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On Understanding Crosstalk in the Face of Small, Quantized, Signals Highly Smeared by Poisson Statistics

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Abstract

As detectors become smaller and more densely packed, signals become smaller and crosstalk between adjacent channels generally increases. Since it is often appropriate to use the distribution of signals in adjacent channels to make a useful measurement, it is imperative that inter-channel crosstalk be well understood. In this paper we shall describe the manner in which Poissonian fluctuations can give counter-intuitive results and offer some methods for extracting the desired information from the highly smeared, observed distributions.

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Introduction

As modern high energy physics experiments try to perform ever more precise measurements, one trend observed is that the granularity of detectors becomes smaller. Wire chambers are giving way to silicon and gas microstrips, and hodoscopes are being abandoned for scintillating fiber ribbons. As the ratio of area to volume increases, edge effects become more important than ever. Since most detectors depend on ionization or other energy loss mechanisms, smaller detectors imply less energy loss, which implies in turn smaller signals. This paper addresses these two future problems, small signals and large edge effects.

In the case of small signals, as in the case of photoelectrons seen in scintillating fibers, typically the signal is small enough that the effects from Poisson statistics become important. Even in the case of perfectly monochromatic dE/dx loss, there will be a significant width associated with Poisson fluctuations.

One edge effect that is becoming more important is crosstalk between adjacent cells. As the effect of crosstalk increases, it becomes more important to understand it. On the face of it, this appears to be a simple problem. One should look at the signal in the 'hit' cell (x) and plot against that the signal in an adjacent cell (y). The slope of the plot $\Delta y/\Delta x$ will be the crosstalk fraction. Such an approach was tried by the DØ Central Preshower Group [1] in their analysis of their prototype module. However, when such a plot was made, a very peculiar curve was observed. While it was expected that their plot would be a single line with an intercept through the origin, actually two distinct slopes were observed (see figure (1).)

The source of this inexplicable behavior stems from fact that the signal in each channel fluctuates independently with a large RMS relative to the mean. A detailed understanding of the effect can be had by investigating the effect of statistical smearing on the observables.

Analysis

Suppose a large number of particles separately transit a detector cell, each losing an identical amount of energy, which corresponds to an average hit cell signal S. Further suppose that the average signal in an adjacent cell (the crosstalk signal) is s and that s is a function of S, $s = s(S)$. Because of statistical fluctuations in a particular event, what would be observed is (Σ, σ) where for a particular S, the hit cell signal Σ occurs with probability $P(\Sigma, S)$ and similarly the adjacent cell signal σ occurs with probability $p(\sigma, s)$. Since S and s are fixed, and $P(\Sigma, S)$ and $p(\sigma, s)$ fluctuate independently, the probability to observe both Σ in the main cell and σ in the adjacent cell is $\text{Prob}(\sigma, \Sigma) = p(\sigma, s)\, P(\Sigma, S)$.

If one allows the energy loss (and associated average signals) to vary, then S and s are no longer fixed. Let the distribution of S be $f(S)$, then one has to sum over contributions from various S when calculating $\text{Prob}(\sigma, \Sigma)$ for this general case :

$$\text{Prob}(\sigma, \Sigma) = \frac{1}{N} \int_{-\infty}^{\infty} p(\sigma, s(S))\, P(\Sigma, S)\, f(S)\, dS$$

where N is the normalization constant for $f(S)$ ($N = \int_{-\infty}^{\infty} f(S)\, dS$.) Similarly, the probability to observe signal Σ in the main cell is

$$\text{Prob}(\Sigma) = \frac{1}{N} \int_{-\infty}^{\infty} P(\Sigma, S)\, f(S) dS.$$

Now given a main cell signal Σ, the average crosstalk signal is

$$<\sigma(\Sigma)> = \sum_{\sigma=1}^{\infty} \sigma\, \text{Prob}(\sigma|\Sigma) = \frac{\sum_{\sigma=1}^{\infty} \sigma\, \text{Prob}(\sigma, \Sigma)}{\text{Prob}(\Sigma)}$$

where $\text{Prob}(\sigma|\Sigma)$ is the conditional probability that one observes crosstalk signal σ in the adjacent cell given that the main cell has signal Σ.

Therefore, for the general case, we reach

$$<\sigma(\Sigma)> = \frac{\int_{-\infty}^{\infty} s(S)\, P(\Sigma, S)\, f(S)\, dS}{\int_{-\infty}^{\infty} P(\Sigma, S)\, f(S)\, dS} \tag{1}$$

after the relation $s(S) = \sum_{\sigma=1}^{\infty} \sigma\, p(\sigma, s(S))$ is used.

While equation (1) is in general true, it is instructive to explore a physically useful case. In the case of close-packed scintillating fibers, photoelectrons arrive in discrete packets and are governed by Poisson statistics. Thus $P(\Sigma, S) = e^{-S} S^{\Sigma} / \Sigma!$. Since the average observed signal (both in the central and adjacent fiber) is proportional to the light emitted in the main fiber, one might expect that $s = kS$, where k is the crosstalk fraction. One might imagine that near the cell edge the main signal might drop while the crosstalk signal rises, thus invalidating the assumption of crosstalk linearity, however it is usually possible to restrict an analysis to a region where the crosstalk proportionality constant k is locally both constant and stable. Making these assignments, and realizing that for Poissonian statistics $S > 0$, equation (1) transforms to:

$$<\sigma(\Sigma)> = k \frac{\int_0^{\infty} e^{-S} S^{\Sigma+1}\, f(S)\, dS}{\int_0^{\infty} e^{-S} S^{\Sigma}\, f(S)\, dS} \tag{2}$$

The remainder of this paper shall explore the behavior of this useful Poisson case, with linear crosstalk. Equation (2) has very interesting properties, which can be illustrated by some extremes. Consider the following three (un-normalized) probability distribution functions:

Case 1

$$f(S) = \delta(S - a), (a > 0) \quad \Rightarrow \quad <\sigma(\Sigma)> = ka$$

Case 2

$$f(S) = \begin{cases} \xi & S > 0 \\ 0 & \text{otherwise} \end{cases} \quad \Rightarrow \quad <\sigma(\Sigma)> = k(\Sigma + 1)$$

Case 3

$$f(S) = \begin{cases} \xi & a < S < b, (a, b > 0) \\ 0 & \text{otherwise} \end{cases} \quad \Rightarrow \quad <\sigma(\Sigma)> = k \frac{[e^{-S} \sum_{r=0}^{\Sigma+1} \frac{(\Sigma+1)!}{(\Sigma+1-r)!} S^{\Sigma+1-r}]|_a^b}{[e^{-S} \sum_{r=0}^{\Sigma} \frac{(\Sigma)!}{(\Sigma-r)!} S^{\Sigma-r}]|_a^b}$$

These three results are plotted in figure (2) and illustrate the scope of the effect. In case 1, the mean S is given a single value from a delta function, which implies that the mean s $(= kS)$ is also single valued. Since both Σ and σ fluctuate independently, the result is a graph with a slope of zero. In case 2, all values of S occur with equal probability (for $S > 0$) and the result most nearly resembles the naive result. Case 3, the square function, is in some sense a compromise between the first two cases and the observed $(\Sigma, <\sigma(\Sigma)>)$ distribution is bracketed by the first two cases. Thus, the function of the underlying signal $f(S)$ significantly affects the observed results. Specifically, although fuzzily defined, the width of the underlying distribution is very important.

So far, the multi-slope behavior observed in figure (1) is not reproduced. Consider a square function with a low signal tail:

$$f(S) = \begin{cases} H & a < S < b \\ h & S < a \end{cases}$$

where $h \ll H$ and the fraction of 'background' $F = (ha)/[H(b-a) + ha]$ is correspondingly small. When this function is put into equation (2) and evaluated, dual slope behavior is observed. See figure (3).

While the nature of the functions used for examples in this paper (square functions) is not typically observed in nature, they do illustrate two important sensitivities in the behavior of equation (2). The first is the 'width' of the underlying function. Skinny functions result in $(\Sigma, <\sigma(\Sigma)>)$ plots that are substantially different than those caused by broader functions. Secondly, the ratio of 'signal' (data in the peak) to 'background' (low probability data far from the peak) has considerable impact on the observed $(\Sigma, <\sigma(\Sigma)>)$ plots.

It is therefore important to get the unobserved, underlying, function $f(S)$ 'right'. What can be *observed* is the function

$$\mathcal{F}(\Sigma) = \int_0^\infty f(S) e^{-S} S^{\Sigma} ds/\Sigma! \tag{3}$$

which is the function $f(S)$ smeared by Poisson statistics. While in general determining $f(S)$ from $\mathcal{F}(\Sigma)$ is quite difficult, it is possible to do this by the use of fitting techniques. If one has a functional form $f(S; a, b, c, \ldots)$, where $a, b, c, \ldots$ are parameters of the function, this function can be smeared by Poisson statistics, compared to $\mathcal{F}(\Sigma)$ and the process iterated until the best parameters are determined. Then the analysis suggested by equation (2) may be done and the crosstalk fraction k can be extracted.

An alternative method for extracting $f(S)$ comes from using inverse Mellin transforms [2]. A Mellin transform is:

$$\mathcal{M}(N) = \int_0^\infty x^{N-1} F(x) dx \tag{4}$$

which is isomorphic to equation (3), if one makes the correspondence $\mathcal{M}(N) = (\Sigma!)\,\mathcal{F}(\Sigma)$ and $F(S) = Se^{-S} f(S)$, and further restricts $f(S) = 0$ for $S < 0$ (the physical case.) Thus one may find the inverse Mellin transform of $(\Sigma!)\,\mathcal{F}(\Sigma)$ and from the resulting $F(S)$ determine $f(S)$.

In the preceeding discussion, three assumptions have been made: (1) the observed distributions have been smeared by Poisson statistics from a true distribution, (2) prior to Poisson smearing, the crosstalk signal is proportional to the main signal ($s = kS$), and (3) the underlying distribution $f(S)$ is both stable and meaningful (e.g. proportional to the Landau distribution of dE/dx loss.) As we have seen, these assumptions are not explicitly required and it is therefore important to understand under what conditions they are valid.

The assumption of Poissonian statistics is most likely valid in all cases where a quantized signal, consisting of small numbers, is present (i.e. in a case where individual photons or electrons are measured.) This represents a vast range of detectors, from scintillating fibers to semiconductor devices. As the size of detectors becomes smaller, the presence of quantized signals will certainly increase.

We have assumed in our examples that $s(S) = kS$. As discussed above, crosstalk is in some sense an edge effect. It is therefore reasonable that near the edge of the main cell, the crosstalk is larger and probably the main signal is smaller, thus invalidating the assumption of crosstalk linearity. Luckily, it is often relatively easy to restrict the signal to a portion of the detector where the linearity assumption is locally valid, which greatly simplifies the analysis. One would then be required to redo the analysis for the different regions.

Assumption (3) simply requires that the main cell signal before Poisson smearing is proportional to the energy loss in the detector. This is generally true. However, it is probable that different channels have somewhat different responses to equal energy loss. Thus it is important to have calibrated each channel properly. In addition, if one wishes to compare two different detectors (say scintillating fibers with two different types of dopants,) one expects different signal for identical energy loss. In order to compare the crosstalk behavior of these two detector types, one must account for these differences. One may do this by finding the slightly different $f(S)$ functions.

Finally, one must interpret the crosstalk fraction k with some care. By definition, $k = s/S$. If both the main cell and the adjacent cell had the same signal in them, then $k = 1$. In order to determine the ratio of cross talk signal to *total* signal, one must solve $[s/(s+S) = k/(k+1)]$. Which result is needed is dependent on the question one is asking.

Results and Conclusions

Thus an important observation regarding detector crosstalk is made. In the region of small, quantized signals, independent Poisson fluctuations in the signals of the main cell and its adjacent cell can substantially alter the data from simple expectations. The details of the underlying signal distribution, $f(S)$, can have a dramatic impact on the observed crosstalk behavior. However, with a judicious model of the underlying signal distribution, it is possible to extract the desired quantity, the fractional crosstalk k. This extraction is quite trivial, as equation (2) is linear in k.

References

[1] M. Adams, et al., *A Detailed Study of the Performance of Plastic Scintillating Strips with Axial Wavelength Shifting Fiber Readout*, FERMILAB-Pub-95/027, March 1995, submitted to Nucl. Instrum. Meth. A.

[2] F. Oberhettinger, *Tables of Mellin Transforms*, Springer-Verlag, New York, 1974.

Figure Captions

1. Figure 1. Shown is observed crosstalk data [1] taken by the DØ Central Preshower group. Contrary to naive expectations, there are two slopes in each of the above plots. The closed circles denote a higher amount of crosstalk, while the open squares denote a case where the crosstalk is purported to be lower.

2. Figure 2. Shown is the effect of the shape of the average signal distribution on the observed $(\Sigma, <\sigma(\Sigma)>)$ plot. Case 1 is a delta function ($a = 20$), case 2 is a uniform probability distribution ($S > 0$), and case 3 is a restricted uniform probability distribution ($f(S)$ constant for $15 < S < 25$, 0 otherwise.) The range of possible observed curves is striking and underscores the effect of the width of the underlying distribution on the observed plots.

3. Figure 3. Shown is the effect of 'background' (i.e. low probability behavior, far from the bulk behavior.) Six curves are shown and correspond to background fractions of 10%, 1%, 0.1%, 0.01%, 0.001%, and 0% respectively. The 10% and 1% curves are explicitly labelled. The lower background curves decrease monotonically as one moves to the left. The curve which does not have the sharp turn over corresponds to 0% background. The bulk behavior $f(S)$ is a constant for ($15 < S < 25$) and 0 otherwise. In all plots, the fraction of crosstalk is 10%.

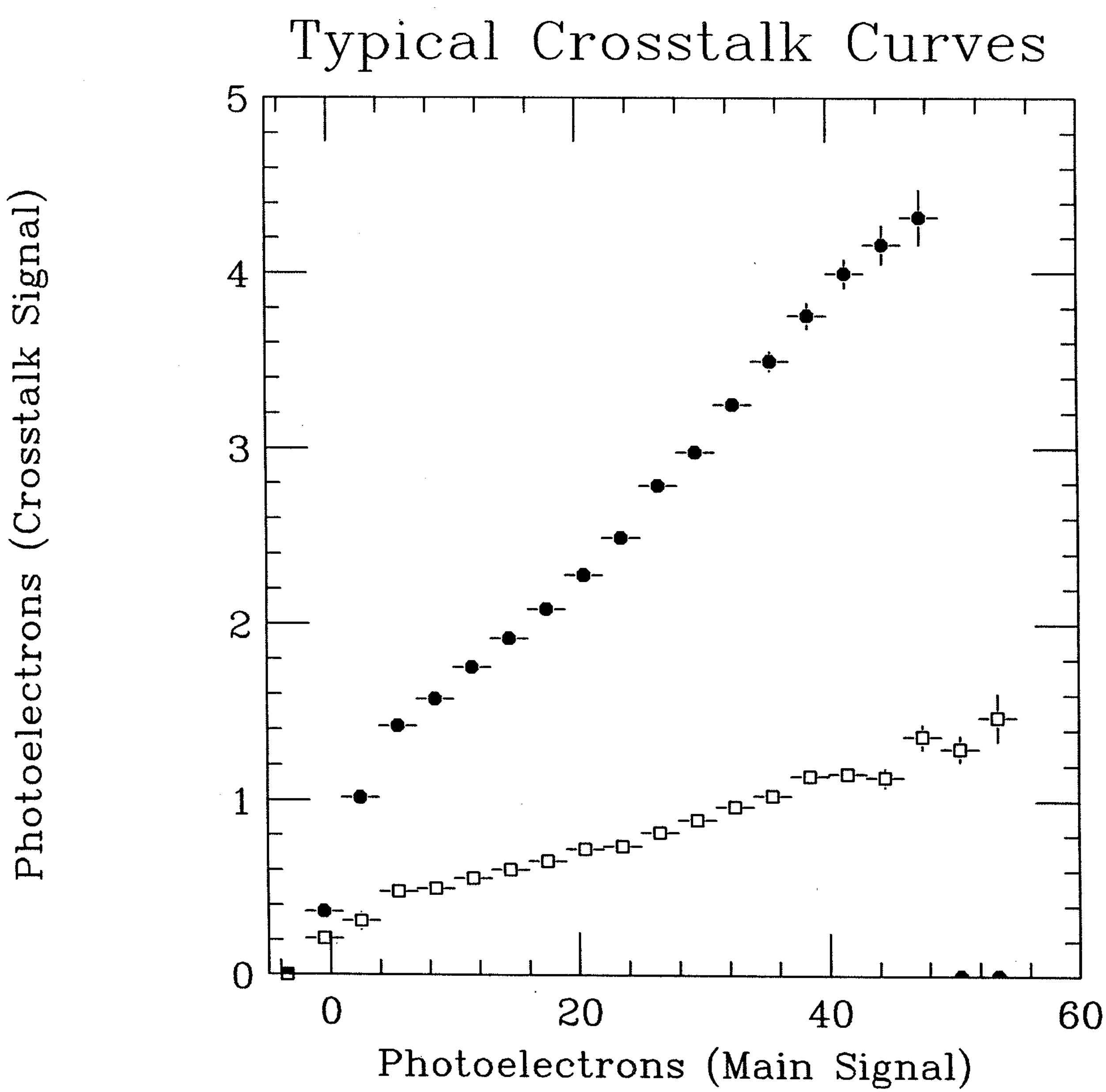
Typical Crosstalk Curves
Photoelectrons (Crosstalk Signal)
0
1
2
3
4
5
0
20
40
60
Photoelectrons (Main Signal)

Figure 1

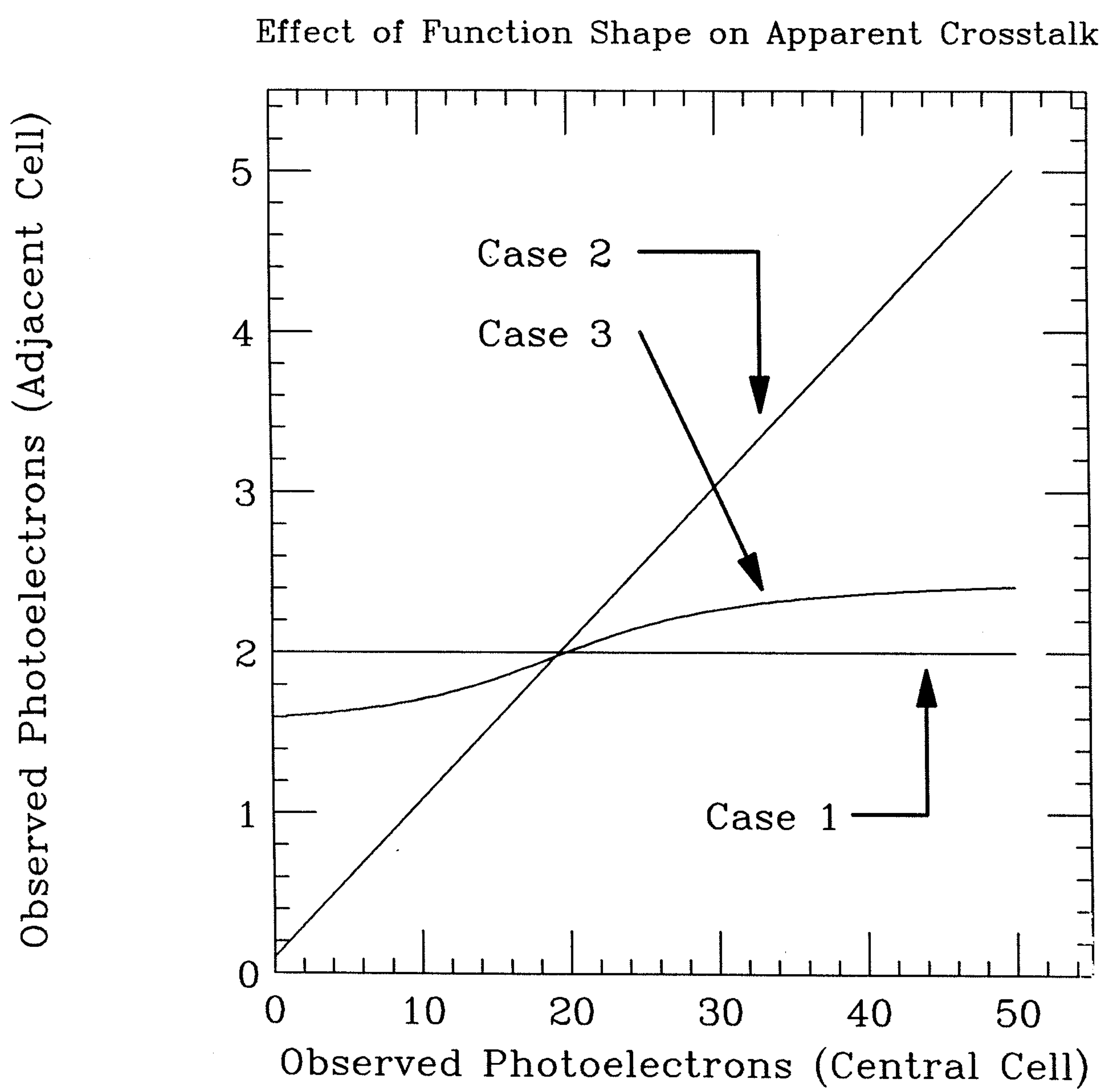
Effect of Function Shape on Apparent Crosstalk
Case 2
Case 3
Case 1
0
1
2
3
4
5
0
10
20
30
40
50
Observed Photoelectrons (Adjacent Cell)
Observed Photoelectrons (Central Cell)

Figure 2

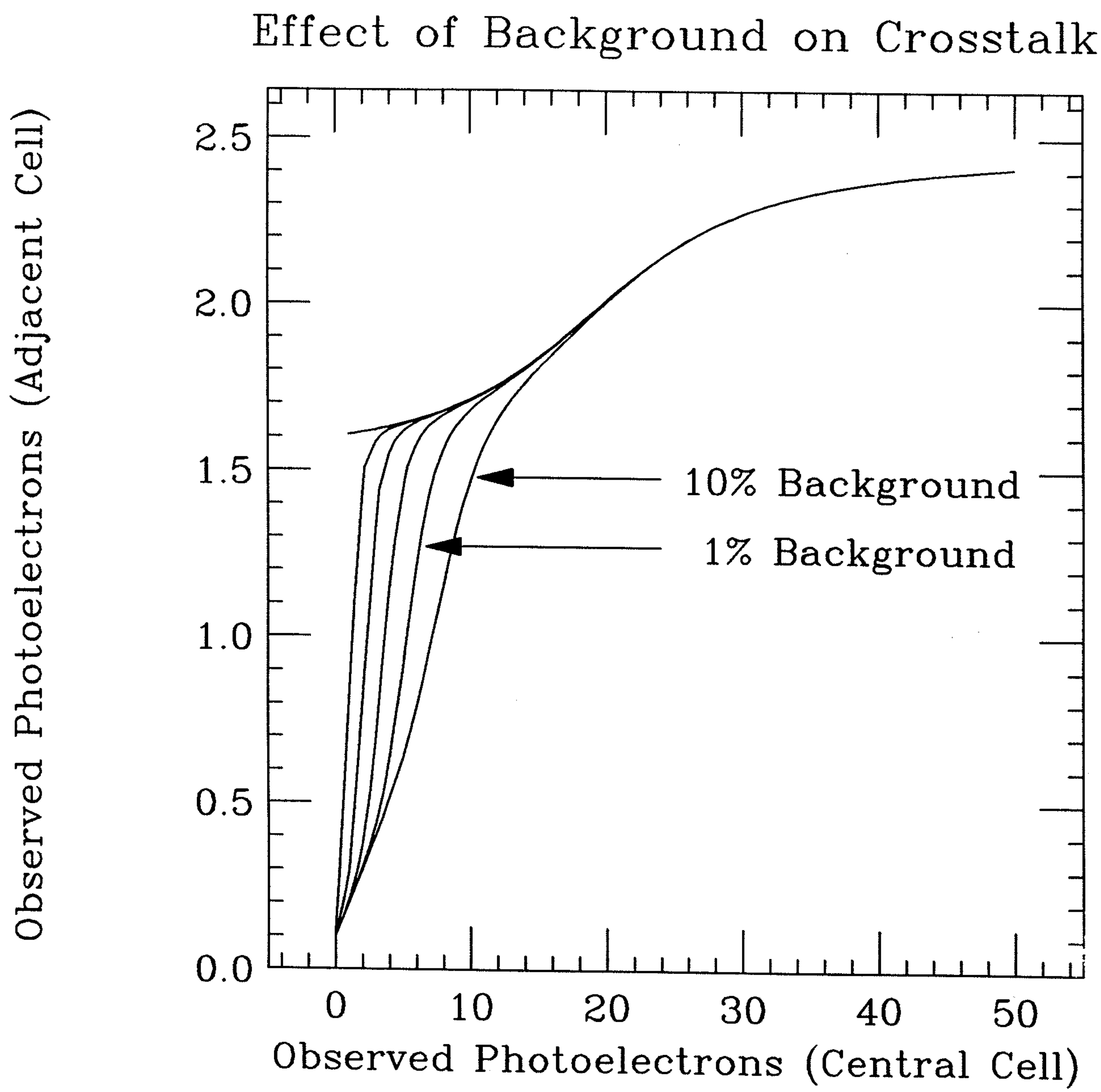
Effect of Background on Crosstalk
Observed Photoelectrons (Adjacent Cell)
2.5
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
50
Observed Photoelectrons (Central Cell)
10% Background
1% Background

Figure 3