2 = -0.055\ \text{fm}^2. \tag{3.177}$$

Comparing with the experimental results in Table 9, we find quite good agreement, though it must be said that the data are perhaps still not definitive.

A second kind of analysis involves the explicit use of constituents. This has been a popular approach since the early days of the quark model (see Gerasimov, 1966). Recent work (Greenberg, et al., 1977 ; Isgur, 1978) has been concentrated on the neutral kaon. In a nonrelativistic description of mesons, we have

$$\langle r_{EM}^2\rangle = \left\langle \sum_{i=q,\bar q} e_i(\underset{\sim}{r}_i - \underset{\sim}{R})^2 \right\rangle, \tag{3.178}$$

where $\underset{\sim}{r}_i$ is the coordinate of the $i^{\underline{th}}$ constituent and

$$\underset{\sim}{R} = (m_q \underset{\sim}{r}_q + m_{\bar q}\underset{\sim}{r}_{\bar q})/(m_q + m_{\bar q}) \tag{3.179}$$

is the CM coordinate. We may also define the relative coordinate

$$\underset{\sim}{\rho} = \underset{\sim}{r}_q - \underset{\sim}{r}_{\bar q}. \tag{3.180}$$

With these definitions, the mean-squared charge radius is

$$\langle r_{EM}^2\rangle_h = \frac{1}{(m_q + m_{\bar q})^2}(e_q m_{\bar q}^2 + e_{\bar q} m_q^2)\langle \underset{\sim}{\rho}^2\rangle_h. \tag{3.181}$$

For the cases of interest, we then have

$$\langle r_{EM}^2\rangle_{\pi^+} = \frac{(2m_d^2 + m_u^2)}{3(m_u + m_d)^2}\langle \underset{\sim}{\rho}^2\rangle_\pi, \tag{3.182}$$

$$\langle r_{EM}^2\rangle_{K^+} = \frac{(2m_s^2 + m_u^2)}{3(m_u + m_s)^2}\langle \underset{\sim}{\rho}^2\rangle_K, \tag{3.183}$$

$$\langle r_{EM}^2\rangle_{K^0} = \frac{(m_d^2 - m_s^2)}{3(m_d + m_s)^2}\langle \underset{\sim}{\rho}^2\rangle_K. \tag{3.184}$$

We see at once that the K^0 charge radius is negative because the planetary d-quark, negatively charged, orbits the more massive s-quark. The numerically successful vector dominance calculation did not give such a transparent explanation. The ratio of charged and neutral kaon charge radii is predicted to be

$$\langle r_{EM}^2\rangle_{K^0}/\langle r_{EM}^2\rangle_{K^+} = \frac{m_d^2 - m_s^2}{2m_s^2 + m_u^2}. \tag{3.185}$$

In the approximation that $m_\rho = m_\omega$, the vector dominance result is

$$\langle r_{EM}^2\rangle_{K^0}/\langle r_{EM}^2\rangle_{K^+} = \frac{m_\rho^2 - m_\varphi^2}{2m_\varphi^2 + m_\rho^2} = -0.16. \tag{3.186}$$

If we set $m_s = m_\varphi/2$ and $m_u = m_\rho/2$, which is consistent with our earlier estimates, the quark model result coincides exactly. Both models thus agree with the experimental result.

The quark model is elementary form is less definite about the comparison of π and K form factors :

$$\langle r_{EM}^2\rangle_{K^+}/\langle r_{EM}^2\rangle_{\pi^+} = \frac{2m_s^2 + m_u^2}{(m_u + m_s)^2}\cdot\frac{(m_u + m_d)^2}{(2m_d^2 + m_u^2)}\frac{\langle \underset{\sim}{\rho}^2\rangle_K}{\langle \underset{\sim}{\rho}^2\rangle_\pi} \tag{3.187}$$

$$\approx \frac{2m_s^2 + m_u^2}{(m_u + m_s)^2}\cdot\frac{4}{3}\cdot\frac{\langle \underset{\sim}{\rho}^2\rangle_K}{\langle \underset{\sim}{\rho}^2\rangle_\pi}. \tag{3.188}$$

with $m_s = m_\varphi/2$ and $m_u = m_\rho/2$, this implies

$$\langle r_{EM}^2\rangle_{K^+}/\langle r_{EM}^2\rangle_{\pi^+} \approx 1.11\langle \underset{\sim}{\rho}^2\rangle_K/\langle \underset{\sim}{\rho}^2\rangle_\pi. \tag{3.189}$$

It is natural to expect that $\langle \underset{\sim}{\rho}^2\rangle_K < \langle \underset{\sim}{\rho}^2\rangle_\pi$, but we require a more specific model to predict the difference. The data imply

$$\langle \underset{\sim}{\rho}^2\rangle_{K^+}/\langle \underset{\sim}{\rho}^2\rangle_{\pi^+} \approx 2/3, \tag{3.190}$$

which is not unreasonable.

Thus, we see that the quark model gives an immediate understanding of the sign of the neutral kaon charge radius, and properly predicts the ratio of charged and neutral kaon form factors. It is poor for observables that rely on some dynamics, such as absolute sizes and π/K ratios. We may nevertheless ask what hadron sizes, as represented by $\langle \rho^2 \rangle$, are required to describe the data. The results are

$$\langle \rho^2 \rangle_{\pi}^{1/2} \approx 2 \langle r_{EM}^2 \rangle_{\pi}^{1/2} = (1.25 \pm 0.06)\ \text{fm}, \qquad (3.191)$$

$$\langle \rho^2 \rangle_{K^+}^{1/2} \approx 1.9 \langle r_{EM}^2 \rangle_{K^+}^{1/2} = (1.00 \pm 0.09)\ \text{fm}, \qquad (3.192)$$

$$\langle \rho^2 \rangle_{K^0}^{1/2} \approx 4.7 \langle r_{EM}^2 \rangle_{K^0}^{1/2} = (1.09 \pm 0.26)\ \text{fm}. \qquad (3.193)$$

which are again reasonable hadronic dimensions. Thus the confined fermion approach of the MIT Bag Model can be expected to yield sensible absolute sizes for the charge radii. The results presented by De Grand, et al. (1975) are somewhat smaller than current measurements require.

4. Orbitally Excited Hadrons :

In the preceding long yet incomplete section, the quark model has been seen to give a creditable acount of the mesons and baryons in the SU(6) ground state : L = 0 configurations of $(q\bar{q})$ for mesons and (qqq) for baryons. In this section, two things are done. First, the spectra of excited mesons and baryons will be shown to match the expectations of the quark model with orbital excitations. This will by necessity not be carried out in infinite detail. For the rich microstructure, I refer the reader to the review articles by Protopopescu and Samios (1979), Rosner (1974a), and Samios et al., (1974) ; to the recent conference talks by Close (1981), Hey (1979), and Montanet (1980) ; and to the proceedings of the latest spectroscopy conferences (Chung 1980 ; Isgur, 1980b), and to chapter 5 of Close (1979). The consonance between quark-model predictions and experimental observations strongly motivates a serious consideration of the quark model as a basis for hadron spectroscopy.

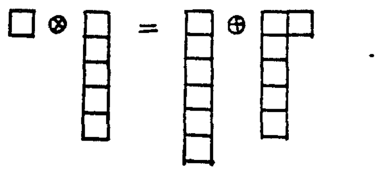
If the quark model is to be taken seriously, it must be made free from internal inconsistencies. Thus the problem of the exclusion principle (i.e. of symmetric fermion wavefunctions), which has been held in abeyance since § 2. 2., must be faced. It is now seen to exist not only for the ground state, but for the excited states as well. This calls for action ; the action taken will be the introduction of color , a degree of freedom not directly observed. A formulation of the color hypothesis and a brief résumé of the evidence for color will make up the second principal topic of this section.

4 . 1. Mesons :

With respect to the flavor-spin symmetry SU(6), the $(q\bar{q})$ meson states lie in the

$$\underset{\sim}{6} \otimes \underset{\sim}{6}^{*} = \underset{\sim}{1} \oplus \underset{\sim}{35} \qquad (4.1)$$

dimensional representations. In the useful notation of Young Tableaux, the arithmetic is



(4.2)

Decomposed with respect to SU(3) $_{\text{flavor}}$ and SU(2) $_{\text{spin}}$, the quark and antiquark states are

$$\underset{\sim}{6} = \{[\underset{\sim}{3}], (\underset{\sim}{2})\}, \qquad (4.3a)$$

$$\underset{\sim}{6}^{*} = \{[\underset{\sim}{3}^{*}], (\underset{\sim}{2})\}, \qquad (4.3b)$$

where the follow our earlier practice (§ 3 .1.) of denoting SU(3) representations by square brackets and SU(2) representations by parentheses. The quark-antiquark product (4.1) is therefore

$$\underset{\sim}{6} \otimes \underset{\sim}{6}^{*} = \{[\underset{\sim}{1}], (\underset{\sim}{1})\} \oplus \{[\underset{\sim}{1}], (\underset{\sim}{3})\} \oplus \{[\underset{\sim}{8}], (\underset{\sim}{1})\} \oplus \{[\underset{\sim}{8}], (\underset{\sim}{3})\}. \qquad (4.4)$$

Consequently the SU(6) $\underset{\sim}{35}$ consists of a spin-triplet nonet (which for the L = 0 ground state is simply the vector mesons $\rho, \omega, K^{*}, \varphi$) and a spin-singlet octet (π, η_8, K). The SU(6) singlet ground state is to be identified with the η_1.

For orbital excitations of the $(q\bar{q})$ systems, it suffices to observe (compare Problem 1) that the discrete quantum numbers are

$$P(q\bar{q})_L = (-1)^{L+1}, \qquad (4.5)$$

$$C(q\bar{q})_L = (-1)^{L+s}, \qquad (4.6)$$

which are of course consistent with the identification of pseudoscalar and vector mesons with the $(q\bar{q})$ ground state. The excited states through L=3 are listed in Table 10, together with the observed meson resonances with which they are identified. Except as otherwise noted, the experimental results are taken from Particle Data Group. (1980). For the isoscalar mesons, the first column contains the dominantly $u\bar{u}+d\bar{d}$ states and the second contains the dominantly $s\bar{s}$ states. In several cases ($^1S_0$, $^3P_0$, $^3D_1$) the amount of mixing is uncertain, and so too are the I=0 assignments. Although numerous openings remain unfilled, the general multiplet structure is quite nicely confirmed by experiment.

Several comments are in order :

(i) The sequence $J^P = 0^+, 1^-, 2^+ \ldots$ for which $P = (-1)^J$ is called natural parity. The $P = -(-1)^J$ sequence of $J^P = 0^-, 1^+, 2^- \ldots$ is known as unnatural parity.

(ii) Some combinations of J^{PC} cannot occur in the $(q\bar{q})$ picture. The state with $J^{PC} = 0^{--}$ would require L=s, to arrive at J=0 , but that implies by (4.6) that C=+1. In addition , the sequence $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \ldots$ must have s=0, in order that CP=−1. This means that J must be equal to L, but then by (4.5) the parity must be unnatural. Hence the sequence cannot be reached in $(q\bar{q})$. These "C-exotic states" would be good signatures for mesons that cannot be accomodated in the $(q\bar{q})$ scheme. None has yet been observed.

(iii) Mixing among the strange particle states with the same J^P (e.g. $^1P_1$, $^3P_1$ or $^1D_2$, $^3D_2$) is not forbidden by the discrete symmetries $\mathbb{C}$ or $\mathbb{G}$. Only SU(3) symmetry breaking is required for the mixing to occur. This phenomenon is observed in the axial strange particles, in which the physical resonances $Q_1(1280)$ and $Q_2(1400)$ are mixtures of the quark-model states Q_A and Q_B (Leith, 1977).

(iv) Radial excitations are also possible, and many are observed in the heavy meson systems known as quarkonium (cf. § 6). It is likely that $\rho'(1600)$ is, or is mixed with , a 2^3S_1 radial excitation of $\rho(776)$, because it is seen prominently in $e^+e^- \to$ hadrons, which suggests a nonvanishing wavefunction at the origin. A $^3D_1$ state would be coupled only weakly to e^+e^-. A similar statement can be made for $\varphi(1634)$ as a radially excited $\varphi(1019)$. A candidate for a radial excitation of the pion, $\pi'(1342) \to \epsilon\pi$ has recently been reported by Bonesini, et al. (1981).

(v) The hyperfine splitting so prominent between the pseudoscalar and vector states is less apparent for the L-excitations. This seems consistent with an elementary picture of hyperfine structure.

(vi) The natural-parity states, including I=1 candidates with $J^P=5^-$ at 2300 MeV/c^2 (Cashmore, 1980) and $J^P=6^+$ at 2515 MeV/c^2 (Cleland, et al., 1980a), lie on linear Regge trajectories of the form

$$J(M^2) = \alpha_0 + \alpha' M^2, \qquad (4.7)$$

as shown in Fig. 19. The data suggest (e.g. Field and Quigg, 1975) the possibility that Regge slopes are not universal, but depend systematically upon the constituent quark masses, as

$$\alpha'_\rho > \alpha'_{K^*} > \alpha'_\varphi > \alpha'_{D^*} > \alpha'_\psi \; . \qquad (4.8)$$

If this trend is not an illusion, it is not obvious whether it is transitory (Igi, 1977) or deeply connected with the flavor-independence of interquark forces (Close, 1981). For a summary of Regge Pole phenomenology , see Collins (1977).

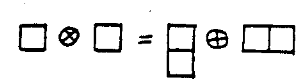
(vii) The scalar mesons do not present an unambiguous picture. While their quantum numbers are those of the $(q\bar{q})$ scheme it is problematical to correlate their masses and decays. Mixing with the vacuum may be pronounced, and may thus be a complicating factor. Jaffe (1977a) has argued that the scalars are in fact $(qq\bar{q}\bar{q})$ states, but at the time of his proposal the s-wave isoscalar state was generally believed to occur at around 700 MeV/c^2. Present fas ion favors 1300 MeV/c^2 and does not as neatly fit the original Jaffe scheme. For additional developments, see Jaffe and Low (1979) and Jaffe (1979).

4 . 2. Baryons :

We now turn to a similarly sketchy review of the baryon resonances. To discuss the SU(6) classification it is necessary to expand the product $\underset{\sim}{6} \otimes \underset{\sim}{6} \otimes \underset{\sim}{6}$. As in Problem 5 for flavorspin SU(4), this will be done explicitly, and in stages. From two quarks, the representations

$$\underset{\sim}{6} \otimes \underset{\sim}{6} = \underset{\sim}{15} \oplus \underset{\sim}{21} \qquad (4.9)$$

may be formed. In Young tableaux, this is



(4.10)

By decomposing $\underset{\sim}{6} \otimes \underset{\sim}{6}$ under $SU(3)_{flavor}$ and $SU(2)_{spin}$,

$$\{[3],(2)\}\otimes\{[3],(2)\} = \{[6],(3)\}\oplus\{[6],(1)\}\oplus\{[3^*],(3)\}\oplus\{[3^*],(1)\}, \tag{4.11}$$

we may identify

$$15 = \{[6],(1)\}\oplus\{[3^*],(3)\}, \tag{4.12}$$

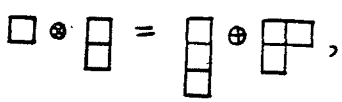
$$21 = \{[6],(3)\}\oplus\{[3^*],(1)\}, \tag{4.13}$$

which reflect the symmetry of the Young tableaux.

The product

$$6\otimes 15 = 20\oplus 70, \tag{4.14}$$

represented as



(4.15)

is expanded as

$$\{[3],(2)\}\otimes\{[6],(1)\} = \{[10],(2)\}\oplus\{[8],(2)\} \tag{4.16}$$

and

$$\{[3],(2)\}\otimes\{[3^*],(3)\} = \{[8],(4)\}\oplus\{[8],(2)\}\oplus\{[1],(4)\}\oplus\{[1],(2)\}. \tag{4.17}$$

This permits the identifications

$$20 = \{[8],(2)\}\oplus\{[1],(4)\} \tag{4.18}$$

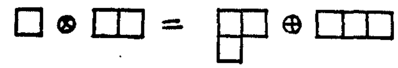
and

$$70 = \{[10],(2)\}\oplus\{[8],(4)\}\oplus\{[8],(2)\}\oplus\{[1],(2)\}. \tag{4.19}$$

Similarly the product

$$6\otimes 21 = 70\oplus 56 \tag{4.20}$$

or



(4.21)

may be expanded as

$$\{[3],(2)\}\otimes\{[6],(3)\} = \{[10],(4)\}\oplus\{[10],(2)\}\oplus\{[8],(4)\}\oplus\{[8],(2)\}, \tag{4.22}$$

and

$$\{[3],(2)\}\otimes\{[3^*],(1)\} = \{[8],(2)\}\oplus\{[1],(2)\}, \tag{4.23}$$

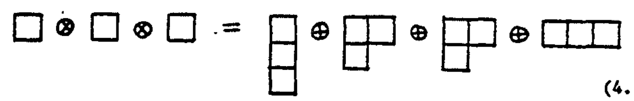
from which we conclude that

$$56 = \{[10],(4)\}\oplus\{[8],(2)\}. \tag{4.24}$$

Three quarks thus lead to the SU(6) representations

$$6\otimes 6\otimes 6 = 20\oplus 70\oplus 70\oplus 56, \tag{4.25}$$

which is written symbolically as



(4.26)

It was found in §3 that the symmetric state 56 describes the ground state baryons. The decomposition (4.24) shows immediately that in the absence of any orbital angular momentum, the 56 contains a spin-$^1/_2$ octet and a spin -$^3/_2$ quartet, as the explicit construction of § 3..1.2 illustrated. Consequently, it is natural to suppose (Greenberg, 1964 ; Dalitz, 1965 ; Greenberg and Resnikoff, 1967), that excited states are given by orbital (or radial) excitations which lead to wavefunctions symmetric under the interchange of any two particles. The flavor-spin (or SU(6)) part of the wavefunction and the spatial part of the wavefunction must therefore have matching symmetry properties.

The permutation group for the three objects has been investigated in Problem 4. For three distinct objects (such as quarks numbered 1, 2, 3), three representations can be constructed. One of these (S) is symmetric under the interchange of any pair. Another (A) is antisymmetric under the interchange of any pair. The last (M) is of mixed symmetry, like the three-quark isospinor wavefunctions of Problem 4. Three-quark wavefunctions with definite permutation symmetry and orbital angular momentum were constructed by Karl and Obryk (1968) for equal-mass quarks. A radial wavefunction is characterized by a polynomial of degree N with specified permutation and rotation (i.e. angular momentum) symmetries times a smooth, symmetric, scalar function of the quark coordinates.

The Karl-Obryk classification of radial wavefunctions has been put to use in constructing Fig. 20, which displays the SU(6) baryon multiplets corresponding to particular values of the degree N, which governs the parity of the state, and angular momentum L. An analogy with atomic physics suggests that baryon masses should be increasing functions of N and L.

Many baryon resonances have been identified as members of these multiplets on the grounds of their masses and their production and decay characteristics. The populations of the first few multiplets are indicated in Table 11, which is based on a similar compilation in Rosner (1981a), supplemented by recent information

(Gopal, 1980 ; Kelly, 1980 ; Kinson, 1980 ; Montanet, 1980). The ground-state $\underline{56}_0{}^+$ is of course completely filled. All of the nucleons and many of the S=-1 hyperons are known for the $\underline{70}_{1}{}^-$. The number of known resonances diminishes rapidly with increasing strangeness. This is undoubtely a consequence of the difficulties in producing hyperons, which must couple to the $\bar{K}N$ entrance channel in formation experiments, and to the general shortcomings of production experiments. There is good reason to hope (Quigg and Rosner, 1976) that high-energy hyperon beams may fill in the gaps, particularly for Ξ - spectroscopy, which seems perpetually in its infancy. See Biagi, et al. (1981).

Absent from Table 11, because no candidates are known, is the $\underline{20}_1{}^+$. There is a specific excuse, in this case, because the $\underline{20}$ cannot be reached in meson-baryon scattering. The product

$$\underline{35} \otimes \underline{56} = \underline{56} \oplus \underline{70} \oplus \underline{700} \oplus \underline{1134} \qquad (4.27)$$

does not contain $\underline{20}$.

There is some evidence for the other N=2 families. Most of the nucleon resonances are known for the $\underline{56}_0{}^+$ and $\underline{56}_2{}^+$ states, and a few candidates can be attributed to the $\underline{70}_0{}^+$ and $\underline{70}_2{}^+$ multiplets (Hey, 1980a). Like the natural parity mesons, the $I={}^1/_2$ and $I={}^3/_2$ nucleon resonances and the Λ states appear to lie on linear Regge trajectories, shown in Fig. 21.

4 . 3. Color at last :

All spectroscopic evidence thus points to the utility of the symmetric quark model as a classification scheme and as a model for the static and dynamical properties of hadrons. Faced with the in-consistency between the model as formulated and the Pauli principle, one has three possible courses of action :

(i) Dismiss the successes of the quark model as coincidence or illusion ;

(ii) Deny that the spin and statistics connection applies to confined particles ;

(iii) Envisage a hitherto unnoticed degree of freedom, in terms of which the wavefunction can be antisymmetrized.

The third course was advocated by Greenberg (1964). Although it seems at first sight arbitrary and extravagant, it has gained experimental support and now forms the foundation of our understanding of the strong interactions.

To reconcile the quark model with the Pauli principle it is necessary that each quark flavor exist in no less than three distinct varieties, so that three-quark wavefunctions can be antisymmetrized. The number of varieties must in fact be precisely three. If there were more, several distinguishable varieties of proton could exist, in violation of common experience and the experimental evidence on specific heats. Other evidence for three species will be produced below. We shall refer to the new degree of freedom as <u>color</u> and label the three colors as

$$C_1 = \text{red} = R, \quad C_2 = \text{blue} = B, \quad C_3 = \text{green} = G. \tag{4.28}$$

If each of the explicit baryon wavefunctions is multiplied by

$$\frac{1}{\sqrt{6}}\,\varepsilon_{ijk}\,C_i C_j C_k = \frac{1}{\sqrt{6}}\,[RBG-RGB+GRB-GBR+BGR-BRG], \tag{4.29}$$

where ε_{ijk} is the antisymmetric three-index symbol (the elements of the antisymmetric representation of the permutation group), the resulting wavefunctions will be antisymmetric with respect to the interchange of any pair of quarks. As an example, consider the Δ^{++} with maximal spin projection, which was in the colorless formulation the most symmetric of states. Its wavefunction is now

$$|\Delta^{++};\, s_z = 3/2\rangle = (uuu)(\uparrow\uparrow\uparrow)\,\varepsilon_{ijk}\,C_i C_j C_k/\sqrt{6}. \tag{4.30}$$

It is useful to consider the idea of a color symmetry group (represented by 3 x 3 matrices $\mathcal{M}_{ij}$) which generates the color transformations

$$C_i \rightarrow C_i' \equiv \mathcal{M}_{ij}\,C_j. \tag{4.31}$$

Under such a transformation, the antisymmetric combination (4.29) becomes

$$\varepsilon_{ijk}\,C_i' C_j' C_k' = \varepsilon_{ijk}\,\mathcal{M}_{im}\mathcal{M}_{jn}\mathcal{M}_{kp}\,C_m C_n C_p. \tag{4.32}$$

If the transformation has unit determinant,

$$|\mathcal{M}| = 1 \tag{4.33}$$

so that

$$\varepsilon_{ijk}\,\mathcal{M}_{im}\mathcal{M}_{jn}\mathcal{M}_{kp} = \varepsilon_{mnp}, \tag{4.34}$$

then

$$\varepsilon_{ijk}\,C_i' C_j' C_k' = \varepsilon_{mnp}\,C_m C_n C_p. \tag{4.35}$$

In other words, the color wavefunction is invariant under color transformations, which is to say that the wavefunction is a color singlet. This would not be the case, had there been more than three colors allowed.

For meson wavefunctions, we assign anticolors to the antiquarks, and construct the color singlet wavefunctions

$$\frac{1}{\sqrt{3}} C_i \bar{C}_i = \frac{1}{\sqrt{3}} \left[R\bar{R} + B\bar{B} + G\bar{G} \right]. \quad (4.36)$$

Depending upon the choice of the color symmetry group, the colors and anticolors may or may not be equivalent. The different implications of the two choices will be commented upon below.

The conclusion of this construction is that a formally consistent picture of mesons and baryons can be obtained by requiring that hadrons be color singlets. What then is the evidence for three quark colors ? A thorough review has been given by Greenberg and Nelson (1977) ; see also Bardeen, et al. (1973). I will therefore be brief.

Apart from the baryon spin and statistics connection, there are two basic observables that rule in favor of the three-color hypothesis. The first category includes the ratio

$$R \equiv \sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-) \quad (4.37)$$

and related quantities. In the quark-parton model (Feynman, 1972), this ratio is given by

$$R = \sum_{\substack{\text{quark} \\ \text{species}}} e_i^2 = N_c \sum e_i^2 , \quad (4.38)$$

where N_c is the number of colors, and the sum runs over the energetically accessible flavors. Thus we predict

$$R(\text{no color}) = \tfrac{2}{3},\ \tfrac{10}{9},\ \tfrac{11}{9} \quad (4.39a)$$

and

$$R(\text{color}) = 2,\ \tfrac{10}{3},\ \tfrac{11}{3} \quad (4.39b)$$

for c.m. energies below the charm, bottom, and top quark thresholds, respectively. The experimental results are shown in Fig. 22 for energies below (a) and above (b) charm threshold. The three-color predictions are decisively favored. Notice that within the framework of quantum chromodynamics, the ratio R receives strong-interaction corrections which are small, positive, and decreasing with c.m. energy.

A related observable is the branching ratio for semileptonic τ decays, which is governed by the branching ratios of the virtual W in the transition $\tau \to \nu_\tau W$. An elementary calculation then leads to

$$B_\ell \equiv \frac{\Gamma(\tau \to \ell \bar{\nu}_\ell \nu_\tau)}{\Gamma(\tau \to \text{all})} = \frac{1}{(1+1+N_c)} = \begin{cases} 1/3 , & N_c = 1, \\ 1/5, & N_c = 3. \end{cases} \quad (4.40)$$

The experimental result (Particle Data Group, 1980),

$$B_\ell = (17.44 \pm 0.85)\% , \quad (4.41)$$

is in accord with the color hypothesis. A refined theoretical estimate (Gilman and Miller, 1978 ; Kawamoto and Sanda, 1978) of the branching ratio, B_ℓ = 17.75%, is in excellent agreement with experiment. It is hoped that direct measurements of the total widths and branching ratios of the intermediate bosons $W^\pm$ and Z^0 will soon be available from high-energy colliders.

The other important measure of the number of quark colors is the $\pi^0 \to \gamma\gamma$ decay rate. A good discussion of the theoretical details is given by Llewellyn Smith (1980). A calculation for π^0 decay via a quark-antiquark loop is straightforward to carry out, although the justification is subtle. The result is

$$\Gamma(\pi^0 \to \gamma\gamma) = \left(\frac{\alpha}{2\pi}\right)^2 \left[N_c\left(e_u^2 - e_d^2\right)\right]^2 \frac{M_\pi^3}{8\pi f_\pi^2} \tag{4.42}$$

where the pion decay constant is normalized so that

$$f_\pi = 130 \text{ MeV}. \tag{4.43}$$

The rate (4.42) is then

$$\Gamma(\pi^0 \to \gamma\gamma) = \begin{cases} 0.86 \text{ eV}, & N_c = 1, \\ 7.75 \text{ eV}, & N_c = 3, \end{cases} \tag{4.44}$$

to be compared with the measured decay rate of (7.86 ± 0.54) eV. We therefore conclude that the hidden color degree of freedom is indeed present. It is tempting then to suppose that color is what distinguishes quarks from leptons, and might play the role of a strong interaction charge.

The mention of charge calls to mind tests of the (average) electric charges of the quarks. In addition to the baryon spectrum itself, there are two checks that are easily explained. The production of massive dileptons in hadron-hadron collisions is viewed (Drell and Yan, 1970, 1971) as the elementary process

$$q\bar{q} \to \gamma \to e^+e^-. \tag{4.45}$$

Consequently the reactions

$$\pi^{\pm} C \to \mu^+\mu^- + \text{anything} \tag{4.46}$$

which entail dilepton production off an isoscalar target correspond to the elementary transitions

$$\pi^+: \quad d\bar{d} \to \mu^+\mu^- \tag{4.47a}$$

$$\pi^-: \quad u\bar{u} \to \mu^+\mu^- \tag{4.47b}$$

The rates for reactions (4.47) are simply proportional to the charge-squared of the annihilating quarks : $^1/_9$ for $\bar{d}d \to \gamma$ and $^4/_9$ for $\bar{u}u \to \gamma$. Thus it is expected that

$$\frac{\sigma(\pi^+ C \to \mu^+\mu^- + \text{anything})}{\sigma(\pi^- C \to \mu^+\mu^- + \text{anything})} = \frac{1}{4}. \tag{4.48}$$

Numerous experiments have by now confirmed this expectation (Pilcher, 1980).

A second, and also model-dependent, test of the quark charges is supplied by the leptonic decay rates of the light vector mesons. As you will show in Problem 13, in a nonrelativistic picture the rate for the decay $V^0 \to e^+e^-$ is given by

$$\Gamma(V^0 \to e^+e^-) = \frac{\text{constant}}{M_V^2} \cdot e_q^2 \, |\Psi(0)|^2, \tag{4.49}$$

where $\Psi(0)$ is the quark wavefunction at zero separation, and e_q^2 is the mean-squared charge of the quarks in V^0. For ρ^0, ω, and φ, this is

$$\langle e_q^2 \rangle_\rho = \left[\frac{1}{\sqrt{2}}\left(\frac{2}{3} + \frac{1}{3}\right)\right]^2 = 1/2, \tag{4.50a}$$

$$\langle e_q^2 \rangle_\omega = \left[\frac{1}{\sqrt{2}}\left(\frac{2}{3} - \frac{1}{3}\right)\right]^2 = 1/18, \tag{4.50b}$$

$$\langle e_q^2 \rangle_\varphi = (1/3)^2 = 1/9. \tag{4.50c}$$

where the ideally mixed wavefunctions (3.5) have been used. If $|\Psi(0)|^2$ is the same for all three of these states, the reduced leptonic widths

$$\tilde{\Gamma}_V \equiv M_V^2 \, \Gamma(V^0 \to e^+e^-) \tag{4.51}$$

will be in the ratio

$$\tilde{\Gamma}_\rho : \tilde{\Gamma}_\omega : \tilde{\Gamma}_\varphi :: 9 : 1 : 2. \tag{4.52}$$

This expectation compares favorably with the experimental result

$$(8.7 \pm 2.9) : 1 : (2.8 \pm 0.8). \tag{4.53}$$

Experimental results are therefore consistent with the canonical charge assignments.

Showing the uniqueness of the fractionally charged quark model is another matter, however. An alternative scheme and some possible experimental distinctions are the subject of Problem 14. The most decisive evidence for fractionally charged quarks comes from an analysis of the decay rate for $\eta' \to \gamma\gamma$ (Chanowitz, 1980).

5. Some Implications of QCD :

In this section we begin to explore the ways in which color can be more than a mere label which serves as a convenient cure for the theoretical ills of the symmetric quark model. Color has been seen in § 4 . 3 to be a hidden quantum number which is manifested, albeit indirectly, in experimental observables. If hadrons must be color singlets, then the nonoccurrence of stable diquarks and other exotic configurations can be understood. In order to understand why hadrons must be color singlets, it is necessary to give color a dynamical standing. This is easily done, at least at a conceptual level, by regarding color as a local gauge symmetry. To do so, it is necessary to choose a color group and so we shall review some arguments in favor of SU(3) color.

Once the theory of colored quarks interacting by means of colored gluons, namely quantum chromodynamics, has been formulated, it is desirable to derive its consequences. This is more easily said than done. The lectures here at Les Houches by Brower (1981) take stock of efforts to solve pure or sourceless QCD on the lattice. All such efforts have until now been restricted to configurations of pure glue, or static configurations of infinitely massive quarks. A direct computation of the hadron spectrum thus lies in the future, although considerable progess appears to have been made recently. In the absence of a complete solution to QCD it is necessary to proceed by means of schematic partial calculations and pictures abstracted from general principles. We shall first verify in the framework of a straightforward "maximally attractive channel" analysis the plausibility of the idea that color singlets are preferred configurations. Then we shall indicate how a string picture might emerge from

QCD, and why it provides an appealing scheme for the spectrum of light hadrons. With those two results in hand, we shall assume that color singlets are confined and investigate QCD-inspired descriptions of the fine and hyperfine structure of hadron spectra.

Discussion of the light-hadron spectrum will be carried out at what seems to be an essentially nonrelativistic level. This is justified in the first instance by faith and subsequently by the results. Many consequences appear likely to survive the transition to a relativistic theory. Indeed, much of the arithmetic to be carried out is common to the nonrelativistic and MIT bag approaches, so I will not meticulously observe the distinctions. A simple implementation of the nonrelativistic reduction of one-gluon exchange gives a generally good account of the ground-state hadrons. It resolves some difficulties of the elementary model of §3, notably the issue of Λ-Σ splitting and the problem of the neutron charge radius. Isoscalar pseudoscalars remain a source of puzzlement, but a systematic understanding of hyperfine splittings is achieved. This success stimulates the extension of the evolving model to multiquark hadrons, lightly bound by the hyperfine interaction alone.

Extensive work has been devoted to the development of QCD-inspired pictures that may give comprehensive descriptions of the complete spectrum of light hadrons. These pictures contain much that is arbitrary, and the implications for QCD of their successes and failures are at best indirect. To the extent that they bring order and understanding to a rich spectrum of hadrons, they command our attention. The history of physics in general and of the quark model in particular contains many reminders that the simplest picture often teaches the most.

Many excellent reviews deal with aspects of QCD that are relevant to the topics in this section. In addition to the specific articles cited below, the following works contain material of general interest : Appelquist (1978), Bjorken (1980), Close (1980), Feynman (1977), Fritzsch (1979), Marciano and Pagels (1978, 1979), Morgan (1981), Quigg (1981), Rosner (1981a), and Wess (1981).

5. 1. Toward QCD :

Having noticed the possibility that color may function as the charge of strong interactions, it is natural to seek to formulate a dynamical theory based on color symmetry. In the present climate it is obvious that what is required is a color gauge theory. Early steps in this direction were taken by Nambu (1966). It was recognized by Greenberg and Zwanziger (1966), and later emphasized by Lipkin (1973, 1979b), and by Fritzsch, Gell-Mann, and Leutwyler (1973), that such a theory might provide a basis for understanding the simple rules that mesons are $(q\bar{q})$ states and baryons are (qqq) states. Let us see why SU(3) is a promising choice for the color symmetry group.

5.1.1. Choosing the Gauge Group :

When we entertain the possibility that the color quantum number reflects a continuous symmetry of the strong-interaction Lagrangian, three candidates for the symmetry group come immediately to mind : SO(3), SU(3), and U(3). Simple arguments discourage the use of SO(3) and U(3), as we shall now see.

Color SO(3) was considered long ago by Tati (1966). In SO(3) there is no difference between color and anticolor, so in the computation of forces there will be no distinction between quarks and antiquarks. The existence of $(q\bar{q})$ mesons thus implies the existence of (qq) diquarks, which will be fractionally charged. Because fractionally charged matter appears to be less commonplace than ordinary mesons, SO(3) does not seem an apt choice for the color symmetry group. One may also be concerned that the asymptotic freedom of SO(3) gauge theory is less secure that that of SU(3). Recently Slansky, Goldman, and Shaw (1981) have proposed that $SU(3)_{color}$ QCD is spontaneously broken down to an SO(3) subgroup to which they refer as "glow," liberating fractionally charged diquarks. Whether this can be accomplished in a manner compatible with experimental limits on the production of fractionally charged objects without upsetting the possibility of grand unification below the Planck mass is left as an exercise for the student.

In U(3) color gauge theory, the color singlet gauge boson which occurs in the product

$$[\underset{\sim}{3}] \otimes [\underset{\sim}{3}^*] = [\underset{\sim}{1}] \oplus [\underset{\sim}{8}], \qquad (5.1)$$

in SU(3) notation, cannot be dispensed with. It would mediate long-range strong interactions between color singlet hadrons, and is thus ruled out by experiment. Thus U(3) is excluded, and we are left with SU(3) as a candidate gauge group.

5. 1. 2. The QCD Lagrangian :

The possibility of constructing a theory based on a local gauge symmetry more complicated than the U(1) phase symmetry of electromagnetism was demonstrated by Yang and Mills (1954), (see also Shaw, 1955) for the flavor-SU(2) symmetry isospin. The problem of extending the Yang-Mills construction to a general gauge symmetry was dealt with by Gell-Mann and Glashow (1961). It is, as the literature attests, one thing to write down a mathematically consistent gauge theory and quite another to choose a gauge symmetry that leads to experimentally acceptable consequences. Having sounded that note of humility, let us formulate the gauge theory of color-triplet quarks that interact by means of vector gluons which belong to the octet representation of color -SU(3). The Lagrangian will have the standard Yang-Mills form,

$$\mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \bar{\psi}_{\alpha}\left(i\gamma^{\mu}\mathcal{D}^{\alpha\beta}_{\mu} - m\delta^{\alpha\beta}\right)\psi_{\beta}, \tag{5.2}$$

where α and β, the color indices for the quark fields ψ, run over the values 1, 2, 3 or R, B, G and $a = 1, 2, \ldots, 8$ is the gluon color label. The field-strength tensor is given by

$$G_{\mu\nu} = (ig)^{-1}\left[\mathcal{D}_{\mu}, \mathcal{D}_{\nu}\right] = \frac{\lambda^{a}}{2} G^{a}_{\mu\nu}, \tag{5.3}$$

and the gauge-covariant derivative is

$$\mathcal{D}_{\mu} = I\partial_{\mu} - \frac{ig}{2}\lambda^{a} B^{a}_{\mu}, \tag{5.4}$$

where B^{a}_{μ} is the color gauge field (the gluon field) and the λ^{a} are the eight 3×3 matrix representations of the $SU(3)_c$ octet :

$$\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{matrix} \bar{R} \\ \bar{B} \\ \bar{G} \end{matrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\begin{matrix} R & B & G \end{matrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{5.5}$$

These are of course the matrices familiar from flavor SU(3), for which the indices 1, 2, 3, correspond to quark flavors u, d, s. In that case the matrices $\lambda_1, \lambda_2, \lambda_3$ are proportional to the generators of $SU(2)_{isospin}$.

With the normalization adopted in eqn. (5.5), the λ-matrices have a number of simple properties, including

$$\mathrm{tr}(\lambda^{a}) = 0, \tag{5.6}$$

$$\mathrm{tr}(\lambda^{a}\lambda^{b}) = 2\delta^{ab}, \tag{5.7}$$

and

$$[\lambda^{a}, \lambda^{b}] = 2if^{abc}\lambda^{c}. \tag{5.8}$$

Using (5.7) and (5.8), it is easy to compute the structure constants

$$f^{abc} = (4i)^{-1}\,\mathrm{tr}\left(\lambda^{c}[\lambda^{a}, \lambda^{b}]\right). \tag{5.9}$$

The field-strength tensor is then

$$G^a_{\mu\nu} = \partial_\nu B^a_\mu - \partial_\mu B^a_\nu - g f^{abc} B^b_\nu B^c_\mu . \qquad (5.10)$$

Although the normalization adopted in (5.5) is a canonical choice, it is not universally employed. For example, Buras (1980) follows a different and also widespread convention in his review of QCD corrections beyond leading order. Forewarned is forearmed !

The form of the QCD Lagrangian (5.2) may compared with that of the QED Lagrangian,

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi , \qquad (5.11)$$

for which

$$\mathcal{D}_\mu = \partial_\mu - i q A_\mu , \qquad (5.12)$$

for fermions of electric charge q, and the field-strength tensor has the familiar form

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu , \qquad (5.13)$$

characteristic of an Abelian gauge theory. The essential difference between the theories is the existence of gluon-gluon interactions in QCD, which contrasts with the absence of photon-photon interactions in QED.

5. 2. Consequences for the Hadron Spectrum :

5. 2. 1. Stability of Color Singlets :

Knowing the QCD Lagrangian, we may now study the properties of the interactions among quarks, at least in a very simplified-perhaps oversimplified-fashion. The point of the exercise is to verify

that color singlets enjoy a preffered status. This encourages the hope that the spectrum of QCD, when it is computed, will display the systematics that inspired the invention of the theory.

The quark-gluon interaction term in the QCD Lagrangian is

$$\mathcal{L}_{inter} = \frac{g}{2} \bar{\psi} \lambda^a \gamma^\mu B^a_\mu \psi , \qquad (5.14)$$

in matrix notation with

$$\psi \equiv \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix} . \qquad (5.15)$$

The Feynman rule for the quark-quark gluon vertex is given in Fig. 23. Thus, the one-gluon-exchange force between quarks is proportional to

$$\mathcal{F} \equiv \frac{1}{4} \sum_a \lambda^a_{\alpha\beta} \lambda^a_{\gamma\delta} \qquad (5.16)$$

for the transition $\alpha + \gamma \to \beta + \delta$. We shall take the quantity (5.16) as representative of the interaction energy between quarks, and proceed to deduce the consequences of QCD for the hadron spectrum, according to that measure. More heuristic, but completely equivalent treatments may be found in the lectures by Feynman (1977) and Quigg (1981).

To compute the interaction it is necessary to evaluate the expectation value of products such as $\frac{1}{4} \underset{\sim}{\lambda}^{(1)} \cdot \underset{\sim}{\lambda}^{(2)}$, where the superscripts label the interacting quarks, and the $\underset{\sim}{\lambda}^{(i)}$ are 8-vectors in color space. The SU(N) techniques are quite standard. An explicit and accessible reference is Rosner (1981), to whose notation I will adhere. Jaffe (1977b) uses a different normalization convention. It will save writing to define the SU(3) generators

$$\underset{\sim}{I} \equiv \frac{1}{2} \underset{\sim}{\lambda} \tag{5.17}$$

and to evaluate the expectation value $\langle \underset{\sim}{I}^2 \rangle$ in various representations of interest.

In SU(N), it is equivalent to average the square of any single generator over the representation, or to perform the sum over the all generators. The former tactic is simpler, and it is particularly convenient to choose I_3 the third component of isospin in the flavor analogy, as the designated generator. Consequently the expectation value $\langle \underset{\sim}{I}^2 \rangle$ in a representation of dimension d is

$$\langle \underset{\sim}{I}^2 \rangle_d = (N^2-1) \sum_{\substack{\text{members} \\ \text{of rep.}}} I_z^2 / d \tag{5.18}$$

where N^2-1 is the number of generators of SU(N). Results for the low-dimensioned representations of SU(3) are given in Table 12. To evaluate $\langle \underset{\sim}{I}^{(1)} \cdot \underset{\sim}{I}^{(2)} \rangle$, we use the familiar identity

$$\langle \underset{\sim}{I}^{(1)} \cdot \underset{\sim}{I}^{(2)} \rangle = \frac{\langle \underset{\sim}{I}^2 \rangle - \langle \underset{\sim}{I}^{(1)2} \rangle - \langle \underset{\sim}{I}^{(2)2} \rangle}{2} \tag{5.19}$$

The "interaction energies" for two-body systems composed of quark-quark and quark-antiquark are given in Table 13. For the $(q\bar{q})$ systems, the one-gluon-exchange contribution is attractive for the color singlet but repulsive for the color octet. Similarly for diquark systems the color triplet is attracted but the color sextet is repelled. Of all the two-body channels, the color singlet $(q\bar{q})$ is the most attractive. On the basis of this analysis, one may choose to believe that colored mesons should not exist , whereas color singlets should be found.

To analyze three (or more) -body systems, let us assume that the interaction is merely the sum of two-body forces, so that

$$\mathcal{E} = \sum_{i<j} \langle \underset{\sim}{I}^{(i)} \cdot \underset{\sim}{I}^{(j)} \rangle , \tag{5.20}$$

which is easily computed as

$$2 \sum_{i<j} \langle \underset{\sim}{I}^{(i)} \cdot \underset{\sim}{I}^{(j)} \rangle = \langle \underset{\sim}{I}^2 \rangle - \sum_i \langle \underset{\sim}{I}^{(i)2} \rangle . \tag{5.21}$$

For three-quark systems, the results in Table 13 show that the color singlet is again the most attractive channel. This is as desired.

Several potentially important effects have been neglected in these calculations :

(i) Multiple gluon exchanges between quarks have been ignored, and we have put forward no arguments for faith in lowest-order perturbation theory.

(ii) Configurations involving the three-gluon vertex, such as the (qqq) color-singlet shown in Fig. 24, have not been taken into account. This is related to the incompleteness of the calculation noted in (i).

(iii) What may be the most serious shortcoming of the toy calculation is its neglect of the energetics associated with the creation of an isolated color non-singlet state. In § 7 we shall review a plausibility argument that implies an infinite cost in energy to isolate a colored system. If that is so, the attraction provided by one-gluon exchange will be insufficient to bind colored states.

In spite of these shortcomings, the elementary calculation we have just completed does make it plausible that color singlets are energetically favored states. In addition, it is easy to see that there is no long-range interaction with color singlets. As an example, consider whether a quark is bound to a baryon. The final entry in Table 13 shows that the interaction energy of the quark .

plus baryon system is precisely that which binds the baryon, with no additional attraction.

It is quite generally believed, but not proved, that QCD is in fact a confining theory, and that colored objects cannot be liberated. Seeking proofs, loopholes, and interpretations of the experimental indications (La Rue, et al., 1981) for fractionally-charged matter is an occupation of key importance.

5. 2. 2. The String Picture of Hadrons :

Suppose that the interaction among quarks is so strong at large distances that a $(q\bar{q})$ pair is always created when the quarks are widely separated, as depicted in Fig. 25. By analogy with the hadronic clusters typically inferred from experiments on multiple production (Dremin and Quigg, 1978), it is reasonable to expect that a quark is accompanied by an antiquark in a typical hadron of mass $\sim 1\ \mathrm{GeV}/c^2$ at a separation of ~ 1fm. That would imply that between every quark and antiquark there is a linear energy density of order

$$\begin{aligned} k &= \Delta E/\Delta r \approx 1\ \mathrm{GeV/fm} \\ &\approx 0.2\ \mathrm{GeV}^2 \approx 5/\mathrm{fm}^2 . \end{aligned} \tag{5.22}$$

This picture is supported by the evidence for linear Regge trajectories of the light hadrons, which have already been displayed in Figs. 19 and 21. For the families of hadrons composed entirely of light quarks, the Regge trajectories are given by

$$J(M^2) = \alpha_0 + \alpha' M^2 , \tag{5.23}$$

with

$$\alpha' \approx (0.8 - 0.9)\ (\mathrm{GeV}/c^2)^{-2} . \tag{5.24}$$

The connection between linear energy density and the linear Regge trajectories is provided by the string model formulated by Nambu (1974).

Consider a massless quark and antiquark connected by a string of length to which is characterized by an energy density per unit length k. The situation is sketched in Fig. 26. For a given value of the length r_0, the largest achievable angular momentum L occurs when the ends of the string move with the velocity of light. In this circumstance , the speed at any point along the string will be

$$\beta(r) = 2r/r_0 . \tag{5.25}$$

The total mass of the system is then

$$\begin{aligned} M &= 2\int_0^{r_0/2} dr\ k\,(1-\beta(r)^2)^{-1/2} \\ &= k r_0 \pi/2 , \end{aligned} \tag{5.26}$$

while the orbital angular momentum of the string is

$$\begin{aligned} L &= 2\int_0^{r_0/2} dr\ k r \beta(r) c\,(1-\beta(r)^2)^{-1/2} \\ &= k c r_0^2 \pi/8 . \end{aligned} \tag{5.27}$$

Using the fact (5.26) that $r_0^2 = 4M^2/k^2\pi^2$, we find that

$$L = M^2/2\pi k , \tag{5.28}$$

which corresponds to a linear Regge trajectory, with

$$\alpha' = 1/2\pi k. \tag{5.29}$$

This connection yields

$$k = \begin{cases} 0.18 \ \mathrm{GeV}^2 \\ 0.20 \ \mathrm{GeV}^2 \end{cases} \quad \text{for} \quad \alpha' = \begin{cases} 0.9 \ \mathrm{GeV}^{-2} \\ 0.8 \ \mathrm{GeV}^{-2}, \end{cases} \tag{5.30}$$

consistent with our heuristic estimate of the energy density. Thus we see that a linear energy density implies linearly rising Regge trajectories, and that the connection makes quantitative sense.

How does a linear energy density arise in quantum chromodynamics ? In fact, I am not certain that it does, but the tendency is indicated in perturbation theory, and there is some similarity between hadron strings and pinned magnetic flux lines in superconductors (Nielsen and Olesen, 1973). [It now seems apparent (Mandelstam, 1979) that the analogy is more properly between the chromoelectric field and the magnetic field. See also the discussion in §7.1.] In ordinary quantum electrodynamics, the pattern of equipotentials and electric field lines is similar at large and small separations between charges. QCD at short distances exhibits a nearly identical flux pattern. At larger separations, a collimated flux tube begins to emerge in QCD. The mechanism for attraction among the chromoelectric flux lines resembles the Biot-Savart effect in classical electrodynamics, the attraction between like currents. Whether this suggestion of an approach to the string picture persists to very large separations is an interesting question.

5. 3. A Picture of Hadron Masses :

In their paper with the perhaps too ambitious title "Hadron masses in a gauge theory," De Rújula, Georgi, and Glashow (1975) proposed to take seriously the proposition that QCD has something to say about hadron masses. Although it would be exaggeration to

claim that the mass of any hadron has yet been computed in QCD, it is plain that QCD-inspired models have provided many important insights into the pattern of hadron masses. The picture put forward by De Rujula, Georgi, and Glashow was an early and influential example of this genre. Their proposal (which is thoroughly discussed in the SLAC Summer School lectures by Jackson, 1976) is that all spin effects arise from the short-distance one-gluon-exchange interaction expected in QCD. This may seem an extreme position and indeed it can readily be relaxed. Let us however see where it leads.

The Fermi-Breit Hamiltonian (toward which Problems 15 - 17 lead) can be transcribed from the familiar QED case. For s-wave states, the structure will be the same as that of eqn. (2.40) which was used to describe hadron electromagnetic mass differences in § 2.3. The "interaction energies" which appeared in the discussion of the stability of color singlets in § 5.2.1 can be recast as Coulomb potentials. With the introduction of a strong coupling constant α_s, we write

$$V(r) = \frac{\alpha_s}{4r} \langle \underset{\sim}{\lambda}^{(1)} \cdot \underset{\sim}{\lambda}^{(2)} \rangle . \tag{5.31}$$

In the specific cases of interest for two-body interactions, this yields

$$V_{q\bar{q} \in [\underset{\sim}{1}]} = -\frac{4}{3}\frac{\alpha_s}{r}, \tag{5.32a}$$

$$V_{q\bar{q} \in [\underset{\sim}{8}]} = \frac{1}{6}\frac{\alpha_s}{r}, \tag{5.32b}$$

$$V_{qq \in [\underset{\sim}{3}^*]} = -\frac{2}{3}\frac{\alpha_s}{r}, \tag{5.32c}$$

$$V_{qq\in[\underset{\sim}{6}]} = \frac{1}{3}\frac{\alpha_s}{r}. \tag{5.32d}$$

We shall assume that the explicit effect of this Coulomb interaction, which is the same for all quark flavors, can be subsumed into the consequences of confinement. Of more immediate concern is the color hyperfine interaction, which takes the form

$$\Delta E_{HFS} = \frac{-\pi\alpha_s}{6m_1m_2}|\Psi_{12}(0)|^2\langle \underset{\sim}{\lambda}^{(1)}\cdot\underset{\sim}{\lambda}^{(2)}\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2\rangle \tag{5.33}$$

between two quarks in a relative s-wave. Thus, for $q\bar{q}$ in a color singlet, relevant for mesons, we have

$$\Delta E_{HFS}(q\bar{q}\in[\underset{\sim}{1}]) = \frac{8\pi\alpha_s}{9m_1m_2}|\Psi_{12}(0)|^2\langle\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2\rangle, \tag{5.34}$$

while for qq in a color antitriplet, relevant for baryons, we have

$$\Delta E_{HFS}(qq\in[\underset{\sim}{3}^*]) = \frac{4\pi\alpha_s}{9m_1m_2}|\Psi_{12}(0)|^2\langle\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2\rangle. \tag{5.35}$$

The intrinsic strength of the color hyperfine interaction is thus half as large in baryons as in mesons.

5. 3. 1. The Light Hadrons :

We recall that in mesons

$$\langle\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2\rangle = \begin{cases}-3, & s=0,\\ 1, & s=1,\end{cases} \tag{2.26}$$

so that the color hyperfine interaction is attractive for pseudoscalars and repulsive for vector mesons. The inverse factors of mass in (5.34) mean that the hyperfine splitting will be smaller between K and K^* than between π and ρ.

In the case of baryons, we have already computed

$$\langle\sum_{i<j}\underset{\sim}{\sigma}_i\cdot\underset{\sim}{\sigma}_j\rangle = \begin{cases}-3, & s=\frac{1}{2},\\ 3, & s=\frac{1}{2}.\end{cases} \tag{2.29}$$

We therefore expect states in the decimet to be more massive than those in the octet. This expectation and the corresponding one for the mesons are in accord with observation. Two other qualitative facts are also correlated with these by the Ansatz (5.33).

First, consider the problem of the Λ-Σ splitting, which electromagnetic considerations decisively failed to explain in § 3 .2. In the $J^P = \frac{1}{2}^+$ Σ-hyperon, the two nonstrange quarks are in a configuration with I=1, color $[\underset{\sim}{3}^*]$, and angular momentum = 0. Under interchange of the two quarks, this state is symmetric × antisymmetric × symmetric = antisymmetric. The spin part of the wavefunction must therefore be symmetric, or in other words s = 1. Thus by (2.26) we conclude that

$$\langle\underset{\sim}{\sigma}_n\cdot\underset{\sim}{\sigma}_n\rangle_\Sigma = 1, \tag{5.36}$$

where the subscript n stands for non-strange. Because according to (2.29) the quantity

$$\langle\sum_{i<j}\underset{\sim}{\sigma}_i\cdot\underset{\sim}{\sigma}_j\rangle_\Sigma = \langle\underset{\sim}{\sigma}_n\cdot\underset{\sim}{\sigma}_n\rangle_\Sigma + 2\langle\underset{\sim}{\sigma}_n\cdot\underset{\sim}{\sigma}_s\rangle_\Sigma = -3, \tag{5.37}$$

evidently

$$\langle\underset{\sim}{\sigma}_n\cdot\underset{\sim}{\sigma}_s\rangle_\Sigma = -2 \tag{5.38}$$

The hyperfine shift in the Σ mass is therefore

$$\Delta E_{HFS}(\Sigma) = \frac{4\pi\alpha_s|\Psi(0)|^2}{9}\left[\frac{1}{m_n^2} - \frac{4}{m_nm_s}\right], \tag{5.39}$$

where we have assumed that $|\Psi(0)|^2$ is the same for any pair of quarks. This cannot be quite right, but we may hope it is not misleading.

In contrast, for the Λ, the nonstrange quarks are in an isoscalar, color-antitriplet, s-wave state, which is antisymmetric $\times$ antisymmetric $\times$ symmetric = symmetric under particle interchange. The spin state must therefore be antisymmetric, s = 0, for which

$$\langle \underset{\sim}{\sigma}_n \cdot \underset{\sim}{\sigma}_n \rangle_\Lambda = -3 , \tag{5.40}$$

by eqn. (2.26). Again according to (2.29) we know that

$$\langle \sum_{i<j} \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \rangle_\Lambda = \langle \underset{\sim}{\sigma}_n \cdot \underset{\sim}{\sigma}_n \rangle_\Lambda + 2\langle \underset{\sim}{\sigma}_n \cdot \underset{\sim}{\sigma}_s \rangle_\Lambda = -3 , \tag{5.41}$$

we deduce that

$$\langle \underset{\sim}{\sigma}_n \cdot \underset{\sim}{\sigma}_s \rangle_\Lambda = 0 . \tag{5.42}$$

As a consequence, the hyperfine shift in the Λ-mass is

$$\Delta E_{HFS}(\Lambda) = \frac{4\pi\alpha_s |\Psi(0)|^2}{9} \left[\frac{-3}{m_n^2}\right] . \tag{5.43}$$

This is more negative than the Σ hyperfine shift, so long as $m_s > m_n$. This is another qualitative success.

The second conspicuous failure of the quark model described in § 2 and 3 was the prediction (2.39) of the vanishing of the neutron's charge radius. In this case as well the color hyperfine interaction provides some enlightenment (Carlitz, Ellis and Savit, 1977). The two down quarks in the neutron must be in an I = 1, color $[\underset{\sim}{3}^*]$, angular momentum zero state, which is symmetric $\times$ antisymmetric $\times$ symmetric = antisymmetric under particle underchange. They must therefore be in a symmetric spin-triplet state, for which the hyperfine interaction is repulsive. Since the overall hyperfine

interaction is attractive, the up-down pairs must attract. Hence the up quark will be drawn to the center of the neutron, while the down quarks are pushed toward the periphery. As a result the neutron's mean-squared charge radius will be negative, in agreement with experiment (Krohn and Ringo, 1973 ; Berard, et al., 1973 ; Borkowski, et al., 1974 ; Koester, et al., 1976). One may, at the price of additional assumptions, attempt to estimate the ratio $\langle r^2_{EM}\rangle_n / \langle r^2_{EM}\rangle_p$. This has been done with reasonable success by Carlitz, et al. (1977) and by Isgur, Karl, and Koniuk (1978). Isgur, Karl, and Sprung (1981) have gone yet further and produced a fit to $G_E^n(q^2)$ within the framwork of a harmonic oscillator model for the confining interaction.

Notice that the mean-squared magnetic radius, defined with respect to the magnetic form factor as

$$\langle r^2_{mag} \rangle = -\partial G_M(q^2)/\partial q^2 \Big|_{q^2=0} \tag{5.44}$$

need not be identical to the charge radius, because the contributions of the quarks are weighted differently. An estimate of the ratio $\langle r^2_{mag}\rangle_p / \langle r^2_{charge}\rangle_p$ has been made by Carlitz, et al. (1977).

The idea of a color hyperfine interaction has been shown to yield a qualitative understanding of the pseudoscalar-vector splitting, the octet-decimet splitting, the Λ-Σ splitting, and the neutron charge radius. Can it also give a quantitative description of the masses of the ground-state hadrons ? To examine this question, we ignore electromagnetic mass differences which are easily restored according to the procedure followed in § 2 .3, 2 .4, 3 .1, and 3 .2.

We first consider baryons. Let us write

$$M = M_o + N_s(m_s - m_u) + m_u^2 \left\langle \sum_{i<j} \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \right\rangle \delta M_{c.m.}, \quad (5.45)$$

where the chromomagnetic hyperfine shift is given by

$$\delta M_{c.m.} = 4\pi\alpha_s |\Psi(0)|^2 / 9m_u^2 . \quad (5.46)$$

M_o is the common unperturbed mass of the octet and decimet baryons, and N_s is the number of strange quarks in a baryon. The hyperfine and strange-quark mass shifts are given in Table 14.

Instead of making a global fit, we determine the parameters as follows :

$$M_o = (M_N + M_\Delta)/2 = 1085.5 \quad MeV/c^2, \quad (5.47)$$

$$\delta M_{c.m.} = (M_\Delta - M_N)/6 = 48.83 \ MeV/c^2, \quad (5.48)$$

$$(m_s - m_u) = (M_\Sigma + M_\Lambda + M_{\Sigma^*})/4 - M_o = 183.75 \, MeV/c^2. \quad (5.49)$$

If we now interpret M_o as three times the up-quark mass, we have

$$m_u = M_o/3 = 361.83 \ MeV/c^2 \quad (5.50)$$

and

$$m_s = 545.6 \quad MeV/c^2. \quad (5.51)$$

These numbers are reasonably consistent with the values of 338 and 510 MeV/c^2 deduced from the fit to baryon magnetic moments in § 3 .4.1. They will enter our calculation of baryon masses only in the ratio and difference, and the results would be effectively

unchanged if we adopted the smaller values.

Overall the agreement between the model and experiment is good, and would be improved slightly if we were to determine parameters by making a global fit. The strange particle hyperfine splittings are just slightly too small, and the m_s-m_u mass difference is slightly too large. These defects are of course correlated. What has been achieved is a unified understanding of the Λ-Σ splitting and the splitting between the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon multiplet This has been done in spite of the fact that we have somewhat cavalierly ignored the possibility of variations in kinetic energies, binding energies, and the wavefunction at the origin. These effects probably tend to reduce the discrepancies we have noted. See, for example, Cohen and Lipkin (1981).

Using the definition (5.46) for the chromomagnetic level shift and the somewhat casual inference from electromagnetic mass splittings in the baryon octet that

$$|\Psi(0)|^2 \simeq 0.0116 \quad GeV^3, \quad (3.77)$$

it is straightforward to compute that

$$\alpha_s \simeq 0.4. \quad (5.52)$$

This value is large enough to be regarded as a strong interaction coupling constant, but not so large as to make the one-gluon-exchange picture seem entirely ridiculous.

A similar analysis can be carried out for the mesons, for which we write

$$M = M_o + N_s(m_s - m_u) + m_u^2 \left\langle \frac{\sigma_q \cdot \sigma_{\bar{q}}}{m_q m_{\bar{q}}} \right\rangle \delta M_{c.m.}, \quad (5.53)$$

where the color-magnetic hyperfine shift is

$$\delta M_{c.m.} = 8\pi\alpha_s|\Psi(0)|^2/9\,m_u^2. \tag{5.54}$$

The hyperfine and strange-quark mass shifts are given in Table. 15. The strange-quark content of the η and η' are determined from the wavefunctions (3.112) deduced from the linear mass formula in §. 3.3.2.

To fit parameters, we proceed as for the baryons :

$$M_0 = (M_\pi + 3M_\rho)/4 = 617\ \mathrm{MeV}/c^2, \tag{5.55}$$

$$\delta M_{c.m.} = (M_\rho - M_\pi)/4 = 160\ \mathrm{MeV}/c^2, \tag{5.56}$$

$$(m_s - m_u) = (M_K + 3M_{K^*})/4 - M_0 = 176\ \mathrm{MeV}/c^2. \tag{5.57}$$

If M_o is interpreted as twice the up-quark mass, we have

$$m_u = M_0/2 = 308.5\ \mathrm{MeV}/c^2, \tag{5.58}$$

and

$$m_s = 484\ \mathrm{MeV}/c^2. \tag{5.59}$$

These numbers, like those deduced for the baryons in (5.50) and (5.51), lie within 10% of the values inferred from baryon magnetic moments. Given that we are playing fast and loose with the idea of a quark mass, the agreement seems quite satisfying.

The fitted masses agree rather well with experiment except for the η and η' masses for which the predictions are disastrous. These states are too heavy for the interpretation of masses that we have given. This problem persists whether or not the pion is regarded as exceptionally light. It is unreasonable in the present framework for the η to outweigh the kaon. A possible description of this phenomenon in terms of communication with (pure gluon) quark-

less channels will be examined in § 8.3 .

Meson electromagnetic mass differences permit a rather crude determination of the mesonic wavefunction at the origin as

$$|\Psi(0)|^2 \approx 0.017\ \mathrm{GeV}^3, \tag{5.60}$$

One may therefore estimate

$$\frac{\delta M_{c.m.}(\text{meson})}{\delta M_{c.m.}(\text{baryon})} \approx \frac{2|\Psi_{\text{meson}}(0)|^2}{|\Psi_{\text{baryon}}(0)|^2} \approx 2.93, \tag{5.61}$$

which compares favorably with the ratio of the values (5.56) and (5.48) , which is 3.28 . The similarity of these numbers is very tantalizing. It underscores the desirability of better experimental determinations of electromagnetic mass differences, which would justify a less casual analysis than I have made here.

5. 3. 2. Extension to Charm and Beauty :

Mesons and baryons may also be formed from the heavy quarks c (charm), b (beauty), and t (truth)—if it exists—either alone or in combination with the light quarks. Characteristics of the heavy quarks are listed in Table 16. The resulting particles may be classified according to an enlarged , and badly broken, flavor symmetry group $SU(6)_{\text{flavor}}$. To display the weight diagrams, it is convenient to decompose

$$SU(6)_{\text{flavor}} \rightarrow SU(4)_{udsc} \otimes U(1)_b \otimes U(1)_t\ . \tag{5.62}$$

The 36 species of mesons are exhibited in Fig. 27. For ground-state baryons, there are 70 spin-$^1/_2$ states, illustrated in Fig. 28, and 56 spin-$^3/_2$ states, shown in Fig. 29. The classic introduction to to charm spectroscopy is the preview by Gaillard, Lee, and Rosner (1975). Some details of the flavor-SU(4) symmetry may be found in the lectures by Einhorn (1975), and in my lectures at the Gomel summer school (Quigg, 1979). See also Rosner (1981).

For charmed mesons, eqn. (5.53) is immediately generalized to

$$M = M_0 + N_s(m_s - m_u) + N_c(m_c - m_u) + m_u^2\left\langle\frac{\sigma_q\cdot\sigma_{\bar{q}}}{m_q m_{\bar{q}}}\right\rangle \delta M_{c.m.} \tag{5.63}$$

where N_c counts the number of charmed quarks. It is then straightforward to evaluate the charmed-quark mass as

$$(m_c - m_u) = (3M_{D^*} + M_D)/4 - (3M_\rho + M_\pi)/4$$
$$= 1354.5 \text{ MeV}/c^2, \tag{5.64}$$

whence

$$m_c \approx 1663 \text{ MeV}/c^2. \tag{5.65}$$

We therefore expect

$$(M_{D^*} - M_D) = (M_{K^*} - M_K)\, m_s/m_c$$
$$= (M_\rho - M_\pi)\, m_u/m_c$$
$$\approx 119 \text{ MeV}/c^2. \tag{5.66}$$

which is roughly in agreement with the observed splitting of approximately 142 MeV/c^2. A similar computation leads to the expectation that

$$(M_{F^*} - M_F) = (M_{D^*} - M_D)\, m_u/m_s = 90 \text{ MeV}/c^2, \tag{5.67}$$

which is to be compared with the experimental hint of approximately 110 MeV/c^2. For the ψ-η_c interval, the same line of argument leads to

$$M_\psi - M_{\eta_c} \approx (M_{D^*} - M_D)^2/(M_\rho - M_\pi) \approx 32 \text{ MeV}/c^2, \tag{5.68}$$

which is far from the observed spacing (Schamberger, 1981) of 115 MeV/c^2. This is a sign that we cannot with impunity ignore the variation of $|\Psi(0)|^2$ with quark mass.

For the charmed baryons, we proceed in analogy with eqn.(5.45). It is convenient to fix the charmed quark mass by comparing the Λ and the charmed Λ (better known as C_0^+), for which the color hyperfine shift is identical up to variations in the wavefunction. This yields

$$(m_c - m_s) = M_{C_0^+} - M_\Lambda \simeq 1169 \text{ MeV}/c^2, \tag{5.69}$$

from which we conclude using (5.51) that

$$m_c \approx 1715 \text{ MeV}/c^2. \tag{5.70}$$

which is not far from (5.65). The hyperfine splitting between the spin-$^1/_2$ charmed baryons C_1 and C_0 (or Σ_c and Λ_c) is then analogous to the Σ-Λ splitting. The appropriate arithmetic yields

$$(M_{C_1} - M_{C_0}) \approx 154 \text{ MeV}/c^2, \tag{5.71}$$

in good agreement with the observed splitting of approximately 155 MeV/c^2. We have as well the simple relation

$$(M_{C_1^*} - M_{C_1}) = (M_{\Sigma^*} - M_\Sigma)\, m_s/m_c$$
$$\approx 62 \text{ MeV}/c^2, \tag{5.72}$$

which cannot yet be given a meaningful test. More extensive discussions of charmed-masses appear in the papers by DeRujula, Georgi, and Glashow(1975); Sakharov(1975); Lee, Quigg, and Rosner(1976); and Copley, Isgur, and Karl(1979).

The procedure to be followed for hadrons with beauty or truth is now obvious. Of most immediate interest is the expectation that

$$(M_{B^*} - M_B) \approx (M_{D^*} - M_D) m_c / m_b \approx 50 \ \mathrm{MeV}/c^2 \tag{5.73}$$

which suggests the decay

$$B^* \to B\gamma \tag{5.74}$$

as a possible experimental tag.

5. 4. Further Applications of Chromomagnetism :

It has been made plausible that color-singlet states correspond to stable hadrons and we have also seen, to a very limited extent, that the dynamical consequences of quark-gluon interactions correspond to reality. Although color-singlet configurations exist that are more complicated than $(q\bar{q})$ or (qqq), the one-gluon-exchange arguments of § 5.2.1. give no reason to expect that these will be appreciably bound by the color force. In ancient times (Rosner, 1968), duality diagrams suggested the necessity of $(qq\bar{q}\bar{q})$ states coupled to the baryon-antibaryon channel. Is it possible that specific configurations of this kind, or $(4q\bar{q})$ or $(6q)$ configurations, might benefit from a large hyperfine interaction and thus be bound ?

The parameter of interest, the expectation value $\langle \sum_{i<j} \underset{\sim}{\lambda}^{(i)} \cdot \underset{\sim}{\lambda}^{(j)} \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \rangle$ in a multiquark state, is conveniently computed using the color-spin technique introduced by Jaffe (1977a,b, 1979). A color ⊗ spin SU(6) algebra can be constructed out of the 35 operators $\lambda_a \otimes \sigma_b$ (24 operators), $\underset{\sim}{1} \otimes \sigma_b$ (3 operators), and $\lambda_a \otimes \underset{\sim}{1}$ (8 operators). It is convenient to define the generators of $SU(6)_{\text{color-spin}}$ as

$$\underset{\sim}{G}^{(6)} \equiv \begin{cases} \lambda_a \otimes \sigma_b / 2\sqrt{2} , \\ \lambda_a \otimes 1_{2\times 2} / 2\sqrt{2} , \\ 1_{3\times 3} \otimes \sigma_b / 2\sqrt{3} , \end{cases} \tag{5.75}$$

which are normalized so that

$$\mathrm{tr}\,(G_i^{(6)} G_j^{(6)}) = \tfrac{1}{2}\,\delta_{ij} \tag{5.76}$$

Similarly, we continue to define the normalized generators of SU(2) and SU(3) as

$$\underset{\sim}{G}^{(2)} = \underset{\sim}{\sigma}/2 \tag{5.77}$$

and

$$\underset{\sim}{G}^{(3)} = \underset{\sim}{\lambda}/2 . \tag{5.78}$$

Again I caution that these normalizations are widely (e.g. Rosner, 1981), but not universally (Buras, 1980 ; Jaffe, 1977ab, 1979) used in the literature.

With the conventions (5.75-78) , it is easy to compute that

$$\underset{\sim}{G}_1^{(6)}\cdot \underset{\sim}{G}_2^{(6)} = \frac{\underset{\sim}{\lambda}^{(1)}\cdot\underset{\sim}{\lambda}^{(2)}\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2}{8} + \frac{\underset{\sim}{\lambda}^{(1)}\cdot\underset{\sim}{\lambda}^{(2)}}{8} + \frac{\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2}{12}, \qquad (5.79)$$

so that

$$\underset{\sim}{\lambda}^{(1)}\cdot\underset{\sim}{\lambda}^{(2)}\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2 = 8\underset{\sim}{G}_1^{(6)}\cdot\underset{\sim}{G}_2^{(6)} - 4\underset{\sim}{G}_1^{(3)}\cdot\underset{\sim}{G}_2^{(3)} - \frac{8}{3}\underset{\sim}{G}_1^{(2)}\cdot\underset{\sim}{G}_2^{(2)}. \qquad (5.80)$$

Each dot-product on the right-hand side can be evaluated as usual as

$$\underset{\sim}{G}_1^{(N)}\cdot\underset{\sim}{G}_2^{(N)} = \frac{1}{2}\left[\left(\underset{\sim}{G}_1^{(N)}+\underset{\sim}{G}_2^{(N)}\right)^2 - \underset{\sim}{G}_1^{(N)2} - \underset{\sim}{G}_2^{(N)2}\right]. \qquad (5.81)$$

Consequently the quantity of interest is

$$\sum_{i<j}\underset{\sim}{\lambda}^{(i)}\cdot\underset{\sim}{\lambda}^{(j)}\underset{\sim}{\sigma}_i\cdot\underset{\sim}{\sigma}_j = 4\left[\underset{\sim}{G}_{tot}^{(6)2} - \sum_i \underset{\sim}{G}_i^{(6)2}\right]$$

$$-2\left[\underset{\sim}{G}_{tot}^{(3)2} - \sum_i \underset{\sim}{G}_i^{(3)2}\right] - \frac{4}{3}\left[\underset{\sim}{G}_{tot}^{(2)2} - \sum_i \underset{\sim}{G}_i^{(2)2}\right]$$

$$= 4\underset{\sim}{G}_{tot}^{(6)2} - 2\underset{\sim}{G}_{tot}^{(3)2} - \frac{4}{3}s(s+1)$$

$$-4\sum_i \underset{\sim}{G}_i^{(6)2} + 2\sum_i \underset{\sim}{G}_i^{(3)2} + \frac{4}{3}\sum_i s_i(s_i+1). \qquad (5.82)$$

For color singlet states, $\langle \underset{\sim}{G}_{tot}^{(3)2}\rangle = 0$, while for a color-triplet, spin-$^1/_2$ quark $\langle \underset{\sim}{G}_i^{(3)2}\rangle = {}^4/_3$, $s_i(s_i+1) = {}^3/_4$, and $\langle \underset{\sim}{G}_i^{(6)2}\rangle = {}^{35}/_{12}$. In passing, I note that a convenient source for properties of group representations is the volume by Patera and Sankoff (1973). The quantity ℓ listed in their Table II is related to the SU(N) Casimir operators required in (5.82) by

$$G^{(N)2} = (N^2-1)\,\ell/2d, \qquad (5.83)$$

for a representation of dimension d. See also McKay and Patera (1981).

For a state of n quarks in a color-singlet, (5.82) becomes

$$\mathcal{C} \equiv \left\langle \sum_{i<j}\underset{\sim}{\lambda}^{(i)}\cdot\underset{\sim}{\lambda}^{(j)}\underset{\sim}{\sigma}_i\cdot\underset{\sim}{\sigma}_j\right\rangle = 4\underset{\sim}{G}_{tot}^{(6)2} - \frac{4}{3}s(s+1) - 8n. \qquad (5.84)$$

It is easy to recover the result of § 5.3.1 for the octet-decimet splitting in this formalism. In s-wave configurations, the $SU(3)_{flavor} \otimes SU(6)_{color-spin}$ wavefunctions must be antisymmetric. The possible antisymmetric combinations for three quarks are shown in Table 17. Note first that there is no appropriate color singlet for the flavor singlet state. It will therefore not exist in the ground state, as we have already seen in § 4.2. For either the flavor octet or the flavor decimet, only a single color-spin representation has the requisite symmetry properties. This is rather generally the case for states of interest. Together with the rarity of color-singlet configurations, it explains much of the practical value of color-spin.

In the remaining cases, the color-spin parameter that controls the strength of the color-hyperfine interaction is

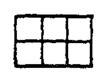
$$\mathcal{C}_{\underline{20}} = -8, \text{ for the flavor } [\underline{10}], \qquad (5.85)$$

and

$$\mathcal{C}_{\underline{70}} = +8, \text{ for the flavor } [\underline{8}]. \qquad (5.86)$$

The color hyperfine interaction is thus equal and opposite in the baryon octet and decimet.

Two color-singlet, octet baryons will have color-spin $2\mathcal{C}_{\underline{70}} = +16$. We have seen before that no residual color force is likely to bind them. Is there a more attractive 6-quark state ? The answer is yes ; the color-spin

$\underline{490}$  $\supset [\underline{1}], (\underline{1})$ (5.87) has color-spin

$$\mathcal{E}_{490} = +24. \tag{5.88}$$

A flavor-singlet $(uds)^2$ state therefore has the possibility of being bound by the excess hyperfine attraction. A ΛΛ bound state was conjectured by Jaffe (1977c) on the basis of these arguments. Its properties were estimated by him in the MIT bag model. A first experimental search (Carroll, et al., 1978) has produced no evidence for such a state.

Further applications of color-spin are made in Problems 18-20. For specific discussions of $(qq\bar{q}\bar{q})$ states, see Jaffe and Johnson (1976), Jaffe (1977a), and Jaffe and Low (1979). A brief review of this approach is given by Hey (1980b). Many other analyses have led to expectations of $(qq\bar{q}\bar{q})$ baryonium states for which there is at the moment no experimental evidence. A comprehensive review of both experiment and theory has been made by Montanet, Rossi, and Veneziano (1980).

At the current stage of understanding of hadronic structure, theory is unable to deny that multiquark states should exist. In many model calculations, such states do emerge. To my knowledge, the only active experimental candidate for a multiquark state is the observation by Amirzadeh, et al. (1979) of a narrow ($\Gamma < 20$ MeV) hyperon structure with a mass of 3.17 GeV/c^2 in the reaction

$$K^- p \to \pi^- Y^{*+} \tag{5.89}$$

at 6.5 and 8.25 GeV/c^2. The narrowness of the state and its propensity for decays into strange particle channels such as $(\Lambda,\Sigma)K\bar{K}$+pions and ΞK+pions are taken as hints that this is a $(4q\bar{q})$ state.

5. 5. Excited Mesons and Baryons :

Some expectations for excited hadron states have been given in § 4. In this section we shall review the status of radially-excited mesons very briefly, and then turn to the question of mass formulas for the orbitally excited states. The last topic deserves a more complete treatment than it will receive here, because it is a principal focal point for the interplay between QCD-based inspiration and experimental results. To maintain the finiteness of these lecture notes, I shall merely summarize the main ideas and provide an entrée to the considerable recent literature.

5. 5. 1. Radially-Excited Mesons :

I have commented in § 4.1 on the likelihood that the vector mesons $\rho'(1600)$ and $\varphi'(1634)$ are, or are considerably mixed with, 2^3S_1 radial excitations of the familiar vector mesons. Essentially pure radial excitations are commonplace in the ψ and T families. These will be treated in some detail in § 6. Increasing attention is being devoted to the study of pseudoscalar states beyond the familiar nonet. If these are indeed $(q\bar{q})$ states, they are necessarily radial excitations, because the quantum numbers $J^{PC} = 0^{-+}$ do not occur in orbitally-excited states. The other likely interpretations of new pseudoscalar levels as glueballs or multiquark states are only tenable if the $(q\bar{q})$ interpretation can be ruled out. This is therefore an issue of more than passing importance.

Cohen and Lipkin (1979) have presented a comprehensive analysis of the pseudoscalars within the framework of two simple models, one of which is in essence that of § 5.3. They argue that the masses of η and η' can be understood if η is identified almost entirely as a member of the ground-state octet, but η' is appreciably mixed with the 2^1S_0 levels. This interpretation also makes less acute the failure of SU(3) sum rules for the peripheral production of η and η'. The remaining isoscalar levels, corresponding roughly to 2^1S_0 levels, are expected in the neighborood of 1280 and 1500 MeV/c^2.

Experimental sightings of unfamiliar pseudoscalars are summarized in Table 18. It is worth emphasizing that not one of these states yet appears in the Particle Data Tables. However, I do not believe the experimental claims to be entirely frivolous, either. In addition to verifying the existence and quantum numbers of these states, it in of some importance to understand whether η'(1400) and ι(1440) are distinct objects. The pseudoscalar masses coincide approximately with the projections by Cohen and Lipkin (1979). Whether they behave as ordinary ($q\bar{q}$) states remains to be understood, particularly in view of the enthusiasm (see § 8.4) for ι(1440) as a glueball candidate. Watch this space !

5.5.2. Orbitally-Excited Mesons :

The hypothesis of De Rujula, Georgi, and Glashow (1975) that spin effects are governed entirely by the short-range Coulomb-like interaction cannot be complete. After the discovery of the charmonium p-states, it was immediately recognized that the spin splittings were not those of a Coulomb potential (Schnitzer, 1976). Therefore, it was reasoned, a study of the ^3P_J intervals might provide insights into the nature of the quarkonium potential (Schnitzer, 1975). It soon became apparent that although the ^3P_J intervals cannot be predicted as such, they can be given a sen-

sible interpretation in terms of a short-range interaction arising from Lorentz vector exchange and a confining potential with a scalar Lorentz structure (Henriques, et al., 1976 ; see the brief summary in Quigg, 1980).

Although a nonrelativistic description of the light mesons cannot so easily be justified, an extensive program to determine the detailed properties of the spin-dependent interactions has been carried out. Accessible reviews, with complete lists of references, have recently been given by Schnitzer (1981ab). Briefly stated, the idea of this line of investigation is to examine the most general form of the spin-dependent Hamiltonian, to determine the various contributions, and to understand the implications for a theory of the interactions . The inverse procedure of deriving the interaction is obviously desirable, but considerably less advanced. (See Eichten and Feinberg, 1979, 198 ; Buchmüller, Ng, and Tye, 1981).

To order $(v/c)^2$, the most general two-body interaction is of the form

$$V(r) = V_{central}(r) + V_{spin}(r)\frac{\underline{\sigma}_1\cdot\underline{\sigma}_2}{m_1 m_2} + V_{tensor}(r)\frac{S_{12}}{m_1 m_2} + \left[\frac{V_1(r)}{m_1 m_2} + V_2(r)\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\right]\underline{L}\cdot\underline{S} + V_2(r)\left(\frac{1}{m_1^2}-\frac{1}{m_2^2}\right)\underline{L}\cdot(\underline{S}_1-\underline{S}_2), \qquad (5.90)$$

where m_1 and m_2 are the quark masses, and the tensor operator is

$$S_{12} \equiv 3\underline{\sigma}_1\cdot\hat{\underline{r}}\;\underline{\sigma}_2\cdot\hat{\underline{r}} - \underline{\sigma}_1\cdot\underline{\sigma}_2 . \qquad (5.91)$$

It is instructive to compare (5.90) with the standard Eisenbud and Wigner (1941) expression for nuclear forces (see alternatively Blatt and Weisskopf, 1952 ; Bohr and Mottelson, 1969 ; or Preston 1962). Given a central potential and its Lorentz structure one may make a nonrelativistic reduction (at least in the absence of non-Abelian complications) and determine V_{spin}, V_{tensor}, and the spin-orbit potentials V_1 and V_2. This has been done most completely by Gromes (1977).

While the meson spectrum portrayed in Table 10 is not as well known as the spectrum of nucleon resonances, the available information on well-established s-wave and p-wave $(q\bar{q})$ states is neatly correlated by the expression (5.90). Schnitzer (1978) has emphasized a characteristic consequence of this nonrelativistic form : the prediction of "inverted multiplets" in light quark-heavy quark systems. If $m_2 >> m_1$, the interaction (5.90) becomes

$$V(r) \longrightarrow V_{central}(r) + \frac{2V_2(r)}{m_1^2} \underline{L} \cdot \underline{s}_1 , \tag{5.92}$$

in which the spin-orbit interaction is determined by the Thomas term V_2. If a scalar confining interaction dominates at medium-to-large distances, then $V_2(r)$ will be repulsive, and one concludes that, for example,

$$M(^3P_0) > M(^3P_1) > M(^3P_2). \tag{5.93}$$

This would be a dramatic reversal of what is expected on the basis of nonspecific intuition, and well worth searching for in the charmed mesons. The general picture of meson masses that inspires this expectation is made more appealing by the fact that a similar scheme leads to many insights in baryon spectroscopy, to which

we now turn.

5.5.3. Orbitally-Excited Baryons :

Because of the availability of direct-channel formation experiments and extensive phase-shift analyses, the baryon spectrum has traditionally been more thoroughly studied than the meson spectrum. In addition to a much larger number of states, the baryons provide a richer variety of information on decay modes and decay amplitudes. That situation continues at the present time, in spite of steady progress in meson spectroscopy. There are, as a comparison of Tables 10 and 11 reveals, more pedigreed p-wave baryons than mesons, and the baryons have been known and studied for a longer time on the average than their mesonic counterparts.

It is thus both natural and desirable that parametrizations inspired by Quantum Chromodynamics should be applied to the excited baryon masses and decay amplitudes, where a pre-existing fabric of flavor-spin SU(6) folklore awaits derivation, extension, and occasional modification (Rosner, 1974ab ; Close 1979). An extensive program of using the Fermi-Breit interaction in combination with basis states arising from a harmonic oscillator confining interaction has been undertaken by Isgur and Karl and collaborators. These efforts typify much of the current work on the subject. For reviews, see Karl (1979), Hey (1979,1980a), Isgur (1980a, c), and Close (1981).

For nonstrange baryons (Isgur and Karl,1977), the Fermi-Breit program amounts to a straightforward variation on a classical theme. In order to treat strange baryons systematically, it is necessary to adopt an unperturbed basis in which the difference in mass between the strange and nonstrange quarks is acknowledged explicitly (Isgur and Karl, 1978b). This breaks the exact permutation symmetry of the spatial wavefunctions described in § 4. 2. Once this has been done, one may hope to derive connections among spin-splittings theoretically, or to deduce them phenomenologically and draw inferences about the underlying interaction.

We have described far too little of the regularities and conundrums of the baryon spectrum to make meaningful a detailed analysis. The following conclusions however seem to me sound. First, the splitting of resonances in the N=1 $\underset{\sim}{70}$ is characteristic of the spin-spin splitting of an s-wave (qq) pair produced by a short-range Coulomb potential due to the exchange of a vector gluon. Second (Isgur and Karl, 1978ab), the inversion of the $J^P = 5/2^-$ states Λ(1830) and Σ(1765) relative to the ground-state $J^P = 1/2^+$ states Λ(1115) and Σ(1193) is readily interpreted as a consequence of the nondegeneracy of the appropriate orbital states. This overcomes the color-hyperfine interaction. Third, a successful overall fit can be made to the baryon spectrum (see also Isgur and Karl, 1979 ; Chao, Isgur and Karl, 1981). The fit requires that the spin-orbit interaction be negligible. As emphasized by Close (1981), it is by no means clear how this circumstance might arise in QCD. However, it seems worthwile to try to understand the remarkable numerical success of these fits, which subsume a great many earlier results of broken -SU(6) phenomenology.

5.6. Hadron-Hadron Interactions ?

If QCD is indeed the complete theory of the strong interactions, it should be possible to derive the interactions among hadrons as collective effects of the interactions among quarks and gluons. How to achieve this worthy goal is not obvious, nor is it apparent that the resulting description should be economical. It is pleasant to realize that the Bardeen-Cooper-Schrieffer (1957) picture of superconductivity must emerge from quantum electrodynamics , but it would be less pleasant to deduce the properties of a specific superconductor from QED. Similar, the hadron exchange picture of nuclear forces may be the neatest parametrization one may hope to find, even after the hadron spectrum has been derived from QCD.

Anticipating the millenium, many people have attempted to apply QCD-inspired pictures of the hadrons to the problem of interactions. These efforts fall into two principal categories : attempts to extend (Detar, 1978,1979,1981ab; Fairley and Squires, 1975) or modify (Brown and Rho, 1979 ; Brown, Rho , and Vento 1979) the MIT bag model ; and applications of quark-potential models to hadronic interactions (Fishbane and Grisaru, 1978 ; Willey, 1978 ; Matsuyama and Miyazawa, 1979 ; Fujii and Mima, 1978 ; Gavela, et al., 1979 ; Stanley and Robson, 1980, 1981). I think it is fair to say that this line of research is still at the groping stage. In particular for the potential-model discussions, insufficient attention to the consequences of t-channel analyticity seems a serious shortcoming. A critical discussion from a different orientation has been given by Greenberg and Lipkin (1981).

6. Quarkonium

We have seen in the preceding sections that many aspects of the spectroscopy of light mesons and baryons have elements in common with nonrelativistic potential descriptions. This is so in spite of the fact that the motion of light quarks cannot be argued to be nonrelativistic, and our understanding of the success of nonrelativistic descriptions is only partial. For systems composed exclusively of heavy quarks, the situation may be quite different, in that a potential model may be both adequate and justifiable.

As Appelquist and Politzer (1975) were first to note, the asymptotic freedom of QCD implies that the strong interaction between quarks becomes feeble at very short distances. For bound states of extremely massive quarks, it is possible to imagine that the natural scale of the system is so small that the quarks are bound by a Coulomb potential characteristic of one gluon exchange. The so-called quarkonium system would then be a nonrelativistic hadronic analog of positronium, the well-known electron-positron bound state. This vision has not been fulfilled, at least not in the sense of meson states that fit a Coulomb spectrum. However, the ψ and Υ families of heavy mesons are systems in which manifestly nonrelativistic techniques of bound-state quantum mechanics are warranted by the kinematics. These methods are also successful, not only in correlating experimental information, but also in bringing to hadron spectroscopy an element of predictive power.

Since the discovery of the ψ/J (Aubert, et al., 1974 ; Augustin, et al., 1974) and of the ψ' (Abrams, et al., 1974), the study of quarkonium physics has flourished. Further stimulus was provided by the discovery (Herb, et al., 1977) of the upsilon resonances. These states have subsequently been the subject of many conference reports (Jackson, 1977b; Gottfried, 1977 ; Eichten, 1978 ; Jackson, Quigg, and Rosner, 1979 ; Quigg, 1980 ; Rosner, 1980a, Gottfried, 1980 ; Berkelman, 1980 ; Schamberger, 1981), summer school lectures (Jackson, 1976 ; Krammer and Krasemann, 1979ab ; Rosner, 1981) and review articles (Novikov, et al., 1978 ; Appelquist, Barnett, and Lane, 1978 ; Quigg and Rosner, 1979 ; Grosse and Martin, 1980). As is the case for many of the topics treated in these lecture notes, quarkonium physics is rich enough that it could fill an entire course of lectures. To be both brief and intelligible, however, I will restrict my attention to some of the most elementary points and to a quick review of the most recent experimental results.

The study of quarkonium levels may be divided into two broad areas in which different methods of analysis are profitably applied. It is to be hoped, of course, that lessons learned in one arena are transferable to the other. The first topic, which I shall not discuss in detail, is the application of perturbative QCD to the strong and electromagnetic decays of quarkonium states. This approach is closely connected with the original motivation of Appelquist and Politzer (1975). The evidence for three-jet events interpreted as

$$\Upsilon \rightarrow g\,g\,g \tag{6.1}$$

has been reviewed by Wiik (1981). We shall return to this interpretation briefly in § 8.3. in connection with the Zweig rule. Another aspect of the perturbative approach is the determination, in either relative or absolute terms, of the rates for various quarkonium decays. Although the analogy with positronium is a powerful tool, quantitative analysis is highly nontrivial because of the importance of QCD radiative corrections and the difficulty of separating them unambiguously from wavefunction effects. For a summary of recent progress, I refer to the rapporteur talk by Buras (1981). The second strategy, to which I shall give an introduction, is the application of nonrelativistic quantum mechanics to the spectra and nonstrong decays of quarkonium states.

The assumption that a nonrelativistic analysis is admissible is an extremely strong one, which must be examined critically. If it is justifiable, it confers a great simplification of the problem as well as the important advantage that rigorous statements can be proved within the framework of potential theory. Work along this line has been divided between efforts to make statements that hold for wide classes of potentials and attempts to determine the nature of the interaction between quarks. Two examples of results which are largely independent of details of the potential will be given below. Within the attempts to determine the potential explicitly, there is yet another division between efforts to derive or deduce the interaction from theory and efforts to infer the interaction from experiment.

The earliest work in this field (e.g. Eichten, et al., 1975) drew upon theoretical inspiration, or theoretical prejudice, in assuming a potential of the form

$$V(r) = -\frac{4}{3}\alpha_s/r + ar. \qquad (6.2)$$

The Coulomb term is inspired by the one-gluon-exchange picture that led to (5.32a). The linear term is contrived to ensure quark confinement, and is consistent with the relativistic string interpretation of the light mesons developed in § 5.2.2. In applications, the coefficients α_s and a and the heavy-quark mass are regarded as parameters. It is important to realize that although the limiting forms of the Coulomb-plus-linear potential (6.2) at short and long distances may be very well motivated, there is no corresponding reason to believe that (6.2) can be relied upon at intermediate distances. The same caution can be raised for subsequent work aimed at using perturbative QCD plus some notions of elegance or simplicity to fix the potential over all values of r (Celmaster and Henyey, 1978 ; Carlitz and Creamer, 1979 ; Levine and Tomozawa, 1979, 1980 ; Richardson, 1979 ; Krasemann and Ono, 1979; Fogleman, Lichtenberg, and Wills, 1979 ; Buchmüller, Grunberg, and Tye, 1980). These caveats stated, it must be said that early work using the Coulomb plus linear form was of great importance in demonstrating the viability of the nonrelativistic approach. In addition, the later work has yielded quite satisfactory descriptions of the ψ and Υ spectra. We shall, however, not discuss this approach further.

I shall stress instead another facet of the work based upon definite potentials, which may be called a model-independent approach. Emphasis is placed on this style of analysis because it permits easy insight into the nature of the interaction between quarks and a test of the self-consistency of the potential model approach. In the end, I believe it is necessary to blend the lessons learned from many techniques. For present purposes, "model-independent" will be taken to mean rather elementary manipulations of the Schrödinger equation. A loftier program based upon an inverse scattering algorithm (Thacker, Quigg, and Rosner, 1978ab ; Grosse and Martin, 1979 ; Quigg, Thacker, and Rosner, 1980 ; Schonfeld, et al., 1980 ; Quigg and Rosner, 1981) leads to similar results. Although I regard this method as especially powerful, it requires an extended introduction which time does not permit here. I shall also not discuss the well-known problem of E1 transition rates in charmonium. Let us await the upsilons !

6.1. Scaling the Schrödinger Equation

For simple potentials, including power-laws and other monotonic wells, rather far-reaching results can be derived using quite elementary techniques. This mode of analysis has been reviewed by Quigg and Rosner (1979), and exploited by many authors. I shall summarize here a few of the results with direct applications to experiment.

6.1.1. Dependence on Constituent Mass and Coupling Constant

The reduced radial Schrödinger equation for a particle with mass μ and angular momentum ℓ moving in a central potential V may be written in the form

$$\frac{\hbar^2}{2\mu} u''(r) + \left[E - V(r) - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right] u(r) = 0, \tag{6.3}$$

subject to the boundary conditions

$$u(0) = 0, \quad u'(0) = R(0). \tag{6.4}$$

A prime is used to denote derivatives with respect to the argument, and the reduced radial wavefunction $u(r)$ is related to the three-dimensional wavefunction

$$\Psi(r) = R(r)\, Y_{\ell m}(\theta, \varphi) \tag{6.5}$$

by

$$u(r) = r R(r). \tag{6.6}$$

The familiar substitution (6.6) places the radial equation in three dimensions in formal correspondence with the one-dimensional Schrödinger equation.

For the special case of a power-law potential,

$$V(r) = \lambda r^{\nu}, \tag{6.7}$$

the equation (6.3) can be divested of all its dimensionful parameters. To see this, we first introduce a scaled measure of length

$$\rho \equiv (\hbar^2/2\mu|\lambda|)^{p}\, r, \tag{6.8}$$

where the exponent p is to be chosen to eliminate dimensions from (6.3). The choice

$$p = -1/(2+\nu), \tag{6.9}$$

when accompanied by the substitutions

$$E \equiv \left(\frac{\hbar^2}{2\mu}\right)\left(\frac{\hbar^2}{2\mu|\lambda|}\right)^{2p} \varepsilon, \tag{6.10}$$

where ε is dimensionless, and

$$w(\rho) \equiv u(r) \tag{6.11}$$

accomplishes precisely this. The ensuing equation is

$$w''(\rho) + \left[\varepsilon - \mathrm{sgn}(\lambda)\,\rho^{\nu} - \ell(\ell+1)/\rho^2\right] w(\rho) = 0, \tag{6.12}$$

which depends only upon pure numbers.

Several consequences follow immediately from this legerdemain. Lengths and quantities with the dimensions of lengths depend upon the constituent mass and coupling strength as

$$L \propto (\mu|\lambda|)^{-1/(2+\nu)}. \tag{6.13}$$

As a result, the particle density at the origin of coordinates behaves as

$$|\Psi(0)|^2 \sim L^{-3} \propto (\mu|\lambda|)^{3/(2+\nu)}. \tag{6.14}$$

Level spacings have a similarly definite behavior, according to (6.10) :

$$\Delta E \propto \mu^{-\nu/(2+\nu)} |\lambda|^{2/(2+\nu)}. \tag{6.15}$$

The limiting behavior of the scaled Schrödinger equation as $\nu \to 0$ is the subject of Problem 21. It is easy to see that the scaling laws (6.13)-(6.15) contain many well-known results. Recall, for example, that in the Coulomb potential, for which $\nu = -1$,

$$\Delta E(\nu = -1) \propto \mu\alpha^2 = \mu|\lambda|^2. \tag{6.16}$$

Likewise, the conclusion that in a linear potential

$$|\Psi(0)|^2\Big|_{\nu=1} \propto \mu|\lambda| \tag{6.17}$$

can be derived at once using the identity (see Problem 22)

$$|\Psi(0)|^2 = \frac{\mu}{2\pi\hbar^2}\left\langle \frac{dV}{dr} \right\rangle. \tag{6.18}$$

The scaling laws (6.13)-(6.15) have many applications in quarkonium physics. For the moment let us merely note that electric multipole matrix elements vary as

$$\langle n'|E_j|n\rangle \sim L^j \propto (\mu|\lambda|)^{-j/(2+\nu)}, \tag{6.19}$$

so that transition rates behave as

$$\Gamma(E_j) \sim k^{2j+1} |\langle n'|E_j|n\rangle|^2, \tag{6.20}$$

where k is the energy of the radiated photon, which is just a level spacing ΔE. Using (6.13) and (6.15) we then deduce that

$$\Gamma(E_j) \propto \mu^{-[2j(1+\nu)+\nu]/(2+\nu)} |\lambda|^{2(j+1)/(2+\nu)} . \tag{6.21}$$

This has the interesting consequence that for fixed potential strength $|\lambda|$, $\Gamma(E_j)$ is a decreasing function of j as $\mu \to \infty$ for potentials less singular than the Coulomb potential.

Using the expression

$$\Gamma(V^0 \to e^+e^-) = \frac{16\pi\alpha^2}{M_V^2} |\Psi(0)|^2 \langle e_q^2 \rangle \tag{6.22}$$

derived in Problem 13, one may easily show that for $\nu > -1$ (for which binding energies are asymptotically negligible)

$$\Gamma(E_j)/\Gamma(V^0 \to e^+e^-) \propto \mu^{-(2j-1)(\nu+1)/(2+\nu)} |\lambda|^{2(j-1)/(2+\nu)} \tag{6.23}$$

which implies the dominance of leptonic over radiative decays as $\mu \to \infty$ for fixed potential strength $|\lambda|$.

6.1.2. Dependence on Principal Quantum Number

To investigate how observables depend upon the principal quantum number with some degree of generality it is convenient to adopt the semiclassical, or JWKB approximation. This turns out to be rather less of a compromise than one might at first surmise. Judiciously applied, the semiclassical approximation is in fact highly accurate for the sort of nonpathological potentials one hopes to encounter for quarkonium. This accuracy is documented in Quigg and Rosner (1979), where additional references may be found.

The semiclassical results all follow from the quantization condition

$$\int_0^{r_c} dr \left[2\mu(E - V(r))\right]^{1/2} = (n - \tfrac{1}{4})\pi\hbar, \tag{6.24}$$

where n is the principal quantum number and the classical turning point r_c is defined through $V(r_c) = E$. Although it is both possible and useful to be more general, it is appropriate to retain the spirit of the preceding Section and specialize to power-law potentials. For s-wave bound states of nonsingular potentials of the form (6.7), eqn. (6.24) can be integrated by elementary means to yield

$$E_n \propto (n - \tfrac{1}{4})^{2\nu/(2+\nu)} \tag{6.25}$$

where with an eye toward the intended applications I have suppressed the dependence on constituent mass and coupling strength given in (6.15). For singular potentials additional care is required near the origin. A simple modification of the usual procedure leads to

$$E_n \propto (n-\gamma(\nu))^{2\nu/(2+\nu)}, \quad -2<\nu<0, \tag{6.26}$$

where

$$\gamma(\nu) = \frac{1}{2}\left(\frac{1+\nu}{2+\nu}\right). \tag{6.27}$$

Similar expressions may be obtained for orbitally-excited states.

By evaluating the expectation value in eqn. (6.18) with JKWB wavefunctions, it is also straightforward to derive

$$|\Psi_n(0)|^2 \propto \begin{cases} (n-\frac{1}{4})^{2(\nu-1)/(2+\nu)}, & \nu>0, \quad (6.28a) \\ (n-\gamma(\nu))^{(\nu-2)/(2+\nu)}, & 0>\nu>-2. \quad (6.28b) \end{cases}$$

A more general result is derived in Problem 23. Generalizations of these results to $\ell \neq 0$ have also been made, but we shall not require them here. Let us now see what can be learned by comparing the simple results of this section with experimental information.

6.2. Inferences from Experiment

6.2.1. Data

The spectrum of $(c\bar{c})$ bound states is summarized in Table 19 and Fig. 30. In addition to refinements in the branching ratios for radiative decays, there are two recent developments of note (Scharre, 1981 ; Schamberger, 1981).

The candidate U(2980) for the 1^1S_0 hyperfine partner of the ψ now appears firmly established (Partridge, et al., 1980 ; Himel, et al. ; 1980b). I therefore designate it as $\eta_c(2983\pm5)$ which corresponds to a hyperfine mass splitting of

$$M_\psi - M_{\eta_c} = 114\pm5 \quad \text{MeV}/c^2 \tag{6.29}$$

The state has a total width $\Gamma(\eta_c\to\text{all})$ = 8-19 MeV, considerably larger than that of the ψ. This is in qualitative accord with the ortho-parapositronium analogy. The η_c is observed in the decays $\psi\to\gamma\eta_c$ and $\psi'\to\gamma\eta_c$ with branching ratios

$$\Gamma(\psi\to\gamma\eta_c)/\Gamma(\psi\to\text{all}) = (0.7\pm1.5)\% \tag{6.30}$$

which implies

$$\Gamma(\psi\to\gamma\eta_c) = 0.44-0.95 \text{ keV}, \tag{6.31}$$

and

$$\Gamma(\psi' \to \gamma \eta_c)/\Gamma(\psi' \to all) = (0.32 \pm 0.05 \pm 0.05)\,\%, \qquad (6.32)$$

which implies

$$\Gamma(\psi' \to \gamma \eta_c) = (0.69 \pm 0.23 \pm 0.11)\ \text{keV}. \qquad (6.33)$$

Where two errors are shown, the first is statistical and the second is systematic.

There is now also a candidate for the radial excitation of η_c, which would be the 2^1S_0 hyperfine partner of ψ'. This state, which I provisionally designate as $\eta_c'(3592 \pm 5)$, has a total width smaller than 9 MeV. The hyperfine splitting is

$$M_{\psi'} - M_{\eta_c'} = 92 \pm 5 \quad \text{MeV}/c^2 \qquad (6.34)$$

which is, as we shall see below in § 6.4., a reasonable value. The state is observed in the inclusive radiative decay $\psi' \to \gamma \eta_c'$ with a branching ratio of

$$\Gamma(\psi' \to \gamma \eta_c')/\Gamma(\psi' \to all) = (0.6 \times 2^{\pm 1})\,\%, \qquad (6.35)$$

which implies a decay rate

$$\Gamma(\psi' \to \gamma \eta_c') = (1.3 \times 2^{\pm 1})\ \text{keV}. \qquad (6.36)$$

No exclusive channels have yet been identified.

In the upsilon spectrum, only vector states have been observed. Their properties are summarized in Table 20 and Fig. 31.

6.2.2. Consequences

The strategy embodied in § 6.1 has been pursued explicitly by several authors (including Quigg and Rosner 1977, 1978a, 1979 ; Quigg, 1980 ; Martin, 1980 ; Rosner, 1980a, 1981a) and implicitly by many others. The conclusion to be drawn from the data is that a potential of the form

$$V(r) = A + B r^{\nu} \qquad (6.37)$$

with $\nu \approx 0.1$ gives a good representation of the ψ and Υ spectra. This is based upon four distinct kinds of evidence.

First, we may note by comparing Figs. 30 and 31 that the level spacings are quite similar in the ψ and Υ families. Indeed, the observation that

$$M_{\Upsilon'} - M_{\Upsilon} \approx M_{\psi'} - M_{\psi} \qquad (6.38)$$

provided an early motivation for the logarithmic potential (Quigg and Rosner, 1977). A more detailed look at the intervals indicates that

$$\Delta E(\Upsilon) \approx 0.95\ \Delta E(\psi). \qquad (6.39)$$

Assuming that the potential strength does not vary between the ψ and Υ systems, this implies a small positive power for the effective potential. The precise value of the exponent depends upon the ratio of quark masses, which is imperfectly known.

The principal-quantum-number dependence of observables within one quarkonium system is free from the assumption that the potential strength λ is the same for different quark flavors. Effective powers may be inferred independently from the ψ and Υ levels and compared for consistency. The level structures $(E_3-E_2)/(E_2-E_1)$ etc. are characteristic of the potential shape. These ratios of intervals are the same for ψ and Υ states, and are again compatible with $\nu \approx 0.1$. Similarly, the 2S-2P spacing, known only for the ψ family, implies a small positive power. Finally, the principal-quantum-number dependence of wavefunctions at the origin, or equivalently of the reduced leptonic widths (4.51), is approximately given by

$$|\Psi_n(0)|^2 \sim 1/(n-\tfrac{1}{4}) \qquad (6.40)$$

for both ψ and Υ. This behavior again corresponds to an effective potential which is a small positive power. It was this observation for the ψ family that led Machacek and Tomozawa (1978) to investigate softer-than-linear confining potentials, including logarithmic forms. Taken together, these results on principal quantum number dependence would seem to exclude the bizarre possibility that the nearly equal spacing in the ψ and Υ families results from a potential strength which varies approximately as

$$\lambda \propto \mu^{\nu/2} . \qquad (6.41)$$

Martin (1980) has shown that careful attention to hyperfine effects does not change the conclusions of this analysis, namely that the interquark potential is flavor-independent (as QCD would have it) and characterized by an effective power-law potential with a small positive exponent. This is also in agreement with the conclusions of all other analyses and fits : In the region of space between 0.1 fm and 1 fm , the interaction between heavy quarks is flavor-independent, and roughly logarithmic in shape (Buchmüller and Tye, 1981 ; Quigg and Rosner, 1981).

6.3. Theorems and Near Theorems

An excellent review of statements about bound-state properties which may be proved rigorously in nonrelativistic potential theory has been given by Grosse and Martin (1980). Many results have been deduced which pertain to the order of levels, inequalities for wavefunctions at the origin, bounds on quark mass differences and so forth. The value of such statements is not only that they are true, but also that they provide a context for computations based upon explicit potentials. It is of great value to understand what must be true for any reasonable potential, or for any potential of a particular class, in order to distinguish the consequences that may be peculiar to a specific model. I shall cite two examples that bear directly upon experimental results.

Considerer a quarkonium potential which is monotonic,

$$dV/dr \geq 0 \qquad (6.42)$$

and concave downward,

$$d^2V/dr^2 \leq 0. \qquad (6.43)$$

The first property is motivated by simplicity, and the second by the expectation that the confining potential rises no fast than linearly. Both are satisfied by the effective power-law potentials just discussed. Then if $m > \mu$ are masses of the constituents of two $Q\bar{Q}$ systems, one may prove (Rosner, Quigg, and Thacker, 1978 ; Leung and Rosner, 1979) that

$$|\Psi_m(0)|^2 \geq (m/\mu)|\Psi_\mu(0)|^2. \qquad (6.44)$$

This result holds for the ground state under the assumptions stated, for all levels in power-law potentials (compare eqn. (6.14)), and for all levels in a general potential satisfying the assumptions, in WKB approximation (Grosse and Martin, 1980). It implies a lower bound on leptonic widths in the more massive system as, in the case at hand,

$$\Gamma(\Upsilon_n \to e^+e^-) \geq \frac{m_b}{m_c} \cdot \frac{e_b^2}{e_c^2} \cdot \frac{M_{\psi_n}^2}{M_{\Upsilon_n}^2}\,\Gamma(\psi_n \to e^+e^-). \qquad (6.45)$$

The lower bounds on upsilon leptonic widths are plotted in Fig. 32, together with the experimental measurements. A b-quark charge of 2/3 is seen to be incompatible with the bound. The conclusion that $|e_b| = 1/3$ is substantiated by the measurements of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ shown in Fig. 22.

A semiclassical near-theorem relates the number of levels below flavor threshold to the mass of the constituents. This would seem to be a question ill-suited to a nonrelativistic approach because it is necessary to compute both quarkonium ($Q\bar{Q}$) masses and the mass of the lightest flavored ($Q\bar{q}$) state. The latter is unlikely to be governed by a potential theory description. However, a key simplifying observation was made by Eichten and Gottfried (1977), who noted that the mass of the light quark-heavy quark state can be written as

$$M(Q\bar{q}) = M(Q) + M(q) + \text{binding} + \text{hyperfine}. \qquad (6.46)$$

Although the binding energy may not be calculable, it is reasonable to suppose that it depends upon the reduced mass of the constituents, which tends to $M(q)$ as $M(Q) \to \infty$. Thus the binding energy must become independent of the heavy quark mass. Furthermore, the hyperfine splitting of the 0^{-+} and 1^{--} ($Q\bar{q}$) levels must certainly (see § 5.3.) vary as $1/M(Q)$. It therefore vanishes as $M(Q) \to \infty$. Hence in the limit of infinite quark mass, the difference

$$\delta(M(Q)) \equiv 2M(Q\bar{q}) - 2M(Q) \to \delta_\infty, \qquad (6.47)$$

independent of the heavy-quark mass.

In the regime in which $\delta(M(Q)) \approx \delta_\infty$ is a good approximation, the number of levels below flavor threshold is easily calculated (Quigg and Rosner, 1978b). Consider any confining potential. In semiclassical approximation the number of levels bound below $E = 2M(Q) + \delta_\infty$ is specified by the quantization condition

$$\int_0^{r_\delta} dr \left[M(Q)(\delta_\infty - V(r))\right]^{1/2} = (n-\tfrac{1}{4})\pi, \qquad (6.48)$$

where to save writing the zero of energy has been set at 2M(Q). The classical turning point r_δ, defined through

$$V(r_\delta) = \delta_\infty \qquad (6.49)$$

is independent of M(Q), so we have by inspection the result that

$$(n-\tfrac{1}{4}) \propto \sqrt{M(Q)} \qquad (6.50)$$

It is likely that the limit (6.47) is already approached within 10 % in the charmonium system, in which two $^3S_1$ levels lie below charm threshold. Thus there should be slightly less than four bound levels in the upsilon family, in agreement with the observation of three narrow vector states. The success of this prediction provides another verification of flavor independence, which was the principal assumption. Many narrow levels are thus to be expected for the next quarkonium family, when it is found, since the next quark mass certainly exceeds 18 GeV/c^2 (Wiik, 1981).

A corollary to the conclusion that the classical turning point of the last narrow level has become independent of quark mass is that the single-channel analysis cannot be extended past about 1 fm. Heavier $(Q\bar{Q})$ systems will extend our knowledge of the interaction to shorter distances

(for some specifics see Moxhay, et al., 1981), but are unlikely to address the nature of the confining potential.

6.4. Remarks on Spin Singlet States

Experimental progress toward establishing the properties of the $^1S_0$ charmonium levels prompts some elementary comments. First let us examine the hyperfine splittings. If they are determined by the mechanism described in § 5.3 for the light hadrons, which should in any case be more trustworthy for quarkonium, we expect

$$M_\psi - M_{\eta_c} \approx \frac{32\pi\alpha_s}{9\, m_c^2} |\Psi(0)|^2 \approx \frac{M_\psi^2 \alpha_s \Gamma(\psi \to e^+e^-)}{2 m_c^2 \alpha^2} \qquad (6.51)$$

where the second equality follows from the Van Royen-Weisskopf formula (6.22). The leptonic width quoted in Table 19 leads to the numerical estimate

$$\alpha_s \approx 1/2, \qquad (6.52)$$

but this is reduced by QCD radiative corrections (Buchmüller, et al., 1981 ; Barbieri, et al., 1981) to

$$\alpha_s \approx 0.3, \qquad (6.53)$$

a plausible value. The hyperfine splitting between the radial excitations may then be estimated as

$$M_{\psi'} - M_{\eta_c'} \approx (M_\psi - M_{\eta_c}) \frac{M_{\psi'}^2 \Gamma(\psi' \to e^+e^-)}{M_\psi^2 \Gamma(\psi \to e^+e^-)} \tag{6.54}$$

$$\approx (75 \pm 20)\ \mathrm{MeV}/c^2 .$$

This is in reasonable agreement with the experimental suggestion (6.34). We have thus encountered no mysteries. The expression (6.51), applied mutatis mutandis to the upsilon ground state, yields the order of magnitude estimate

$$M_\Upsilon - M_{\eta_b} \approx M_\psi - M_{\eta_c} . \tag{6.55}$$

To the extent that they are known, the $M1$ transition rates to the ${}^1S_0$ states also seem reasonable. In complete analogy to the treatment of § 3.4.4 , we may compute

$$\Gamma(\psi \to \gamma\eta_c) = 4\alpha\, e_c^2 \omega^3 / 3 m_c^2 . \tag{6.56}$$

Bearing in mind the uncertainty in the charmed quark mass, but choosing m_c = 1.5 GeV/c^2 for illustration, we expect

$$\Gamma(\psi \to \eta_c \gamma) \approx 2.7\ \mathrm{keV} \tag{6.57}$$

and

$$\Gamma(\psi' \to \eta_c' \gamma) \approx 1.5\ \mathrm{keV} \tag{6.58}$$

for the observed photon energies. The predicted ground-state rate is somewhat too high, but the 2S prediction agrees well with the preliminary data. [Joy over the latter success is tempered by the discovery

of a confusion between per cent and fractions in the ordinate of Fig. 11(b) of Quigg (1980)] . It will be important to check, as data improve, the ω^3 -dependence embodied in (6.56). At the present stage of quarkonium theory, one can only guess at the hindered $M1$ rate for the transition $\psi' \to \gamma\eta_c$. The standard estimates for the branching ratio,

$$\Gamma(\psi' \to \gamma\eta_c)/\Gamma(\psi' \to \mathrm{all}) \approx 10^{-2} - 10^{-3} , \tag{6.59}$$

do not disagree with the measurement (6.32).

6.5. Relation to Light-Quark Spectroscopy

Several authors, especially Lipkin (1978ab, 1979b) , Cohen and Lipkin (1980), Martin (1981), and Richard (1981), have attempted to extend the successful description of the quarkonium spectra to light mesons and baryons. This may be done either by abstracting the scaling laws from the ψ and Υ states or by transplanting the quarkonium potential to what would seem a manifestly relativistic regime. This activity requires more courage than I can summon, but the resulting numerical correlations are suggestive indeed. As food for thought I present in Fig. 33 a highly speculative spectrum of $(s\bar{s})$ states. Many of the assignments are uncertain, but the resemblance to the ψ and Υ spectra is remarkable, as Martin (1981) and Close (1981) have also commented. Whether this spectrum (if correct !) shows that nonrelativistic analysis has a wider-than-expected range of validity, or that a deeper principle of hadron dynamics awaits recognition, I do not know. The parallel between Figs. 30, 31, and 33 and the nuclear level schemes shown in Figs. 1-3 is haunting.

7. Toward the Bag

We have already mentioned in § 3.4.2 some consequences of quark confinement, in the context of an extremely stylized description of confinement : the boundary condition that the Dirac wavefunctions vanish on a static spherical surface. The static cavity approximation, as it is called, is a principal technical assumption in the formulation of the MIT bag model (Chodos, et al., 1974 ; Chodos, Jaffe, Johnson, and Thorn, 1974 ; De Grand, Jaffe, Johnson, and Kiskis, 1975). Of the bag model itself, which has been extremely influential in hadron spectroscopy, there exist several fine reviews, including those by Johnson (1975, 1977, 1979); De Tar (1980), Hasenfratz and Kuti (1978), Jaffe and Johnson (1977), and Jaffe (1977d, 1979) as well as the summary in Close (1979). A different but related picture of quark confinement, known as the SLAC bag, was put forward by Bardeen, et al. (1975) ; see also Giles (1976). Our interest here is much more restricted : to understand how the mechanism of quark confinement (see Wilson, 1974 ; Nambu, 1976 ; Mandelstam, 1980 ; 't Hooft, 1980 ; Adler, 1981 ; Bander, 1981) thought to operate in QCD may give rise to hadronic bags. In the absence of a compelling argument, I follow the usual practice of giving two incomplete arguments. The heuristic discussions are themselves quite standard, and can be found in similar form in many places, including Kogut and Susskind (1974), Lee (1980) and Gottfried and Weisskopf (1981).

7.1. An Electrostatic Analog

It is typical in field theories that the coupling constant depend upon the distance scale. This dependence can be expressed in terms of a dielectric constant ε . We define

$$\varepsilon(r_0) \equiv 1 \tag{7.1}$$

and write

$$g^2(r) = g^2(r_0)/\varepsilon(r). \tag{7.2}$$

We assert that the implication of asymptotic freedom (Gross and Wilczek, 1973 ; Politzer, 1973 ; see also 't Hooft, 1973ab and Khriplovich, 1969) is that in QCD the effective color charge decreases at short distances and increases at large distances. In other words, the dielectric "constant" will obey

$$\varepsilon(r) > 1, \quad \text{for} \quad r < r_0, \tag{7.3a}$$

$$\varepsilon(r) < 1, \quad \text{for} \quad r > r_0. \tag{7.3b}$$

Indeed, to second order in the strong coupling we may write

$$\varepsilon(r) = \left[1 + \frac{1}{2\pi}\frac{g^2(r_0)}{4\pi}(11 - 2n_f/3)\ln(r/r_0) + O(g^4)\right]^{-1} \tag{7.4}$$

in QCD, where n_f is the number of active quark flavors.

Let us now consider an idealization based upon electrodynamics. In Quantum Electrodynamics, we choose

$$\varepsilon_{\text{vacuum}} = 1, \tag{7.5}$$

and can show (e.g. Landau and Lifshitz, 1960) that physical media have $\varepsilon > 1$. The displacement field is

$$\underset{\sim}{D} = \underset{\sim}{E} + 4\pi \underset{\sim}{P} \tag{7.6}$$

and atoms are polarizable with $\underset{\sim}{P}$ parallel to the applied field $\underset{\sim}{E}$, so that $|\underset{\sim}{D}| \geqslant |\underset{\sim}{E}|$. Since the dielectric constant is defined through $\underset{\sim}{D} = \varepsilon \underset{\sim}{E}$ in these simple circumstances, we conclude that $\varepsilon > 1$. For a thorough treatment, see Dolgov, Kirzhnits, and Maksimov (1981).

Now let us consider, in contrast to the familiar situation, the possibility of a dielectric medium with

$$\varepsilon_{\text{medium}} = 0, \tag{7.7}$$

a perfect dia-electric, or at least

$$\varepsilon_{\text{medium}} \ll 1, \tag{7.8}$$

a very effective dia-electric medium. We can easily show that if a test charge is placed within the medium, a hole will develop around it.

To see this, consider the arrangement depicted in Fig. 34(a), a positive charge distribution ρ_+ placed in the medium. Suppose that a hole is formed. Then because the dielectric constant of the medium is less than unity, the induced charge on the inner surface of the hole will also be positive. The test charge and the induced charge thus repel, and the hole is stable against collapse. In normal QED, the induced charge will be negative, as indicated in Fig. 34(b), and will attract the test charge. The hole is thus unstable against collapse.

The radius R of the hole can be estimated on the basis of energetics. Within the hole the electrical energy W_{in} is finite and independent of the dielectric constant of the medium. The displacement field is radial and hence continuous across the spherical boundary. Thus it is given outside the hole by

$$\underline{D}_{out}(r>R) = \hat{\underline{r}}\, Q/r^2, \tag{7.9}$$

where Q is the total test charge. The induced charge density on the surface of the hole is

$$\begin{aligned}\sigma_{induced} &= (1-\varepsilon)|\underline{D}(R)|/4\pi\varepsilon \\ &= (1-\varepsilon)Q/4\pi\varepsilon R^2,\end{aligned} \tag{7.10}$$

which has the same sign as Q, as earlier asserted. Outside the hole, the electric field is determined by the total interior charge

$$Q + (1-\varepsilon)Q/\varepsilon = Q/\varepsilon, \tag{7.11}$$

so that

$$\underline{E}_{out}(r>R) = \hat{\underline{r}}\, Q/\varepsilon r^2 \tag{7.12}$$

The energy stored in electric fields outside the hole is then

$$\begin{aligned}W_{out} &= \frac{1}{8\pi}\int d^3\underline{r}\ \ \underline{D}_{out}(r)\cdot\underline{E}_{out}(r) \\ &= \frac{1}{2}\int_R^\infty r^2 dr\, Q^2/\varepsilon r^4 = Q^2/2\varepsilon R.\end{aligned} \tag{7.13}$$

As the dielectric constant of the medium approaches zero, W_{out} becomes large compared to W_{in}, so that the total electrical energy

$$W_{el} \equiv W_{in} + W_{out} \rightarrow W_{out}, \quad \text{as} \quad \varepsilon \rightarrow 0. \tag{7.14}$$

One must consider as well the energy required to hew such a hole out of the medium. For a hole of macroscopic size, it is reasonable to suppose that

$$W_{hole} = \frac{4\pi R^3}{3}v + 4\pi R^2 s + \cdots, \tag{7.15}$$

where v and s are non-negative constants. The total energy of the system,

$$W = W_{el} + W_{hole} \tag{7.16}$$

can now be minimized with respect to R . In the regime where the volume term dominates W_{hole} , the minimum occurs at

$$R = \left(\frac{Q^2}{2\varepsilon} \cdot \frac{1}{4\pi v} \right)^{1/4} \neq 0, \tag{7.17}$$

for which

$$W_{el} \approx \left(\frac{Q^2}{2\varepsilon} \right)^{3/4} (4\pi v) \tag{7.18}$$

and

$$W_{hole} \approx \frac{1}{3} \left(\frac{Q^2}{2\varepsilon} \right)^{3/4} (4\pi v)^{1/4}, \tag{7.19}$$

so that

$$W \approx \frac{4}{3} \left(\frac{Q^2}{2\varepsilon} \right)^{3/4} (4\pi v)^{1/4}. \tag{7.20}$$

Thus, in a very effective dia-electric medium, a test charge will induce a bubble or hole of finite radius. Notice, however, that in the limit of a perfect dia-electric medium

$$W \to \infty \qquad \text{as} \qquad \varepsilon \to 0. \tag{7.21}$$

An isolated charge in a perfect dia-electric thus has infinite energy. This is the promised analog of the argument used in § 5.2.1. to wish away isolated colored objects.

If instead of an isolated charge we place a test dipole within the putative hole in the medium, we can again show that the minimum energy configuration occurs for a hole of finite radius about the test dipole. In this case, however, the field lines need not extend to infinity, so the hole radius remains finite as $\varepsilon \to 0$, and so does the total energy of the system. The analogy between the exclusion of chromoelectric flux from the QCD vacuum and the exclusion of magnetic flux from a superconductor is now obvious. To separate the dipole charges to $\pm\infty$ requires an infinite amount of work, as shown in the previous example. This is the would-be analog of quark confinement. For a recent attempt to deduce an effective dia-electric theory from QCD, see Nielsen and Patkós (1981).

Two issues arise in this line of reasoning. One is the question of quark (or as we have phrased it here, charge) confinement. The other is what form does the sourceless QCD vacuum take if it is analogous to a perfect, or very effective, dia-electric medium ? Is the QCD vacuum unstable against the formation of domains containing dipole pairs in the electrostatic model, corresponding to gluons in color-singlet spin-singlet configurations ?

7.2. A String Analog

Suppose, as discussed in § 5.2.2 , that color-electric flux lines are squeezed into a flux tube. This effect can be parametrized by the statement that a region of space of volume

$$V = \sigma \cdot r \tag{7.22}$$

containing color-electric flux contributes a term

$$W_{bag} = B \cdot V = B\sigma r \tag{7.23}$$

to the total energy of the world, where B is a positive constant. The effect of the "bag pressure" B will be to compress the flux lines as much as possible.

The region of color-electric field emanating from a source of charge Q contains an energy density $E^2/8\pi$, where the electric field strength is

$$E = 4\pi Q/\sigma \tag{7.24}$$

if the flux lines are confined within an area σ. The energy stored in the field is thus

$$W_{field} = \frac{E^2}{8\pi} V = \left(\frac{4\pi Q}{\sigma}\right)^2 \cdot \frac{\sigma r}{8\pi}$$

$$= 2\pi Q^2 r/\sigma. \tag{7.25}$$

The total bag plus field energy is

$$W = W_{bag} + W_{field} = \left(B\sigma + \frac{2\pi Q^2}{\sigma}\right) r, \tag{7.26}$$

which can be minimized with respect to the area σ, whereupon

$$\sigma_0 = Q(2\pi/B)^{1/2}. \tag{7.27}$$

At this minimum, the energy density per unit length is

$$k = B\sigma_0 + 2\pi Q^2/\sigma_0 = 4\pi Q^2/\sigma_0 . \tag{7.28}$$

For a quark-antiquark pair, the replacement

$$Q^2 \rightarrow 4\alpha_s/3 \tag{7.29}$$

(compare (5.31a)) leads to

$$k = 16\pi\alpha_s/3\sigma_0 \tag{7.30}$$

or

$$\sigma_0 = 16\pi\alpha_s/3k \equiv \pi d^2. \tag{7.31}$$

Recalling from (5.21) that

$$k^{-1/2} \approx \tfrac{1}{2}\ \text{fm} \tag{7.32}$$

we find that

$$d \approx 2\ \text{fm}\ (\alpha_s/3)^{1/2}. \tag{7.33}$$

For a strong coupling constant $\alpha_s \approx 1$, the radius of the flux tube is

$$d \simeq 1\ \text{fm}, \tag{7.34}$$

a reasonable hadronic dimension. We have therefore contrived a situation in which a flux tube of finite radius is a stable configuration. It remains to show that this situation actually obtains in QCD.

Both in the present discussion and in § 5.3.2. we have neglected quark masses. Their inclusion is interesting as a matter of principle and is of some practical importance for particles composed of heavy quarks. Within the framework of an extended bag, the problem has been addressed by Johnson and Thorn (1976) and by Johnson and Nohl (1976) ; see also Chodos and Thorn (1974). Their work suggests that the Regge trajectories of particles composed of massive quarks should be shallow at low spins, but should approach a universal slope as $J \to \infty$. Some evidence for the first half of this statement was noted in § 4.1 , in connection with Fig. 19.

If chromoelectric confinement is indeed the origin of the string picture, we also gain an understanding of the equality of the Regge slopes of the mesons and baryons, which is apparent from Figs. 19 and 20. In an elongated bag, both mesons :

$$q \text{ ——————— } \bar{q} \tag{7.35}$$

and baryons :

$$q \text{ ——————— } (qq) \tag{7.36}$$

are $[\underset{\sim}{3}]-[\underset{\sim}{3}^*]$ color configurations. They must therefore have the same chromoelectric flux density, hence the same amount of

stored energy per unit area, hence the same Regge slope. It will thus be of considerable interest to learn the Regge trajectories of baryons containing several strange quarks or a heavy quark.

7.3. Quark Nonconfinement ?

If we assume in view of the heuristic arguments reviewed above that unbroken QCD is indeed a confining theory, how might we accommodate the observation of free quarks ? At first sight it seems straightforward to consider a spontaneously broken color symmetry which endows gluons with small masses and permits quark liberation. This has been explored by De Rujula, Giles, and Jaffe (1978), for example. Georgi (1980) has countered that a small mass term in the Lagrangian need not, in the face of strong quantum corrections, lead to a spontaneous symmetry breakdown. This possibility is open to discussion (De Rujula, Giles, and Jaffe, 1980). Okun and Shifman (1981) have argued that this style of partial confinement is incompatible with the known evidence for asymptotic freedom and with the absence of fractionally-charged hadrons. A different pattern of spontaneous symmetry breaking has been advocated by Slansky, Goldman, and Shaw (1981). Evidently the experimental search for fractionally-charged matter and the theoretical search for proofs or evasions of confinement are research topics of no little importance.

8. Glueballs and Related Topics

The possibility of quarkless states, composed entirely of gluons, would seem to be unique to a non-Abelian field theory such as QCD—as opposed to the elementary quark model. In this short introduction to glueballs I shall try to explore the four important questions :

i/ Should glueballs exist in QCD ?

ii/ What are their properties ?

iii/ How can they be found ?

iv/ Are they found in nature ?

Since most of what we believe to be the solution to QCD is abstracted from the elementary quark model, and because the quark model provides no guidance for quarkless states, the answers given to all of these questions will be partial and frustratingly vague. In the course of explaining these partial answers, one naturally encounters some other issues of significance : violations of the Zweig rule, deviations from ideal mixing, and the continuing problem of the pseudoscalar masses. A common thread will be seen to run through all these topics, and to tie them to the properties of glueballs.

The search for quarkless states has become intense, and several candidate states have appeared. I am not prepared to endorse any of these claims, at least not yet, but I shall have a little bit to say about the experimental situation. This will include some general and specific suggestions for experimental studies.

The subject of glueballs is a newly active one, which remains to be distinctly defined by experimental observations and by theoretical predictions of greater clarity. The modest aim of this Section is merely to underline the importance of the topic, and to introduce some of the issues involved. As for multiquark states, understanding the role of quarkless states in hadron spectroscopy remains in the future.

8.1. The Idea of Glueballs

If color is confined, color singlet states composed entirely of glue may exist as isolated hadron resonances. This is in essence the argument for the existence of quarkless states, as emphasized quite early by Fritzsch and Gell-Mann (1972). If one assumes the existence of gluons, the gauge interaction among gluons, and the confinement of color, this conclusion cannot easily be challenged. After the recognition of asymptotic freedom and the increasingly explicit formulation of QCD (among them Fritzsch, Gell-Mann, and Leutwyler, 1973 ; Gross and Wilczek, 1973b ; Weinberg, 1973), many authors have analyzed, in one or another framework, the possibilities for glueballs. A partial bibliography includes the papers by Freund and Nambu (1975), Fritzsch and Minkowski (1975), Bolzan, Palmer, and Pinsky (1976), Jaffe and Johnson (1976), Willemsen (1976), Kogut, Sinclair, and Susskind (1976), Veneziano (1976), Robson (1977), Roy and Walsh (1978), Koller and Walsh (1978), Ishikawa (1979ab), Bjorken (1979, 1980), Novikov, et al. (1979, 1980abcd, 1981), Zakharov (1980ab), Suura (1980), Donoghue (1980,1981), Roy (1979,1980), Soni (1980), Berg (1980), Coyne et al. (1980), Carlson, et al. (1980, 1981), Bhanot and Rebbi (1981), Bhanot (1981), Shifman (1981), Barnes (1981).

Bjorken (1979, 1980) has emphasized the apparent inevitability of color singlet, flavor singlet multigluon states within QCD. In pure (sourceless) QCD, with no fermions, the existence of glueballs follows at once from our assumptions stated above. This may be argued in any of the pictures we have discussed before. A "most attractive channel" analysis is implicit in the work of Barnes (1981). Bag arguments, of the sort given in § 7, lead to the conclusion that the color-singlet configuration is energetically favored, whereas colored states require infinite energy. The string picture of § 5.2.2 is also easily transplanted, with gluon sources replacing quark sources and glueballs replacing $(q\bar{q})$ mesons. The larger flux density between octet sources (cf. Table 12) than between triplets implies flatter Regge trajectories and hence a smaller level density for glueballs than for $(q\bar{q})$ states. In pure QCD there will be among the glueballs a lightest glueball state which, it is reasonable to expect, must be stable.

The introduction of massive quarks (stage II of Bjorken, 1980) does little but provide new sources of glueball production. Quarkonium states may now decay according to

$$^1S_0(Q\bar{Q}) \rightarrow gg, \qquad (8.1)$$

a colorless, $J^{PC} = 0^{-+}$ final state, and

$$^3S_1(Q\bar{Q}) \rightarrow \begin{cases} ggg, & (8.2a) \\ gg\gamma & (8.2b) \end{cases}$$

colorless hadronic states with $J^{PC} = 1^{--}$ for the three-gluon semifinal state and $J^{PC} = 0^{++}, 0^{-+}, 2^{++}$, etc. for the $gg\gamma$ semifinal state. Again the lightest gluon will be stable, because all $(Q\bar{Q})$ states are— by assumption— extremely massive.

Extending QCD to the light-quark sector raises two questions that go directly to the heart of the matter : what is the mass scale for glueballs, and how prominently will they appear in the spectrum of hadrons ? Given the small mass of the pion it is essentially a certainty that the lightest glueball will be unstable. We must then ask whether the quarkless states will become so broad as to be lost in a general continuum, whether they will mix so strongly with $(q\bar{q})$ and $(q\bar{q}g)$ states as to lose their identity, or whether they will remain relatively pure glue states of modest width. Until definite theoretical predictions can be given, we may conclude only that the observation of glueballs would support the notion that gluons exist and interact among themselves. Not finding glueballs, at the present level of understanding of QCD, has a less obvious significance.

8.2. The Properties of Glueballs

Some characteristics of quarkless states such as their flavor properties are unambiguous, but many others including masses and decay widths are predicted rather indecisively. It is reasonable to attempt to enumerate few-gluon states by analogy with Landau's (1948) classification of two-photon states, which incorporates the restrictions of Yang's (1950) theorem. This has been done by Fritzsch and Minkowski (1975), Barnes (1981), and within the bag model by Donoghue (1980). A pair of massless vector particles can be combined to yield states with

$$J^{PC} = \begin{cases} (\text{even} \geq 0)^{++} , & (8.3a) \\ (\text{even} \geq 0)^{-+} , & (8.3b) \\ (\text{even} \geq 2)^{++} , & (8.3c) \\ (\text{odd} \geq 3)^{++} . & (8.3d) \end{cases}$$

Many papers (e.g. Robson, 1977 ; Coyne, et al., 1980) treat the gluons as massive vectors and arrive at longer lists of two-gluon states. Similarly, extra states may arise in the bag model unless spurious modes associated with the empty bag are eliminated (Donoghue, Johnson, and Li, 1981). The lowest-lying two-gluon configurations should therefore include $J^{PC} = 0^{++}, 2^{++}, 0^{-+}$, and 2^{-+} states.

A variety of estimates of varying degrees of sophistication have been made for the masses of these states. Keeping in mind that the scalar ground state has precisely the quantum numbers of the vacuum and may therefore be appreciably mixed or even subsumed into the vacuum, let us list some representative predictions. The bag model (Donoghue, 1981) suggests that

$$M(0^{++}) \approx M(2^{++}) \approx 1 \text{ GeV}/c^2, \qquad (8.4)$$

neglecting hyperfine effects, and that

$$M(0^{-+}) \approx M(2^{-+}) \approx 1.3 \text{ GeV}/c^2, \qquad (8.5)$$

again neglecting hyperfine effects. The QCD sum rules of the ITEP Group (Novikov, et al., 1979, 1980abcd, 1981 ; Zakharov, 1980ab ; Shifman, 1981) lead to slightly larger values :

$$M(0^{++}) \approx M(2^{++}) \approx 1.2\text{–}1.4 \text{ GeV}/c^2, \qquad (8.6)$$

and

$$M(0^{-+}) \approx 2\text{–}2.5 \text{ GeV}/c^2, \qquad (8.7)$$

but with an important gluon component in η' (958). The effective potential calculations of Suura (1980) and Barnes (1981) lead to degenerate pseudoscalar and scalar states, with masses supposed to be on the order of 1–2 GeV/c^2. Barnes (1981) concludes that

$$M(2^{++})/M(0^{++}) \approx 1.8, \qquad (8.8)$$

with his description of hyperfine forces.

Three-gluon bound states are more complicated to analyze, especially in terms of the dynamics. It will suffice to give one estimate (Donoghue, 1981) of the masses, obtained in the bag model upon neglecting spin-spin forces :

$$M(0^{++}) \approx M(1^{+-}) \approx 1.45\ \mathrm{GeV}/c^2, \qquad (8.9)$$

and

$$M(0^{-+}) \approx M(1^{-+}) \approx M(1^{--}) \approx M(2^{-+}) \approx M(2^{--}) \approx 1.8\ \mathrm{GeV}/c^2. \qquad (8.10)$$

The general conclusion is that a host of states are to be expected, and that it is plausible that many exist in the region between 1 and 2 GeV/c^2. However all calculations have at least some degree of arbitrariness in the overall mass scale.

A simple lattice argument has also been presented (Kogut, Sinclair, and Susskind, 1976) for glueball masses in the 1-2 GeV/c^2 region. Figure 35(a) shows the minimal lattice configuration for a meson : a single link. On the other hand, on a rectangular lattice the minimal quarkless state consists of a closed loop made up of four links, as shown in Fig. 35(b). Consequently one may suppose that the mass of a typical ground-state glueball is approximately four times the mass of a typical ground-state meson and thus on the order of 1-2 GeV/c^2.

With respect to quantum numbers let us note that apart from the 1^{-+} level, which cannot occur as a $(q\bar{q})$ state, all of the glue states

resemble ordinary mesons. Their distinctive property is that pure glue states must be flavor singlets. With this restriction, the allowed decay modes follow from standard selection rules, although branching ratios may be strongly influenced by phase space effects and by the preëminence of quasi-two body final states.

It is quite possible, as we shall now discuss, that glueballs may be narrower structures than $(q\bar{q})$ mesons of comparable mass. This suspicion is tied up with the validity of the so-called Zweig rule and the mixing of glueballs with ordinary mesons, to which we now turn.

8.3. Gluons and the Zweig Rule

In § 3.3.2 we concluded on the basis of simple mass formulas, that φ (1019) is essentially a pure $(s\bar{s})$ state. This conclusion is sustained by an examination of the decay modes of φ , which are collected in Table 21. The total width is

$$\Gamma(\varphi) = 4.1 \pm 0.2\ \mathrm{MeV}. \qquad (8.11)$$

Decays into $K\bar{K}$ are inhibited by the limited phase space available, and the relative rates for the charged and neutral final states are understood in terms of p-wave kinematics. For the suppression of the 3π $(\pi\rho)$ mode, however, a dynamical explanation must be sought.

The suppression of nonstrange decays can be accounted for, if not explained from first principles, by the rule (Okubo, 1963 ; Zweig, 1964ab ; Iizuka, Okado, and Shito, 1966 ; Iizuka, 1966) that decays which correspond to connected quark-line diagrams are allowed, but those which correspond to disconnected diagrams are not. This is made concrete for the case of φ decay in Fig. 36. The dissociation and subsequent dressing of the $(s\bar{s})$ pair is allowed (a), but the quarkless semifinal state reached by $(s\bar{s})$ annihilation is not. One may attribute the small observed rate for $\varphi \to 3\pi$ either to light-quark impurities in the φ wavefunction or to violations of the Zweig rule.

Additional evidence in favor of the rule comes from the remarkable metastability of ψ(3097), for which (Particle Data Group, 1980)

$$\Gamma(\psi \to \text{hadrons}) = 45 \text{ keV}, \qquad (8.12)$$

and of Υ (9433), for which (Schamberger, 1981)

$$\Gamma(\Upsilon \to \text{hadrons}) = 28 \text{ keV}. \qquad (8.13)$$

The Zweig rule thus provides a notable mnemonic for forbidden decays. It is of interest to ask whether there is a dynamical basis for the rule within QCD, and whether there may be other manifestations of violations of the rule.

To this end, recall the outstanding failure of our description of meson masses : the problem of the η and η' masses. In the

language of singlet and octet mixing we found it possible in § 3.3.2 to parametrize $M(\eta)$ and $M(\eta')$ in terms of two free parameters : a flavor-singlet mass M_1 and a mixing angle θ . The resulting wavefunctions imply relations between decay and reaction rates that are imperfectly respected by the data, as noted in § 5.5.1. In the otherwise successful quark language we were not able to understand the $\pi^0-\eta-\eta'$ splitting or the high mass of the η' . If we interpret the failure as pertaining only to isoscalar states, it is sensible to consider the possibility that virtual annihilations into glue states

$$q\bar{q} \to \text{glue} \to q'\bar{q}' \qquad (8.14)$$

may influence the masses of $(q\bar{q})$ states.(See among others De Rujula, Georgi, and Glashow, 1975, Isgur, 1976). Such transitions of course cannot affect flavored states.

In the $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ basis the mass matrix of the pseudoscalar mesons can be written as

$$\mathcal{M} = \begin{pmatrix} 2m_u - 3\delta M_{c.m.} + A & A & A \\ A & 2m_u - 3\delta M_{c.m.} + A & A \\ A & A & 2m_s - 3(m_u/m_s)^2 \delta M_{c.m.} + A \end{pmatrix} \qquad (8.15)$$

in the notation of eqns. (5.52, 53, 57, 58), where A represents the flavor-independent amplitude for the process (8.14). Recognizing that virtual annihilations cannot affect the π^0 mass, we recast (8.15) in a basis of $(u\bar{u} \mp d\bar{d})/\sqrt{2}$, $s\bar{s}$ as

$$\mathcal{M}=\begin{pmatrix} 2m_u-3\delta M_{c.m.} & 0 & 0 \\ 0 & 2m_u-3\delta M_{c.m.}\ 2A & \sqrt{2}\,A \\ 0 & \sqrt{2}\,A & 2m_s-3(m_u/m_s)^2\delta M_{c.m.}+A \end{pmatrix}, \qquad (8.16)$$

which retains the expected result $M(\pi^{\pm})=M(\pi^0)$.

The remaining two-by-two isoscalar mass matrix suggests a common origin for the $\pi^0-\eta-\eta'$ splitting and the deviations from ideal mixing. With the parameters of § 5.3., the sum of η and η' masses is reproduced with the choice

$$A = 198 \ \text{MeV}/c^2, \qquad (8.17)$$

for which

$$M(\eta) = 408 \ \text{MeV}/c^2 \qquad (8.18a)$$

and

$$M(\eta') = 1109 \ \text{MeV}/c^2. \qquad (8.18b)$$

The wavefunctions implied are

$$|\eta'\rangle = 0.44\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right) + 0.81\ s\bar{s}, \qquad (8.19a)$$

and

$$|\eta\rangle = 0.81\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right) - 0.44\ s\bar{s}, \qquad (8.19b)$$

which are similar to those (3.112) given by the linear mass formula in the singlet-octet picture. The masses (8.18) are considerably improved over those produced in § 5.3 , but they are still not perfect. At any rate, we have succeeded in raising the η-η' center of gravity by invoking virtual annihilations, and have thus been able to begin to reconcile the constituent picture with the symmetry approach. Note also that if physical glue states do exist, the mass matrix must be enlarged and the mixing pattern may be considerably more complicated.

The success of our earlier description of the vector meson masses argues that no appreciable annihilation is required there. For heavy mesons such as ψ and Υ , the analogy with ortho- and para-positronium seems apt. A coupling constant argument then suggests that in the asymptotically free regime

$$\Gamma(^3S_1\rightarrow \text{glue})/\Gamma(^1S_0\rightarrow \text{glue})=\alpha_s \times \text{numerical factors} \qquad (8.20)$$

A power of small coupling constant may inhibit mixing of vector states with gluons in quarkonium, but this is a tenuous argument for the light mesons. In the following § 9 we shall review an argument in favor of the Zweig rule that does not depend upon powers of the coupling constant.

Among the orbitally-excited mesons, there is also room for virtual annihilations. One should in general be alert for the possibility whenever a breakdown of ideal mixing is signalled by the nondegeneracy of the isovector and would-be $(u\bar{u}+d\bar{d})/\sqrt{2}$ states. For a recent look at the 1^{++} and 2^{++} nonets, see Schnitzer (1981ab).

If virtual transitions such as (8.14) occur, they may account for violations of the Zweig rule. This mechanism for Zweig-rule violations naturally suggests the pattern

$$\Gamma(Q\bar{Q}\to\text{hadrons}) < \Gamma(\text{glue}\to\text{hadrons}) < \Gamma(q\bar{q}\to\text{hadrons}), \quad (8.21)$$

where the Zweig-inhibited quarkonium decay rate is of order A^2, the decay rate of a glueball into light quarks is of order A^1, and the rate for Zweig-allowed dissociation of a light quark pair is of order A^0. The possibility therefore exists that a pure glue state will be relatively more stable than a light-quark meson of comparable mass.

8.4. Searching for Glueballs

As strongly-interacting particles, glueballs should be produced routinely in hadronic collisions, where they may be sought out using the techniques of traditional meson spectroscopy. Special kinematic selections may enhance the glueball signal over ordinary mesonic background. An obvious choice with the CERN $p\bar{p}$ collider at hand is an investigation of "Double-Pomeron events", which yield hadronic states in the central region of rapidity with vacuum quantum numbers. If, as Freund and Nambu (1975) and others have suggested, there is a deep connection between the Pomeron and quarkless states, such a selection may be of more than merely kinematical benefit.

Another favorable situation may be in the decays of heavy quarkonium according to (8.2b), leading to transitions of the form

$$\psi\to\gamma+G \quad (8.21)$$

where G denotes a glueball. This is not only a case in which the general arguments of § 8.1 lead us to expect that glueballs may be produced, but also one that permits inclusive as well as exclusive searches and lends itself to comparison with

$$\psi\to(\omega,\varphi) + \text{anything} \quad (8.22)$$

in which the anything is presumably composed of quarks.

Interest in quarkonium decay has been increased by the recent observation of suggestive structures in

$$\psi\to\gamma + \text{hadrons}. \quad (8.23)$$

According to Scharre (1981), there is evidence in the Crystal Ball Experiment at SPEAR for two new states. The first, named iota (1440) is seen in the cascade decay

$$\psi\to\gamma+\iota(1440) \quad (8.24)$$
$$\hookrightarrow K\bar{K}\pi \neq KK^*$$

with a mass $M_\iota = 1440^{+10}_{-15}$ MeV/c^2 and a width of $\Gamma_\iota = 50^{+30}_{-20}$ MeV. The state has $J^{PC} = 0^{-+}$ and the combined branching ratio for the cascade is

$$B(\psi \to \gamma\iota)\, B(\iota \to K\bar{K}\pi) \approx 4 \times 10^{-3}. \tag{8.25}$$

The second is seen in

$$\psi \to \gamma + \theta(1640) \quad \hookrightarrow \eta\eta, \tag{8.26}$$

with a mass M_θ = 1650 ± 50 MeV/c^2 and a width of $\Gamma_\theta = 220^{+100}_{-70}$ MeV. The decay angular distribution favors J^{PC} = 2^{++}, and the product of branching ratios is

$$B(\psi \to \gamma\theta)\, B(\theta \to \eta\eta) \approx 5 \times 10^{-4}. \tag{8.27}$$

An upper limit exists for the decay of θ into $\pi^\circ\pi^\circ$:

$$B(\theta \to \pi\pi) \lesssim B(\theta \to \eta\eta). \tag{8.28}$$

We have seen above that scalar, pseudoscalar, and tensor glueballs are to be expected in this mass range. In addition, an analysis (Billoire, et al., 1979) of the spin-parity content of the gluon pair in (8.2b) suggests that 2^{++} formation is favored with equal but smaller probabilities for 0^{++} and 0^{-+} configurations. At the same time, radial excitations of the low-lying mesons are to be expected in precisely this region (Cohen and Lipkin, 1979). Thus it is easily possible that any new states be traditional ($q\bar{q}$) states, or mixtures of ($q\bar{q}$) with glueballs, or other exotic possibilities (Close, 1981), as well as states of pure glue. How can these possibilities be distinguished ?

Without going into details, let us note that Chanowitz (1981), Ishikawa (1981), Donoghue, Johnson, and Li (1981), Lipkin (1981b), and Cho, Cortes, and Pham (1981) have examined the case that ι(1440) is a glueball. Opinion is divided. Chanowitz (1981) has shown that a large number of seemingly contradictory experiments may be reconciled if, in addition to the 1^{++} E(1420) there is a nearby pseudoscalar state for which ι(1440) is the obvious candidate. He further argues that ι(1440) has the characteristics of a glueball, but does not concern himself with the η'(1400) - see Table 18. If we accept the spin-parity assignments, then there are at least two isoscalar states around 1400 MeV/c^2. The conclusion that ι(1440) is pseudoscalar and not axial (as E(1420)) removes a potential embarrassment for the two-gluon glueball interpretation. Lipkin (1981), on the other hand, argues that the absence of an appreciable $\iota \to \eta\pi\pi$ signal is inconsistent with a flavor singlet assignment. Obviously there are many experimental questions to settle, among them the spin-parity assignments and the relationship between η' and ι.

Another obvious test may be available in two-photon reactions

$$e^+e^- \to e^+e^- + \text{hadrons}, \tag{8.29}$$

at least for the pseudoscalar state (s). A $(q\bar{q})$ state should decay into two photons, whereas a pure glue state should not, in lowest order. This inspires a search for the reactions

$$e^+e^- \to e^+e^- \eta'(1400) \atop \hookrightarrow \eta\pi\pi \tag{8.30a}$$

$$e^+e^- \to e^+e^- \iota(1440) \atop \hookrightarrow K\bar{K}\pi \tag{8.30b}$$

Given an estimate for the two-photon decay rate, standard techniques (described in Quigg, 1980) lead to the two-photon production cross section. The rate for production of a $J^{PC} = 0^{-+}$ $\iota(1440)$ is shown in Fig. 37 under the assumption that

$$\Gamma(\iota \to \gamma\gamma) = 1 \text{ keV}. \tag{8.31}$$

At the energies accessible at PEP and PETRA, an ample cross section is to be expected.

If a prominent signal is observed, one may conclude that the hadronic state is not an axial vector meson and that it is not dominantly a glueball. But if no signal is found, what then ? I see four possibilities :

i/ the hadron is an axial state,

ii/ the hadron is a glueball,

iii/ the hadron is a $(q\bar{q})$ state with a small width for two-photon decay,

iv/ the hadron is a mixed $(q\bar{q})$-glueball state.

The first and second points are self-evident. The third is more problematical. I believe a two-photon width of 0.1-1 keV is reasonable for a radially-excited pseudoscalar, but I cannot convince myself that this represents the full range of "reasonable" possibilities. If glueballs exist, the fourth possibility seems to me the most reasonable one. It has been studied in some detail by Donoghue, Johnson, and Li (1981), by Rosner (1981b), and by Cho, Cortes, and Pham (1981).

In general we may expect some degree of mixing between nearby (or overlapping) hadrons. The simplest case of one glueball and one $(q\bar{q})$ state can be parametrized as

$$|h_1\rangle = |\underline{q\bar{q}}\rangle \cos\theta + |G\rangle \sin\theta , \tag{8.32a}$$

$$|h_2\rangle = -|\underline{q\bar{q}}\rangle \sin\theta + |G\rangle \cos\theta , \tag{8.32b}$$

in an obvious notation. The decay $\psi \to \gamma$ + glue would then lead to a line shape characteristic of

$$|h_1\rangle \sin\theta + |h_2\rangle \cos\theta , \tag{8.33}$$

whereas two-photon collisions would excite

$$|h_1\rangle \cos\theta - |h_2\rangle \sin\theta . \tag{8.34}$$

Thus we are led to ask whether, for example, the f^{*}(1270) seen in the decay $\psi \rightarrow \gamma + f^{o}$ is identical with that observed in two-photon collisions. To answer such questions requires careful measurements of line shapes and branching ratios in both kinds of reactions, as well as in peripheral and central hadron collisions. There is much to be learned here, but the experimental work called for is demanding and meticulous.

Before leaving the subject of glue, let us note that there may be other manifestations of degrees of freedom beyond those of quark and antiquark. The specific possibility of "vibrational modes" has been raised by Giles and Tye (1977, 1978) and by Buchmüller and Tye (1980). States with constituent gluons ($q\bar{q}g$) states have been examined by Horn and Mandula (1978) and by de Viron and Weyers (1981) ; see also Close (1981). Glue-Bearing baryons (qqq g) have been considered by Bowler, et al. (1980).

9. The Idea of the 1/N Expansion in QCD

The search for small parameters which can play the part of expansion parameters is a central element of the process of approximation and model making that is theoretical physics. In many physical situations, extremes of energy or distance suggest highly accurate and readily improved approximation schemes. In classical electrodynamics the indispensable far-field approximation is applicable when the size of a radiator is negligible compared to the distance between the radiator and receiver. The Born approximation for the scattering of charged-particle beams from atomic electrons is trustworthy for beam energies greatly in excess of the atomic binding energy. In Quantum Chromodynamics, a perturbative treatment (which is to say an expansion in powers of the strong coupling parameter $\alpha_s(Q^2)$) is expected to be reliable when the invariant momentum transfer Q^2 is large compared to a characteristic mass scale denoted by Λ^2 .

For the problem of hadron structure, no similar expansion is applicable. All of the relevant energies of the problem are on the order of the naturally occurring scale. In a typical hadron, the separation of the quarks is simply the hadronic size of approximately 1 fm — hardly a regime in which perturbative QCD is likely to make any sense. We may, of course, simply await the day when a very heavy quarkonium family is found, and then happily apply conventional perturbative measures. That insouciant course however leaves untouched the problem of the structure of all the hadrons now known, so other actions are called for.

The strategy of the 1/N expansion is a familiar one. When confronted with a problem we cannot solve, we invent a related problem that we can solve . If this is done adroitly, the new problem will not only be simpler but will also capture the physical essence of the original one. More specifically, the 1/N expansion represents an attempt to introduce a parameter that permits a simplification of the calculation at hand. Problem 24 introduces an elementary example.

For QCD, this simplification is achieved ('t Hooft, 1974ab) by generalizing the color gauge group from $SU(3)_c$ to $SU(N)_c$ and considering the limit in which N becomes very large. Although SU(N) is in general more complicated than SU(3), the hadron structure problem is simplified by two observations :

i/ At any order in the strong coupling constant, some classes diagrams are found to be combinatorially negligible.

ii/ The remaining diagrams have common consequences, in large-N perturbation theory.

This technique does not entirely free us from the constraints of perturbative analysis. Since we shall find, by inspection, that entire classes of combinatorially-favored diagrams have common features to all orders in the coupling constant, we shall have to assume that the content of the theory is accurately represented by the set of all diagrams. For QCD, the reliability of the 1/N expansion is inferred

from the fact that $SU(N)_c$ QCD seems to resemble the world we observe. Clear introductions to the method, with allusions to other physical situations, are given by Coleman (1980), and by Witten (1979b, 1980 ab).

The combinatorial analysis of $SU(N)_c$ QCD is most transparent in terms of the double line notation introduced for this purpose by 't Hooft (1974a), which is illustrated in Fig. 38. Several examples will suffice to make the main points.

Consider first the lowest-order vacuum polarization contributions to the gluon propagator, the quark loop illustrated in Fig. 39(a) and the gluon loop pictured in Fig. 39(b), in conventional notation. These are redrawn in the double line notation in Fig. 39 (c,d). For an initial gluon of type $i\bar{j}$, only a single color configuration is possible for the quark loop intermediate state : a quark of color i and an antiquark of color $\bar{j}$. For the gluon loop, however, the index k is free to take on any value 1, 2, ..., N. Thus the gluon loop diagram has a combinatoric factor N associated with it. This illustrates the general rule that gluon loops dominate over quark loops by a factor of N, as $N \rightarrow \infty$.

The presence of the factor N would seem to imply that the gluon loop diagram diverges as $N \rightarrow \infty$. This can be cured by choosing the coupling constant to be $g/\sqrt{N}$, with g fixed as $N \rightarrow \infty$. Then for any value of N, the contribution of the gluon loop goes as

$$\left(\frac{g}{\sqrt{N}}\right)^2 \times N \longrightarrow g^2, \qquad (9.1)$$

a smooth limit.

That this device solves the divergence problem in general is indicated by an analysis of diagrams with more than one loop. The two-loop diagram depicted in Fig. 40 in (a) standard and (b) two-loop notation is immediately seen to be proportional to

$$\left(\frac{g}{\sqrt{N}}\right)^4 \times N^2 \longrightarrow g^4. \qquad (9.2)$$

Similarly, the three-loop diagram of Fig. 41 obviously goes as

$$\left(\frac{g}{\sqrt{N}}\right)^6 \times N^3 \longrightarrow g^6. \qquad (9.3)$$

The situation is different for nonplanar graphs, however. The simplest such graph is shown in Fig. 42. The double-line notation makes it apparent that this graph contains but a single, tangled color loop, and therefore goes as

$$\left(\frac{g}{\sqrt{N}}\right)^6 \times N \longrightarrow g^6/N^2, \qquad (9.4)$$

and is therefore suppressed by $1/N^2$ compared to its planar counterpart at the same order in g^2. It is generally the case that nonplanar graphs are reduced by $1/N^2$, as $N \to \infty$.

These combinatorial arguments select planar graphs as an important subclass. To evaluate and sum all the graphs thus selected is no trivial task. Instead, we may identify their common features and speculate that these survive confinement. It is possible in this way to establish the following results in the large-N limit :

i/ Mesons are free, stable, and noninteracting. For each allowed combination of J^{PC} and flavor quantum numbers, there are an infinite number of resonances.

ii/ Zweig's rule is exact. Singlet-octet mixing (through virtual annihilations) and meson-glue mixing are suppressed. Mesons are pure $(q\bar{q})$ states, with no quark-antiquark sea.

iii/ Meson-meson bound states, which would include particles with exotic quantum numbers, are absent.

iv/ Meson decay amplitudes are proportional to $1/\sqrt{N}$, so mesons are narrow structures.

v/ The meson-meson elastic scattering amplitude is proportional to $1/N$ and is given, as in Regge theory, by an infinite number of one-meson exchange diagrams.

vi/ Multibody decays of unstable mesons are dominated by resonant, quasi-two body channels whenever they are open. The partial width of an intrinsically k-body final state goes as $1/N^{k-1}$.

vii/ For each allowed J^{PC} there are infinitely many glueball states, with widths of order $1/N^2$. They are thus more stable than $(q\bar{q})$ mesons, interact feebly with $(q\bar{q})$ mesons, and mix only weakly with $(q\bar{q})$ states.

Until QCD is actually solved, we will not know how closely the $N \to \infty$ limit of $SU(N)_c$ resembles the case of interest, which is color-SU(3). The preceding list of large-N results does bear, however, a quite striking resemblance to the world described earlier in these lectures. To the extent that the 1/N expansion faithfully represents the consequences of QCD, much of the foregoing phenomenology is explained, and many of the model approximations are justified.

To see how conclusions (i)-(vii) may be reached, let us consider the 1/N derivation of the Zweig rule. A possible mechanism for the Zweig-forbidden decay of $(q\bar{q})$ state is shown in Fig. 43, the process

$$(q\bar{q}) \longrightarrow gg \longrightarrow q'\bar{q}' \longrightarrow \text{mesons}\,. \qquad (9.5)$$

This is shown in standard notation in Fig. 43(a), and in double-line notation in Fig. 43(b). In the latter case I have tied together the ends of the quark and antiquark lines in mesons to emphasize that the mesons are color singlets. The Zweig-forbidden decay amplitude contains a single color loop. It therefore goes as

$$\left(\frac{g}{\sqrt{N}}\right)^4 \times N \sim g^4/N. \qquad (9.6)$$

At the same order in the strong coupling constant, the allowed decay is illustrated in Fig. 44. In the double-line representation, it is seen to contain two color loops. The allowed amplitude is therefore proportional to

$$\left(\frac{g^2}{\sqrt{N}}\right)^4 \times N^2 \sim g^4. \qquad (9.7)$$

Thus at each order in perturbation theory, the Zweig-forbidden decay is down by a power of 1/N in amplitude compared with the Zweig-allowed decay. Since this reasoning does not rely upon the smallness of the strong coupling constant, which may well be appropriate for the ψ and Υ families, it is an appealing argument for the inhibition of $\varphi \to \rho\pi$. The 1/N expansion has also been applied to the problem of baryon structure by Witten (1979b).

To close this brief section on the 1/N expansion, let us briefly return to the difficulty of understanding the η' mass. A clear statement of the puzzle of the flavor-singlet pseudoscalar meson, which is known as the U(1) problem, was given by Gell-Mann, Oakes, and Renner (1968) and by Weinberg (1975). What seems a promising phenomenological explanation is the influence of virtual states composed of glue alone, as described in § 8.3. A formal solution to the U(1) problem was given by 't Hooft (1976), who argued that the U(1) current has an anomaly which leads to a physical nonconservation of the U(1) charge. This removes the raison d'être for a ninth light pseudoscalar. The relationship between the intuitive and formal approaches was exhibited in the context of the 1/N expansion by Witten (1979a, 1980c), Di Vecchia (1979), and Veneziano (1979b).

10. Regrets

One cannot reach the end of a course such as this without contemplating what might have been, or what should have been. There are a number of subjects that I have been forced by the pressure of time to omit. Here I attempt to make amends by providing a brief bibliography for some of the topics I had hoped to discuss.

10.1. The Masses of Quarks

At various points in the analysis of hadron masses we have had occasion to refer to the effective masses of confined quarks. Several important issues have thus been swept under the rug, or at best talked around. One is how QCD behaves in the limit of vanishing quark masses, for which the Lagrangian will have an exact SU(n) $\otimes$ SU(n) chiral symmetry operating independently on the left-and right-handed parts of the quark fields for the n massless flavors. That this is approximately so in Nature is evidenced by the success of soft-pion theorems (see Adler and Dashen, 1968 ; Renner, 1968 ; Lee, 1972). The Lagrangian will also have the chiral U(1) symmetry which leads to the puzzle of the η' mass dealt with in § 9.

In the limit of zero up-, down-, and strange-quark masses, QCD possesses an octet of exactly conserved axial currents. It is believed that the corresponding chiral symmetry must be spontaneously broken along the lines described by Nambu and

Jona-Lasinio (1961ab). Accordingly, in the world of three massless quark flavors there should be eight massless Goldstone (1961) bosons which we identify with the pseudoscalar octet. See also Nambu (1960). The pattern of the spontaneous breaking of the chiral symmetry of QCD has been discussed from the point of view of the 1/N expansion by Coleman and Witten (1980).

Nonzero values of the quark masses which appear in the Lagrangian are thought to arise from the spontaneous breakdown of the $SU(2) \otimes U(1)$ gauge symmetry of the electroweak interactions by means of the Higgs (1964ab) mechanism (for an elementary discussion, see Quigg, 1981), or through dynamical symmetry breaking (Weinberg, 1976, 1977 ; Susskind, 1979 ; Farhi and Susskind, 1981). It then follows that the π , K , and η are only approximately massless, although they are presumed to retain some memory of their chiral origin. The Lagrangian ("current quark") masses have been studied by Leutwyler (1974ab), Pagels (1975), and Langacker and Pagels (1979), among others.

If the masses of the up and down quarks are not identical — a possibility we have entertained in connection with electromagnetic mass differences of hadrons — there may be a number of observable violations of isospin symmetry. The effect upon $\rho\omega$ mixing was mentioned in passing in § 3.1.3 , and many other applications are discussed by Gross, Treiman, and Wilczek (1979), Isgur, Rubinstein, Schwimmer, and Lipkin (1979), Langacker (1979, 1980), and Shifman, Vainshtein, and Zakharov (1979d).

For recent attempts to understand the chiral nature of the pion within the framework of QCD and confinement, consult Pagels (1979), Pagels and Stokar (1979), Donoghue and Johnson (1980), Goldman and Haymaker (1981), and Haymaker and Goldman (1981).

10.2. Decays and Interactions of Hadrons

Important support for flavor-SU(3) symmetry and for specific multiplet assignments derives from the systematic study of hadron decay rates and hadron-hadron reaction rates. The quark model, with or without specific dynamical assumptions, makes many predictions that are sharper than those of SU(3) alone. Entry to the extensive literature on these subjects may be gained via the lecture notes by Rosner (1981a) and the book by Close (1979).

10.3. QCD Sum Rules

A very different and extremely provocative approach to hadron spectroscopy has been pioneered by a group from the Institute for Theoretical and Experimental Physics in Moscow. I regard my omission of their method of analysis as particularly unfortunate. For the students at Les Houches, although not for posterity, this void was filled by informative seminars by John Bell and Eduardo de Rafael (but see in part Bourrely, Machet, and de Rafael, 1981). A short course is provided by the following articles : Shifman, Vainshtein, and

Zakharov (1979abcd) ; Voloshin (1979) ; Leutwyler (1981) ; Reinders, Rubinstein, and Yazaki (1981) ; Bell and Bertlmann (1981) ; and Ioffe (1981).

10.4. Relation to Other Pictures of Hadrons

Finally, and still more telegraphically, I wish to note a few articles which pertain to other approaches to hadron structure and their connections with the schemes I have discussed. Renormalization group techniques for quarks and strings are reviewed by Kadanoff (1977). The theory of dual models and strings is summarized by Scherk (1975). Parallels between QCD, especially in the $1/N_{color}$ expansion, dual theories, and the Reggeon calculus are drawn by Veneziano (1976, 1979a).

Acknowledgements

Many individuals and institutions have contributed to the existence of these lecture notes. Mary K. Gaillard invited me to teach a course on hadrons at Les Houches. I thank her and Raymond Stora for their warm encouragement and for the pleasant atmosphere I found there. Jonathan Rosner has taught me much about spectroscopy ; in addition I have benefited enormously in planning this course from hearing his St. Croix lectures on quark models. Students at Les Houches and in a related course at the University of Chicago in the Spring of 1981 have asked many provocative questions, not all of which have been answered here. The writing of these notes was facilitated by the hospitality of Jacques Prentki and his colleagues in the Theoretical Studies Division at CERN and by that of Jean-Loup Gervais and other members of the Laboratoire de Physique Théorique de l'Ecole Normale Supérieure. No less important has been the generosity of Leon Lederman and the Trustees of Universities Research Association in granting the leave of absence during which this work was completed. Special thanks are owed to Valérie Lecuyer and Yolande Ruelle for typing the lengthy manuscript under unusual conditions. Finally, I thank my family for a long season of tolerant support.

PROBLEMS

1. Consider bound states composed of fundamental scalar particles (denoted σ). The quantum numbers of σ are $J^{PC} = 0^{++}$. For $(\sigma\sigma)$ composites,

a) Show that a bound state with angular momentum L (i.e. an orbital excitation) must have quantum numbers

$$C = (-1)^L \; ; \quad P = (-1)^L \; .$$

b) Allowing for both orbital and radial excitations, construct a schematic mass spectrum of $(\sigma\sigma)$ bound states. Label each state with its quantum numbers J^{PC}.

c) Now suppose that the fundamental scalars have isospin I. Compute C, P, and G for $(\sigma\sigma)$ bound states, and redo part b).

2. Consider bound states composed of fundamental spin-1/2 particles (denoted f), with isospin = 1/2. For $(f\bar{f})$ composites,

a) Show that a bound state with angular momentum L must have quantum numbers

$$C = (-1)^{L+s} \; ; \quad P = (-1)^{L+1} \; ; \quad G = (-1)^{L+s+I} \; ,$$

where s is the spin of the composite system, and I is its isospin.

b) Allowing for both orbital and radial excitations, construct a schematic mass spectrum of $(f\bar{f})$ bound states. Label each state with its quantum numbers J^{PC}.

3. The η-meson (550 MeV/c^2) has quantum numbers $J^{PC} = 0^{-+}$ and isospin zero. Its principal decay modes, and branching fractions, are

$\gamma\gamma$	38 %
$\pi^0\pi^0\pi^0$	30 %
$\pi^+\pi^-\pi^0$	24 % .

We wish to understand the surprising competition of photonic and hadronic decay modes. Show that the hadronic decays are isospin-violating. Analyze the $3\pi^0$ and the $\pi^+\pi^-\pi^0$ decays separately. What qualitative explanation can be offered for the relative decay rates ?

4. The permutation group on three objects admits three representations : symmetric (S), antisymmetric (A), and mixed (M). For the first two, the group elements are

Representation \ Element	I	(12)	(13)	(23)	(123)	(132)
S	1	1	1	1	1	1
A	1	-1	-1	-1	1	1

When baryon wavefunctions are constructed from three isospin quarks, $|I = 1/2, I_z = \pm 1/2\rangle$, the antisymmetric representation cannot be formed. Consider the M representation of I = 1/2 final states, which may be built by first coupling quarks 1 and 2 to isospin 0 or 1, and then coupling the third quark. Use as a basis the two states $|1/2, 1/2\rangle_1$ (symmetric in (12)) and $|1/2, 1/2\rangle_0$ (antisymmetric in (12)). Denote these states as $|1\rangle$ and $|0\rangle$, and use $\begin{pmatrix} |1\rangle \\ |0\rangle \end{pmatrix}$ as a basis vector for M. Find the 2 x 2 matrices representing the action of the permutations listed in the table for the M representation.

5. The flavor symmetry $SU(2)_{isospin}$ and the rotational symmetry $SU(2)_{spin}$ may be combined systematically in the group SU(4). In nuclear physics, this symmetry group provides the basis for classification into "Wigner supermultiplets". The fundamental representation of SU(4) is

$$\underset{\sim}{4} \equiv \begin{pmatrix} u_\uparrow \\ d_\uparrow \\ u_\downarrow \\ d_\downarrow \end{pmatrix} .$$

Using the notation ($\underset{\sim}{2I+1}$, $\underset{\sim}{2s+1}$) for the isospin x spin decomposition of SU(4) representations, we may write

$$\underset{\sim}{4} = (\underset{\sim}{2}, \underset{\sim}{2}),$$

which shows that the $\underset{\sim}{4}$ of SU(4) transforms as a doublet under isospin rotations and as a doublet under spin rotations.

a) Using the techniques for SU(N) computations developed for example in Chapter 3 of Close (1979) or in Bacry (1967), work out the SU(4) content of the product

$$\underset{\sim}{4} \otimes \underset{\sim}{4} \otimes \underset{\sim}{4} .$$

Characterize each SU(4) representation in the product by its Young tableau, symmetry properties, and dimension.

b) Give the ($\underset{\sim}{2I+1}$, $\underset{\sim}{2s+1}$) content of each of the SU(4) representations in your expansion of $\underset{\sim}{4} \otimes \underset{\sim}{4} \otimes \underset{\sim}{4}$.

Reference : Lipkin (1966).

6. Compute the magnetic moments of $\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$. Assume that the magnetic moment of a quark is given by

$$\mu_i = e_i \hbar / 2 m_i c$$

where $i = u, d$ and e_i is the quark charge in units of $|e|$. Further assume that $m_u = m_d$.

References : Close (1979), Chapter 4 ; Kokkedee (1969), Chapter 11.

7. Work out the explicit SU(6) wavefunctions for the strange members of the baryon octet :

The expressions will be the analogs of

$$|p\uparrow\rangle = (1/\sqrt{18})(2u_\uparrow d_\downarrow u_\uparrow - u_\downarrow d_\uparrow u_\uparrow - u_\uparrow d_\uparrow u_\downarrow$$
$$- d_\uparrow u_\downarrow u_\uparrow + 2d_\downarrow u_\uparrow u_\uparrow - d_\uparrow u_\uparrow u_\downarrow$$
$$- u_\uparrow u_\downarrow d_\uparrow - u_\downarrow u_\uparrow d_\uparrow + 2u_\uparrow u_\uparrow d_\downarrow)$$

and

$$|n\uparrow\rangle = (-1/\sqrt{18})(2d_\uparrow u_\downarrow d_\uparrow - d_\downarrow u_\uparrow d_\uparrow - d_\uparrow u_\uparrow d_\downarrow$$
$$- u_\uparrow d_\downarrow d_\uparrow + 2u_\downarrow d_\uparrow d_\uparrow - u_\uparrow d_\uparrow d_\downarrow$$
$$- d_\uparrow d_\downarrow u_\uparrow - d_\downarrow d_\uparrow u_\uparrow + 2d_\uparrow d_\uparrow u_\downarrow)$$

8. Using your explicit wavefunctions, express the magnetic moments of the strange baryons in terms of μ_u, μ_d, μ_s.

9. In the SU(3) symmetry limit, the quark magnetic moments are proportional to quark charge :

$$\mu_d = \mu_s = -\tfrac{1}{2}\mu_u .$$

Using the proton moment,

$$\mu_p = 2.793 \text{ n.m.}$$

as input, predict the numerical values of the magnetic moments of the octet baryons. Compare with the measured values given by the Particle Data Group (1980).

Reference for problems 7-9 : Thirring (1966).

10. Consider the weak decay of a Λ-hyperon (with four-momentum p_Λ) into a proton (with four-momentum p) and a π^- (with four-momentum q). In general, the Feynman amplitude for the decay will have vector and axial vector terms. We write the general form for the amplitude as

$$\mathcal{M} = \bar{u}_p(p)(A + B\gamma_5)\gamma_\mu u_\Lambda(p_\Lambda)\cdot q^\mu .$$

Work in the rest-frame of the Λ ($\underset{\sim}{p}_\Lambda = 0$) and let the proton momentum lie along the $\hat{z}$-direction. Compute the decay angular distribution for a Λ with net polarization $\mathcal{P}_\Lambda$ along an arbitrary direction $\hat{\underline{n}}$. Show that it takes the form

$$\frac{d\sigma}{d\Omega} = \text{constant} \times (1 + \alpha \mathcal{P}_\Lambda \hat{\underline{q}}\cdot\hat{\underline{n}}),$$

so a measurement of the decay angular distribution determines $\alpha\mathcal{P}_\Lambda$. Express the asymmetry parameter α in terms of A, B, M_p, and M_Λ .

11. Now consider the decay of an unpolarized Λ . Show that a measurement of the proton's helicity leads to a determination of the asymmetry parameter α .

References for Problems 10 and 11 : Gasiorowicz (1966), c.33 ; Cronin and Overseth (1963) ; Okun (1965).

12. The magnetic dipole transitions among charmed mesons may be relatively immune from recoil effects, because of the large masses and small mass differences.

a) Neglecting the small phase space difference, and approximating $\mu_c = 0$, calculate the ratio $\Gamma(D^{*0} \to D^0\gamma)/\Gamma(D^{*+} \to D^+\gamma)$.

b) Redo your calculation assuming $\mu_c = -\frac{2}{3}\mu_s = 0.41$. Continue to use $\mu_u = -2\mu_d = \frac{2}{3}\mu_p$.

c) Now using the masses, branching ratios, and momenta given in the Particle Data Group (1980) meson table, compare your predictions with experiment. You will need to use isospin invariance for the strong decay amplitudes, and to correct the strong decay rates for phase space differences.

d) Assume the masses of the charmed-strange mesons are F^+ : 2030 MeV/c^2 and F^{*+} = 2140 MeV/c^2. Using μ_c and μ_s as in part b), estimate the absolute width for the decay $F^* \to F\gamma$. What branching ratio do you expect ?

13. Derive the connection between $|\Psi(0)|^2$ and the leptonic decay rate of a $(q\bar{q})$ vector meson . It is convenient to proceed by the following steps :

a) Compute the spin-averaged cross section for the reaction $q\bar{q} \to e^+e^-$. Show that it is

$$\sigma = \frac{\pi\alpha^2 e_q^2}{12E^2} \cdot \frac{\beta_\ell}{\beta_q}(3-\beta_\ell^2)(3-\beta_q^2),$$

where E is the c.m. energy of a quark and β is the speed of a particle.

b) The annihilation rate in a $^3S_1$ vector meson is the density x relative velocity x $\frac{4}{3}$ (to undo the spin average) x σ , or

$$\Gamma = |\Psi(0)|^2 \times 2\beta_q \times \frac{4}{3} \times \sigma.$$

c) How is the result modified if the vector meson wavefunction is

$$|V^0\rangle = \sum_i c_i |q_i\bar{q}_i\rangle \ ?$$

d) Now neglect the lepton mass and the quark binding energy and assume the quarks move nonrelativistically. Show that

$$\Gamma(V^0 \to e^+e^-) = \frac{16\pi\alpha^2}{3M_V^2}|\Psi(0)|^2\left(\sum c_i e_i\right)^2.$$

e) How is the result modified if quarks come in N_c colors and hadrons are color singlets ?

References : Van Royen and Weisskopf (1967) ; Pietschmann and Thirring (1966), Jackson (1976).

14. The Han-Nambu (1965) model is an integer-charge alternative to the fractional-charge quark model, with quark charges assigned as

color \ flavor	u	d	s
R	0	-1	-1
G	1	0	0
B	1	0	0

a) Show that below the threshold for color liberation, the ratio

$$R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

is R = 2, as in the fractional-charge model, and that R = 4 if color can be liberated.

b) Consider the reaction

$$\gamma\gamma \to \text{hadrons} \ ,$$

viewed as $\gamma\gamma \to q\bar{q}$. Show that with fractionally charged quarks

$$\sigma(\gamma\gamma \to \text{hadrons}) \propto \sum_i e_i^4 = \frac{2}{3} \ ,$$

and that in the Han-Nambu model

$$\sigma(\gamma\gamma \to \text{hadrons}) \propto \begin{cases} 2 & \text{below color threshold} \\ 4 & \text{above color threshold} \end{cases}$$

References : Close (1979), c.8 ; Chanowitz (1975) ; Lipkin (1979a) ; see also Okun, Voloshin, and Zakharov (1979).

15. Consider the electromagnetic interaction of two classical charged particles, with charges of q_1 and q_2, masses m_1 and m_2, and positions $\underline{r}_1$ and $\underline{r}_2$. In the static limit the interaction Lagrangian is the familiar Coulomb Lagrangian,

$$\mathcal{L}_{int;NR} = -q_1q_2/r$$

where $\underline{r} \equiv \underline{r}_1 - \underline{r}_2$ is the relative coordinate. Derive the interaction Lagrangian through order $(v/c)^2$, and show that it may be written in the form obtained by Darwin in 1920 :

$$\mathcal{L}_{int} = -\frac{q_1q_2}{r}\left\{1 - \frac{1}{2c^2}\left[\underline{v}_1\cdot\underline{v}_2 + (\underline{v}_1\cdot\hat{\underline{r}})(\underline{v}_2\cdot\hat{\underline{r}})\right]\right\}.$$

The derivation is most gracefully carried out in the Coulomb gauge.

Reference : Jackson (1975), c. 12.

16. a) Show that the magnetic field due to a classical particle with magnetic dipole moment $\underline{\mu}$ at the origin of coordinates is

$$\underline{B}(\underline{r}) = \frac{8\pi}{3}\underline{\mu}\,\delta^3(\underline{r}) + \frac{3\hat{\underline{r}}(\hat{\underline{r}}\cdot\underline{\mu}) - \underline{\mu}}{r^3}$$

b) Now consider the (classical) interaction of a static nucleus with magnetic moment $\underline{\mu}_N$, fixed at the origin, with an electron (with magnetic moment $\underline{\mu}_e$ and electric charge e) orbiting about it with angular momentum $\underline{L}$. Show that the interaction energy is given by the hyperfine Hamiltonian

$$\mathcal{H}_{HFS} = -\frac{8\pi}{3}\, \underset{\sim}{\mu}_e \cdot \underset{\sim}{\mu}_N\, \delta^3(\underset{\sim}{r}) + \frac{1}{r^3}\left[\underset{\sim}{\mu}_e \cdot \underset{\sim}{\mu}_N - 3(\hat{\underset{\sim}{r}}\cdot\underset{\sim}{\mu}_e)(\hat{\underset{\sim}{r}}\cdot\underset{\sim}{\mu}_N) - \frac{e}{mc}\, \underset{\sim}{L}\cdot\underset{\sim}{\mu}_N\right].$$

Discuss the origin of each term.

References : Fermi (1930), Jackson (1975), c. 5.

17. The Darwin Lagrangian for two charged particles is given by the interaction Lagrangian $\mathcal{L}_{int}$ of Problem 15 plus the free-particle Lagrangian expanded to order $1/c^2$,

$$\mathcal{L}_{free} = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2) + \frac{1}{8c^2}(m_1 v_1^4 + m_2 v_2^4).$$

a) Introduce relative coordinates $\underset{\sim}{r} = \underset{\sim}{r}_1 - \underset{\sim}{r}_2$ and $\underset{\sim}{v} = \underset{\sim}{v}_1 - \underset{\sim}{v}_2$ and c.m. coordinates. Write out the Lagrangian $\mathcal{L}_{Darwin} = \mathcal{L}_{free} + \mathcal{L}_{int}$ in the reference frame in which the velocity of the center of mass vanishes and evaluate the canonical momentum components $p_x = \partial\mathcal{L}/\partial v_x$, etc.

b) Compute the Hamiltonian to first order in $1/c^2$ and show that it is

$$\mathcal{H} = \frac{p^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) + \frac{q_1 q_2}{r} - \frac{p^4}{8c^2}\left(\frac{1}{m_1^3} + \frac{1}{m_2^3}\right) + \frac{q_1 q_2}{2m_1 m_2 c^2}\left(\frac{p^2 + (\underset{\sim}{p}\cdot\hat{\underset{\sim}{r}})^2}{r}\right).$$

Compare with the various terms in eqn. (42.1) on p. 193 of Bethe and Salpeter (1957). Discuss the agreements and disagreements.

References : Jackson (1975), problem 12.12 ; Berestetskii, Lifshitz, and Pitaevski (1971), pp. 280-284 ; Breit (1930) ; Heisenberg (1926).

18. By coupling together first the quarks and antiquarks separately, show that the colorspin for a collection of n constituents is given by

$$\sum_{i<j} \langle \underset{\sim}{\lambda}_i\cdot\underset{\sim}{\lambda}_j\, \underset{\sim}{\sigma}_i\cdot\underset{\sim}{\sigma}_j\rangle = -4\underset{\sim}{G}^{(6)\,2}_{tot} + \frac{4}{3}s_{tot}(s_{tot}+1) + 8\left[\underset{\sim}{G}^{(6)\,2}_{quarks} + \underset{\sim}{G}^{(6)\,2}_{antiq}\right] - 4\left[\underset{\sim}{G}^{(3)\,2}_{quarks} + \underset{\sim}{G}^{(3)\,2}_{antiq}\right] - \frac{8}{3}\left[s_{quarks}(s_{quarks}+1) + s_{antiq}(s_{antiq}+1)\right] - 8n\,,$$

where the labels (quarks, antiquarks) refer to the collective representations of quarks and antiquarks. Verify that for a state composed only of quarks you recover (5.81).

19. Consider quark-antiquark states. Using SU(6) techniques, identify the colorspin representations containing color singlets, and compute the expectation value of the colorspin operator. Compare with the results in Table 15 for $0^- - 1^-$ splitting.

20. Enumerate the $SU(6)_{colorspin}$ representations that can be formed out of two quarks and two antiquarks. Give the $SU(3) \otimes SU(2)$ decomposition of each. Compute $\underset{\sim}{C}^{(6)2}$ for each representation.

Reference for Problems 18-20 : Jaffe (1977ab).

21. Show that quarkonium level spacings independent of the constituent mass occur in a logarithmic potential, $V(r) = \lambda \log(r/r_0)$.

Reference : Quigg and Rosner (1977).

22. Using the Schrödinger equation (6.3), prove the identity

$$\frac{|u'(0)|^2}{4\pi} = |\Psi(0)|^2 = \frac{\mu}{2\pi\hbar^2}\left\langle\frac{dV}{dr}\right\rangle$$

for a system with reduced mass μ .

Reference : This result is apparently due to Fermi and to Schwinger, in unpublished work. A general derivation appears in § 2.2 of Quigg and Rosner (1979).

23. By evaluating the identity just derived in semiclassical approximation, show that for a general nonsingular potential

$$|\Psi_n(0)|^2 \simeq \left(\frac{2\mu}{\hbar^2}\right)^{3/2} \frac{E_n^{1/2}}{4\pi^2}\frac{\partial E_n}{\partial n} .$$

References : Krammer and Léal Ferreira (1976) ; Quigg and Rosner (1978c) ; Bell and Pasupathy (1979).

24. Consider the Schrödinger equation for s-wave bound states of a 1/r potential in N space dimensions:

$$[\nabla^2 + 2\mu(E + \alpha/r)]\Psi(r) = 0 \quad . \qquad (1)$$

(a) Show that the radial equation is

$$\left[\frac{d^2}{dr^2} + \frac{(N-1)}{r}\frac{d}{dr} + 2\mu(E + \alpha/r)\right]\Psi(r) = 0 \quad . \qquad (2)$$

(b) Now take the limit of large N, so that $(N-1) \to N$. Introduce a reduced radial wavefunction

$$u = r^{N/2}\psi \qquad (3)$$

and a scaled radial coordinate

$$R = r/N^2 \quad . \qquad (4)$$

Show that the Schrödinger equation becomes

$$\left[\frac{1}{N^2}\frac{d^2u}{dR^2} - \frac{u}{4R^2} + 2\mu(N^2E + \alpha/R)u\right] = 0 \quad . \qquad (5)$$

(c) Apart from the factor N^2 which sets the scale of E, this equation describes a particle with effective mass μN^2 moving in an effective potential

$$V_{eff} = \frac{1}{8\mu R^2} - \frac{\alpha}{R} \quad . \qquad (6)$$

Find the energy of the ground state in the limit as $N \to \infty$, for which the kinetic energy vanishes. Show that it is given by the absolute minimum of V_{eff}, so that

$$E_{N\to\infty} = -2\mu\alpha^2/N^2 \qquad . \qquad (7)$$

Corrections to (7) may be computed by expanding V_{eff} about the minimum and treating the additional terms as perturbations.

(d) The exact solution to the exact eigenvalue problem (2) is easily verified to be

$$E_{exact} = -2\mu\alpha^2/(N-1)^2 \qquad . \qquad (8)$$

Show that the exact eigenvalue can be recast in the form of an expansion in powers of 1/N as

$$E_{exact} = -\frac{2\mu\alpha^2}{N^2}\sum_{j=1}^{\infty} jN^{1-j}$$

$$= E_{N\to\infty}\left\{1 + \sum_{j=2}^{\infty} jN^{1-j}\right\} \quad ,$$

so that the $N\to\infty$ result may form the basis for a systematic approximation scheme. How many terms must be retained to obtain a 1% approximation for N = 3?

Reference: Mlodinow and Papanicolaou (1980).

REFERENCES

Abrams, G. S., et al., 1974, Phys. Rev. Lett. 33, 1453.

Abrams, G. S., et al., 1979, Phys. Rev. Lett. 43, 477.

Ademollo, M., and Gatto, R., 1964, Phys. Rev. Lett. 13, 264.

Adler, S. L., 1981, "The Mechanism for Confinement in Massive Quark QCD," Institute for Advanced Study preprint.

Adler, S. L., and Dashen, R., 1968, Current Algebras (Benjamin, New York).

Ajzenberg-Selove, F., 1975, Nucl. Phys. A248, 1.

Ajzenberg-Selove, F., 1976, Nucl. Phys. A268, 1.

Ajzenberg-Selove, F., 1979, Nucl. Phys. A320, 1.

Akhiezer, A. I., and Berestetskii, V. B., 1965, Quantum Electrodynamics (Wiley, New York), §11.

Allen, E., 1975, Phys. Lett. 57B, 263.

Amaldi, U., Jacob, M., and Matthiae, G., 1976, Ann. Rev. Nucl. Sci. 26, 385.

Amirzadeh, J., et al., 1979, Phys. Lett. 89B, 125.

Appelquist, T., 1978, "Chromodynamic Structure and Phenomenology," in: Particles and Fields, 1977, Banff Summer Institute, ed. D. H. Boal and A. N. Kamal (Plenum, New York and London), p. 33.

Appelquist, T., Barnett, R. M., and Lane, K., 1978, Ann. Rev. Nucl. Part. Sci. 28, 387.

Appelquist, T., and Politzer, H. D., 1975, Phys. Rev. Lett. 34, 43.

Armstrong, T., et al., 1981, "Study of the K^+K^- System Produced in $K^-p \to K^+K^-(\Lambda,\Sigma)$ at 18.5 GeV/c," CERN Report contributed to the Lisbon Conference.

Aston, D., et al., 1981, SLAC-PUB-2759.

Aubert, J. J., et al., 1974, Phys. Rev. Lett. 33, 1404.

Augustin, J. E., et al., 1974, Phys. Rev. Lett. 33, 1406.

Bacry, H., 1967, Leçons sur la Théorie des Groupes et les Symétries des Particules Elémentaires (Gordon and Breach/Dunod, Paris and New York).

Bander, M., 1981, Phys. Rep. 75C, 205.

Bander, M., Chiu, T.-W., Shaw, G. L., and Silverman, D., 1981, Phys. Rev. Lett. 47, 549.

Barbieri, R., Gatto, R., and Remiddi, E., 1981, CERN preprint TH.3144.

Bardeen, J., Cooper, L. N., and Schrieffer, J. R., 1957, Phys. Rev. 108, 1175.

Bardeen, W. A., Chanowitz, M., Drell, S. D., Weinstein, M., and Yan, T.-M., 1975, Phys. Rev. D11, 1094.

Bardeen, W. A., Fritzsch, H., and Gell-Mann, M., 1973, in: Scale and Conformal Symmetry in Hadron Physics, ed. R. Gatto (Wiley, New York), p. 139.

Barnes, T., 1981, Z. Phys. C10, 275.

Bell, J. S., and Bertlmann, R., 1981, Nucl. Phys. B187, 285.

Bell, J. S., and Pasupathy, J., 1979, Phys. Lett. 83B, 389.

Berard, R. W., et al., 1973, Phys. Lett. 47B, 355.

Berestetskii, V. B., Lifshitz, E. M., and Pitaevski, L. P., 1971, Relativistic Quantum Theory, part 1, transl. J. B. Sykes and J. S. Bell (Pergamon, Oxford, 1971, 1979).

Berg, B., 1980, Phys. Lett. 97B, 401.

Berg, D., et al., 1980a, Phys. Rev. Lett. 44, 706.

Berg, D., et al., 1980b, in: High Energy Physics-1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 537.

Berg, D., et al., 1981, Phys. Lett. 98B, 119.

Berkelman, K., 1980, in: High Energy Physics-1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 1499.

Bethe, H., and Salpeter, E. E., 1957, Quantum Mechanics of One- and Two-Electron Atoms (Springer-Verlag, Berlin).

Bhanot, G., 1981, Phys. Lett. 105B, 95.

Bhanot, G., and Rebbi, C., 1981, Nucl. Phys. B180, 469.

Biagi, S. F., et al., 1981, Z. Phys. C9, 305.

Billoire, A., Lacaze, R., Morel, A., and Navelet, H., 1979, Phys. Lett. 80B, 381.

Binnie, D. M., et al., 1979, Phys. Lett. 83B, 141.

Bjorken, J. D., 1979, in: Proceedings of the European Physical Society International Conference on High Energy Physics, Geneva, 27 June - 4 July 1979 (CERN, Geneva), p. 245.

Bjorken, J. D., 1980, "Elements of Quantum Chromodynamics," in: Quantum Chromodynamics, Proceedings of the 7th SLAC Summer Institute on Particle Physics, 1979, ed. A. Mosher (SLAC, Stanford), p. 219.

Blatt, J. M., and Weisskopf, V. F., 1952, Theoretical Nuclear Physics (Wiley, New York).

Bogoliubov, P. N., 1967, Ann. Inst. Henri Poincaré 8, 163.

Bohm, A., and Teese, R. B., 1981, "Fits of the Baryon Magnetic Moments," Lett. Nuovo Cim. 32, 122.

Bohr, A., and Mottelson, B. R., 1969, Nuclear Structure, vol. 1 (Benjamin, New York).

Bolzan, J. F., Palmer, W. F., and Pinsky, S. S., 1976, Phys. Rev. D14, 3202.

Bonesini, M., et al., 1981, Phys. Lett. 103B, 75.

Borkowski, F., et al., 1974, Nucl. Phys. A222, 269.

Bouchiat, C., and Meyer, Ph., 1964, Nuovo Cim. 34, 1122.

Bourrely, C., Machet, B., and de Rafael, E., 1981, Nucl. Phys. B189, 157.

Bowler, K. C., Corvi, P. J., Hey, A. J. G., and Jarvis, P. D., 1980, Phys. Rev. Lett. 45, 97.

Brandenburg, G., et al., 1976, Phys. Rev. Lett. 36, 1239.

Breit, G., 1930, Phys. Rev. 36, 383.

Breit, G., Condon, E. U., and Present, R. D., 1936, Phys. Rev. 50, 846.

Brower, R. C., 1981, Les Houches lectures, this session.

Brown, G. E., and Rho, M., 1979, Phys. Lett. 82B, 177.

Brown, G. E., Rho, M., and Vento, V., 1979, Phys. Lett. 84B, 383.

Buchmüller, W., Grunberg, G., and Tye, S.-H. H., 1980, Phys. Rev. Lett. 45, 103, 587(E).

Buchmüller, W., Ng, Y. J., and Tye, S.-H. H., 1981, "Hyperfine Splittings in Heavy Quark Systems," Fermilab-Pub-81/46-THY.

Buchmüller, W., and Tye, S.-H. H., 1980, Phys. Rev. Lett. 44, 850.

Buchmüller, W., and Tye, S.-H. H., 1981, Phys. Rev. D24, 132.

Buras, A. J., 1980, Rev. Mod. Phys. 52, 199.

Buras, A. J., 1981, invited talk at the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn.

Cabibbo, N., 1963, Phys. Rev. Lett. 10, 531.

Cahn, R. N., and Einhorn, M. B., 1971, Phys. Rev. D4, 3337.

Carlitz, R., and Creamer, D., 1979, Ann. Phys. (NY) 118, 429.

Carlitz, R. D., Ellis, S. D., and Savit, R., 1977, Phys. Lett. 68B, 443.

Carlson, C., Coyne, J., Fishbane, P., Gross, F., and Meshkov, S., 1980, Phys. Lett. 98B, 110.

Carlson, C., Coyne, J., Fishbane, P., Gross, F., and Meshkov, S., 1981, Phys. Lett. 99B, 353.

Carroll, A. S., et al., 1978, Phys. Rev. Lett. 41, 777.

Carruthers, P., 1966, Introduction to Unitary Symmetry (Interscience, New York).

Cashmore, R. J., 1980, in: Proceedings of the 1980 Conference on Experimental Meson Spectroscopy, Brookhaven, ed. S.-U. Chung (American Institute of Physics, New York), p. .

Cassen, B., and Condon, E. U., 1936, Phys. Rev. 50, 846.

Celmaster, W., and Henyey, F., 1978, Phys. Rev. D18, 1688.

Chanowitz, M., 1975, "Color and Experimental Physics," in: Particles and Fields - 1975, ed. H. J. Lubatti and P. M. Mockett (University of Washington, Seattle), p. 448.

Chanowitz, M. S., 1980, Phys. Rev. Lett. 44, 59.

Chanowitz, M. S., 1981, Phys. Rev. Lett. 46, 981.

Chao, K.-T., Isgur, N., and Karl, G., 1981, Phys. Rev. D23, 155.

Chew, G. F., 1965, The Analytic S-Matrix, in: High Energy Physics, 1965 Les Houches Lectures, ed. C. DeWitt and M. Jacob (Gordon and Breach, New York), p. 187.

Chew, G. F., and Frautschi, S. C., 1961, Phys. Rev. Lett. 7, 394.

Chew, G. F., and Low, F. E., 1959, Phys. Rev. 113, 1640.

Cho, Y. M., Cortes, J. L., and Pham, X. Y., 1981, Université Pierre et Marie Curie preprint PAR/LPTHE 81/08.

Chodos, A., Jaffe, R. L., Johnson, K., and Thorn, C. B., 1974, Phys. Rev. D10, 2599.

Chodos, A., Jaffe, R. L., Johnson, K., Thorn, C. B., and Weisskopf, V. F., 1974, Phys. Rev. D9, 3471.

Chodos, A., and Thorn, C. B., 1974, Nucl. Phys. B72, 509.

Chou, T. T., and Yang, C. N., 1968, Phys. Rev. 170, 1591.

Chounet, L.-M., Gaillard, J.-M., and Gaillard, M. K., 1972, Phys. Rep. 4C, 199.

Chung, S.-U. (ed.), 1980, Proceedings of the 1980 Conference on Experimental Meson Spectroscopy, Brookhaven (American Institute of Physics, New York).

Cleland, W. E., et al., 1980a, "Observation of a Spin-6 Isospin-1 Boson Resonance in the Charged $K\bar{K}$ System," Université de Genève preprint UGVA-DPNC 1980/07-101.

Cleland, W. E., et al., 1980b, Phys. Lett. 97B, 465.

Close, F. E., 1979, An Introduction to Quarks and Partons (Academic, New York).

Close, F. E., 1980, "The First Lap in QCD," Rutherford Laboratory preprint RL-80-063.

Close, F. E., 1981, "Glueballs, Hermaphrodites, and QCD Problems for Baryon Spectroscopy," Rapporteur talk at the European Physical Society Conference on High Energy Physics, Lisbon; Rutherford Laboratory preprint RL-81-066/T295.

Cohen, I., and Lipkin, H. J., 1979, Nucl. Phys. B151, 16.

Cohen, I., and Lipkin, H. J., 1980, Phys. Lett. 93B, 56.

Cohen, I., and Lipkin, H. J., 1981, Phys. Lett. 106B, 119.

Coleman, S., 1980, "1/N," lectures presented at the International School of Subnuclear Physics: Pointlike Structures Inside and Outside Hadrons, Erice, Italy, 1979; SLAC-PUB-2484.

Coleman, S., and Glashow, S. L., 1961, Phys. Rev. Lett. 6, 423.

Coleman, S., and Glashow, S. L., 1964, Phys. Rev. 134, B671.

Coleman, S., and Witten, E., 1980, Phys. Rev. Lett. 45, 100.

Collins, P. D. B., 1977, An Introduction to Regge Theory and High Energy Physics (Cambridge University Press, Cambridge).

Copley, L. A., Isgur, N., and Karl, G., 1979, Phys. Rev. D20, 768.

Cox, P. T., 1981, seminar at this Les Houches session.

Cox, P. T., et al., 1981, Phys. Rev. Lett. 46, 877.

Coyne, J., Fishbane, P., and Meshkov, S., 1980, Phys. Lett. 91B, 259.

Cronin, J., and Overseth, O., 1963, Phys. Rev. 129, 1795.

Dalitz, R. H., 1965, Quark Models for the Elementary Particles, in: High Energy Physics, 1965 Les Houches Lectures, ed. C. DeWitt and M. Jacob (Gordon and Breach, New York), p. 251.

Dalitz, R. H., 1977, in: Fundamentals of Quark Models, Scottish Universities Summer School in Physics, 1976, ed. I. M. Barbour and A. T. Davies (SUSSP, Edinburgh), p. 151.

Dalitz, R. H., and Sutherland, D. G., 1966, Phys. Rev. 146, 1180.

Dally, E. B., et al., 1977, Phys. Rev. Lett. 39, 1176.

Dally, E. B., et al., 1980a, "Elastic Scattering Comparison of the Negative Pion Form Factor," contribution to the XX International Conference on High Energy Physics, Madison.

Dally, E. B., et al., 1980b, "Direct Comparison of the π^- and K^- Form Factors," contribution to the XX International Conference on High Energy Physics, Madison.

Dally, E. B., et al., 1980c, Phys. Rev. Lett. 45, 232.

Dankowych, J. A., et al., 1981, Phys. Rev. Lett. 46, 580.

DeGrand, T., 1980, "Thinking about Baryon Excited States with Bags," in: Baryon 1980, IVth International Conference on Baryon Resonances, ed. N. Isgur (University of Toronto, Toronto), p. .

DeGrand, T., Jaffe, R. L., Johnson, K., and Kiskis, J., 1975, Phys. Rev. D12, 2060.

DeRujula, A., Georgi, H., and Glashow, S. L., 1975, Phys. Rev. D12, 147.

DeRujula, A., Giles, R., and Jaffe, R., 1978, Phys. Rev. D17, 285.

DeRujula, A., Giles, R., and Jaffe, R., 1980, Phys. Rev. D22, 227.

DeTar, C., 1978, Phys. Rev. D17, 302, 323; D19, 1028(E)(1979).

DeTar, C., 1979, Phys. Rev. D19, 1451.

DeTar, C., 1980, "The MIT Bag Model," in: Quantum Flavordynamics, Quantum Chromodynamics, and Unified Theories, ed. K. T. Mahanthappa and J. Randa, NATO Advanced Studies Institute Series B, Physics, vol. 54 (Plenum, New York), p. 393.

DeTar, C. E., 1981a, Phys. Rev. D24, 752.

DeTar, C. E., 1981b, Phys. Rev. D24, 762.

deViron, F., and Weyers, J., 1981, Nucl. Phys. B185, 391.

DiVecchia, P., 1979, Phys. Lett. 85B, 357.

Dolgov, A. D., Okun, L. B., Pomeranchuk, I. Ya., and Soloviev, V. V., 1965, Phys. Lett. 15, 84.

Dolgov, O. V., Kirzhnits, D. A., and Maksimov, E. G., 1981, Rev. Mod. Phys. 53, 81.

Donoghue, J. F., 1980, in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 35.

Donoghue, J. F., 1981, "Glueballs in the Bag," Massachusetts preprint UMHEP-147, to be published in the proceedings of Orbis Scientiae.

Donoghue, J. F., Golowich, E., and Holstein, B. R., 1975, Phys. Rev. D12, 2875.

Donoghue, J. F., and Johnson, K., 1980, Phys. Rev. D21, 1975.

Donoghue, J. F., Johnson, K., and Li Bing-An, 1981, Phys. Lett. 99B, 416.

Dorsaz, P.-A., 1981, Université de Genève Thesis UGVA-Thèse-1994.

Dothan, Y., 1981, "Linear Symmetric Quark Theories of Baryon Magnetic Moments," University of Minnesota preprint.

Drell, S. D., and Yan, T.-M., 1970, Phys. Rev. Lett. 25, 316.

Drell, S. D., and Yan, T.-M., 1971, Ann. Phys. (NY) 66, 578.

Dremin, I. M., and Quigg, C., 1978, Science 199, 937.

Duinker, P., 1981, "Review of e^+e^- Experiments at PETRA, DESY preprint 81-012, Rev. Mod. Phys. (to be published).

Dydak, F., et al., 1977, Nucl. Phys. B118, 1.

Eichten, E., 1978, in: New Results in High-Energy Physics, Vanderbilt Conference, ed. R. S. Panvini and S. E. Csorna (American Institute of Physics, New York), p. 252.

Eichten, E., and Feinberg, F., 1979, Phys. Rev. Lett. 43, 1205.

Eichten, E., and Feinberg, F., 1981, Phys. Rev. D23, 2724.

Eichten, E., and Gottfried, K., 1977, Phys. Lett. 66B, 286.

Eichten, E., Gottfried, K., Kinoshita, T., Kogut, J., Lane, K. D., and Yan, T.-M., 1975, Phys. Rev. Lett. 34, 369.

Einhorn, M. B., 1975, "Introduction to SU(4) and the Properties of Charmed Particles," Fermilab-Lectures-75/1-THY.

Eisenbud, L., and Wigner, E. P., 1941, Proc. Nat. Acad. Sci. (US) 27, 281.

Ellis, J., Gaillard, M. K., Nanopoulos, D. V., and Rudaz, S., 1981, Phys. Lett. 99B, 101.

Estabrooks, P., 1977, "$K\pi$ and $\pi\pi$ Data and Partial Wave Analyses," in: Experimental Meson Spectroscopy 1977, 5th International Conference, Boston, ed. E. von Goeler and R. Weinstein (Northeastern University Press, Boston), p. 185.

Fairley, G. T., and Squires, E. J., 1975, Nucl. Phys. B93, 56.

Farhi, E., and Susskind, L., 1981, Phys. Rep. 74C, 277.

Feld, B., 1969, Models of Elementary Particles (Blaisdell, Waltham, Mass.).

Fermi, E., 1930, Mem. Accad. d'Italia I, (Fis.) 139; reprinted in: E. Fermi, Collected Papers, eds. E. Segrè, et al. (University of Chicago Press, Chicago, 1962), vol. 1, paper 57a.

Fermi, E., and Yang, C. N., 1949, Phys. Rev. 76, 1739.

Feynman, R. P., 1972, Photon-Hadron Interactions (Benjamin, Reading, Mass.).

Feynman, R. P., 1974, Science 183, 601.

Feynman, R. P., 1977, "Gauge Theories," in: Weak and Electromagnetic Interactions at High Energy, ed. R. Balian and C. H. Llewellyn Smith (North-Holland, Amsterdam), p. 120.

Fiarman, S., and Hanna, S. S., 1975, Nucl. Phys. A251, 1.

Field, R. D., and Quigg, C., 1975, "Estimates of Associated Charm Production Cross Sections," Fermilab-75/15-THY.

Fishbane, P. M., and Grisaru, M. T., 1978, Phys. Lett. 74B, 98.

Fogleman, G., Lichtenberg, D. B., and Wills, J. G., 1979, Lett. Nuovo Cim. 26, 369.

Fox, G. C., and Quigg, C., 1973, Ann. Rev. Nucl. Sci. 23, 219.

Frauenfelder, H., and Henley, E. M., 1974, Subatomic Physics (Prentice-Hall, Englewood Cliffs, N.J.).

Frazer, W. R., 1966, Elementary Particles (Prentice-Hall, Englewood Cliffs, N.J.).

Freund, P. G. O., and Nambu, Y., 1975, Phys. Rev. Lett. 34, 1645.

Fritzsch, H., 1979, "Chromodynamics," in: Proceedings of the 1978 International Meeting on Frontier of Physics, Singapore, ed. K. K. Phua, C. K. Chew, and Y. K. Lim (Singapore National Academy of Science, Singapore), p. 1005.

Fritzsch, H., and Gell-Mann, M., 1972, in: Proceedings of the XVI International Conference on High Energy Physics, ed. J. D. Jackson, A. Roberts, and R. Donaldson (Fermilab, Batavia), p. II-135.

Fritzsch, H., Gell-Mann, M., and Leutwyler, H., 1973, Phys. Lett. 47B, 365.

Fritzsch, H., and Minkowski, P., 1975, Nuovo Cim. 30A, 393.

Fujii, Y., and Mima, K., 1978, Phys. Lett. 79B, 138.

Gaillard, M. K., Lee, B. W., and Rosner, J. L., 1975, Rev. Mod. Phys. 47, 277.

Gasiorowicz, S., 1966, Elementary Particle Physics (Wiley, New York).

Gavela, M. B., et al., 1979, Phys. Lett. 82B, 431.

Geffen, D. A., and Wilson, W., 1980, Phys. Rev. Lett. 44, 370.

Gell-Mann, M., 1953, Phys. Rev. 92, 833.

Gell-Mann, M., 1961, California Institute of Technology Synchrotron Laboratory Report CTSL-20, reprinted in Gell-Mann and Ne'eman (1964), p. 11.

Gell-Mann, M., 1962, Phys. Rev. 125, 1067.

Gell-Mann, M., 1964, Phys. Lett. 8, 214.

Gell-Mann, M., and Glashow, S. L., 1961, Ann. Phys. (NY) 15, 437.

Gell-Mann, M., and Ne'eman, Y., 1964, The Eightfold Way (Benjamin, New York).

Gell-Mann, M., Oakes, R. J., and Renner, B., 1968, Phys. Rev. 175, 2195.

Georgi, H., 1980, Phys. Rev. D22, 225.

Gerasimov, S. B., 1966, Zh. Eksp. Teor. Fiz. 50, 1559; Eng. transl.: Sov. Phys. - JETP 23, 1040.

Gidal, G., Goldhaber, G., Guy, J. G., et al., 1981, Mark II Collaboration report presented at the EPS Conference on High Energy Physics, Lisbon, by G. Goldhaber.

Giles, R. C., 1976, Phys. Rev. D13, 1670.

Giles, R. C., and Tye, S.-H. H., 1977, Phys. Rev. D16, 1079.

Giles, R. C., and Tye, S.-H. H., 1978, Phys. Lett. 73B, 30.

Gilman, F., and Karliner, I., 1974, Phys. Rev. D10, 2194.

Gilman, F., and Miller, D. H., 1978, Phys. Rev. D17, 1846.

Glashow, S. L., 1961, Phys. Rev. Lett. 7, 469.

Goebel, C., 1958, Phys. Rev. Lett. 1, 337.

Goldhaber, A. S., Fox, G. C., and Quigg, C., 1969, Phys. Lett. 30B, 249.

Goldhaber, G., 1970, "Experimental Results on the ω-ρ Interference Effect," in: Experimental Meson Spectroscopy, ed. C. Baltay and A. H. Rosenfeld (Columbia University Press, New York and London), p. 59.

Goldman, T., and Haymaker, R. W., 1981, Phys. Rev. D24, 724.

Goldstone, J., 1961, Nuovo Cim. 19, 154.

Golowich, E., 1975, Phys. Rev. D12, 2108.

Golub, R., et al., 1979, Scientific American 240, no. 6, p. 106 (June).

Gopal, G. N., 1980, in: Baryon 1980, IVth International Conference on Baryon Resonances, ed. N. Isgur (University of Toronto, Toronto), p. 159.

Gottfried, K., 1977, in: Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, ed. F. Gutbrod (DESY, Hamburg), P. 667.

Gottfried, K., 1980, in: High Energy e^+e^- Interactions, Vanderbilt Conference, ed. R. S. Panvini and S. E. Csorna (American Institute of Physics, New York), p. 88.

Gottfried, K., and Weisskopf, V. F., 1981, textbook in preparation.

Gourdin, M., 1967, Unitary Symmetries and Their Applications to High Energy Physics (North-Holland, Amsterdam).

Gourdin, M., 1974, Phys. Rep. 11C, 29.

Greenberg, O. W., 1964, Phys. Rev. Lett. 13, 598.

Greenberg, O. W., 1978, Ann. Rev. Nucl. Part. Sci. 28, 327.

Greenberg, O. W., and Lipkin, H. J., 1981, Fermilab-Pub-81/45-THY.

Greenberg, O. W., and Nelson, C. A., 1977, Phys. Rep. 32C, 69.

Greenberg, O. W., Nussinov, S., and Sucher, J., 1977, Phys. Lett. 70B, 465; reprinted as Phys. Lett. 72B, 87 (1977).

Greenberg, O. W., and Resnikoff, M., 1967, Phys. Rev. 163, 1844.

Greenberg, O. W., and Zwanziger, D., 1966, Phys. Rev. 150, 1177.

Greene, G. L., et al., 1979, Phys. Rev. D20, 2139.

Gromes, D., 1977, Nucl. Phys. B131, 80.

Gross, D. J., Treiman, S. B., and Wilczek, F., 1979, Phys. Rev. D19, 2188.

Gross, D. J., and Wilczek, F., 1973a, Phys. Rev. Lett. 30, 1343.

Gross, D. J., and Wilczek, F., 1973b, Phys. Rev. D8, 3633.

Grosse, H., and Martin, A., 1979, Nucl. Phys. B148, 413.

Grosse, H., and Martin, A., 1980, Phys. Rep. 60C, 341.

Halprin, A., Anderson, C. M., and Primakoff, H., 1966, Phys. Rev. 152, 1295.

Hamermesh, M., 1963, Group Theory (Addison-Wesley, Reading, Mass.).

Han, M.-Y., and Nambu, Y., 1965, Phys. Rev. 139B, 1006.

Handler, R., et al., 1980, in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 539.

Hasenfratz, P., and Kuti, J., 1978, Phys. Rep. 40C, 75.

Haymaker, R. W., and Goldman, T., 1981, Phys. Rev. D24, 743.

Heisenberg, W., 1926, Z. Phys. 39, 499.

Heisenberg, W., 1932, Z. Phys. 77, 1.

Hendry, A. W., and Lichtenberg, D. B., 1978, Rep. Prog. Phys. 41, 1707.

Henriques, A. B., Kellett, B. H., and Moorhouse, R. G., 1976, Phys. Lett. 64B, 85.

Herb, S. W., et al., 1977, Phys. Rev. Lett. 39, 252.

Hey, A. J. G., 1979, "Particle Systematics," in: Proceedings of the European Physical Society Conference on High Energy Physics, Geneva (CERN, Geneva), p. 523.

Hey, A. J. G., 1980a, "Quark Models of Baryons," in: Baryon 1980, IVth International Conference on Baryon Resonances, Toronto, ed. N. Isgur (University of Toronto , Toronto), p. 223.

Hey, A. J. G., 1980b, "Multiquark States and Exotics," in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 22.

Higgs, P. W., 1964a, Phys. Rev. Lett. 12, 132.

Higgs, P. W., 1964b, Phys. Rev. Lett. 13, 508.

Himel, T. M., et al., 1980a, Phys. Rev. Lett. 44, 920.

Himel, T. M., et al., 1980b, Phys. Rev. Lett. 45, 1146.

Hofstadter, R., 1963, Electron Scattering and Nuclear and Nucleon Structure (Benjamin, New York).

't Hooft, G., 1973a, Phys. Lett. 61B, 455.

't Hooft, G., 1973b, Phys. Lett. 62B, 444.

't Hooft, G., 1974a, Nucl. Phys. B72, 461.

't Hooft, G., 1974b, Nucl. Phys. B75, 461.

't Hooft, G., 1976, Phys. Rev. Lett. 37, 8.

't Hooft, G., 1980, Scientific American 242, no. 6, p. 104 (June).

Horn, D., and Mandula, J., 1978, Phys. Rev. D17, 898.

Igi, K., 1977, Phys. Rev. D16, 196.

Iizuka, J., 1966, Suppl. Prog. Theoret. Phys. 37-38, 21.

Iizuka, J., Okada, K., and Shito, O., 1966, Prog. Theoret. Phys. 35, 1061.

Ioffe, B. L., 1981, Nucl. Phys. B188, 317.

Isgur, N., 1976, Phys. Rev. D13, 122.

Isgur, N., 1978, Phys. Rev. D17, 369.

Isgur, N., 1980a, "Baryons with Chromodynamics," in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 30.

Isgur, N. (ed.), 1980b, Baryon 1980, IVth International Conference on Baryon Resonances, Toronto (University of Toronto , Toronto).

Isgur, N., 1980c, "Soft QCD: Low Energy Hadron Physics with Chromodynamics," in: New Aspects of Subnuclear Physics, International School of Physics "Ettore Majorana", 1978, ed. A. Zichichi (Plenum, New York).

Isgur, N., and Karl, G., 1977, Phys. Lett. 72B, 109.

Isgur, N., and Karl, G., 1978a, Phys. Lett. 74B, 353.

Isgur, N., and Karl, G., 1978b, Phys. Rev. D18, 4187.

Isgur, N., and Karl, G., 1979, Phys. Rev. D19, 2653.

Isgur, N., and Karl, G., 1980, Phys. Rev. D21, 3175.

Isgur, N., Karl, G., and Koniuk, R., 1978, Phys. Rev. Lett. 41, 1269; 45, 1738(E)(1980).

Isgur, N., Karl, G., and Sprung, D. W. L., 1981, Phys. Rev. D23, 163.

Isgur, N., Rubinstein, H. R., Schwimmer, A., and Lipkin, H. J., 1979, Phys. Lett. 89B, 79.

Ishikawa, K., 1979a, Phys. Rev. D20, 731.

Ishikawa, K., 1979b, Phys. Rev. D20, 2903.

Ishikawa, K., 1981, Phys. Rev. Lett. 46, 978.

Jackson, J. D., 1966, in: Proceedings of the XIIIth International Conference on High Energy Physics, Berkeley, ed. M. Alston-Garnjost (University of California Press, Berkeley and Los Angeles), p. 149.

Jackson, J. D., 1975, Classical Electrodynamics, second edition (Wiley, New York).

Jackson, J. D., 1976, "Lectures on the New Particles," in: Proceedings of the 4th SLAC Summer Institute on Particle Physics, ed. Martha C. Zipf, SLAC-198.

Jackson, J. D., 1977a, "The Nature of Intrinsic Magnetic Dipole Moments," CERN Yellow Report 77-17.

Jackson, J. D., 1977b, in: Proceedings of the 1977 European Conference on Particle Physics, Budapest, ed. L. Jenik and I. Montvay (Central Research Institute for Physics, Budapest), vol. 1, p. 603.

Jackson, J. D., Quigg, C., and Rosner, J. L., 1979, in: Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978, ed. S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo), p. 391.

Jaffe, R. L., 1977a, Phys. Rev. D15, 267.

Jaffe, R. L., 1977b, Phys. Rev. D15, 281.

Jaffe, R. L., 1977c, Phys. Rev. Lett. 38, 195.

Jaffe, R. L., 1977d, Nature 268, 201.

Jaffe, R. L., 1979, "The Bag," lectures presented at the 1979 Erice Summer School "Ettore Majorana", MIT preprint CTP-814.

Jaffe, R. L., and Johnson, K., 1976, Phys. Lett. 60B, 201.

Jaffe, R. L., and Johnson, K., 1977, Comments Nucl. Part. Phys. 7, 107.

Jaffe, R. L., and Low, F. E., 1979, Phys. Rev. D19, 2105.

Jensen, D. A., et al., 1980, in: High Energy Physics-1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 364.

Jensen, T., 1980, University of Rochester thesis, UR-747.

Jensen, T., 1981, seminar at this Les Houches session.

Johnson, K., 1975, Acta Phys. Polon. B6, 865.

Johnson, K., 1977, in: Fundamentals of Quark Models, Scottish Universities Summer School in Physics, 1976, ed. I. M. Barbour and A. T. Davies (SUSSP, Edinburgh).

Johnson, K., 1979, Scientific American 241, no. 1, p. 112 (July).

Johnson, K., and Nohl, C., 1979, Phys. Rev. D19, 291.

Johnson, K., and Thorn, C. B., 1976, Phys. Rev. D13, 1934.

Jones, L. W., 1977, Rev. Mod. Phys. 49, 717.

Kadanoff, L. P., 1977, Rev. Mod. Phys. 49, 267.

Kane, G. L., 1972, Acta Phys. Polon. B3, 845.

Karl, G., 1979, in: Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978, ed. S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo), p. 135.

Karl, G., and Obryk, E., 1968, Nucl. Phys. B8, 609.

Kawamoto, N., and Sanda, A. I., 1978, Phys. Lett. 76B, 446.

Kelly, R. L., 1980, in: Baryon 1980, IVth International Conference on Baryon Resonances, Toronto, ed. N. Isgur (University of Toronto, Toronto), p. 149.

Khriplovich, I. B., 1969, Yad. Fiz. 10, 409; Eng. transl.: SJNP 10, 235 (1970).

Kinson, J. B., 1980, in: Baryon 1980, IVth International Conference on Baryon Resonances, Toronto, ed. N. Isgur (University of Toronto, Toronto), p. 263.

Koester, L., et al., 1976, Phys. Rev. Lett. 36, 1021.

Kogut, J., Sinclair, D., and Susskind, L., 1976, Nucl. Phys. B114, 199.

Kogut, J. B., and Susskind, L., 1974, Phys. Rev. D9, 3501.

Kokkedee, J. J. J., 1969, The Quark Model (Benjamin, Reading, Mass.).

Koller, K., and Walsh, T. F., 1978, Nucl. Phys. B140, 449.

Krammer, M., and Krasemann, H., 1979a, "Quarkonium," in: New Phenomena in Lepton-Hadron Physics, NATO Advanced Study Institutes Series B, Physics, vol. 49, ed. D. E. C. Fries and J. Wess (Plenum, New York and London), p. 161.

Krammer, M., and Krasemann, H., 1979b, "Quarkonia," in: Quarks and Leptons as Fundamental Particles, ed. P. Urban, Acta Phys. Austriaca, Suppl. XXI (Springer-Verlag, New York), p. 259.

Krammer, M., and Léal Ferreira, P., 1976, Rev. Bras. Fis. 6, 7.

Krasemann, H., and Ono, S., 1979, Nucl. Phys. B154, 283.

Krohn, V. E., and Ringo, G. R., 1973, Phys. Rev. D8, 1305.

Landau, L. D., 1948, Dokl. Akad. Nauk SSSR 60, 207.

Landau, L. D., and Lifshitz, E. M., 1960, Electrodynamics of Continuous Media (Addison-Wesley, Reading, Mass.), §14.

Langacker, P., 1979, Phys. Rev. D20, 2983.

Langacker, P., 1980, Phys. Lett. 90B, 447.

Langacker, P., 1981, Phys. Rep. 72C, 185.

Langacker, P., and Pagels, H., 1979, Phys. Rev. D19, 2070.

LaRue, G. S., Phillips, J. D., and Fairbank, W. M., 1981, Phys. Rev. Lett. 46, 967.

Lee, B. W., 1972, Chiral Dynamics (Gordon and Breach, New York).

Lee, B. W., Quigg, C., and Rosner, J. L., 1977, Phys. Rev. D15, 157.

Lee, T. D., 1980, "QCD and the Bag Model of Hadrons," in: Proceedings of the 1980 Guangzhou Conference on Theoretical Physics (Science Press, Beijing), p. 32.

Leith, D. W. G. S., 1977, "Progress in K^* Spectroscopy," in: Experimental Meson Spectroscopy 1977, Proceedings of the 5th International Conference, Boston, ed. E. von Goeler and R. Weinstein (Northeastern University Press, Boston), p. 207.

Leung, C. N., and Rosner, J. L., 1979, J. Math Phys. 20, 1435.

Leutwyler, H., 1974a, Phys. Lett. 48B, 45.

Leutwyler, H., 1974b, Nucl. Phys. B76, 413.

Leutwyler, H., 1981, Phys. Lett. 98B, 447.

Levine, R., and Tomozawa, Y., 1979, Phys. Rev. D19, 1572.

Levine, R., and Tomozawa, Y., 1980, Phys. Rev. D21, 840.

Levinson, C. A., Lipkin, H. J., and Meshkov, S., 1962, Phys. Lett. 1, 44.

LeYaouanc, A., et al., 1977, Phys. Rev. D15, 844.

Lichtenberg, D. B., 1978, Unitary Symmetry and Elementary Particles (Academic Press, New York).

Lichtenberg, D. B., 1981, Z. Phys. C7, 143.

Lipkin, H. J., 1966, Lie Groups for Pedestrians, second edition (North-Holland, Amsterdam).

Lipkin, H. J., 1973a, Phys. Rep. 8C, 173.

Lipkin, H. J., 1973b, Phys. Rev. D7, 846.

Lipkin, H. J., 1973c, Phys. Lett. 45B, 267.

Lipkin, H. J., 1978a, Phys. Lett. 74B, 399.

Lipkin, H. J., 1978b, Phys. Rev. Lett. 41, 1629.

Lipkin, H. J., 1979a, Nucl. Phys. B155, 104.

Lipkin, H. J., 1979b, "A Quasi-Nuclear Colored Quark Model for Hadrons," in: Common Problems in Low- and Medium-Energy Nuclear Physics, Banff Summer School, ed. B. Castel, B. Goulard, and F. C. Khanna (Plenum, New York), p. 173.

Lipkin, H. J., 1981a, Phys. Rev. D24, 1437.

Lipkin, H. J., 1981b, Phys. Lett. 106B, 114.

Lipkin, H. J., and Tavkhelidze, A., 1965, Phys. Lett. 17, 331.

Llewellyn Smith, C. H., 1980, "Topics in QCD," in: Quantum Flavordynamics, Quantum Chromodynamics, and Unified Theories, NATO Advanced Studies Institute Series B, Physics, vol. 54, ed. K. T. Mahanthappa and J. Randa (Plenum, New York), p. 59.

Lyons, L., 1981, "Current Status of Quark Search Experiments," in: Progress in Particle and Nuclear Physics, vol. 7, ed. D. Wilkinson (Pergamon Press, Oxford), p. 169.

Machacek, M., and Tomozawa, Y., 1978, Ann. Phys. (NY) 110, 407.

Mandelstam, S., 1979, Phys. Rev. D19, 2391.

Mandelstam, S., 1980, Phys. Rep. 67C, 109.

Marciano, W. J., and Pagels, H., 1978, Phys. Rep. 36C, 137.

Marciano, W. J., and Pagels, H., 1979, Nature 279, 479.

Marshak, R., Okubo, S., and Sudarshan, E. C. G., 1957, Phys. Rev. 106, 599.

Martin, A., 1980, Phys. Lett. 93B, 338.

Martin, A., 1981, Phys. Lett. 100B, 511.

Matsuyama, S., and Miyazawa, H., 1979, Prog. Theoret. Phys. 61, 942.

McKay, W. G., and Patera, J., 1981, Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Lie Algebras, vol. 69 in Lecture Notes in Pure and Applied Mathematics (Marcel Dekker, New York and Basel).

Mistretta, C., et al., 1969, Phys. Rev. 184, 1487.

Mlodinow, L. D., and Papanicolaou, N., 1980, Ann. Phys. (NY) 128, 314.

Molzon, W. R., et al., 1978, Phys. Rev. Lett. 41, 1213.

Montanet, L., 1980, in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 1196.

Montanet, L., Rossi, G. C., and Veneziano, G., 1980, Phys. Rep. 63C, 149.

Morgan, D., 1981, Acta Phys. Polon. B12, 43.

Moxhay, P., Rosner, J. L., and Quigg, C., 1981, Phys. Rev. D23, 2638.

Nakano, T., and Nishijima, K., 1955, Prog. Theoret. Phys. 10, 581.

Nambu, Y., 1960, Phys. Rev. Lett. 4, 380.

Nambu, Y., 1966, "A Systematics of Hadrons in Subnuclear Physics," in: Preludes in Theoretical Physics in Honor of V. F. Weisskopf, ed. A. De-Shalit, H. Feshbach, and L. Van Hove (North-Holland, Amsterdam; Wiley, New York), p. 133.

Nambu, Y., 1974, Phys. Rev. D10, 4262.

Nambu, Y., 1976, Scientific American 235, no. 5, p. 48 (November).

Nambu, Y., and Jona-Lasinio, G., 1961a, Phys. Rev. 122, 345.

Nambu, Y., and Jona-Lasinio, G., 1961b, Phys. Rev. 124, 246.

Ne'eman, Y., 1961, Nucl. Phys. 26, 222.

Nefkens, B. M. K., et al., 1978, Phys. Rev. D18, 3911.

Nielsen, H. B., and Olesen, P., 1973, Nucl. Phys. B61, 45.

Nielsen, H. B., and Patkós, A., 1981, "Effective Diaelectric Theory from QCD," NORDITA preprint 81/32.

Novikov, V. A., Okun, L. B., Shifman, M. A., Vainshtein, A. I., Voloshin, M. B., and Zakharov, V. I., 1978, Phys. Rep. 41C, 1.

Novikov, V. A., Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1979, Phys. Lett. 86B, 347.

Novikov, V. A., Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1980a, Nucl. Phys. B165, 55.

Novikov, V. A., Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1980b, Nucl. Phys. B165, 67.

Novikov, V. A., Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1980c, ITEP, Moscow preprint ITEP-87.

Novikov, V. A., Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1980d, ITEP, Moscow preprint ITEP-88.

Novikov, V. A., Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1981, Acta Phys. Polon. B12, 399.

O'Donnell, P. J., 1981, Rev. Mod. Phys. 53, 673.

Ohshima, T., 1980, Phys. Rev. D22, 707.

Okubo, S., 1962a, Prog. Theoret. Phys. 27, 949.

Okubo, S., 1962b, Prog. Theoret. Phys. 28, 24.

Okubo, S., 1963, Phys. Lett. 5, 165.

Okun, L. B., 1965, Weak Interactions of Elementary Particles (Addison-Wesley, Reading, Mass.), c. 13.

Okun, L. B., and Shifman, M. A., 1981, Z. Phys. C8, 17.

Okun, L. B., Voloshin, M. B., and Zakharov, V. I., 1979, "On the Price of Integer Charge Quarks," ITEP, Moscow preprint ITEP-79.

Oreglia, M., et al., 1980, Phys. Rev. Lett. 45, 959.

Pagels, H., 1975, Phys. Rep. 16C, 219.

Pagels, H., 1979, Phys. Rev. D19, 3080.

Pagels, H., and Stokar, S., 1979, Phys. Rev. D20, 2947.

Particle Data Group, 1980, Rev. Mod. Phys. 52, S1.

Partridge, R., et al., 1980, Phys. Rev. Lett. 45, 1150.

Patera, J., and Sankoff, D., 1973, Tables of Branching Rules for Representations of Simple Lie Algebras (Université de Montréal, Montréal).

Perkins, D. H., 1972, Introduction to High-Energy Physics (Addison-Wesley, Reading, Mass.).

Perl, M. L., 1974, High Energy Hadron Physics (Wiley, New York).

Pietschmann, H., and Thirring, W., 1966, Phys. Lett. 21, 713.

Pilcher, J., 1980, in Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, ed. T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia), p. 185.

Politzer, H. D., 1973, Phys. Rev. Lett. 30, 1346.

Preston, M. A., 1962, Physics of the Nucleus (Addison-Wesley, Reading, Mass.).

Primakoff, H., 1951, Phys. Rev. 81, 899.

Protopopescu, S., and Samios, N. P., 1979, Ann. Rev. Nucl. Part. Sci. 29, 339.

Quigg, C., 1979, "Lectures on Charmed Particles," in: Proceedings of the XIth International School for Young Scientists on High Energy Physics and Relativistic Nuclear Physics, Gomel, Byelorussia, 1977 (JINR, Dubna), p. 203.

Quigg, C., 1980, "Bound States of Heavy Quarks and Antiquarks," in: Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, ed. T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia), p. 239.

Quigg, C., 1981, "Introduction to Gauge Theories of the Strong, Weak, and Electromagnetic Interactions," in: Techniques and Concepts of High Energy Physics, St. Croix, 1980, ed. T. Ferbel (Plenum, New York), p. 143.

Quigg, C., and Rosner, J. L., 1976, Phys. Rev. D14, 160.

Quigg, C., and Rosner, J. L., 1977, Phys. Lett. 71B, 153.

Quigg, C., and Rosner, J. L., 1978a, Comments Nucl. Part. Phys. 8, 11.

Quigg, C., and Rosner, J. L., 1978b, Phys. Lett. 72B, 462.

Quigg, C., and Rosner, J. L., 1978c, Phys. Rev. D17, 2364.

Quigg, C., and Rosner, J. L., 1979, Phys. Rep. 56C, 167.

Quigg, C., and Rosner, J. L., 1981, Phys. Rev. D23, 2625.

Ramsey, N. F., 1956, Molecular Beams (Oxford University Press, Oxford).

Reinders, L. J., Rubinstein, H. R., and Yazaki, S., 1981, Nucl. Phys. B186, 109.

Renner, B., 1968, Current Algebras and Their Applications (Pergamon Press, London).

Richard, J. M., 1981, Phys. Lett. 100B, 515.

Richardson, J. L., 1979, Phys. Lett. 82B, 272.

Roberts, B. L., et al., 1975, Phys. Rev. D12, 1232.

Robson, D., 1977, Nucl. Phys. B130, 328.

Rosner, J. L., 1968, Phys. Rev. Lett. 21, 950, 1422(E).

Rosner, J. L., 1974a, Phys. Rep. 11C, 189.

Rosner, J. L., 1974b, in: Proceedings of the XVIIth International Conference on High Energy Physics, London, ed. J. R. Smith (Rutherford Laboratory, Chilton, UK), p. II-171.

Rosner, J. L., 1980a, in: Particles and Fields - 1979, ed. B. Margolis and D. G. Stairs (American Institute of Physics, New York), p. 325.

Rosner, J. L., 1980b, in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 540.

Rosner, J. L., 1981a, "Quark Models," in: Techniques and Concepts of High Energy Physics, St. Croix, 1980, ed. T. Ferbel (Plenum, New York), p. 1.

Rosner, J. L., 1981b, Phys. Rev. D24, 1347.

Rosner, J. L., Quigg, C., and Thacker, H. B., 1978, Phys. Lett. 74B, 350.

Roy, P., 1979, Nature 280, 106.

Roy, P., 1980, "The Glueball Trail," Rutherford Laboratory report RL-80-007.

Roy, P., and Walsh, T. F., 1978, Phys. Lett. 78B, 62.

Rutherglen, J., 1969, in: Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energies, Liverpool, ed. D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury), p. 163.

Sakata, S., 1956, Prog. Theoret. Phys. 16, 686.

Sakharov, A. D., 1975, ZhETF Pis. Red. 21, 554; Eng. transl.: JETP Letters 21, 258.

Sakharov, A. D., 1980, "Mass Formula for Mesons and Baryons," Gorki preprint, SLAC-TRANS-0191.

Sakurai, J. J., 1964, Invariance Principles and Elementary Particles (Princeton University Press, Princeton).

Samios, N. P., Goldberg, M., and Meadows, B. T., 1974, Rev. Mod. Phys. 46, 49.

Schachinger, L. C., et al., 1978, Phys. Rev. Lett. 41, 1348.

Schamberger, R. D., 1981, "Spectroscopy of Heavy Quark-Antiquark Bound States," invited talk at the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn.

Scharre, D., 1981, invited talk at the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn.

Scherk, J., 1975, Rev. Mod. Phys. 47, 123.

Schnitzer, H. J., 1975, Phys. Rev. Lett. 35, 1540.

Schnitzer, H. J., 1976, Phys. Rev. D13, 74.

Schnitzer, H. J., 1978, Phys. Rev. D18, 3482.

Schnitzer, H. J., 1981a, "Spin Structure in Meson Spectroscopy," Brandeis preprint.

Schnitzer, H. J., 1981b, "Spin-Dependence and Gluon Mixing in Ordinary Meson Spectroscopy," Brandeis preprint.

Schonfeld, J. F., Kwong, W., Rosner, J. L., Quigg, C., and Thacker, H. B., 1980, Ann. Phys. (NY) 128, 1.

Settles, R., et al., 1979, Phys. Rev. D20, 2154.

Shaw, R., 1955, "The Problem of Particle Types and Other Contributions to the Theory of Elementary Particles," Cambridge University Thesis.

Shifman, M. A., 1981, Z. Phys. C9, 347.

Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1979a, Phys. Rev. Lett. 42, 297.

Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1979b, Nucl. Phys. B147, 385.

Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1979c, Nucl. Phys. B147, 448.

Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I., 1979d, Nucl. Phys. B147, 519.

Shrock, R., and Wang, L.-L., 1978, Phys. Rev. Lett. 41, 1692.

Slansky, R., Goldman, T., and Shaw, G. L., 1981, Phys. Rev. Lett. 47, 887.

Soni, A., 1980, Nucl. Phys. B168, 147.

Spinetti, M., 1980, in: Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, ed. T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia), p. 506.

Stanley, D. P., and Robson, D., 1980, Phys. Rev. Lett. 45, 235.

Stanley, D. P., and Robson, D., 1981, Phys. Rev. D23, 2776.

Stanton, N., et al., 1979, Phys. Rev. Lett. 42, 346.

Susskind, L., 1979, Phys. Rev. D20, 2619.

Suura, H., 1980, Phys. Rev. Lett. 44, 1319.

Tati, T., 1966, Prog. Theoret. Phys. 35, 126.

Teese, R. B., 1981, Phys. Rev. D24, 1413.

Thacker, H. B., Quigg, C., and Rosner, J. L., 1978a, Phys. Rev. D18, 274.

Thacker, H. B., Quigg, C., and Rosner, J. L., 1978b, Phys. Rev. D18, 287.

Thirring, W., 1966, Acta Phys. Austriaca Suppl. II, 205; reprinted in Kokkedee (1969), p. 183.

Tsyganov, E., 1979, in: Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978, ed. S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo), p. 315.

Van Royen, R., and Weisskopf, V. F., 1967, Nuovo Cim. 50, 617; 51, 583.

Veneziano, G., 1976, Nucl. Phys. B117, 519.

Veneziano, G., 1979a, "Dynamics of Hadron Reactions," in: Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978, ed. S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo), p. 725.

Veneziano, G., 1979b, Nucl. Phys. B159, 213.

Voloshin, M. B., 1979, Nucl. Phys. B154, 365.

Weber, G., 1967, in: Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies, Stanford, ed. S. M. Berman (SLAC, Stanford), p. 59.

Weinberg, S., 1973, Phys. Rev. Lett. 31, 494.

Weinberg, S., 1975, Phys. Rev. D11, 3583.

Weinberg, S., 1976, Phys. Rev. D13, 974.

Weinberg, S., 1977, "The Problem of Mass," in: A Festschrift for I. I. Rabi, ed. L. Motz, Trans. NY Acad. Sci. Series II, 38, 185.

Wess, J., 1981, Les Houches lectures, this session.

Wicklund, A. B., et al., 1980, Phys. Rev. Lett. 45, 1469.

Wigner, E. P., 1937, Phys. Rev. 51, 106.

Wiik, B., 1981, Les Houches lectures, this session.

Willemsen, J. F., 1976, Phys. Rev. D13, 1327.

Willey, R. S., 1978, Phys. Rev. D18, 270.

Williams, P. K., and Hagopian, V. (eds.), 1973, $\pi\pi$ Scattering - 1973, Tallahassee Conference (American Institute of Physics, New York).

Willis, W., and Thompson, J., 1968, "Leptonic Decays of Elementary Particles," in: Advances in Particle Physics, ed. R. L. Cool and R. E. Marshak (Interscience, New York), vol. 1, p. 295.

Wilson, K., 1974, Phys. Rev. D10, 2445.

Witten, E., 1979a, Nucl. Phys. B156, 269.

Witten, E., 1979b, Nucl. Phys. B160, 57.

Witten, E., 1980a, Phys. Today 33, no. 7, p. 38 (July).

Witten, E., 1980b, "The 1/N Expansion in Atomic and Particle Physics," in: Recent Developments in Gauge Theories, Cargèse, 1979, NATO Advanced Studies Institute Series B, Physics, vol. 59, ed. G. 't Hooft, et al. (Plenum, New York).

Witten, E., 1980c, Ann. Phys. (NY) 128, 363.

Yang, C. N., 1950, Phys. Rev. 77, 242.

Yang, C. N., and Mills, R. L., 1954, Phys. Rev. 96, 191.

Zakharov, V. I., 1980a, in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 1027.

Zakharov, V. I., 1980b, in: High Energy Physics - 1980, XX International Conference, Madison, ed. L. Durand and L. G. Pondrom (American Institute of Physics, New York), p. 1234.

Zee, A., 1972, Phys. Rep. 3C, 127.

Zel'dovich, Ya. B., and Sakharov, A. D., 1966, Yad. Fiz. 4, 395; Eng. transl.: Sov. J. Nucl. Phys. 4, 283(1967).

Zweig, G., 1964a, CERN Report 8182/TH.401.

Zweig, G., 1964b, CERN Report 8419/TH.412; reprinted in Developments in the Quark Theory of Hadrons, vol. 1: 1964-1978, ed. D. B. Lichtenberg and S. P. Rosen (Hadronic Press, Nonantum, Mass., 1980), p. 22.

Zweig, G., 1964c, "Fractionally Charged Particles and SU(6)," in: Symmetries in Elementary Particle Physics, 1964 International School of Physics "Ettore Majorana", ed. A. Zichichi (Academic Press, New York, 1965), p. 192.

Zweig, G., 1980, "Origins of the Quark Model," in: Baryon 1980, IVth International Conference on Baryon Resonances, Toronto, ed. N. Isgur (University of Toronto, Toronto), p. 439.

Table 1

Contributions to Electromagnetic Mass Differences in Nonstrange Baryons

Particle	N_d	$\sum_{i<j} e_i e_j$	$\langle \sum_{i<j} e_i e_j \sigma_i \cdot \sigma_j \rangle$
p	1	0	4/3
n	2	-1/3	1
Δ^{++}	0	4/3	4/3
Δ^{+}	1	0	0
Δ^{0}	2	-1/3	-1/3
Δ^{-}	3	1/3	1/3

Table 2

Contributions to Electromagnetic Mass Differences in Nonstrange Mesons

Particle	N_d	$\langle e_q e_{\bar{q}} \rangle$	$\langle e_q e_{\bar{q}} \sigma_q \cdot \sigma_{\bar{q}} \rangle$
"η"	1	-5/18	5/6
π^{+}	1	2/9	-2/3
π^{0}	1	-5/18	5/6
π^{-}	1	2/9	-2/3
ω	1	-5/18	-5/18
ρ^{+}	1	2/9	2/9
ρ^{0}	1	-5/18	-5/18
ρ^{-}	1	2/9	2/9
$\langle \rho^0 \| M \| \omega \rangle$	1	1/6	1/6

Table 3

Properties of the Quarks

Quark	I	I_3	S	B	Y=B+S	$Q=I_3+Y/2$
u	1/2	1/2	0	1/3	1/3	2/3
d	1/2	-1/2	0	1/3	1/3	-1/3
s	0	0	-1	1/3	-2/3	-1/3

Table 4

Contributions to Meson Masses for particles containing strange quarks

Particle	N_s	N_d	$\langle e_q e_{\bar{q}} \rangle$	$\langle e_q e_{\bar{q}} \underset{\sim}{\sigma}_q \cdot \underset{\sim}{\sigma}_{\bar{q}} \rangle$
K^+	1	0	2/9	-2/3
K^o	1	1	-1/9	1/3
$\bar{K}^o$	1	1	-1/9	1/3
K^-	1	0	2/9	-2/3
K^{*+}	1	0	2/9	2/9
K^{*o}	1	1	-1/9	-1/9
$\bar{K}^{*o}$	1	1	-1/9	-1/9
K^{*-}	1	0	2/9	2/9
φ	2	0	-1/9	-1/9

Table 5

Baryon masses (MeV/c^2)

Particle	Model		Experiment	
p	938.28		938.28	
n	939.58		939.57	
Σ^+	1125.95		1189.36	
		−7.98		-7.98 ± 0.08
Σ^-	1133.93		1197.34	
		4.88		4.88 ± 0.06
Σ^0	1129.05		1192.46	
Λ	1127.84		1115.60	
Ξ^0	1314.90		1314.9	
Ξ^-	1321.60		1321.62	

Table 6. Magnetic Moments of the Baryon Octet. Numerical values are given in nuclear magnetons. Underlined quantities are inputs.

Baryon	Quark Model	Exact SU(3) $\mu_u=-2\mu_d=-2\mu_s=2\mu_p/3$	$\mu_u=-2\mu_d$ $\mu_s\equiv\mu_\Lambda$	$\mu_u\neq-2\mu_d$ $\mu_s\equiv\mu_\Lambda$	Measured values
p	$(4\mu_u-\mu_d)/3$	μ_p = $\underline{2.793}$	$\underline{2.793}$	$\underline{2.793}$	2.793
n	$(4\mu_d-\mu_u)/3$	$-\frac{2}{3}\mu_p$ = −1.862	−1.862	$\underline{-1.913}$	−1.913
Λ	μ_s	$-\mu_p/3$ = −0.931	$\underline{-0.614}$	$\underline{-0.6138}$	-0.6138 ± 0.0047 a) -0.6129 ± 0.0045 b)
Λ-Σ^0	$(\mu_d-\mu_u)/\sqrt{3}$	$-\mu_p/\sqrt{3}$ = −1.612	−1.612	−1.633	$-1.82^{+0.18}_{-0.25}$ c)
Σ^+	$(4\mu_u-\mu_s)/3$	μ_p = 2.793	2.687	2.673	2.33 ± 0.13 d)
Σ^0	$(2\mu_u+2\mu_d-\mu_s)/3$	$\mu_p/3$ = 0.931	0.825	0.791	0.46 ± 0.28 e)
Σ^-	$(4\mu_d-\mu_s)/3$	$-\mu_p/3$ = −0.931	−1.037	−1.091	-1.41 ± 0.25 f)
Ξ^0	$(4\mu_s-\mu_u)/3$	$-2\mu_p/3$ = −1.862	−1.439	−1.436	-1.250 ± 0.014 b)
Ξ^-	$(4\mu_s-\mu_d)/3$	$-\mu_p/3$ = −0.931	−0.508	−0.494	-0.75 ± 0.06 g)

a) Schachinger, et al. (1978)
b) Cox, et al. (1981)
c) Dydak, et al. (1977)
d) Settles, et al. (1979) and Particle Data Group (1980)
e) Defined by eq. (3.59)
f) Roberts, et al. (1975) and Particle Data Group (1980)
g) Handler, et al. (1980)

Table 7

Axial Charges g_A/g_V in Semileptonic Decays of Baryons

Decay	Cabibbo/SU(3)	Quark Model	Experiment
$n \to pe\nu$	D+F	5/3	1.254±0.007 a)
$\Lambda \to pe\nu$	F+D/3	1	0.62±0.05 a) ±(0.734±0.03) b)
$\Sigma^- \to \Sigma^0 e\nu$	F	2/3	
$\Sigma^\pm \to \Lambda e\nu$	pure axial	pure axial	g_V/g_A =−0.10±0.22 a)
$\Sigma^- \to ne\nu$	F−D	−1/3	±(0.385±0.070) a)
$\Xi^- \to \Xi^0 e\nu$	F−D	−1/3	
$\Xi^- \to \Lambda e\nu$	F−D/3	1/3	

a) Particle Data Group (1980)
b) Jensen, et al. (1980)

Table 8. M1 Transitions in Mesons

Process	Photon Energy ω (GeV)	Decay Rate Γ (keV)	$3\pi\Gamma/\omega^3\mu_N^2$	$\lvert\mathcal{M}_{\text{flavor-spin}}\rvert^2$ Quark model	SU(3) $3\pi\Gamma/\omega^3\mu_N^2\theta$	θ	$\mu_s \neq \mu_d$ $3\pi\Gamma/\omega^3\mu_N^2$	θ
$\omega \to \pi^0\gamma$	0.380	889±50 a) 789±92	5.87±0.35 5.21±0.61	$(\mu_u - \mu_d)^2$	7.80	0.75±0.04 0.67±0.08		
$\rho \to \pi\gamma$	0.375	67±7 b)	0.461±0.038	$(\mu_u + \mu_d)^2$	0.87	0.53±0.04		
$K^{*+} \to K^+\gamma$	0.309	62±14 c)	0.753±0.170	$(\mu_u + \mu_s)^2$	0.87	0.87±0.20	1.56	0.48±0.11
$K^{*0} \to K^0\gamma$	0.309	75±35 a)	0.921±0.422	$(\mu_d + \mu_s)^2$	3.47	0.27±0.12	2.39	0.39±0.18
$\varphi \to \pi^0\gamma$	0.501	5.7±2 a)	0.017±0.006	0	0	-		
$\omega \to \eta\gamma$	0.199	$3^{+2.5}_{-1.8}$ a)	$0.138^{+0.115}_{-0.084}$	Q: $(\mu_u+\mu_d)^2/2$ L: $3(\mu_u+\mu_d)^2/4$	0.43 0.65	$0.32^{+.28}_{-.20}$ $0.21^{+.18}_{-.13}$		
$\rho^0 \to \eta\gamma$	0.194	50±13 a)	2.48±0.64	Q: $(\mu_u-\mu_d)^2/2$ L: $3(\mu_u-\mu_d)^2/4$	3.90 5.85	0.63±0.17 0.42±0.11		
$\varphi \to \eta\gamma$	0.362	62±9 a)	0.47+0.07	Q: $2\mu_s^2$ L: μ_s^2	1.73 0.87	0.27+0.04 0.54±0.08	0.75 0.38	0.63±0.09 1.24±0.18
$\varphi \to \eta'\gamma$	0.060			Q: $2\mu_s^2$ L: $3\mu_s^2$	1.73 2.60		0.75 1.13	
$\eta' \to \rho^0\gamma$	0.164	83±30 a,d)	6.81±2.46	Q: $3(\mu_u-\mu_d)^2/2$ L: $3(\mu_u-\mu_d)^2/4$	11.70 5.85	0.58±0.21 0.29±0.11		
$\eta' \to \omega\gamma$	0.159	7.6±3 a,d)	0.68±0.27	Q: $3(\mu_u+\mu_d)^2/2$ L: $3(\mu_u+\mu_d)^2/4$	1.30 0.65	0.53±0.21 1.05±0.41		

a) Particle Data Group (1980) b) Berg, et al. (1980a) c) Berg, et al. (1980b,1981)

d) Adjusted by Rosner (1980) using total η' width measured by Binnie, et al. (1979) and by Abrams, et al. (1979)

Table 9. Electromagnetic Charge Radii of Mesons

Particle	Beam Momentum (GeV/c)	$\langle r^2_{EM} \rangle$, fm^2	Reference
π^-	100	0.31 ± 0.04	Dally, et al. (1977)
	250	0.43 ± 0.03	Dally, et al. (1980a)
	Combined fit	0.39 ± 0.04	Dally, et al. (1980a)
K^-	250	0.28 ± 0.05	Tsyganov (1979) ; Dally, et al. (1980c)
	π/K Comparison	0.25 ± 0.05	Dally, et al. (1980b)
K^0	30 - 100	-0.054 ± 0.026	Molzon, et al. (1978)

Table 10. Light Mesons as Quark-Antiquark Bound States

State	Mixing ?	J^{PC}	I = 1	I = 0		I = 1/2
$^1S_0$		0^{-+}	$\pi(140)$	$\eta(549)$	$\eta'(958)$	K(496)
$^3S_1$	$^3D_1$	1^{--}	$\rho(776)$	$\omega(784)$	$\varphi(1019)$	$K^*(892)$
$^1P_1$		1^{+-}	B(1231)	H(1190)[a]		$Q_B(1355)$[b]
$^3P_0$		0^{++}	$\delta(981)$	$\epsilon(1300)$?[h]	$S^*(980)$	$\chi(1500)$?
$^3P_1$		1^{++}	$A_1(1240)$[a]	D(1285)	E(1418)	$Q_A(1340)$[b]
$^3P_2$	$^3F_2$	2^{++}	$A_2(1317)$	$f^0(1273)$	$f'(1516)$	$K^{**}(1430)$
$^1D_2$		2^{-+}	$A_3(1660)$			L(1765)?
$^3D_1$	$^3S_1$	1^{--}	$\rho'(1600)$		$\varphi'(1634)$	$K^*(1650)$?
$^3D_2$		2^{--}				
$^3D_3$	$^3G_3$	3^{--}	g(1700)	$\omega(1670)$	$\varphi_3(1870)$[c]	$K^*(1753)$[d]
$^1F_3$		3^{+-}				
$^3F_2$	$^3P_2$	2^{++}	$f\pi(1700)$[e]		$\theta(1640)$[g]?	
$^3F_3$		3^{++}				
$^3F_4$		4^{++}	$K^+K_s(2060)$[f]	h(2040)		$K^*(2070)$[d]

a) Dankowych, et al. (1981)
b) Leith (1977)
c) Armstrong, et al. (1981)
d) Aston, et al. (1981) ; Cleland et al.(1980b);Dorsaz(1981)
e) Cashmore (1980) : see also Montanet (1980)
f) Cleland, et al. (1980a) ; see also Montanet (1980)
g) Seen in $\psi \rightarrow \gamma\eta\eta$; Scharre (1981)
h) According to Wicklund,et al.(1980)the pole lies at 1425MeV/c^2

Table 11

SU(6) Classification of the Baryon Resonances

N	$SU(6)_L{}^P$	$^{(2J+1)}SU(3)$	Members
0	$56_0{}^+$	^{2}[8]	N(939), Λ(1115), Σ(1193), Ξ(1318)
		^{4}[10]	Δ(1232), Σ(1385), Ξ(1533), Ω(1672)
1	$70_1{}^-$	^{2}[1]	Λ(1405)
		^{4}[1]	Λ(1520)
		^{2}[8]	N(1535), Λ(1670), Σ(1750), Ξ(1684) ?
		^{2}[8]	N(1700), Λ(1870)
		^{4}[8]	N(1520), Λ(1690), Σ(1670), Ξ(1820) ?
		^{4}[8]	N(1700) Σ(1940) ?
		^{6}[8]	N(1670), Λ(1830), Σ(1765)
		^{2}[10]	Δ(1650)
		^{4}[10]	Δ(1670)
2	$56_2{}^+$	^{4}[8]	N(1810), Λ(1860)
		^{6}[8]	N(1688), Λ(1815), Σ(1915), Ξ(2030) ?
		^{2}[10]	Δ(1910)
		^{4}[10]	
		^{6}[10]	Δ(1890)
		^{8}[10]	Δ(1950), Σ(2030)
	$56_0{}^+$	^{2}[8]	N(1470) Σ(1660)
		^{4}[10]	Δ(1690)

Table 12

Value of the Color Casimir Operator in Small Representations of SU(3)

Representation	$\langle T^2 \rangle$
[1]	0
[3] or [3*]	4/3
[6] or [6*]	10/3
[8]	3
[10] or [10*]	6
[27]	8

Table 13

"Interaction energies" for few-quark systems

Configuration	$\left\langle \sum_{i<j} \underset{\sim}{T}^{(i)} \cdot \underset{\sim}{T}^{(j)} \right\rangle$
$(q\bar{q})\,[\underset{\sim}{1}]$	$-4/3$
$(q\bar{q})\,[\underset{\sim}{8}]$	$+1/6$
$(qq)\,[\underset{\sim}{3}^*]$	$-2/3$
$(qq)\,[\underset{\sim}{6}]$	$1/3$
$(qqq)\,[\underset{\sim}{1}]$	-2
$(qqq)\,[\underset{\sim}{8}]$	$-1/2$
$(qqq)\,[\underset{\sim}{10}]$	$+1$
$(qqqq)\,[\underset{\sim}{3}]$	-2

Table 14

Baryon masses including the color hyperfine interaction.
For definitions see (5.44) and (5.45).

Baryon	$\Delta E_{HFS}/\delta M_{c.m.}$	N_s	Fitted mass (MeV/c^2)
N(939)	-3	0	939
Λ(1116)	-3	1	1123
Σ(1193)	$1-4\,m_u/m_s$	1	1189
Ξ(1318)	$-4\,m_u/m_s + m_u^2/m_s^2$	2	1345
Δ(1232)	$+3$	0	1232
Σ^*(1384)	$1+2\,m_u/m_s$	1	1383
Ξ^*(1533)	$2\,m_u/m_s + m_u^2/m_s^2$	2	1539
Ω(1672)	$3\,m_u^2/m_s^2$	3	1701

Table 15

Meson masses including the color hyperfine interaction.
For definitions see (5.52) and (5.53).

Meson	$\Delta E_{HFS} / \delta M_{c.m.}$	N_s	Fitted mass (MeV/c^2)
π(138)	-3	0	138
K(496)	$-3\, m_u/m_s$	1	489
η(549)	$-\frac{9}{4} - \frac{3}{4}\left(\frac{m_u}{m_s}\right)^2$	1/2	297
η'(958)	$-\frac{3}{4} - \frac{9}{4}\left(\frac{m_u}{m_s}\right)^2$	3/2	616
ρ (776)	1	0	776
ω(784)	1	0	776
K^* (892)	m_u/m_s	1	894
φ(1020)	m_u^2/m_s^2	2	1034

Table 16

Some properties of the Heavy Quarks

Quark	I	Q	Charm	Beauty	Truth
c	0	2/3	1	0	0
b	0	-1/3	0	1	0
t	0	2/3	0	0	1

Table 17

Symmetry properties of flavor and color-spin wavefunctions for three quarks

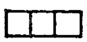
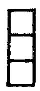
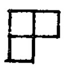
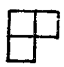
Flavor SU(3)		Color-spin SU(6)		$G^{(6)2}$	$SU(3)_{color} \otimes SU(2)_{spin}$
[10] 	S	20 	A	21/4	[1], (4) [8], (2)
[8] 	M	70 	M	33/4	[1], (2) [8], (4) ⊕ [8], (2) [10], (2)
[1] 	A	56 	S	45/4	[8], (2) [10], (4)

Table 18

Candidates for Radially-Excited Pseudoscalars

State	I	Seen In	Remarks
$\pi'(1342)$	1	$\varepsilon\pi$	Bonesini, et al. (1981)
$\eta(1275)$	0	$\eta\pi\pi$	Stanton, et al. (1979)
$\eta'(1400)$	0	$\eta\varepsilon$	Stanton, et al. (1979) via Close (1981)
$\iota(1440)$	0	$\psi \to \gamma + (K\bar{K}\pi)$	Scharre (1981)
K'(1400)	1/2	$K\pi\pi$ $(K\varepsilon)$	Brandenburg, et al. (1976) Aston, et al. (1981)

Table 19

Some properties of the $^3S_1$ ψ states (from Particle Data Group, 1980)

Level	$\Gamma(\psi \to e^+e^-)$, keV	Γ_{tot}, keV
$\psi(3097)$	4.60 ± 0.42	63 ± 9
$\psi'(3685)$	2.05 ± 0.23	215 ± 40
$\psi(4029)$	0.75 ± 0.15	52 ± 10 MeV
$\psi(4159)$	0.77 ± 0.23	78 ± 20 MeV
$\psi(4415)$	0.49 ± 0.13	42 ± 10 MeV

Table 20

Some properties of the $^3S_1$ Υ states (from the review by Schamberger, 1981)

Level	$\Gamma(\Upsilon \to e^+e^-)$, keV	Γ_{tot}, keV
Υ (9433)	1.17 ± 0.05	35.5^{+8}_{-6}
Υ' (9993)	0.54 ± 0.03	$\approx \Gamma(\Upsilon)$
Υ'' (10323)	0.37 ± 0.03	
Υ''' (10546)	0.27 ± 0.02	~ 15 MeV

Table 21

Decay modes of the $(s\bar{s})$ state, $\varphi(1020)$ (Particle Data Group, 1980)

Channel	Branching Fraction (%)	q_{max} (MeV/c)
K^+K^-	48.6 ± 1.2	127
$K_L K_S$	35.2 ± 1.2	111
$\pi^+\pi^-\pi^0$	14.7 ± 0.7	462
$\eta\gamma$	1.5 ± 0.2	362
$\pi^0\gamma$	0.14 ± 0.05	501
e^+e^-	0.031 ± 0.001	510
$\mu^+\mu^-$	0.025 ± 0.003	499

CAPTIONS

Fig. 1 : Energy levels for A = 7 nuclei (from Ajzenberg-Selove, 1979). The diagrams for individual isobars have been shifted vertically to eliminate the neutron-proton mass difference and the Coulomb energy, taken as $E_C = (0.6\ \mathrm{MeV})Z(Z-1)/A^{1/3}$. Energies in square brackets represent the approximate nuclear binding energy $E_N = M(Z,A) - ZM_p - (A-Z)M_n - E_C$, minus the corresponding quantity for ^7Li. Note the one-to-one correspondence between levels of the mirror nuclei ^7Li and ^7Be.

Fig. 2 : Energy levels for A = 11 nuclei (from Ajzenberg-Selove, 1975). Notation as in Fig. 1, with binding energies referred to ^{11}B.

Fig. 3 : Energy levels for A = 14 nuclei (from Ajzenberg-Selove, 1976). Notation as in Fig. 1, with binding energies referred to ^{14}N.

Fig. 4 : The isospin quarks.

Fig. 5 : Isospin assignments of the nucleons and nucleon resonances.

Fig. 6 : The weight diagram for the fundamental $[\underset{\sim}{3}]$ representation of SU(3).

Fig. 7 : Weight diagram for the vector meson nonet = $[\underset{\sim}{1}] \oplus [\underset{\sim}{8}]$.

Fig. 8 : Decompositions of the fundamental quark triplets with respect to the SU(2) subgroups U-spin and V-spin.

Fig. 9 : The $J^P = 3/2^+$ baryon decimet.

Fig. 10 : The $J^P = 1/2^+$ baryon octet.

Fig. 11 : Action of the I-spin, U-spin, and V-spin raising and lowering operators on the fundamental triplets of quarks $[\underset{\sim}{3}]$ and antiquarks $[\underset{\sim}{3}^*]$.

Fig. 12 : Properties of the lowest mode of a fermion confined within a rigid sphere. (a) Fermion momentum as a function of its mass m and the sphere radius R. (b) Ratio of the fermion mass to the energy of its lowest confined mode.

Fig. 13 : Lowest-mode energy of a massless fermion confined to a rigid, static sphere of radius R (see eqn. (3.145)).

Fig. 14 : Lowest-mode energy of a fermion of mass m confined within a rigid, static sphere of radius 4/3 fm.

Fig. 15 : Magnetic moment of the confined fermion in units of the Dirac moment for a free fermion with mass equal to the energy ω of the confined fermion.

Fig. 16 : Axial charge of a nucleon composed of equal-mass quarks confined within a rigid spherical cavity, as a function of the dimensionless parameter mR.

Fig. 17 : Overlap factor ϑ defined in eqn. (3.155) as measured in various M1 decays of mesons. The dashed entry for $\omega \to \pi^0 \gamma$ is for the reanalysis by Ohshima (1980).

Fig. 18 : Method of pole extrapolation for studying the interactions of unstable target particles.
(a) Pion scattering from a virtual pion.
(b) Measurement of the electromagnetic form factor of a virtual pion. Additional diagrams are required to contribute by gauge invariance.

Fig. 19 : Regge trajectories of the natural-parity mesons. Uncertain states are indicated by open circles.

Fig. 20 : Expected SU(6) multiplets of baryons.

Fig. 21 : Regge trajectories of the nucleon, Δ , and Λ resonances.

Fig. 22 : The ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ compared with the predictions of the quark-parton model.
(a) $W < 4$ GeV (after Spinetti, 1979) ; (b) 8 GeV $< W <$ 40 GeV (after Schamberger, 1981).

Fig. 23 : The quark-quark-gluon interaction in QCD.

Fig. 24 : A baryon configuration which is not considered in the sum over two-body forces.

Fig. 25 : Attempting to separate a quark and antiquark results in the creation of a quark-antiquark pair from the vacuum, so that color is always neutralized locally.

Fig. 26 : A massless quark and antiquark connected by a linear string.

Fig. 27 : Meson states in flavor SU(6), decomposed into $SU(4)_{udsc} \otimes U(1)_b \otimes U(1)_t$. The additive quantum numbers are denoted by B(beauty) and T(truth).

Fig. 28 : $J^P = 1/2^+$ baryon states in flavor SU(6). The circled states occur twice, as do those that lie in both $[\underset{\sim}{6}]$ and $[\underset{\sim}{3}^*]$ of $SU(3)_{uds}$. There are 70 states in all.

Fig. 29 : $J^P = 3/2^+$ baryon states in flavor SU(6). There are 56 states in all.

Fig. 30 : The spectrum of charmonium ($c\bar{c}$). Branching fractions (in percent) are shown for the important classes of decays (Particle Data Group, 1980 ; Himel, et al., 1980a ; Oreglia, et al., 1980 ; Schamberger, 1981 ; Scharre, 1981). Charm threshold is indicated at twice the D meson mass.

Fig. 31 : The spectrum of upsilon ($b\bar{b}$) states. Branching fractions (in percent) are shown for the important classes of identified decays (Schamberger, 1981). Beauty threshold is indicated schematically.

Fig. 32 : Lower bounds for leptonic decays of Υ and Υ' (after Rosner, et al., 1978) together with the data cited in Table 20. The bounds are computed from eqn. (6.45) using ψ leptonic widths 1σ below the central values and assuming $m_b/m_c \gtrsim 3$.

Fig. 33 : A possible spectrum of strangeonium ($s\bar{s}$) levels. Identification of E(1418) and φ(1634) as pure $s\bar{s}$ states may be disputed. The dotted 0^{-+} entry is impressionistic, having been invented from the φ mass and the $\pi-\rho$ splitting, appropriately rescaled.

Fig. 34 : Charge induced by a positive test charge placed at the center of a hole in a dielectric medium. (a) Dia-electric case $\varepsilon_{medium} < 1$ hoped to resemble QCD ; (b) Dielectric case $\varepsilon_{medium} > 1$ of normal electrodynamics.

Fig. 35 : (a) A single link between two quarks in lattice gauge theory. (b) The smallest closed loop, corresponding to a quarkless excitation.

Fig. 36 : The Okubo-Zweig-Iizuka rule applied to φ-decay. The connected diagram (a) is allowed ; the disconnected diagram (b) is forbidden.

Fig. 37 : Cross sections for the two-photon reactions $e^+e^- \longrightarrow e^+e^- +$ hadrons. The cross section for excitation of $i(1440)$ is computed under the assumption that $\Gamma(i \rightarrow \gamma\gamma) = 1$ keV, and so should be multiplied by $\Gamma(i \rightarrow \gamma\gamma)$ /(1 keV). The cross section for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ ("one unit of R") is shown for reference.

Fig. 38 : Double-line notation for quarks, gluons, and their interactions useful for $1/N_c$ analyses.

Fig. 39 : Lowest order vacuum polarization contributions to the gluon propagator. (a) quark loop ; (b) gluon loop ; (c) quark loop in the double-line notation ; (d) gluon loop in the double-line notation.

Fig. 40 : A two-loop diagram in (a) conventional and (b) double-line notation.

Fig. 41 : A three-loop diagram in (a) conventional and (b) double-line notation.

Fig. 42 : A nonplanar graph in (a) conventional and (b) double-line notation.

Fig. 43 : A mechanism for OZI-forbidden decay, at order g^4, in (a) conventional and (b) double-line notation.

Fig. 44 : OZI-allowed decay of a meson, at order g^4, in (a) conventional and (b) double-line notation.

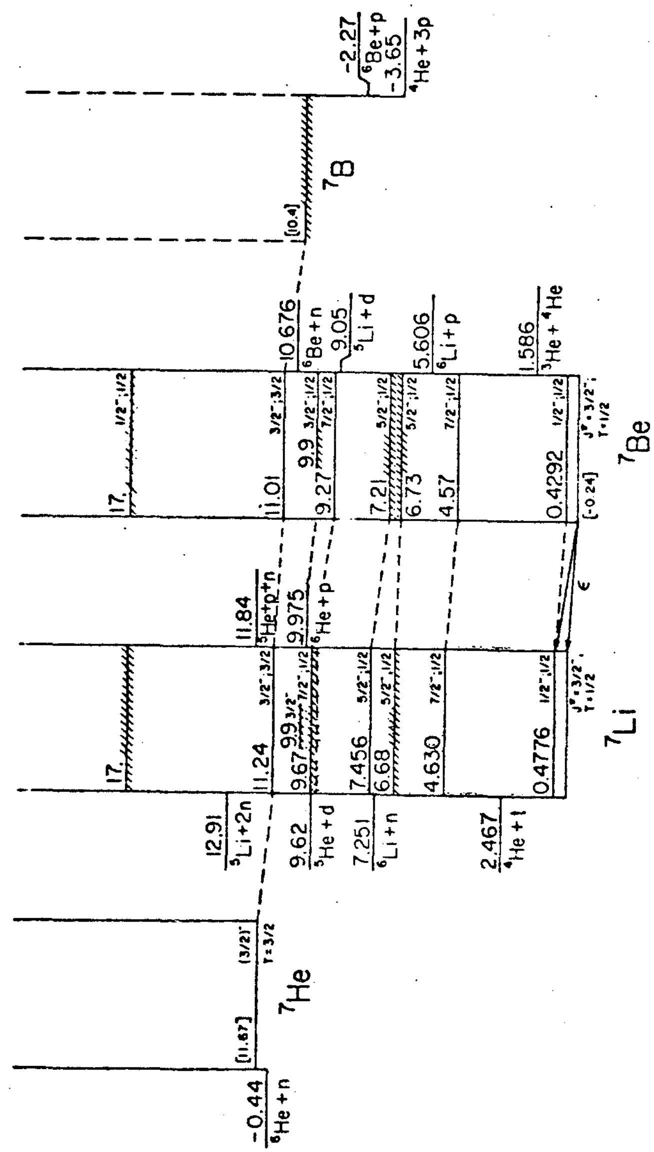
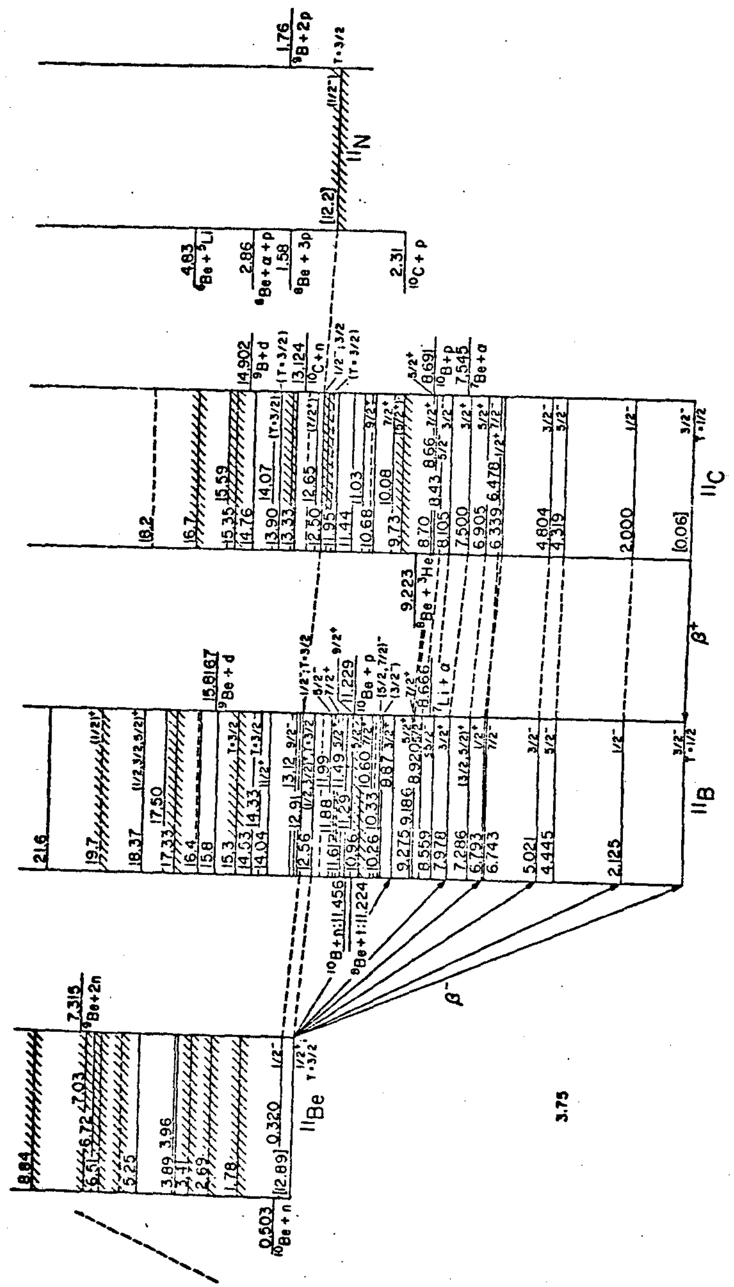
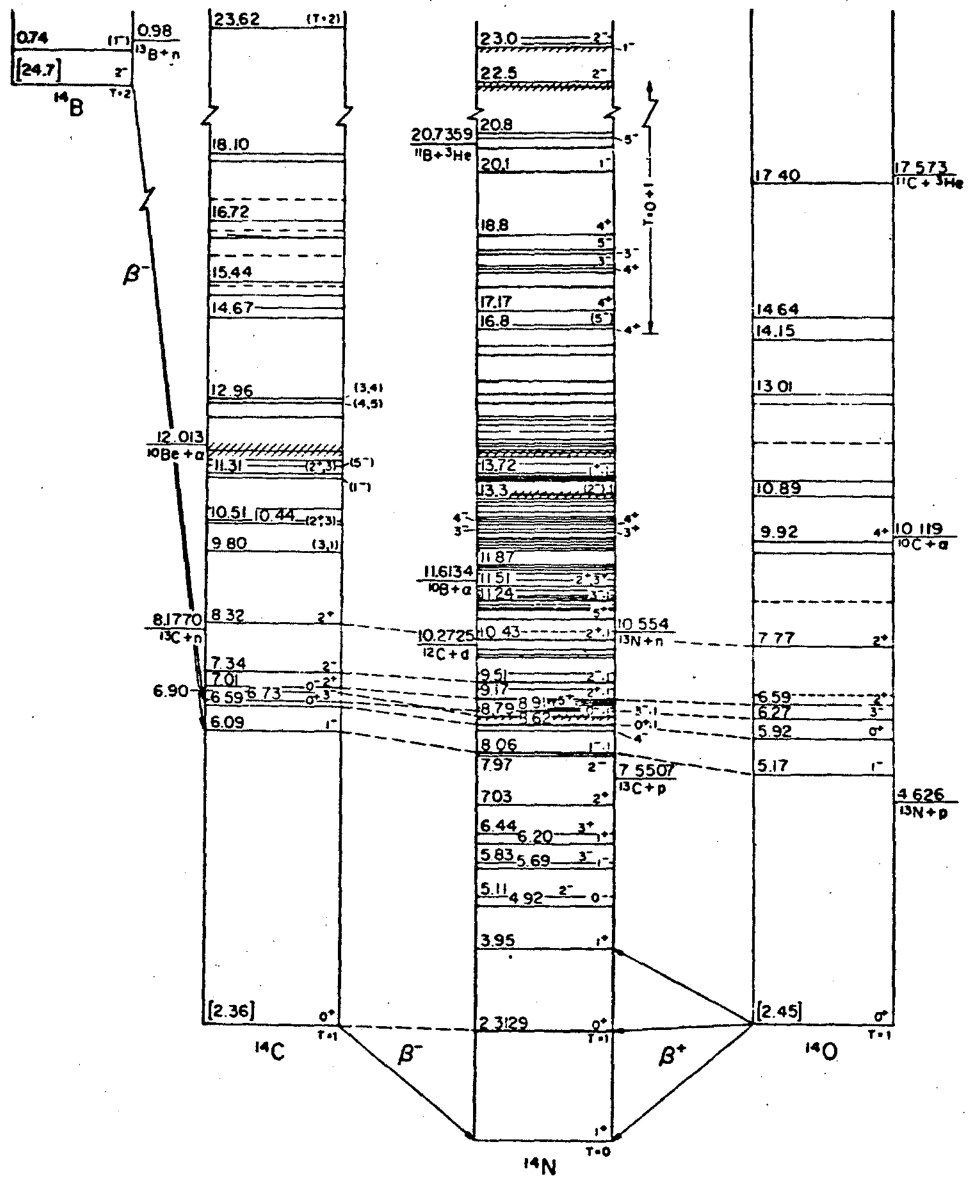


Fig. 1



3.75

Fig. 2



4.76

Fig. 3

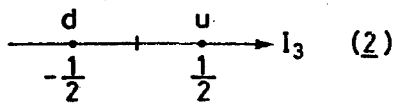


Fig. 4

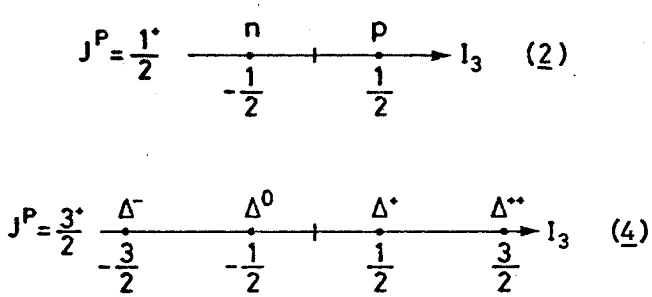


Fig. 5

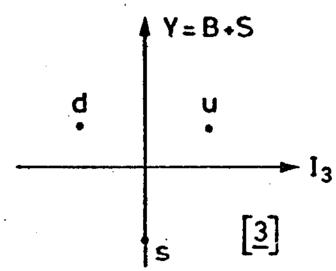


Fig. 6

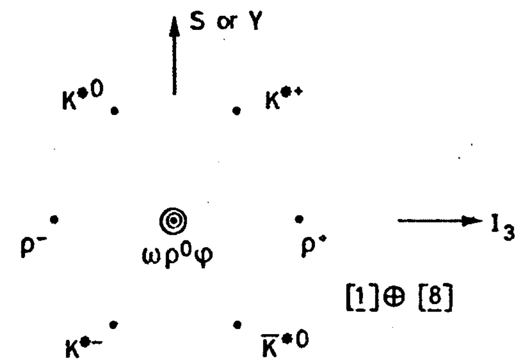


Fig. 7

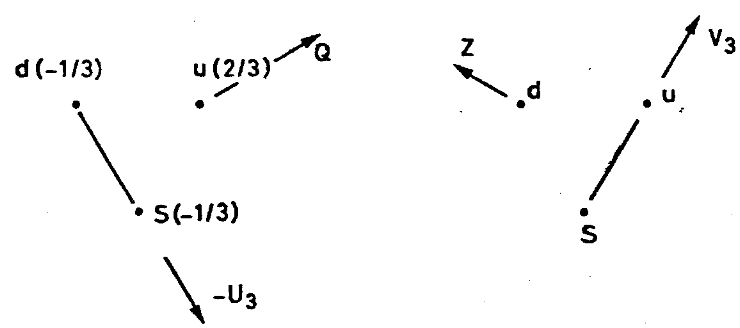


Fig. 8

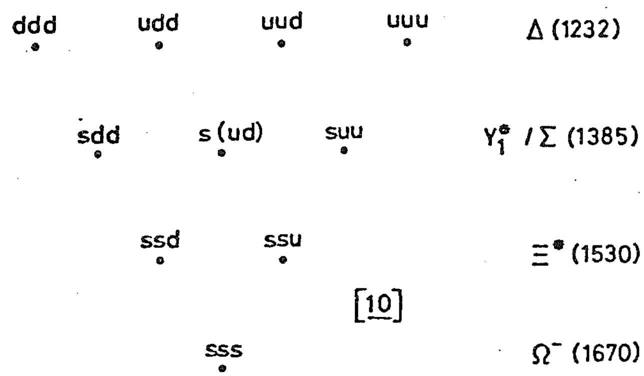


Fig. 9

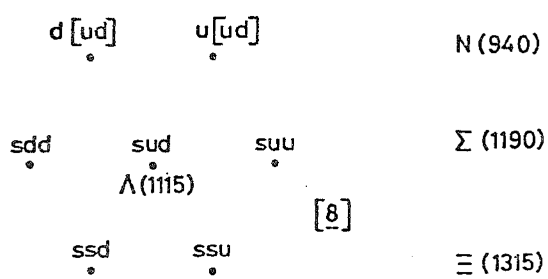


Fig. 10

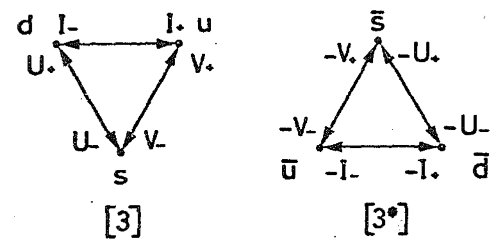


Fig. 11

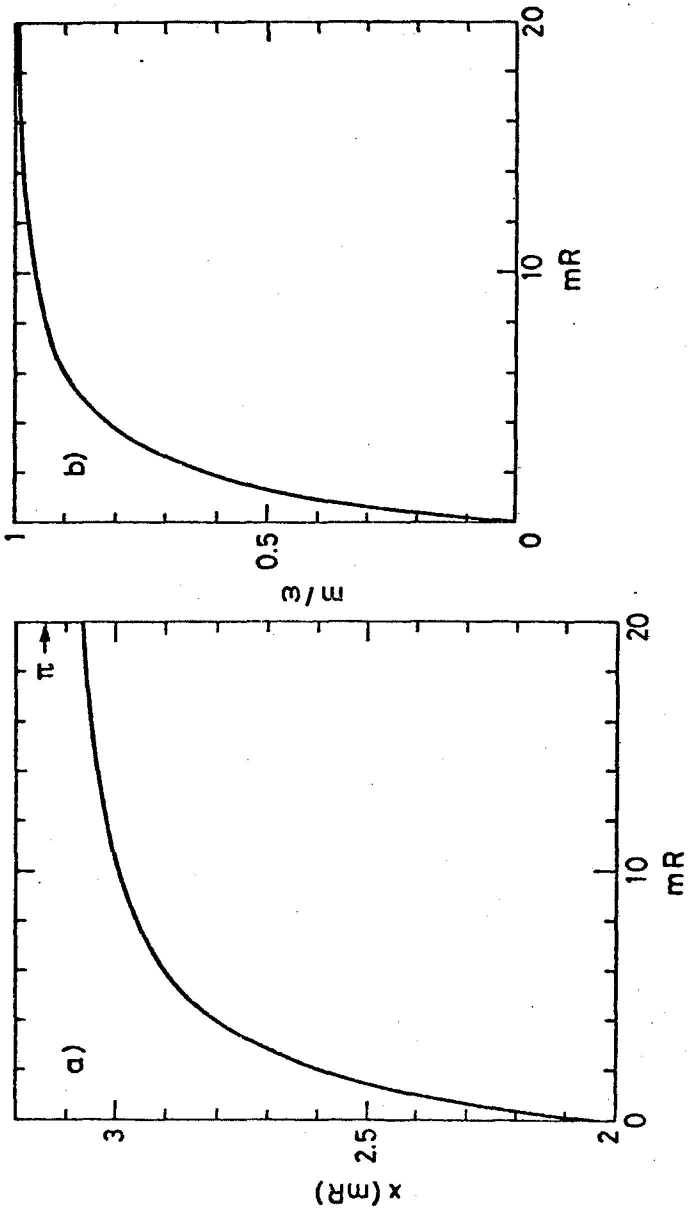


Fig. 12

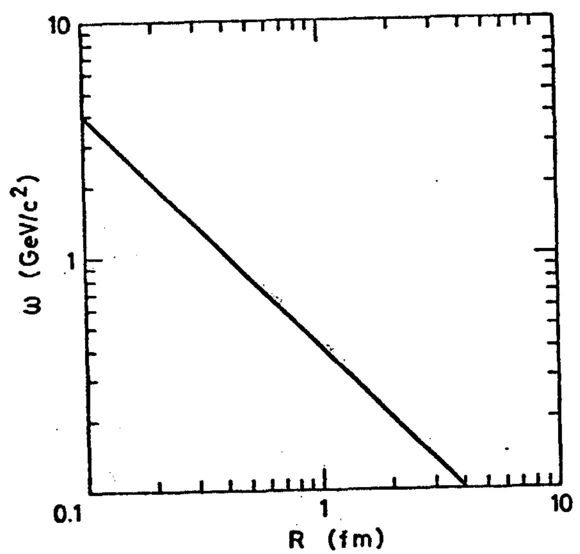


Fig. 13

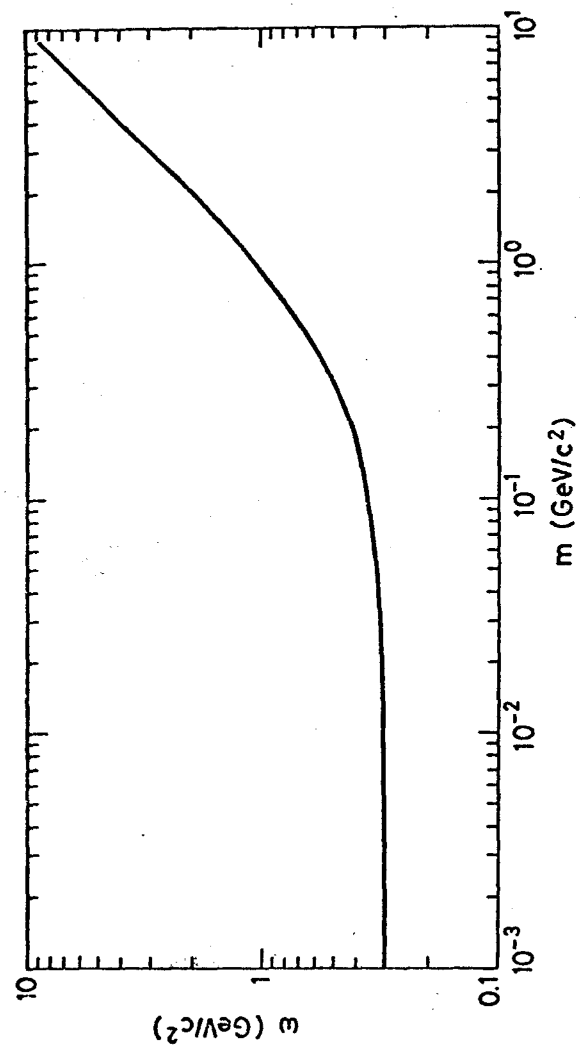


Fig. 14

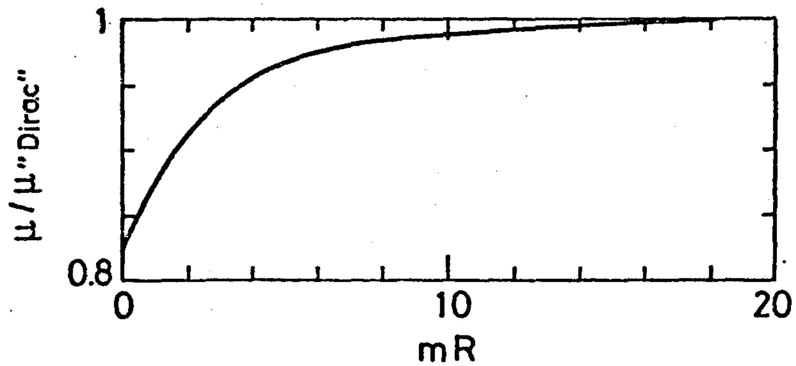


Fig. 15

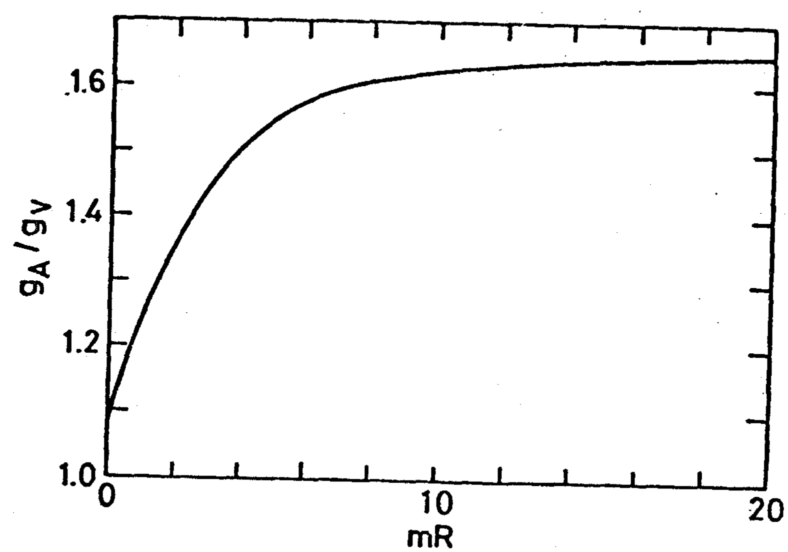


Fig. 16

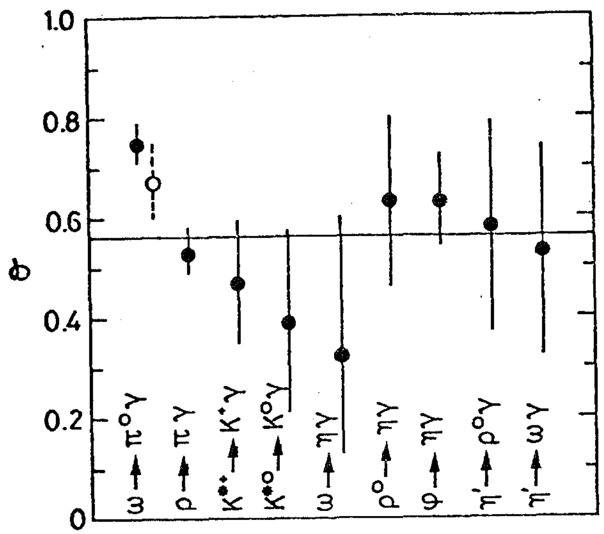


Fig. 17

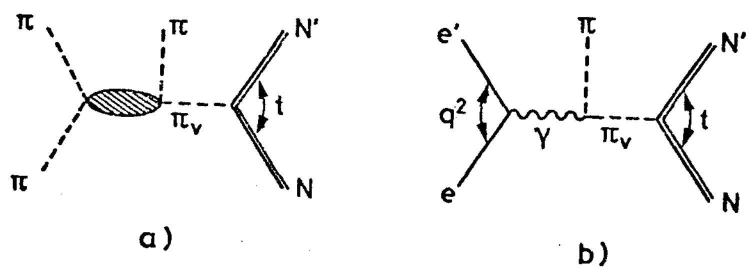


Fig. 18

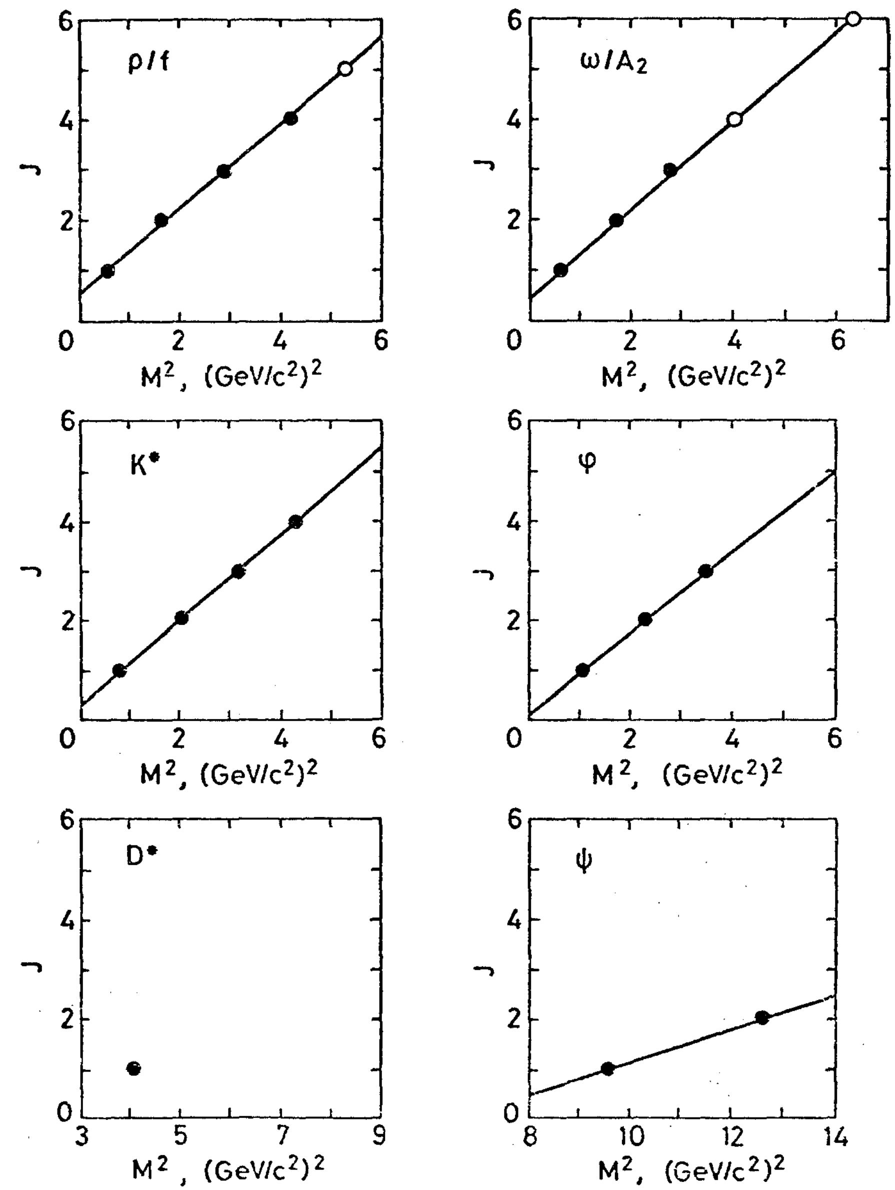
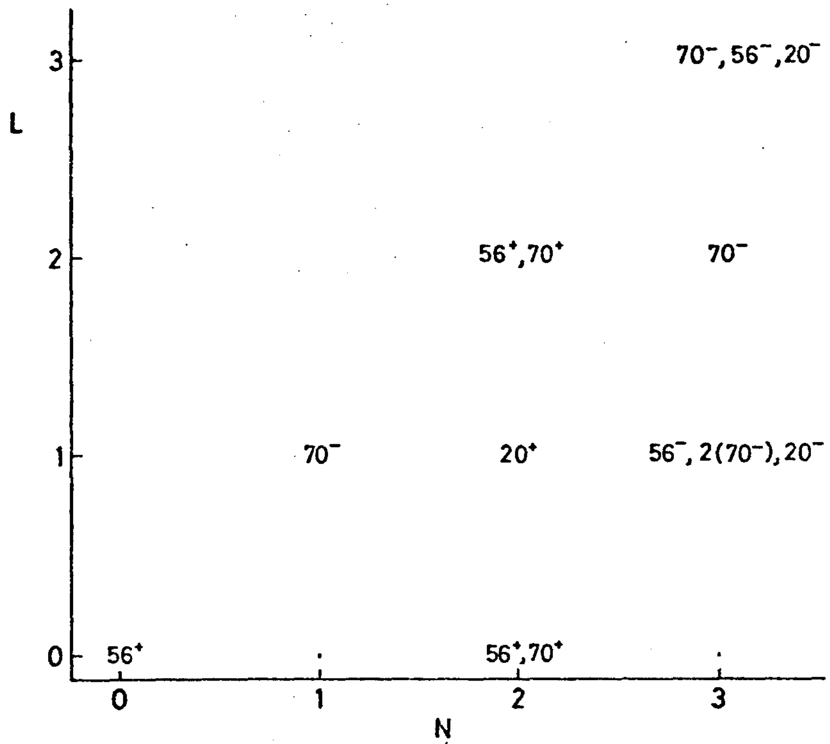
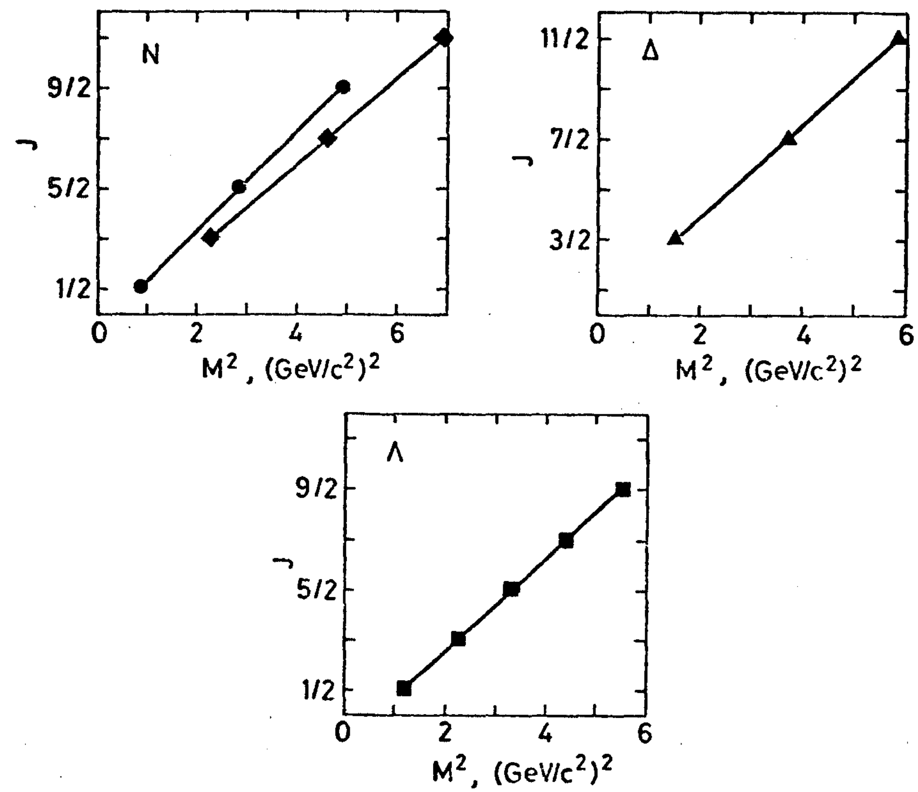


Fig. 19



L
N
3
2
1
0
0
1
2
3
70⁻, 56⁻, 20⁻
56⁺, 70⁺
70⁻
70⁻
20⁺
56⁻, 2(70⁻), 20⁻
56⁺
56⁺, 70⁺

Fig. 20



N
J
9/2
5/2
1/2
0
2
4
6
M², (GeV/c²)²
Δ
11/2
7/2
3/2
0
2
4
6
M², (GeV/c²)²
Λ
9/2
5/2
1/2
0
2
4
6
M², (GeV/c²)²

Fig. 21

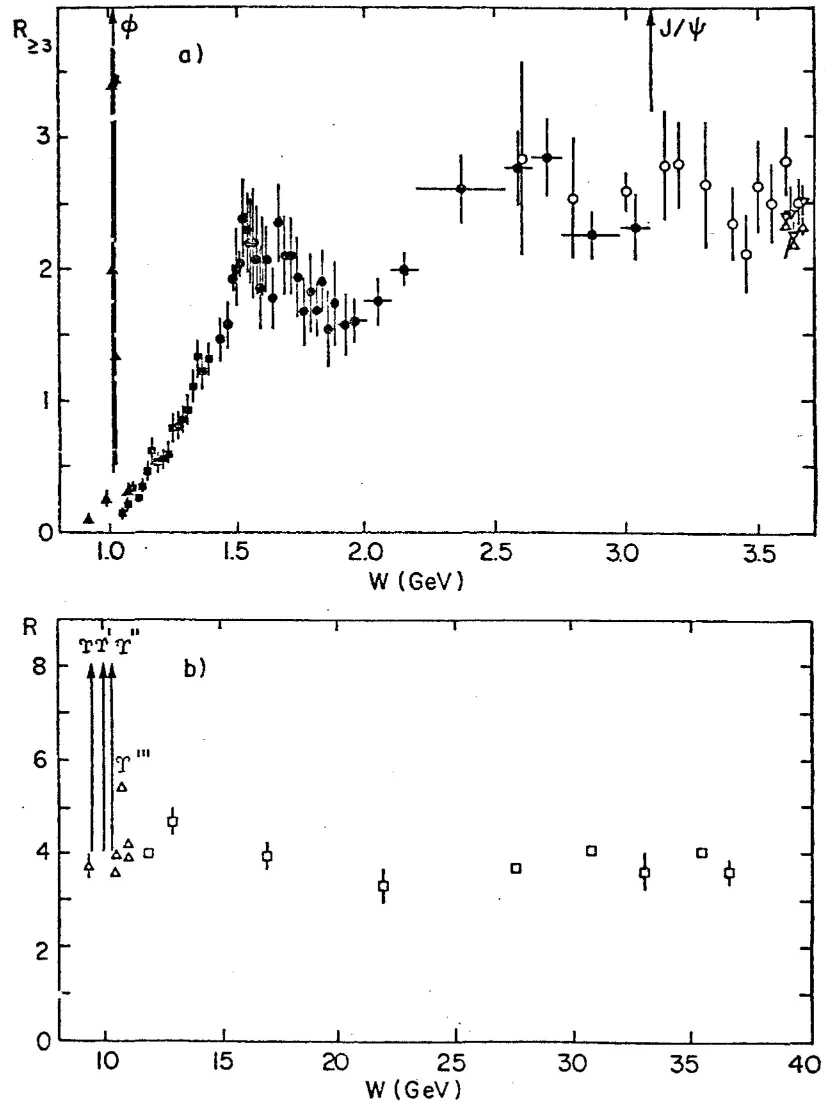


Fig. 22

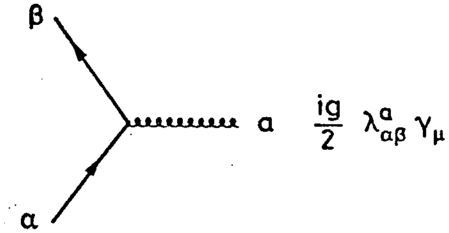


Fig. 23

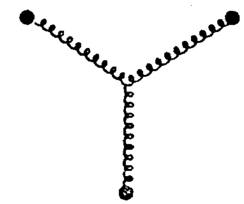


Fig. 24

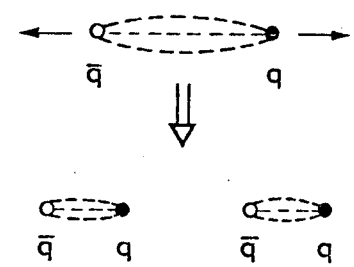


Fig. 25

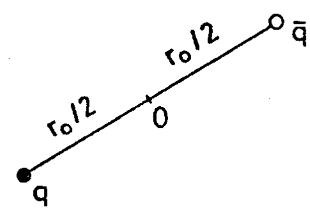


Fig. 26

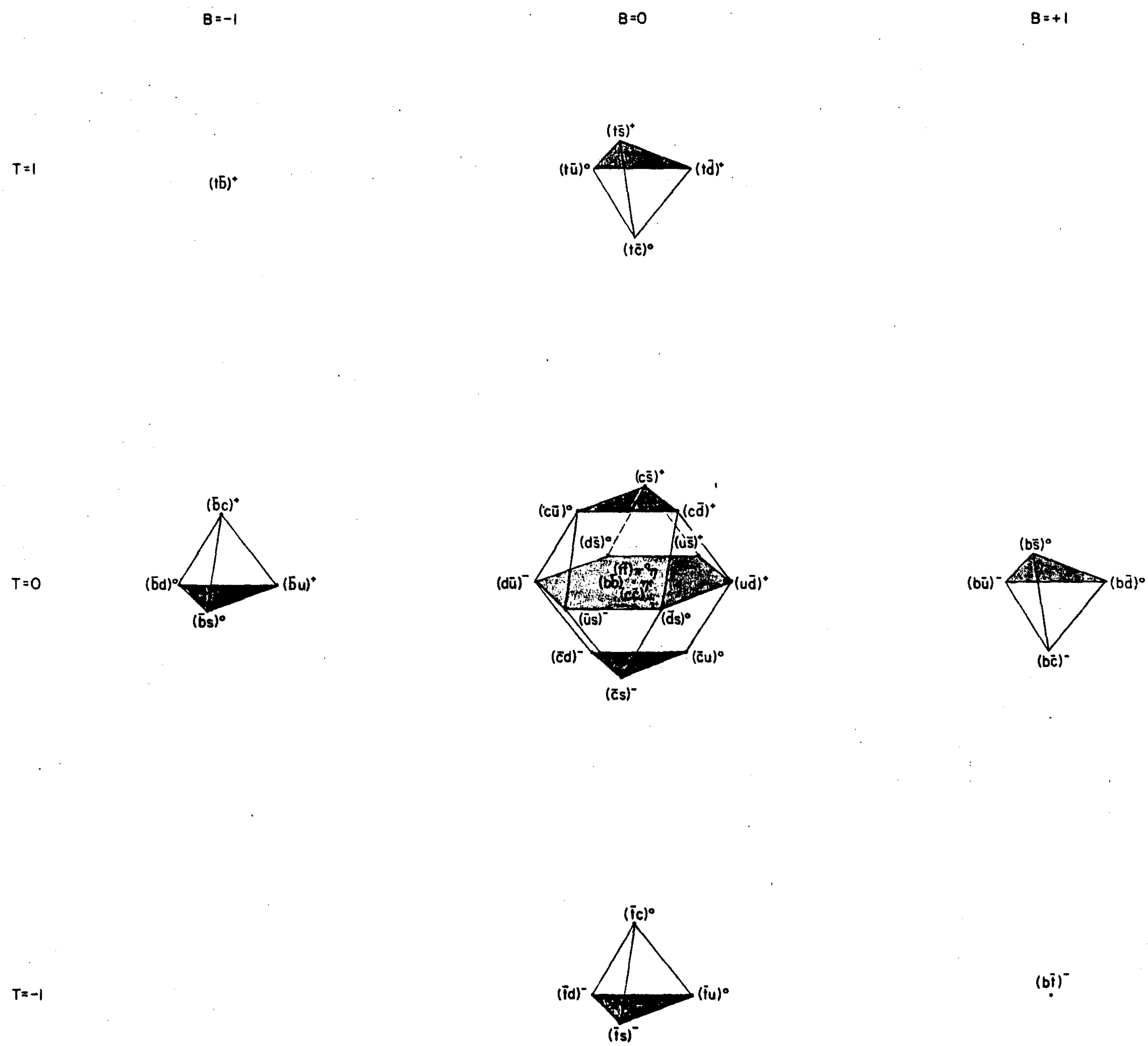

B=-1
B=0
B=+1
T=1
T=0
T=-1
$(t\bar{b})^+$
$(t\bar{s})^+$
$(t\bar{u})^0$
$(t\bar{d})^+$
$(t\bar{c})^0$
$(\bar{b}c)^+$
$(\bar{b}d)^0$
$(\bar{b}u)^+$
$(\bar{b}s)^0$
$(c\bar{s})^+$
$(c\bar{u})^0$
$(c\bar{d})^+$
$(d\bar{s})^0$
$(u\bar{s})^+$
$(d\bar{u})^-$
$(u\bar{d})^+$
$(\bar{u}s)^-$
$(\bar{d}s)^0$
$(\bar{c}d)^-$
$(\bar{c}u)^0$
$(\bar{c}s)^-$
$(b\bar{s})^0$
$(b\bar{u})^-$
$(b\bar{d})^0$
$(b\bar{c})^-$
$(\bar{t}c)^0$
$(\bar{t}d)^-$
$(\bar{t}u)^0$
$(\bar{t}s)^-$
$(b\bar{t})^-$

Fig. 27

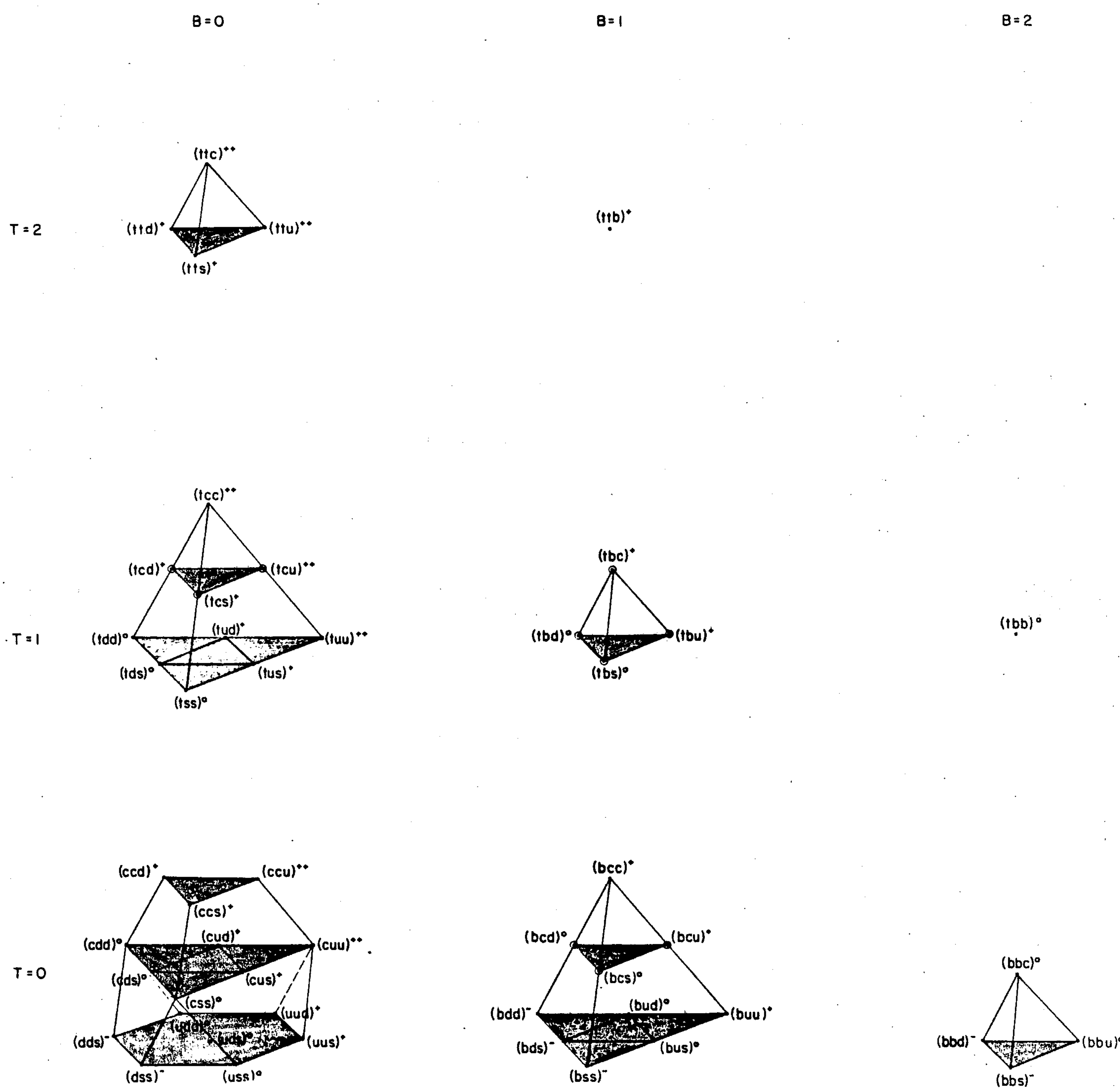

B=0
B=1
B=2
T=2
(ttc)++
(ttd)+
(ttu)++
(tts)+
(ttb)+
T=1
(tcc)++
(tcd)+
(tcu)++
(tcs)+
(tdd)°
(tud)+
(tuu)++
(tds)°
(tus)+
(tss)°
(tbc)+
(tbd)°
(tbu)+
(tbs)°
(tbb)°
T=0
(ccd)+
(ccu)++
(ccs)+
(cdd)°
(cud)+
(cuu)++
(cds)°
(cus)+
(css)°
(uud)+
(dds)-
(udd)°
(uds)°
(uus)+
(dss)-
(uss)°
(bcc)+
(bcd)°
(bcu)+
(bcs)°
(bdd)-
(bud)°
(buu)+
(bds)-
(bus)°
(bss)-
(bbc)°
(bbd)-
(bbu)°
(bbs)-

Fig. 28

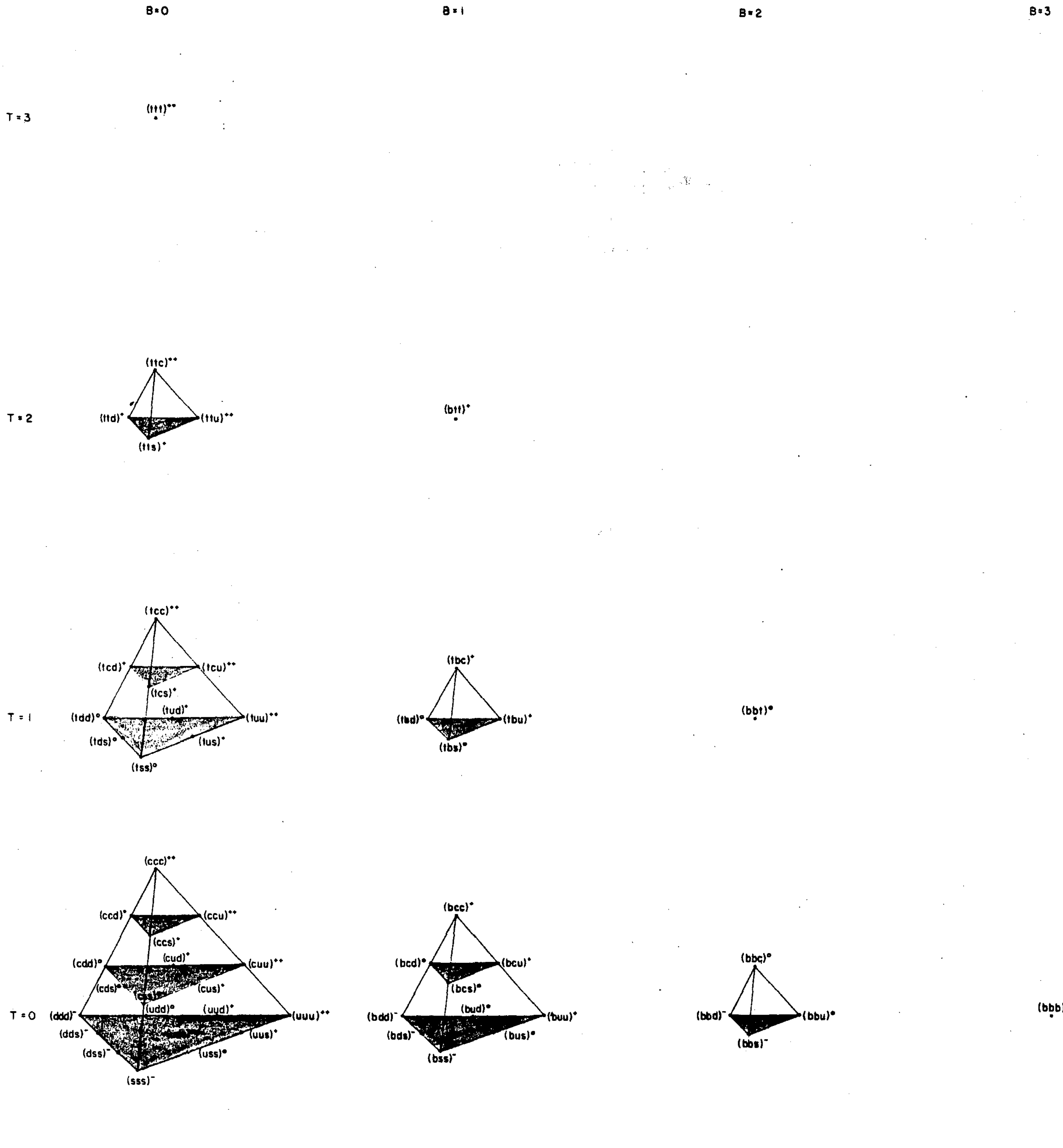

B=0
B=1
B=2
B=3
T=3
(ttt)++
T=2
(ttc)++
(ttd)+
(ttu)++
(tts)+
(btt)+
T=1
(tcc)++
(tcd)+
(tcu)++
(tcs)+
(tud)+
(tdd)°
(tuu)++
(tds)°
(tus)+
(tss)°
(tbc)+
(tbd)°
(tbu)+
(tbs)°
(bbt)°
T=0
(ccc)++
(ccd)+
(ccu)++
(ccs)+
(cud)+
(cdd)°
(cuu)++
(cds)°
(cus)+
(udd)°
(uud)+
(ddd)-
(uuu)++
(dds)-
(uus)+
(dss)-
(uss)°
(sss)-
(bcc)+
(bcd)°
(bcu)+
(bcs)°
(bud)°
(bdd)-
(buu)+
(bds)-
(bus)°
(bss)-
(bbc)°
(bbd)-
(bbu)°
(bbs)-
(bbb)-

Fig. 29

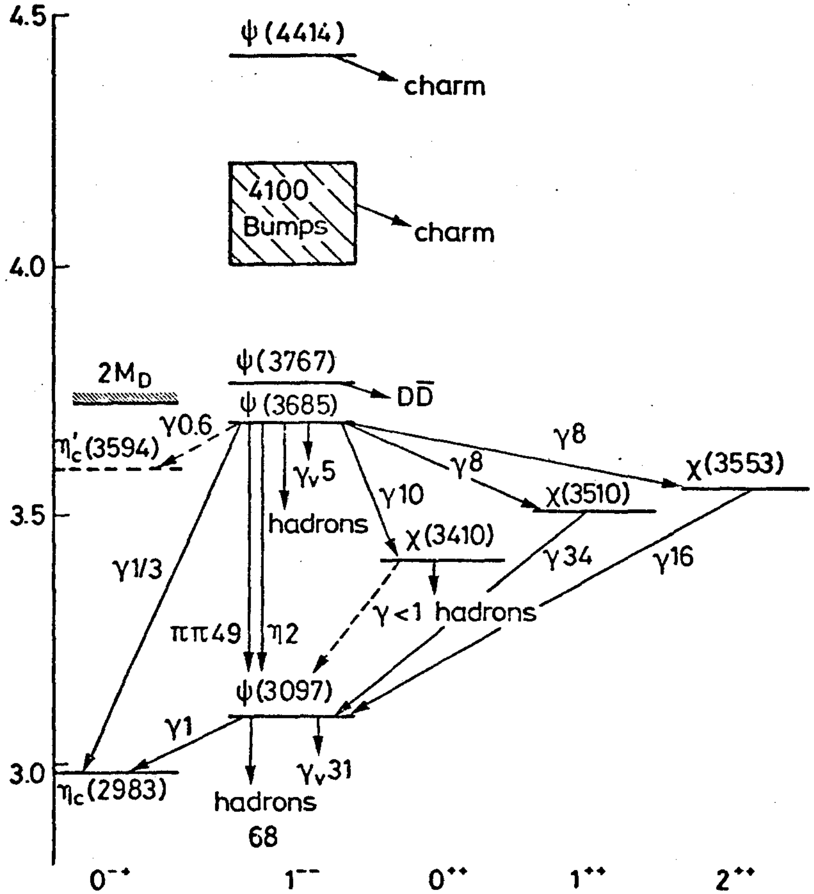

4.5
4.0
3.5
3.0
ψ(4414)
charm
4100
Bumps
charm
2M_D
ψ(3767)
ψ(3685)
DD̄
γ0.6
η'c(3594)
γ8
γ8
χ(3553)
γv5
hadrons
γ10
χ(3510)
χ(3410)
γ1/3
γ34
γ16
γ<1 hadrons
ππ49
η2
ψ(3097)
γ1
ηc(2983)
hadrons
68
γv31
0⁻⁺
1⁻⁻
0⁺⁺
1⁺⁺
2⁺⁺

Fig. 30

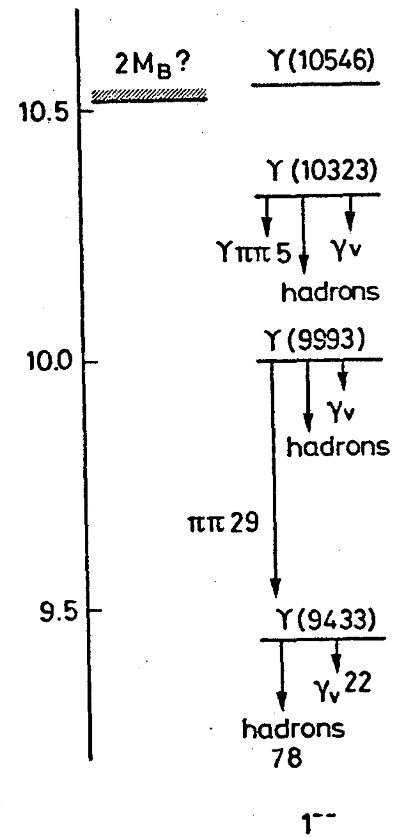
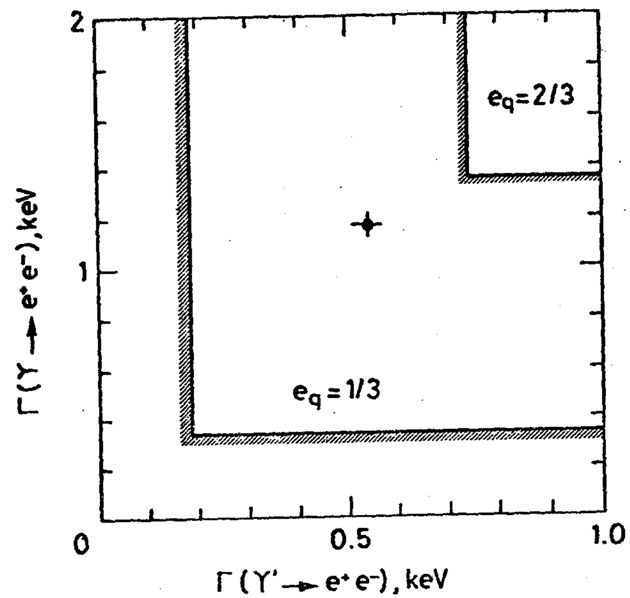
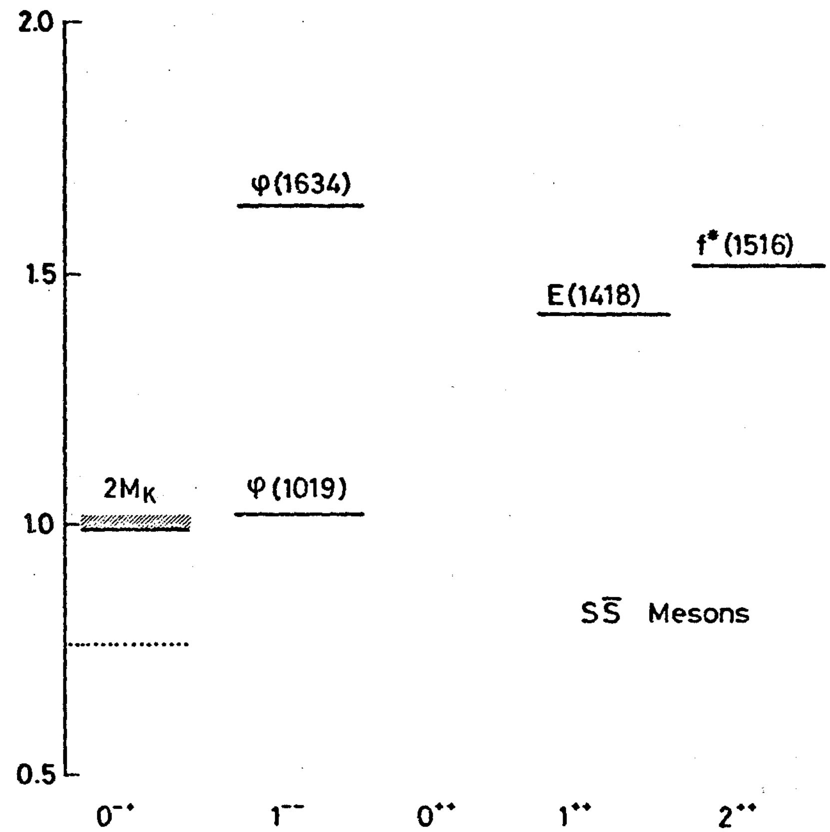

2M_B?
Υ(10546)
10.5
Υ(10323)
Υππ 5
γv
hadrons
10.0
Υ(9993)
γv
hadrons
ππ 29
9.5
Υ(9433)
γv 22
hadrons
78
1⁻⁻

Fig. 31



2
1
0
0.5
1.0
$e_q = 2/3$
$e_q = 1/3$
Γ (Υ → e⁺e⁻), keV
Γ (Υ' → e⁺e⁻), keV

Fig. 32



2.0
1.5
1.0
0.5
φ(1634)
f*(1516)
E(1418)
2M_K
φ(1019)
SS̄ Mesons
0⁻⁺
1⁻⁻
0⁺⁺
1⁺⁺
2⁺⁺

Fig. 33

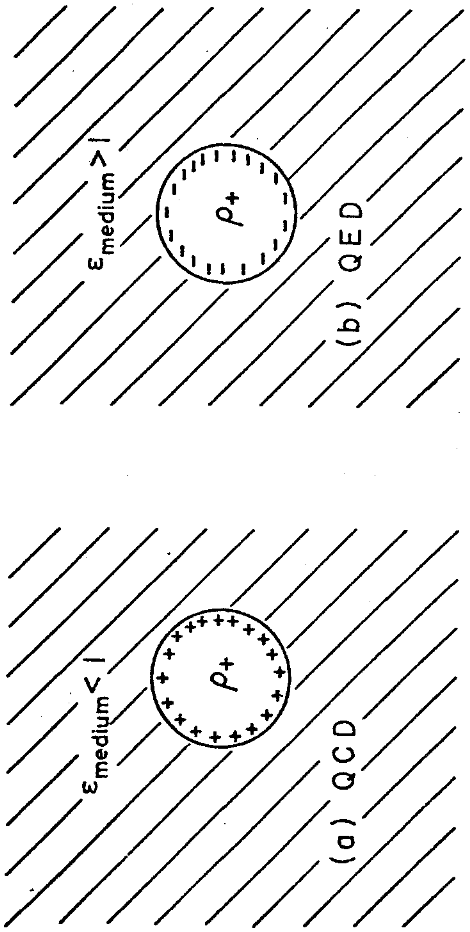


Fig. 34

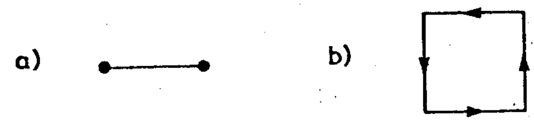


Fig. 35

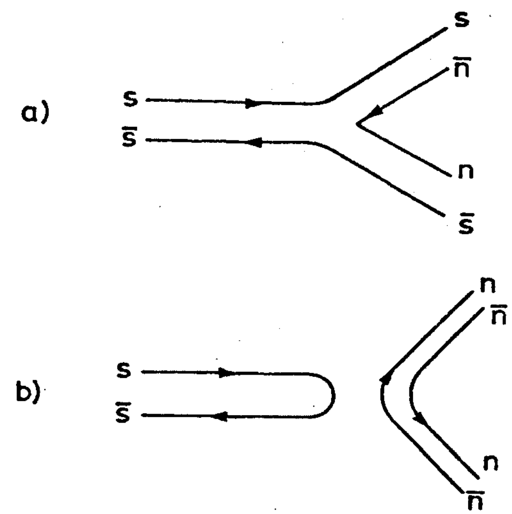
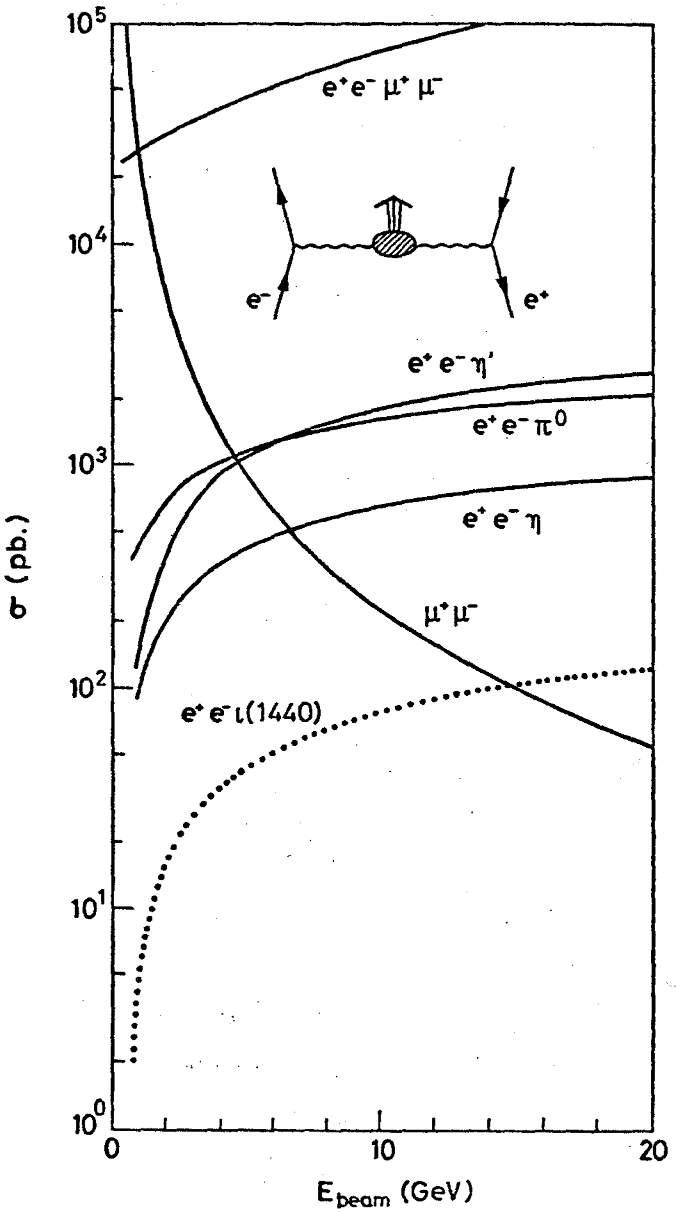
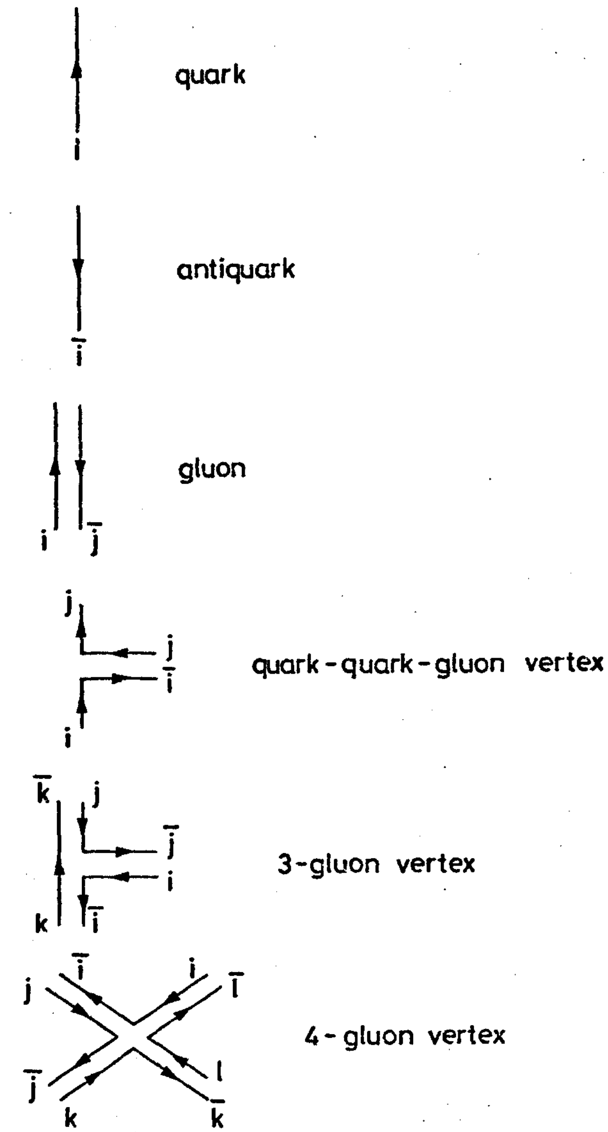


Fig. 36



10^5
10^4
10^3
10^2
10^1
10^0
σ (pb.)
$e^+e^-\mu^+\mu^-$
e^-
e^+
$e^+e^-\eta'$
$e^+e^-\pi^0$
$e^+e^-\eta$
$\mu^+\mu^-$
$e^+e^-\iota(1440)$
0
10
20
E_{beam} (GeV)

Fig. 37



quark
antiquark
gluon
quark-quark-gluon vertex
3-gluon vertex
4-gluon vertex

Fig. 38

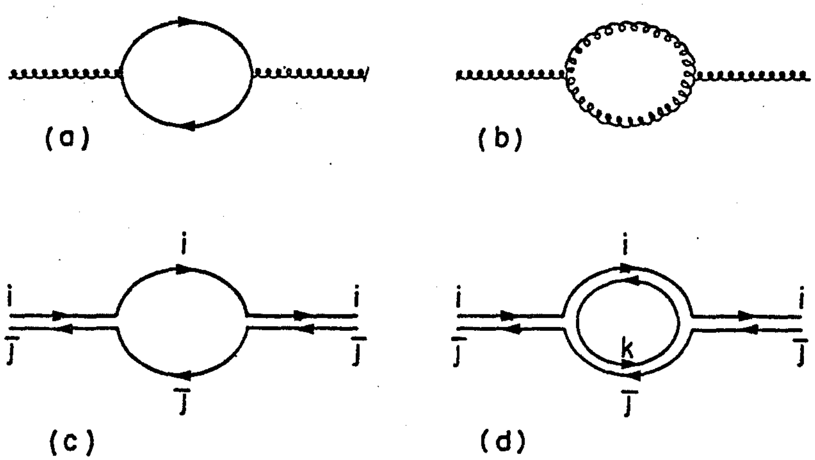


Fig. 39

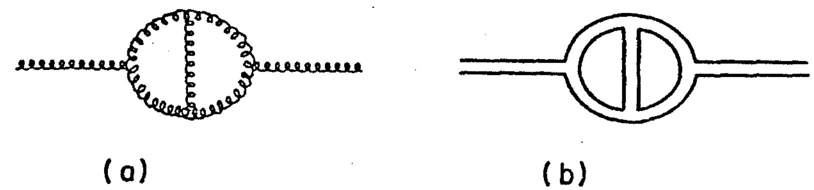


Fig. 40

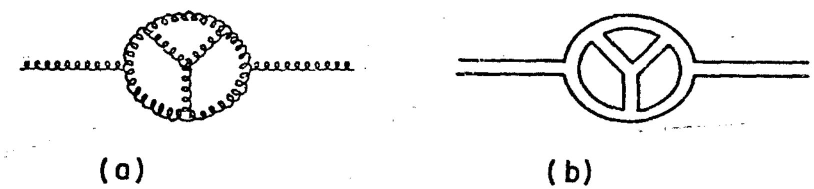


Fig. 41

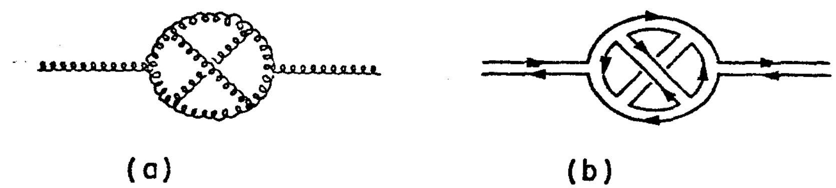


Fig. 42

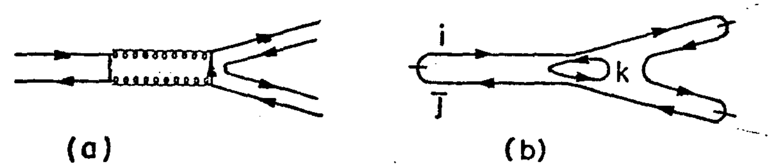


Fig. 43

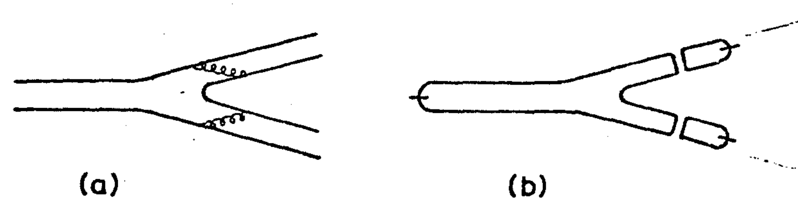


Fig. 44